REED'S PRACTICAL MATHEMATICS

FOR

MARINE ENGINEERS

FIRST AND SECOND CLASS

SIXTH EDITION

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PREFACE

The object of this text book is to present the principles upon which the Board of Trade base the questions in Practical Mathematics, for First and Second-Class Certificates for Engineers.

The questions set for First-Class in this subject are of a higher standard than those set for Second-Class. The syllabus covered and the principles underlying the questions are, however, practically the same for both examinations. The text has therefore been written to cover completely the ground work for both the examinations, and has, in some cases, been extended somewhat beyond the present requirements.

The large number of failures in this subject seems to be due to the fact that candidates do not understand the principles involved, but merely attempt to remember solutions to great numbers of questions.

The correct method of study is to read carefully through each Chapter, working out the examples given in the text; then when the principles have been mastered, to work carefully through the examples set at the end of the Chapter. Every Chapter should be carefully studied before attempting the examination papers given as a final test in the latter part of the book.

Students should prepare themselves for these examinations by study at sea, and if the contents of these Chapters are mastered, there should be no difficulty in passing in this important part of the syllabus. It is necessary to obtain sixty per cent. of the total marks to pass, and as this is a high standard, students must be thoroughly familiar with the subject.

The time spent at school, while ashore, will be greatly curtailed for those students who have worked carefully through this book.

A copy of four figure Mathematical Tables, containing Logarithms, Natural Sines, Cosines and Tangents, Logarithmic Sines Cosines and Tangents should be procured. As Antilogarithms are not provided at the examinations, the student should work from a table of Logarithms only.

PREFACE TO THIRD EDITION

Since the publication of the second edition of this book, considerable alterations have been made to the examinations for the certificates of competency. In order to cover the scope of the new syllabus, much new matter has been introduced in this edition. The examination questions have been arranged under their respective headings, and this should assist the candidate who is taking a part only of the examination. The authors express their appreciation of the reception of the previous editions.

FOURTH EDITION, April, 1942.

FIFTH EDITION, May, 1943.

PREFACE TO SIXTH EDITION

In presenting the Sixth Edition of Reed's Practical Mathematics for Engineers, the authors express their thanks for the reception of previous editions.

All the text matter has been carefully revised, and many additions have been made to the examination questions for both grades.

October, 1944.

CONTENTS

Sections of the text, and test examples marked by an asterisk, may be omitted by students for the Second-class Certificate.

Slide—I

CONTENTS

CHAPTER XX.

EXPANSION OF SOLIDS AND GASES; HEAT ENGINE CYCLES

CHAPTER XXI.

PROPERTIES OF STEAM

CHAPTER XXII.

THE EXPANSION OF STEAM

CHAPTER XXIII.

THE SLIDE VALVE · SIZE OF PORTS, ETC.

CHAPTER XXIV.

COMBUSTION OF FUEL

CHAPTER XXV.

PROPULSION OF SHIPS

CHAPTER XXVI.

REED'S PRACTICAL MATHEMATICS FOR ENGINEERS

CHAPTER I.

OPERATIONS IN SIMPLE ARITHMETIC

Decimal Fractions.

The quantity 0·7654 is a decimal fraction. The dot in front of the figure 7 is called the decimal point. The figure 7 is said to be in the first decimal place, the figure 6 is in the second decimal place, and so on. The first decimal denotes tenths, the second denotes hundredths and the third denotes thousandths.

Therefore 0·7654 really means $\frac{7}{10} + \frac{6}{100} + \frac{5}{1000} + \frac{4}{10000}$

and 0·0076 really means $\frac{0}{10} + \frac{0}{100} + \frac{7}{1000} + \frac{6}{10000}$

To Convert a Decimal into a Vulgar Fraction.

$$0{\cdot}5675 = \frac{5675}{10000}$$

Divide throughout by 5

$$\frac{5675}{10000} = \frac{1135}{2000}$$

Divide by 5 again.

$$\frac{1135}{2000} = \frac{227}{400}$$

Place the figures in the numerator without the decimal point. In the denominator place 1, followed by as many noughts (0) as there are figures after the decimal point. Reduce to the lowest terms by dividing top and bottom by the same number.

Addition and Subtraction of Decimals.

Add 21·2, 25·63, 281·651,

```
 21·2
 25·63
281·651
  3·6271

        Ans.
```

Proceed as in ordinary addition, but place the decimal points in the same vertical line. The decimal point in the Answer must be in the same vertical line as those in the quantities to be added.

Subtract 49·1257 from 221·3754.

```
221·3754
 49·1257
--------
172·2497   Ans.
```

Proceed as in ordinary subtraction but keep all decimal points in the same vertical line.

Multiplication of Decimals.

37·75 × 41·65

```
   37·75
   41·65
   -----
   18875
  22650
  3775
 15100
---------
 1572·2875   Ans.
```

Follow the same rules as in ordinary multiplication, but the number of decimal places in the answer is equal to the sum of the number of decimal places in the two quantities being multiplied. Here 2 + 2 = 4, therefore there are 4 decimal places in the answer.

0·0375 × 0·00039

```
 0·0375
 0·00039
 -------
    3375
   1125
 ---------
 ·000014625   Ans.
```

Here there are 4 + 5 = 9 decimal places in the answer.

Division of Decimals.

52·8312 ÷ 17·32.

```
1732·)5283·12(3·0503  Ans.
      5196
      ----
        8712
        8660
        ----
          5200
          5196
          ----
```

Make the divisor a whole number by moving the decimal point to the end. Now move the point the same number of places in the dividend, and proceed as in ordinary division. The decimal point in the answer occurs after bringing down the first decimal figure in the dividend.

52·8312 ÷ 17·32 means $\dfrac{52\cdot8312}{17\cdot32}$ multiplying top and bottom by 100 gives $\dfrac{5283\cdot12}{1732}$ as above.

$27{\cdot}532 \div 0{\cdot}00053.$

```
53·)2753200(51947·1698  Ans.
    265
    ---
     103
      53
     ---
      502
      477
      ---
       250
       212

        380
        371
        ---
          90
          53
          --
          370
          318
          ---
           520
           477

             430
             424

               6
```

To make 0·00053 a whole number move the decimal 5 places to the right. Then move the decimal point 5 places to the right in 27·532.

NOTE.—We may add any number of noughts on to the end of a decimal quantity without altering its value.

$\dfrac{0{\cdot}000658}{2{\cdot}31}$

```
1231·)·06580(·0000534  Ans.
        6155
        ----
         4250

         5570
         4924

          646
```

The last figure used in the dividend in the first step is 0, in the 5th decimal place, therefore the 5 in the answer must be in the 5th decimal place, i.e., there must be 4 noughts before the 5.

Fractions.

A proper fraction represents a quantity less than unity or One. Thus: $\frac{1}{2}$, $\frac{10}{11}$, $\frac{11}{23}$, are proper fractions. An improper fraction represents a quantity greater than one, and it may be expressed as a whole number plus a fraction. Thus $\frac{15}{8}$, $\frac{4}{3}$, $\frac{28}{15}$, are improper fractions, and these may be written $1\frac{7}{8}$, $1\frac{1}{3}$, $1\frac{13}{15}$, and when in this form these quantities are called mixed fractions.

To find the **Least Common Multiple** of a number of quantities. Find the L.C.M. of 3, 5, 12, 48, 54. Divide by the smallest number which is a factor of more than one term.

$$\begin{array}{r|rrrrr} 3 & 3, & 5, & 12, & 48, & 54 \\ \hline 4 & 1, & 5, & 4, & 16, & 18 \\ \hline 2 & 1, & 5, & 1, & 4, & 18 \\ \hline & 1, & 5, & 1, & 2, & 9 \end{array}$$

Divide first by 3, writing down again all the numbers not divisible by 3. Next divide by 4, writing down again those numbers not divisible by 4. Repeat this step again, dividing this time by 2.

L.C.M. $= 3 \times 4 \times 2 \times 1 \times 5 \times 1 \times 2 \times 9 = 2160$. Ans.

This is the least number into which all the given numbers will divide exactly.

To add Fractions.

$\frac{3}{4} + \frac{1}{5} + \frac{3}{8}$

Find the L.C.M. of 4, 5 and 8. Divide the L.C.M. by each denominator and multiply by the numerator.

$$\frac{(10 \times 3) + (8 \times 1) + (5 \times 3)}{40} \quad \frac{30 + 8 + 15}{40} = \frac{53}{40}$$

$5\frac{1}{3} + 4\frac{1}{2} + 7\frac{1}{6} \qquad \frac{16}{3} + \frac{9}{2} + \frac{43}{6}$, L.C.M. $= 6$.

$$\frac{32 + 27 + 43}{6} = \frac{102}{6} = \frac{51}{3} = 17. \quad \text{Ans.}$$

$\frac{7}{8} + 2\frac{1}{6} + \frac{15}{16} = \frac{7}{8} + \frac{13}{6} + \frac{15}{16}$, L.C.M. $= 48$.

$$\frac{42 + 104 + 45}{48}$$

Subtraction of Fractions.

$\frac{1}{3} - \frac{1}{5}$, L.C.M. $= 15$.

The method is the same generally as in addition, but careful must be taken of the signs.

$$\frac{5 - 3}{15} = \frac{2}{15}. \quad \text{Ans.}$$

$\frac{1}{3} - \frac{1}{6} + \frac{1}{5} - \frac{5}{7} + \frac{7}{8}$, L.C.M. $= 840$.

$$\frac{280 - 140 + 168 - 600 + 735}{840} = \frac{4}{8} \qquad \text{Ans.}$$

To Multiply Fractions.

$\frac{3}{4} \times \frac{7}{8}$, this is the same as $\dfrac{3 \times 7}{4 \times 8} = \frac{21}{32}$. Ans.

i.e., multiply the numerators together and then the denominators.

$$\frac{\cancel{3}}{\cancel{4}} \times \frac{\overset{5}{\cancel{20}}}{\underset{7}{\cancel{21}}} \times \frac{5}{11},$$

$= \frac{25}{77}$. Ans.

Cancel numerators out into denominators when possible, and finally multiply out as shown.

To Divide Fractions.

$\frac{15}{16} \div \frac{3}{4}$

$$\frac{\overset{5}{\cancel{15}}}{\underset{4}{\cancel{16}}} \times \frac{\cancel{4}}{\cancel{3}} = \tfrac{5}{4} \text{ or } 1\tfrac{1}{4}. \text{ Ans.}$$

To divide one fraction by another, invert the dividing fraction and then multiply out, cancelling when possible.

$$\frac{\frac{5}{6} \times \frac{1}{8} \times \frac{3}{5} \times \frac{1}{4}}{\frac{1}{6} \times \frac{3}{10} \times \frac{7}{8}} = \frac{\cancel{5} \times 1 \times \cancel{3} \times 1 \times \cancel{6} \times 10 \times \cancel{8}}{\cancel{6} \times \cancel{8} \times \cancel{5} \times 4 \times 1 \times \cancel{3} \times 7}$$

$= \frac{10}{28} = \frac{5}{14}$. Ans.

To Convert Vulgar Fractions to Decimal Fractions.

$\frac{1}{8} = 8)1{\cdot}0$

$0{\cdot}125$. Ans.

Simply divide the numerator by the denominator.

$\frac{11}{12} = 12)11{\cdot}0$

$0{\cdot}91666$

$= 0{\cdot}91\dot{6}$. Ans.

Note that the figure six repeats endlessly, no matter to how many places the division is taken. This is called a repeating or recurring decimal, and it is written as shown with a dot above the 6.

To Convert Repeating Decimals to Vulgar Fractions.

If all the figures repeat after the decimal point this is called a pure recurring decimal. If some of the figures after the decimal do not repeat, it is called a mixed recurring decimal.

$0{\cdot}\dot{4}$

$\frac{331}{999}$

$0{\cdot}\dot{7}\dot{3} = \frac{73}{99}$.

Write the figures after the decimal point as the numerator. In the denominator write as many nines as there are figures after the decimal point. This is for a pure recurring decimal.

$0{\cdot}25\dot{7}2\dot{1}$

$\cdot 25721$
$\quad 25$ subtract

$\cdot 25696$

$\frac{25696}{} =$ Ans.

$0{\cdot}09\dot{7}$, $\quad \cdot 097$
subtract

$\cdot 088$
$= \frac{}{225}$. Ans.

If all the decimal figures do not repeat, subtract the non-repeating part from the whole decimal, and the result gives the numerator for the required fraction. For the denominator write first as many nines as there are repeating decimals, then add a nought for each non-repeating decimal. If a recurring decimal such as $0{\cdot}25\dot{7}2\dot{1}$ is written as $0{\cdot}25\dot{7}2\dot{1}$, we mean that the figures from 7 to 1 all recur. Thus $0{\cdot}\dot{2}3\dot{4}$ is sometimes written $0{\cdot}\dot{2}3\dot{4}$.

To Express a Fraction as a Percentage.

Express $\frac{5}{14}$ as a percentage, multiply the fraction by 100.

$$\frac{5}{14} \times 100 = \frac{500}{14} = 35{\cdot}714 \text{ per cent.} \quad \text{Ans.}$$

Note in dividing 500 by 14, it is convenient to divide first by 2 and then by 7, thus :—

$$\begin{array}{r} 2)500 \\ \hline 7)250 \\ \hline 35{\cdot}714 \end{array}$$

Examples involving Decimals and Fractions.

Our British system of money is not expressed in the decimal form. Sometimes it is convenient to express sums of money in the decimal form before multiplying them, or dividing them.

Convert £117 17s. 7d. to the decimal system.

7 pence is $\frac{7}{12}$ or $0{\cdot}58\dot{3}$ of a shilling.

We have now £117 $17{\cdot}58\dot{3}$ shillings.

$$17{\cdot}583 \text{ shillings is } \frac{17{\cdot}58\dot{3}}{20} \text{ pounds} = 0{\cdot}8791\dot{6} \text{ pound (£).}$$

Therefore £117 17s. 7d. $= 117 + 0{\cdot}8791\dot{6} = 117{\cdot}8791\dot{6}$ pounds. Ans.

Express 3 tons, 5 cwts., 1 qr., 18 lbs. as tons and a decimal of a ton.

$$\frac{18}{28} = 0{\cdot}6428 \text{ qrs.} \qquad \frac{1{\cdot}6428}{} = 0{\cdot}4107 \text{ cwts.,}$$

$$\frac{5{\cdot}4107}{20} = 0{\cdot}2705 \text{ ton.} \quad \text{The answer is } 3{\cdot}2705 \text{ tons.}$$

What amount is $\frac{7}{8}$ of £10 6s. 7d. ?

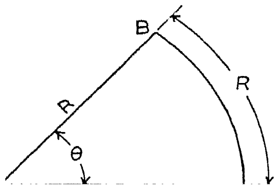
£10	6s.	7d.	Multiply first by 7, and then divide by 8.
		7	
8)72	6	1	Ans. £9 0s. $9\frac{1}{8}$d.
9	0	$9\frac{1}{8}$	

Find the value of $\frac{1}{3} + \frac{1}{2} \times \frac{1}{8} + \frac{1}{3} \times \frac{2}{5} - \frac{2}{3}$,

Always multiply and divide before adding or subtracting. If there are any terms in brackets, work out the brackets first.

$\frac{1}{3} \quad \frac{1}{16} + \frac{2}{15} - \frac{2}{3}$

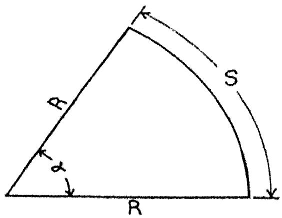
$\frac{15 + 32 -}{240} \qquad \frac{33}{240} \qquad \frac{11}{80}$. Ans.



The radian is the unit of circular measure.

Let ABC be a sector of a circle. Then the angle θ is said to be one radian, when the distance BC, measured along the arc, is equal to the radius.

If we divide the length of the arc of a sector of a circle by the radius, we express the angle of the sector in radians.



Thus α (in radians) $= \dfrac{}{\mathbf{R}}$

There are 2π radians in one circle, because :—

Length of circumference $= 2\pi r$, radius $= r$

$$\text{Angle at the centre} = \frac{2\pi r}{r} = 2\pi \text{ radians.}$$

Note that a radian is a ratio of two lengths, and is therefore a pure number.

$$\text{Number of degrees in one radian} = \frac{360}{2\pi} = 57{\cdot}3 \text{ degrees.}$$

If a body moves along a circular path of radius R, with a uniform velocity of V feet per second, then the number of radians turned through per second is $\frac{V}{R}$. This is called the angular velocity of the body and its symbol is ω, meaning radians per second.

The length of an arc of a circle is 10 feet, and the radius is 2 feet. Give the circular measure of the angle.

Angle at centre of the arc $= \frac{10}{2} = 5$ radians. Ans.

If we want to express this in degrees, we multiply by 57·3.

A body moves with a speed of 25 feet per sec. in a circular path. Find its angular velocity and its speed in revolutions per minute, if the radius is 4·5 feet.

$$\text{Angular velocity } (\omega) = \frac{25}{4{\cdot}5} = \frac{25 \times 2}{9} = 5{\cdot}\dot{5} \text{ radians per second.}$$

$5{\cdot}\dot{5}$ radians per sec. is $\frac{5{\cdot}\dot{5}}{2\pi}$ revolutions per sec., because there are 2π radians in one revolution.

and $\frac{5{\cdot}\dot{5}}{2\pi}$ revolutions per second is $\frac{5{\cdot}\dot{5}}{2\pi} \times 60 = 53$ revolutions per min.

Calculation of Wages.

The time of duration of a voyage, whether estimated for the purposes of wages payment, or as qualifying time for a certificate is reckoned by the calendar month. The calendar month is the time included between any given day in any month and the preceding day of the following month, both inclusive. The

number of complete months should be estimated in this manner, after which the odd days should be counted. The day upon which the agreement commences as well as that on which it terminates should both be included, and all odd days should be added together and reckoned at 30 days to the month.

FOR INSTANCE.—From 17th January to 14th February—here we have (31 — 17) = 14 days left in January but 15 days are really paid for, as we count the 17th as 1 day. Then we have 14 days to add for the time served in February, making a total of 15 + 14 = 29 days. This means being paid $\frac{29}{30}$ of one month's wage. From 22nd February to 17th March—here we have (28 — 22) = 6 days left in February, and, adding 1, gives 7 days worked in February. Then adding the 17 days worked in March gives a total of 24 days. From 3rd February to 21st August—from 3rd February to 2nd August is 6 whole months, and from 3rd August to 21st August is 19 days, making a total of 6 months 19 days.

What is the amount of the following wages account ?

From 4th January to 8th July at £28 0s. 0d. per month.

From 5th January to 3rd July at £20 10s. 0d.

From 4th March to 7th August at £16 0s. 0d.

From 4th January to 3rd July is 6 months, and 5 days in July is 6 months 5 days. From 5th January to 4th June is 5 months, 26 days in June + 3 in July, 5 months 29 days. From 4th March to 3rd August is 5 months, and 4 days in August is 5 months 4 days.

		£	s.	d.
$28 \times 6\frac{5}{30} = 28 \times 6\frac{1}{6} = 28 \times$	$= £172\frac{2}{3}$	$=$ 172	13	4
$20\frac{1}{2} \times 5\frac{29}{30} = \frac{41}{2} \times \frac{179}{30} = £122$		$=$ 122	6	4
$16 \times 5\frac{4}{30} = 16 \times \frac{154}{30} =$		$=$ 82	2	8
		£377	2	4

Simple Proportion ; Ratios.

In order to find the "Ratio" of two quantities, we find what fraction the first quantity is of the second. Thus the ratio of 5 to 8 is $\frac{5}{8}$. Find the ratio of 2 feet 2 inches to 5 feet, the

ratio is $\dfrac{\text{26 inches}}{\text{60 inches}} = \frac{13}{30}$, this is a pure number.

If 2·25 tons of a material cost £1·6, how much will 1·55 tons cost ?

1 ton costs $\dfrac{1\cdot 6}{2\cdot 25}$ £, and this multiplied by 1·55 is the answer.

$$\text{Ans.} = \frac{1\cdot 6}{2\cdot 25} \times 1\cdot 55 = 1\cdot 102\ £.$$

An equality of Ratios is called a Proportion.

Thus the ratio $\dfrac{1\cdot 55}{2\cdot 25}$ $\dfrac{1\cdot 102}{1\cdot}$ or the numbers 1·55, 2·25, and 1·102, 1·6 are in proportion. In any proportion the product of the extremes is equal to the product of the means. The extremes are here 1·55 and 1·6, and the means 2·25 and 1·102.

Degree of Accuracy.

In the measuring of physical quantities it is impossible to achieve absolute accuracy. Two students measure a flat test specimen, independently. One gives the dimensions as 2·375 inches wide, 0·375 inches thick, 10 inches long. The other gives the dimensions as 2·376 inches wide, 0·375 inches thick, 10·01 inches long. From the first set of figures the volume is 8·90625 cubic inches, and from the second set 8·91891 cubic inches. The first student might have given his volume as 8·906 cubic inches, and the second as 8·919 cubic inches. The given volumes differ because of the inaccuracy of the measuring instrument, and the errors of observation. As the observations are only taken to four figures, all figures in the calculated volumes after the fourth are of no value. Let us suppose that the actual volume is the mean of 8·906 and 8·919, i.e., 8·9125. Then the error is (8·9125 — 8·906) = 0·0065 cubic inch in 8·9125 cubic inches, or $\dfrac{0\cdot 0065}{8\cdot 9125} = \dfrac{65}{89125}$ or, roughly, 1 in 1,370. It is necessary to see that the actual magnitude of the error is not important, but that the ratio of the error to the quantity measured is important. Here the error is unimportant, 1 in 1370 being a very small error.

TEST EXAMPLES I.

1. Find the value of $\frac{7}{8} - \frac{1}{2} \times 2\frac{15}{16} + \quad \times 5\frac{1}{3}$.

Ans.

2. A bar of iron, 17 feet 3 inches long, is to be cut up into lengths of 1 foot $3\frac{1}{4}$ inches each. How many pieces of this

length will there be, allowing for a saw cut $\frac{1}{16}$ inch thick, and what is the length of the remaining piece ?

Ans. 13 pieces and $7\frac{13}{16}$ inches left.

3. Your steamer has a speed of $8\frac{1}{4}$ knots, but you are told that a certain railway train runs at $6\frac{1}{4}$ times that speed. How many statute miles per hour does the train run ?

Ans. 59·37 statute miles.

4. The distance between two places is known to be 3,500 nautical miles. Your steamer does this in 18 days 12 hours 25 minutes. What has been her mean speed per hour ?

Ans. 7·875 nautical miles per hour.

5. At 3-45 p.m. the counter stands at 259898. At noon it stood at 246849. What will it indicate at 4 p.m., and what has been the mean speed of the engines if the revolutions remain the same.

Ans. 260768. 57·995 revs. per min.

6. A ship burns 280 tons of Newcastle coal in 12 days. If she steams at the same horse power, how far would 245 tons of Welsh coal take her, if Newcastle coal is inferior to Welsh in the ratio of 19 to 21 ?

Ans. 11·6 days' steaming.

7. The crank shaft of a paddle engine turns through an angle of 21·47 radians. The effective diameter of the paddle is 18 feet and the slip is 8 per cent. Find the distance moved by the ship.

Ans. 177·77 feet.

8. By measurement and calculation two men find the weight of a body to be 15·72 lb. and 15·59 lb. If the mean of these two figures is correct, what is the percentage error in each ?

Ans. 0·415 per cent.

9. A rough rule, used by gunners, is that 1 inch at 100 yards subtends an angle of 1 minute. What is the percentage error of this rule ?

Ans. 4·5 per cent.

10. A shaft revolves at 825 revolutions per minute. Find its angular velocity in radians per second.

Ans. 86·43 radians per sec.

11. 27 cubic inches of copper weigh 125 ozs., and 72 cubic inches of iron weigh 312·5 ozs. Find the ratio of the weights of equal volumes of the two metals.

Ans. 16 : 15.

12. Find the amount of the following wages account :—
From January 5th to August 10th, at £24 0s. 0d. per month.
From February 17th to August 12th, at £20 0s. 0d. per month.
From April 20th to August 24th, at £16 0s. 0d. per month.

Ans. £357 9s. 4d.

CHAPTER II.

THE ELEMENTS OF ALGEBRA.

Algebra is the symbolic form of computation. The symbols used are letters, and they represent quantities. By means of these symbols and the various signs used, the relation between quantities may be stated in brief and concise form. An algebraic formula is really an expression in a convenient form of shorthand.

If a and b represent any quantities whatever, if we wish to add them, we write $a + b$; if we wish to subtract them we write $a - b$; if we wish to multiply them together we write $a \times b$ or more often $a\,b$; if we divide them we write $a \div b$, or $\frac{a}{b}$ or sometimes a/b. Note that $5\,x$ means $5 \times x$, and that $3\,ab$ means $3 \times a \times b$. If $a = 10$, $b = 3$, $c = 5$, find the value of $5\,abc$.

$$5\,a\,b\,c = 5 \times a \times b \times c = 5 \times 10 \times 3 \times 5 = 750.$$

An algebraic expression may contain any number of terms, and these may be written in any order, but the sign in front of a term must be retained. Thus: $a + 5\,ab - 3\,bc - c$, this expression may have its term re-arranged and written $-3\,bc + a + 5\,ab - c$. Observe also that if we write x we mean $+ x$, i.e., the plus sign, if not definitely written, is supposed to be there. But if we wish to express "minus x," we must always write it as $- x$.

Addition.

To add algebraic expressions place the expressions under one another, arranging similar terms in the same vertical column. Then add each column separately.

Add together $3\,a - 2\,b + 5\,c$; $2\,a + 3\,c + b$.

$$\left.\begin{array}{l} 3\,a - 2\,b + 5\,c \\ 2\,a + b + 3\,c \end{array}\right\}\text{Add.}$$

$$5\,a - b + 8\,c \quad \text{Ans.}$$

Note.—The terms in the second expressions have been rearranged before adding.

In the first expression the figures 3, —2, 5 are called co-efficients.

The co-efficient is the number which multiplies or divides a term.

In $\frac{x}{3}$, $3\,y$, $5\,a\,b$, $\frac{c}{7}$, the co-efficients are $\frac{1}{3}$, 3, 5, $\frac{1}{7}$.

Subtraction.

the sign of the quantity which is to be subtracted and then add the two quantities. Now if we subtract 3 from 7, we write $7 - 3 = 4$, here we have really changed the sign of 3 from $+$ to $-$ and then added.

Note. $\begin{array}{r} +7 \\ +3 \\ \hline 10 \end{array}$, but $\begin{array}{r} +7 \\ -3 \\ \hline 4 \end{array}$, we *add* in both cases. Again, subtract -7 from $+3$, here we change the sign of -7 and write $7 + 3$, which is 10.

Subtract $3c - 2a + b$ from $6a + 5b + c$.

$$\begin{array}{rrr} 6a & +5b & +c \\ -2a & +b & +3c \\ \hline 8a & +4b & -2c \end{array}$$

Change the signs mentally and add.

Ans.

Any expression which contains more than one term of the same kind, should be simplified before adding or subtracting.

For instance, $x + xy + 3x + 4y + 2xy = (x + 3x) + (xy + 2xy) + 4y$, because things of the same kind may always be added together. This expression then becomes $4x + 3xy + 4y$.

Subtract $5x^2 + 3xy + y^2$ from $6x^2 + 3xy + x^2 - 5y^2$, the second expression becomes $(6x^2 + x^2) + 3xy - 5y^2 = 7x^2 + 3xy - 5y^2$.

$$\begin{array}{rrr} 7x^2 & +3xy & -5y^2 \\ 5x^2 & +3xy & +y^2 \\ \hline 2x^2 & 0 & -6y^2 \end{array}$$

Change the signs mentally and add.

Ans. $2x^2 - 6y^2$.

Note. $3xy - 3xy = 0$.

Indices.

Powers and roots have the same meaning as in arithmetic.

The first power of x is merely x itself. It might be written as x^1, but we generally write it as x.

The second power of x, is $x \times x$, written x^2.

The third power of x is $x \times x \times x$, written x^3.

The second power of x is often called "the square of x"

The third power of x is often called "the cube of x."

The small figure representing the power is called the Index of the power; the plural of Index is Indices.

First Index Law. To multiply powers of the same quantity, add their Indices.

Thus $a^4 \times a^3 = a^{4+3} = a^7$. The law is explained as follows :— Since a^4 means $a \times a \times a \times a$, and a^3 means $a \times a \times a$, it follows that $a^4 \times a^3$ means $(a \times a \times a \times a) \times (a \times a \times a)$, and this is the continuous product of 7 'a's, and must be written a^7. Find the value of $3\,ab \times a^3$, this is the same as $3 \times a \times (a \times a \times a) \times b = 3\,a^4\,b$. Find the value of $\frac{x^2}{3} \times \frac{x\,y^3}{2}$, this means $\frac{1}{3} \times \frac{1}{2} \times x \times x \times x \times y^3 = \frac{x^3\,y^3}{6}$, note that the co-efficients, $\frac{1}{3}$ and $\frac{1}{2}$, are multiplied exactly as in arithmetic.

To divide two powers of the same quantity, subtract their indices.

It is shown above that $a^4 \times a^3 = a^7$, it must therefore follow that $a^7 \div a^4 = a^3$, or a^7 The result, in each case, has been obtained by subtracting the indices. Combining the rules for multiplication and division in the following example $a^4 \times a^5 \div a^6$, we get $a^{4+5-6} = a^3$. Find the value of $\frac{a^7 \times a^3 \times a}{a^3 \times a^2}$, here we have $a^{7+3+1-3-2} = a^6$.

Note that $\frac{x}{x} = 1$, but $\frac{x}{x}$ means x^{1-1}, or x^0, therefore $x^0 = 1$.

Second Index Law. To find the value of a power of a letter or quantity raised to a power, multiply the indices.

Evaluate $(a^3)^5 = a^{3 \times 5} = a^{15}$.

Now $(a^3)^5$ means $a^3 \times a^3 \times a^3 \times a^3 \times a^3$, and as we multiply by adding the indices, the result is a^{15}. Thus $(a^2\,b)^3$ means $a^2\,b \times a^2\,b \times a^2\,b$, and this is $a^2 \times a^2 \times a^2 \times b \times b \times b = a^6\,b^3$.

$$(a^{\frac{1}{3}})^5 = a^{\frac{5}{3}}, \text{ or } a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}}.$$
$$= a^{\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}+\frac{1}{3}} = a^{\frac{5}{3}}.$$

$$(a^{\frac{1}{2}})^{\frac{1}{3}} = a^{\frac{1}{2} \times \frac{1}{3}} = a^{\frac{1}{6}}.$$

$$(a^{-2})^2 = a^{-2 \times 2} = a^{-4} = \frac{1}{a^4}$$

The process is generally done mentally.

To find the meaning of the negative Index.

By the first index law, $a^5 \div a^7$, means a^{5-7} or a^{-2}

but $a^5 \div a^7 = \frac{a^5}{a^7}$. Divide by $\quad \frac{a^5}{a^{7-5}} \quad \frac{a^{5-5}}{} \quad \frac{a^0}{} \quad \frac{1}{}$

Therefore if $a^5 \div a^7 = a^{-2}$

and $a^5 \div a^7 = \frac{1}{a^2}$ then $a^{-2} = —$

We may write then that $x^{-3} =$ And therefore $x^{-n} =$

–, where n has any value.

Surds.

A surd is a root which does not work out exactly. $\sqrt{3}$ and $\sqrt{2}$ are surds, their value being 1·732 + . . ., 1·414 + . . .
$\sqrt{a} \times \sqrt{a}$ may be written $a^{\frac{1}{2}} \times a^{\frac{1}{2}} = a$, and $\sqrt{3} \times \sqrt{3} = 3$.
$\sqrt{a} \times \sqrt{a} \times \sqrt{a} = a \times \sqrt{a}$ and $\sqrt{3} \times \sqrt{3} \times \sqrt{3} = 3\sqrt{3}$.

The object of manipulating surds, is to make the expression easier to evaluate.

Now $\sqrt{9} \times \sqrt{4}$ is the same as $\sqrt{36}$, because $\sqrt{9} \times \sqrt{4} = 6$ and $\sqrt{36} = 6$, it follows then that $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$. Find the value of $\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a}$, this is $a^{\frac{1}{3}} \times a^{\frac{1}{3}} \times a^{\frac{1}{3}}$, and is or a.

Examples. $\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5}$; $\sqrt{1210} = \sqrt{121} \times \sqrt{10} = 11\sqrt{10}$:

$\times\ 2 = \sqrt[3]{64} \times \sqrt[3]{2} = 4 \times \sqrt[3]{2}$;

$= \sqrt{2} \times \sqrt{9} = 3 \times \sqrt{2}$.

$\times\ 5\sqrt{yz} = 15\sqrt{xy \times yz} = 15\sqrt{xy^2z} = 15\ y\ \sqrt{xz}$, if we take y^2 from under the square root sign we must extract its square root, which is y. It is useful to remember that $\sqrt{2} = 1{\cdot}414 + \ldots$, $\sqrt{3} = 1{\cdot}732 + \ldots$, $\sqrt{5} = 2{\cdot}236 + \ldots$, $\sqrt{10} = 3{\cdot}1622 + \ldots$, these sometimes help us in our work.

$\sqrt{18} = 3\sqrt{2} = 3 \times 1{\cdot}414 = 4{\cdot}242$; $\sqrt{20} = \sqrt{4} \times \sqrt{5} = 2\sqrt{5} = 4{\cdot}472$; $\sqrt{12} = \sqrt{4} \times \sqrt{3} = 2 \times \sqrt{3} = 3{\cdot}464$; $\sqrt{1000} = \sqrt{100} \times \sqrt{10} = 10\sqrt{10} = 31{\cdot}622$.

Fractions involving Surds.

$\frac{6}{\sqrt{3}}$, if we multiply this fraction by $\frac{\sqrt{3}}{\sqrt{3}}$ we do not alter its value, since $\frac{\quad}{\sqrt{3}}$ is 1.

$$\begin{array}{lll} 6 & \sqrt{3} & 6\sqrt{3} \\ & & \qquad\qquad 2\sqrt{3}, \text{ which is easier to solve.} \\ & \sqrt{3} & 3 \end{array}$$

$$\begin{array}{lllll} & = \dfrac{7 \times \sqrt{22}}{2\sqrt{22} \times \sqrt{22}} & & 7\sqrt{22} & \\ 2\sqrt{22} & & & 2 \times 22 & 44 \\ 3\sqrt{3} & 3 \times \sqrt{3} \times \sqrt{5} & & 3\sqrt{15} & \\ 5\sqrt{5} & 5 \times \sqrt{5} \times \sqrt{5} & & 5 \times 5 & 25 \end{array}$$

Always endeavour to get rid of the surd in the denominator.

In the last example, the final form $\frac{3\sqrt{15}}{25}$ is easy to solve, since it is much easier to divide by 25 than by $5\sqrt{5}$.

Multiplication.

If a positive quantity be multiplied by another positive one the result is positive, but if a positive quantity is multiplied by a negative one the result is negative. If a negative quantity be multiplied by a negative one the result is positive.

$$\begin{array}{rl} \text{Thus:} & 2\,a \times 4\,b = +\,8\,a\,b \\ & 2\,a \times (-\,4\,b) = -\,8\,a\,b \\ & -\,2\,a \times (-\,4\,b) = +\,8\,a\,b \end{array} \Bigg\} \text{ This is important.}$$

Multiply $3\,x + 7\,y$ by $2\,x - 5\,y$

$$\begin{array}{l} 3\,x + 7\,y \\ 2\,x - 5\,y \\ \hline 6\,x^2 + 14\,xy \\ \qquad -\,15\,xy - 35\,y^2 \\ \\ \qquad\quad -\,xy - 35 \end{array}$$

Ans.

Multiply every term in $3\,x + 7\,y$, first by $2\,x$, then by $-\,5\,y$, write terms of the same kind und each other and add.

Note that $(3x + 7y)$ and $\quad -\,5y)$ are called the "factors" of $6\,x^2 - xy - 35\,y^2$.

Multiply $3\,a + 4\,b + 2\,c$ by $a - 3\,b$.

$$\begin{array}{l}
3\,a + 4\,b + 2\,c \\
\quad a - 3\,b \\
\hline
3\,a^2 + 4\,ab + 2\,ac \\
\qquad - 9\,ab \qquad\qquad - 12\,b^2 - 6\,bc \\
\\
3\,a^2 - 5\,ab + 2\,ac - 12\,b^2 - 6\,bc \quad \text{Ans.}
\end{array}$$

Note here that b^2 and bc do not appear in the first line of the multiplication, therefore $-12b^2$ and $-6bc$ must be written in separate columns in the second line before adding.

Multiply $a + b$ by $a + b$, or square $(a + b)$, this may be written $(a + b)^2$.

$$\begin{array}{l}
a + b \\
a + b \\
\hline
a^2 + ab \\
\quad + ab + b^2 \\
\hline
a^2 + 2\,ab + b^2 \quad \text{Ans.}
\end{array}$$

To square the sum of two quantities, write down the squares of the two quantities, add them together, then multiply the two quantities together, double the term obtained and add to the first two terms.

Thus $(a + b)^2 = a^2 + b^2 + 2\,ab$, the terms may be arranged in any order.

Thus $(x + 1)^2 = x^2 + 1^2 + (2 \times x \times 1) = x^2 + 2\,x + 1$. We note, in passing, that $x + 1$ is the square root of $x^2 + 2x + 1$, and because the square root has a complete and definite value we call $x^2 + 2\,x + 1$ a "complete square."

Multiply $(a + b)$ by $(a - b)$.

$$\begin{array}{l}
a + b \\
a - b \\
\hline
a^2 + ab
\end{array}$$

If we have to evaluate the difference of two squares such as $(a^2 - b^2)$, it is generally easier to multiply $(a - b)$ by $(a + b)$.

$a^2 \quad 0 \quad - b^2$ Ans. is $a^2 - b^2$.

The factors of $a^2 - b^2$ are $(a + b)$ and $(a - b)$.

Evaluate $(10^2 - 6^2)$ this is the same as $(10 + 6) \times (10 - 6) = 64$.

Evaluate $(16\tfrac{1}{2})^2 - (10\tfrac{1}{2})^2$ this is the same as $(16\tfrac{1}{2} + 10\tfrac{1}{2})$ $- 10\tfrac{1}{2}) = 27 \times 6 = 162$.

It is often easier to proceed in this manner than to multiply the squares out and subtract.

Multiply $(x + 3)$ by $(x + 5)$.

$$
\begin{array}{l}
x + 3 \\
x + 5 \\
\hline
x^2 + 3\,x \\
\phantom{x^2 +{}} 5\,x + 15 \\
\hline
x^2 + 8\,x + 15 \quad \text{Ans.}
\end{array}
$$

First term x^2
Second term $(3 + 5) \times x$
Last term 3×5

This is an easy example, but it is important. If the first term of both expressions is x, and if the second term in both is a number, then the following rule holds. Square the first term (x), and write it down. For the second term in the answer, add the two numbers together and multiply the result by x. The final term in the answer is the product of the two numbers.

Multiply $(x + 3)$ by $(x - 5)$,

Ans. $x^2 - 2x - 15$.

First term is x^2, second term is $[3 + (-5)]\,x$ or — [illegible], third term is $3 \times ($— [illegible] 15.

Now we sometimes need the factors of expressions such as $x^2 + 8\,x + 15$ or $x^2 - 2\,x - 15$. We can often find the factors by mere inspection, because we get used to the fact that 8 is the sum of 3 and 5, and that 15 is the product of 5 and 3, in the first expression. In the second expression we know that -2 is the sum of -5 and 3, and that -15 is the result of multiplying -5 by 3. The factors in these cases can be guessed. However, it is only sometimes that the factors may be determined by inspection.

Division.

Divide $x^2 + 8\,x + 15$ by $x + 3$.

$$
\begin{array}{l}
x + 3)x^2 + 8\,x + 15(x + 5. \quad \text{Ans.} \\
x^2 + 3\,x \\
\hline
\phantom{x + 3)x^2 + {}} 5\,x + 15 \\
\phantom{x + 3)x^2 + {}} 5\,x + 15 \\
\hline
\end{array}
$$

Now x goes into x^2, x times. Write x in the answer; multiply the divisor $(x + 3)$ by x; this gives $x^2 + 3\,x$, subtract from the dividend (note, change the signs and add), this gives $5\,x$; now bring down the 15; x into $5x$ goes 5 times, multiply $(x + 3)$ by 5, this gives $5\,x + 15$, set down and subtract.

Divide $2\,x^3 - x^2 - 13\,x + 5$ by $2\,x + 5$.

$$
\begin{array}{l}
2\,x + 5)2\,x^3 - x^2 - 13\,x + 5(x^2 - 3\,x + 1. \quad \text{Ans.} \\
2\,x^3 + 5\,x^2 \\
\hline
 - 6\,x^2 - 13\,x + 5 \\
 - 6\,x^2 - 15\,x \\
\hline
 2\,x + 5 \\
 2\,x + 5
\end{array}
$$

$2x$ goes into $2x^3$, x^2 times. Multiply $(2x + 5)$ by x^2, set down and subtract. This gives $-6x^2$. Now bring down $-13x + 5$, and repeat the whole process until the division is complete.

Quantities and Expressions in Brackets.

Simplify $(3x + 5) + 2(x - 3) - (10x - 2)$.

If we have a multiplier in front of a bracket, every term in the bracket must be multiplied. If we have a minus sign in front of a bracket, every term must have its sign reversed when the bracket is taken away.

Thus $2(x - 3)$ is $2x - 6$, and $-(10x - 2)$ is $-10x + 2$. $(3x + 5) + 2(x - 3) - (10x - 2) = 3x + 5 + 2x - 6 - 10x + 2$.

Collecting terms, $3x + 2x - 10x + 5 + 2 - 6 = -5x + 1$.

Simplify $(x + 2)[(x + 5)^2 - (2x + 1)(x + 3)]$. Solve the inner brackets first.

$(x + 2) \quad 10x + 25 - (2x^2 + 7x + 3)]$

$(x + 2) \quad 10x + 25 - 2x^2 - 7x - 3]$, remove the bracket and change the signs.

$= (x + 2)[3x - x^2 + 22]$, the final step would be to multiply the two expressions together.

Note.—In mixed expressions, we must always perform multiplication and division first before we add or subtract.

Thus: $x + 3 \times x + 2x^2 \div x = x + 3x + 2x = 6x$.

This is the same as we do in arithmetic, for instance,

$12 + 10 \div 2 - 12 \div 4 + 5 \times 2 = 12 \quad \cdot 3 + 10 = 24$.

If all the terms in an expression have a common multiplier or divisor, the common factor may be taken out of each term and written outside a bracket. Thus:—

$ab + ab^2 + ac = a(b + b^2 + c)$.

$$\frac{b^2}{a} + \frac{a^2 b^2}{a} + \frac{c}{a^2} = \frac{1}{a}\left(b^2 + a^2 b^2 + \frac{c}{a}\right),$$

if we multiply out again, we must get the same expression as we had at the beginning.

TEST EXAMPLES II.

1. Add $3\,x^2 + 5\,xy + 3\,y^2$ to $4\,x^2 - 3\,xy + y^2$. Subtract $3\,x^2 - 10$ from $5\,x^3 + 6\,x^2 - 12$.

2. Simplify $(x + 5)^2 + 3(x^2 + 2\,x + 1) - x\,(x + 3)$

3. Divide $x^3 + x^2 + 2x - 16$ by $x - 2$.

4. Multiply $3\,a^2 + 5\,b + 3$ by $a - b$.

5. Find the factors of $x^2 + 13\,x + 30$; $x^2 + 2\,x \quad 3$;

6. Divide $(a^3 - b^3)$ by $\quad b)$.

7. Square the following, $(a \quad b)$, $(a^2 \quad b)$, $3\,a\,b\,c$, $\dfrac{3\,a\,b}{\quad^2}$

8. Solve $(x + 1)\,[x + 3 - (x + 5)\,(2\,x + 3)\,]$.

9. Evaluate $3\sqrt{48}$; $\sqrt{98}$; $\dfrac{3\sqrt{54}}{\sqrt{2}}$ $\dfrac{4 \times \sqrt{72}}{\sqrt{3}}$

10. Find the value of $\sqrt{\dfrac{a^2\,b^2}{4\,x^3\,y^3}}$, and evaluate it when $a = 2$, $y = 3$, $x = 6$, $b = 4$.

CHAPTER III.

THE SOLUTION OF SIMPLE EQUATIONS; SIMULTANEOUS EQUATIONS.

The Solution of Simple Equations.

A simple equation is one in which there is only one unknown term, always of the first power, to determine.

Solve the equation $5x + 12 = 22$, here x is the only unknown term.

We are given that $5x$, if added to 12, makes a total of 22.

$5x$ therefore must equal 10, and $x = 2$; or we may proceed as follows:—

$5x + 12 = 22$

$5x + 12 - 12 = 22 - 12$

This is the same as—

$5x = 22 - 12$,

or $5x = 10$

and $x = 2$. Ans.

If we subtract 12 from the right hand side, and omit 12 from the left hand side, we have really subtracted 12 from both sides, and the equality is not destroyed.

Therefore we have the rule:—If a term, or a quantity is transposed, i.e., taken away from one side of the equation to the other side, the sign of the term must be reversed. Collect all terms which contain the unknown, on the left hand side of the equation, taking all other terms to the right hand side.

Solve the equation $2(x + 3) - (3 - 5x) = (5x + 12)$.

Simplify by removing the brackets, $2x + 6 - 3 + 5x = 5x + 12$.

Collect terms by transposing, $2x + 5x - 5x = 12 - 6 + 3$, note how the signs have been changed.

$$2x = 9,$$

$$x = 4{\cdot}5. \quad \text{Ans.}$$

If $3a + b - 3c = a + 3b + 6c$, find the value of a.

$3a - a = 3b + 6c - b + 3c$.

$2a = 2b + 9c$

$$a = \frac{2b + 9c}{2}$$

First arrange all terms in a, on the left hand side, taking all other terms to the right hand side. Note the change of signs.

The next step needs no explanation. Finally, to get the value of a, divide both sides of the equation by 2. Note carefully that both terms $2b$ and $9c$ are divided by 2.

Generally.—If we add a term, or a quantity to one side of an equation we must add the same term to the other side. The same applies in the case of subtraction. Also, if we multiply

or divide one side of an equation by a term or a number, we must perform the same operation to the other side. The equality is preserved in this manner. We have dealt so far with equations where quantities are only added or subtracted, until the final step.

Solve the equation $\dfrac{x+2}{4} - \dfrac{x}{3} = \dfrac{x+6}{6} - 5.$

L.C.M. is 12; multiply every term by 12.

$$\frac{(x+2)}{\cancel{4}} \times \overset{3}{\cancel{12}} - \frac{x}{\cancel{3}} \times \overset{4}{\cancel{12}} = \frac{(x+6)}{\cancel{6}} \times \overset{2}{\cancel{12}} - 5 \times 12$$

$$3\,(x+2) - 4\,x = 2\,(x+6) - 60$$
$$3\,x + 6 - 4\,x = 2\,x + 12 - 60$$
$$3\,x - 4\,x - 2\,x = 12 - 60 - 6$$
$$-3\,x = -54, \text{ divide both sides by } -3,$$
$$x = 18.$$

Remember that multiplication and division must always be performed before addition and subtraction of terms.

Find the value of x if $\dfrac{a-b}{x-y} = \dfrac{p\,q}{c\,z}$

First multiply both sides by $c\,z$; $\dfrac{(a-b)\,c\,z}{x-y} = \dfrac{p\,q}{\cancel{c\,z}} \times$

Next multiply both by $(x-y)$

$$\frac{(a-b)\,c\,z}{\cancel{(x-y)}} \times \cancel{(x-y)} = p\,q \times (x-y)$$

We have then $(a-b)\,c\,z = p\,q\,(x-y)$.

These two steps are generally performed in one operation.

Multiply the numerator of each side by the denominator of the other side, and write the two results obtained as the two sides of a new equation.

$\dfrac{a-b}{x-y} = \dfrac{p\,q}{c\,z}$, cross multiply as suggested and we have

$$(a-b) \times cz = (x-y) \times pq.$$
$$(x \quad y) = (a-b)\,\frac{c\,z}{p\,q}$$
$$x = (a-b)\,\frac{\quad}{p\,q} + y$$

Now take $(x-y)$ to the left side, and divide both sides by $p\,q$. Note, if we transpose a multiplier it must appear on the other side as a divisor, that is to say, it goes from the top of one side to the bottom of the other. The reason for these changes has been demonstrated above.

Find S in the following equation $\frac{C\ (T + 1)^2}{S - 6} = P$

Cross multiply, $P\ (S - 6) = C\ (T + 1)^2$

$$S - 6 = \frac{C\ (T + 1)^2}{P}, \text{ or } S = \frac{C\ (T + 1)^2}{P} + 6. \quad \text{Ans}$$

Find T in the same equation

$$\frac{C\ (T + 1)^2}{S - 6} = P, \qquad (T + 1)^2\ C = P\ (S - 6)$$

$$(T + 1)^2 = \frac{P\ (S - 6)}{C}$$

Take the square root of both sides, $T + 1 = \sqrt{\frac{P\ (S - 6)}{C}}$

$$\text{or } T = \sqrt{\frac{P\ (S - 6)}{C}} - 1. \quad \text{Ans.}$$

TEST EXAMPLES III.

Solve the following equations :—

1. $3\ x - 21 = x + 5.$
2. $3\ x + 9 = 7\ x - 3.$
3. $\frac{2\ x + 2}{4} \quad \frac{4}{3}\ x \quad \frac{x + 5}{6}$
4. $5\ (x + 2) + 6 = 3\ (x + 10).$
5. $2\ (x^2 + 3) = 3\ (x^2 - 1).$
6. Find x, if $\frac{(p - q)}{(a + b)} = \frac{x\ y}{z}$
7. Find p in the above equation (No. 6).
8. Find the value of a, if $\frac{p \times c}{d \times e} = \frac{k \times q}{a \times b}$
9. Solve $0{\cdot}3\ (x + 3) = 0{\cdot}5\ (x + 1).$

Examples in Solution of Formulæ

1. $H. = \frac{2\ P\ A\ L\ N}{33000}$, **Find L, when H = 1500, P 28 N = 65, A = 2530.**

2.

Find S, when $H_n = 450$, $P = 216$, $D = 80$ and $H = 13{,}950$.

3. $P = \dfrac{(D - d)\,T \times 28{,}000}{W \times D}$ Find d, when $D = 4\frac{1}{2}$, $T = \frac{3}{4}$, $P = 225$, and $W = 25$.

4. $P = \dfrac{99{,}000\,t^2}{(L + 1)\,D}$ Find L, when $P = 180$, $D = 40$, $t = \frac{13}{16}$.

5. $P = \dfrac{9900\,T}{3\,D}\left[5 - \dfrac{C + 12}{60 \times T}\right]$ Find P, when $T = \frac{3}{4}$, $D = 41$, and $C = 29$.

6. $S^3 = \dfrac{C\,P\,D^2}{f\left(2 + \dfrac{D^2}{d^2}\right)}$ By transposing this formula, give the value of d in algebraic form.

Simultaneous Equations.

To find the value of two unknown terms we must be given two equations, each containing both the unknown terms.

In the equation, $6\,x + 5\,y = 43$, we cannot find the value of either x or y. We can, however, express either in terms of the other. Thus $5\,y = 43 - 6\,x$, and $6\,x = 43 - 5\,y$. But if we are given two equations, both of which are true for the two unknowns, x and y can be determined.

Thus $6\,x + 5\,y = 43$, $10\,x - 3\,y = 15$, Set down as shown.

$$6\,x + 5\,y = 43\,; \times 3$$
$$10\,x - 3\,y = 15\,; \times 5$$

$$18\,x + \cancel{15\,y} = 129 \quad \text{add}$$
$$50\,x - \cancel{15\,y} = 75$$

$$68\,x = 204,$$
$$x = 3. \quad \text{Ans.}$$

$$6\,x + 5\,y = 43, \text{ put } = 3$$
$$(6 \times 3) + 5\,y = 43$$
$$5\,y = 43 - 18$$
$$5\,y = 25$$
$$y = 5. \quad \text{Ans.}$$

Method. Multiply the top equation right across by 3. Multiply the other equation right across by 5. Write the new equations down again and add. The 15 y's will cancel out. Then solve for the value of x. Finally put the value of x now found in either of the first equations, and solve for the value of y. In this method we are said to "eliminate" one of the unknown terms. Here we have eliminated y.

Solve and find the values of x and y from the following :—

$$5\,x + 6\,y = 92\,; \; 3\,y = 41 - 2\,x.$$

Rearrange the terms in the second equation and set down as shown.

$5x + 6y = 92$;
$2x + 3y = 41$; $\times 2$
we get
$5x + 6y = 92$
$4x + 6y = 82$

$x = 10$. Ans.

To eliminate y; multiply the second equation right across by 2. Then subtract as shown, i.e., change the signs and add.

Now take either equation, put in the value of x and solve for y.

$$2x + 3y = 41, \text{ write } x = 10$$
$$(2 \times 10) + 3y = 41$$
$$3y = 41 - 20, \; 3y = 21$$
$$y = 7. \quad \text{Ans.}$$

Problems leading to Simple Equations.

Divide 25 into 2 parts, one of which is $1\frac{1}{2}$ times the other.

Let x = the lesser part,
then $25 - x$ = the greater part.

Now the condition of the problem is that $1\frac{1}{2}$ times the lesser part must equal the greater part.

$$\therefore x \times 1\tfrac{1}{2} = 25 - x$$
$$2\tfrac{1}{2}x = 25, \text{ or } x = \frac{25}{2\frac{1}{2}}$$
$$x = 25 \times \tfrac{2}{5} = 10.$$

The parts are therefore 10 and 15. Ans.

Find the number whose third part exceeds its fourth part by 5.

Let x = the number, its third part is $\frac{x}{3}$, its fourth $\frac{x}{4}$,

Now $\frac{x}{3}$ is greater than $\frac{x}{4}$ by 5.

$\therefore \frac{x}{3} = 5 + \frac{x}{4}$, multiply throughout by 12,

$$4x = 60 + 3x, \text{ transposing,}$$
$$x = 60. \quad \text{Ans.}$$

Note that the question could also be set down

as $\frac{x}{3} - \frac{x}{4} = 5$, and solved.

Find two numbers which differ by 20, and whose ratio is as 3 : 5.

Let x = the lesser, then $\frac{5}{3}x$ is the greater.

These must differ by 20.

$\therefore \frac{5}{3}x - x = 20$, multiply throughout by 3,

$5x - 3x = 60$.

$2x = 60,\ x = 30$.

The numbers are 30 and 50. Ans.

One ship is 3 times as old as another, but in 10 years' time will be only twice as old. Find the ages of the ships.

Let x = present age of the newer ship.

then $3x$ = present age of the old ship.

In ten years' time $x + 10$, and $3x + 10$, will be the respective ages.

Then $2(x + 10) = 3x + 10$

$2x + 20 = 3x + 10,$

$-x = -10$, or $x = 10$.

One ship is now 10 years' old and the other 30 years. Ans.

Two ships sail on the same course, from the same port. Their speeds are $10\frac{1}{2}$ knots and 12 knots respectively. If the slower vessel has half an hour's start, how far from port will she be overtaken ?

Let x = distance from port when overtaken.

$$\frac{\text{distance}}{\text{speed}} = \text{time}.$$

Time to do x miles $= \dfrac{x}{10\frac{1}{2}}$ for slow ship, and $\dfrac{x}{12}$ for fast ship (hrs.) But the time of the fast ship is less by $\frac{1}{2}$ hour than time of the slow ship.

$$\therefore \frac{x}{10\frac{1}{2}} - \frac{1}{2} = \frac{x}{12}, \text{ L.C.M. is } 10\tfrac{1}{2} \times 12$$

$$12x - 63 = 10\tfrac{1}{2}x$$

$$1\tfrac{1}{2}x = 63, \text{ or } x = 63 \times \tfrac{2}{3}$$

$$x = 42 \text{ nautical miles. Ans.}$$

TEST EXAMPLES IV.

Problems on Simple Equations.

1. A certain number, divided by 3, is equal to half the number minus 4. What is the number ? Ans. 24.

2. A number consists of two digits, the first being twice the value of the second. If the digits are reversed, the number so formed is 27 less than the first number. Find the digits. Ans. Digits 6 and 3. Number 63.

3. The sum of the ages of two men is 73 years. Twice one man's age is 3 times the other man's age. Find their ages. Ans. 43·8 and 29·2 years.

4. Two numbers are in the ratio 2 : 3, and when each is increased by 15, they are in the ratio 5 : 6. Find the numbers. Ans. 10 and 15.

5. 100 lb. of bronze contains 85 per cent. copper and 15 per cent. tin. With how much copper must it be melted to obtain a bronze containing 92 per cent. copper ? Ans. 87·5 lb.

6. A man's salary increases each year in the ratio 5 : 6. In three years he receives altogether £728. Find his salary in the first year. Ans. £200.

Simultaneous Equations.

7. Find two quantities such that the first exceeds half the second by 0·6, while one-quarter of the second exceeds one-fifth of the first by 0·42. Ans. 2·4 and 3·6.

8. Find the ages of two men if 10 years ago their ages were in the ratio of 5 : 3, and 5 years hence will be in the ratio of 4 : 3. Ans. 35 and 25.

QUADRATIC

Quadratic Equations are those in which we deal with the *square* of the unknown quantity. Equations of this kind are easily solved if we have only x^2 in them ; the solution merely involves taking the square root of both sides, but because when we take the square root of 9, our answer may be either $+ 3$ or $- 3$, since $(+ 3)^2 = 9$, and $(- 3)^2 = 9$, we always write our answer as $\pm$, meaning plus or minus.

Thus $\sqrt{9} = \pm 3$, and if $x^2 = 49$, then $x = \pm 7$.

If, however, the quadratic contains both x^2 and x, we may proceed along the following lines.

Suppose $x^2 - 3x = 18$, write it as $x^2 - 3x - 18 = 0$.

Now the factors are $(x - 6)$ and $(x + 3)$, because :—

$(x - 6)(x + 3) = x^2 - 3x - 18$, as already explained.

$$\text{Therefore if } x^2 - 3x - 18 = 0,$$
$$\text{It follows that } (x - 6)(x + 3) = 0.$$

Now how can this expression be 0 ? It can be 0 if $(x - 6) = 0$, or if $(x + 3) = 0$, because :—

Dividing both sides by $(x + 3)$, we get—

$$\frac{(x - 6)(x + 3)}{(x + 3)} \qquad \frac{0}{(x + 3)}$$

This is the same as $(x - 6) = \dfrac{0}{(x + 3)}$

Now 0 divided by any quantity is 0,

$$\therefore x - 6 = 0, \text{ or } x = 6.$$

By the same method, $x + 3 = 0$ or $x = - 3$.

The answers, or as we call them the "roots" of the equation are $+ 6$ and $- 3$. Note that either of these values the equation, since

$x^2 - 3x = 18$, if $x = 6$, then putting $x = 6$

$6^2 - 3 \times 6 = 18$

$36 - 18 = 18$ which is correct, or if we put x

$(- 3)^2 - 3 \times (- 3) = 18$

$9 + 9 = 18$ which is also correct. It should be pointed out, however, that while either value is correct for the equation, only one of them may apply in the case of the solution of a problem ; but there are problems in which both values are used as answers.

Solve the following equations by factorizing.

$x^2 - 13\,x = 30$, write it as $x^2 - 13\,x - 30 = 0$; by inspection, the factors are $(x - 15)\,(x + 2)$, $\therefore x - 15 = 0$, and $x + 2 = 0$.

The roots are therefore $+ 15$ and $- 2$. Ans.

$x^2 - 3{\cdot}8\,x = 11{\cdot}6$, or $x^2 - 3{\cdot}8\,x - 11{\cdot}6 = 0$

The factors are $(x - 5{\cdot}8)\,(x + 2)$ and therefore

if $(x - 5{\cdot}8)\,(x + 2) = 0$

$x = 5{\cdot}8$, or $- 2$. Ans.

We get equations in x^2 and x, however, in which the factors cannot be got by inspection ; in fact, the number of quadratics to which we can apply the above method is small.

Method of completing the Square.

We observe that $(x + 2)^2 = x^2 + 4\,x + 4$, by methods already shown.

$$\sqrt{x^2 + 4\,x + 4} = x + 2.$$

Now $x^2 + 4\,x + 4$ is a "complete square." But if we are given that $x^2 + 4\,x = 21$, we cannot at once take the square root of both sides, because we do not know any quantity which when squared will give $x^2 + 4\,x$; but we can "complete the square" of the left hand side of the equation by adding 4 to it. We must add 4 to the right hand side as well, to preserve the equality. Note that 4 is equal to the square of 2, and here 2 is half the co-efficient of x. The rule to complete the square is :—Add to each side of the equation, the square of half the co-efficient of x. For instance,

$x^2 + 6\,x = 16$. To complete the square add $(\frac{6}{2})^2$ to each side.

$$x^2 + 6\,x + (\tfrac{6}{2})^2 = 16 + (\tfrac{6}{2})^2\,;$$

or, $x^2 + 6\,x + 9 = 16 + 9$, take the square root of both sides

$$x + 3 = \pm\sqrt{25}$$

$$x + 3 = \pm 5, \text{ or } x = \pm 5 - 3$$

$$x = -8 \text{ or } + 2. \quad \text{Ans.}$$

Solve $3\ x^2 + 5\ x = 31{\cdot}25$.

To solve by the last method, we first make the co-efficient of x^2 unity. Divide throughout by 3.

This gives:

$$x^2 + \tfrac{5}{3}\ x = \frac{31{\cdot}25}{3}$$

Half the co-efficient of x is $\tfrac{5}{6}$ and its square is $(\tfrac{5}{6})^2$, add this value to both sides, and take the square root of both sides.

$$x^2 + \tfrac{5}{3}\ x + (\tfrac{5}{6})^2 = \frac{31{\cdot}25}{3} + (\tfrac{5}{6})^2$$

$$x + \tfrac{5}{6} = \pm \sqrt{\frac{31{\cdot}25}{3} + \tfrac{25}{36}} = \pm \sqrt{\frac{375 + 25}{36}}$$

$$= \pm \sqrt{\frac{400}{36}}$$

$$x + \tfrac{5}{6} = \pm \sqrt{\frac{400}{36}}$$

$$x = \pm \tfrac{20}{6} - \tfrac{5}{6},\ x = + \tfrac{15}{6} \text{ or } - \tfrac{25}{6}$$

$$x = + 2\tfrac{1}{2} \text{ or } - 4\tfrac{1}{6}. \quad \text{Ans.}$$

Solve $2\ x^2 - 5\ x = 2{\cdot}8$, proceeding as above:—

$$x^2 - \tfrac{5}{2}\ x = 1{\cdot}4$$

$$x^2 - \tfrac{5}{2}\ x + (\tfrac{5}{4})^2 = 1{\cdot}4 + (\tfrac{5}{4})^2$$

$$x - \tfrac{5}{4} = \pm \sqrt{1{\cdot}4 + \tfrac{25}{16}} = \pm \sqrt{1{\cdot}4 + 1{\cdot}5625}$$

$$x - \tfrac{5}{4} = \pm \sqrt{2{\cdot}9625}$$

$$x - \tfrac{5}{4} = \pm 1{\cdot}721$$

$$x = \pm 1{\cdot}721 + 1{\cdot}25$$

$$x = + 2{\cdot}971 \text{ or } - 0{\cdot}471$$

Note that the square root of $x^2 - \tfrac{5}{2}\ x + (\tfrac{5}{4})^2 = x - \tfrac{5}{4}$, and the square root of $x^2 + \tfrac{5}{2}\ x + (\tfrac{5}{4})^2 = x + \tfrac{5}{4}$.

Attention to these *signs* is important.

In the above method of solving these equations, the student soon learns to complete the square mentally, and write down the square root of the left side of the equation in one step.

The quadratic formula is obtained by the previous method.

If a is the co-efficient of x^2, and if b is the co-efficient of x, c being the third term, then the general form is :—

$$a\ x^2 + b\ x = - c, \text{ or } ax^2 + b\ x + c = 0 \quad . \quad . \quad \text{I.}$$

this becomes, $x^2 + \frac{b}{a}\ x = - \frac{c}{a}$, i.e., dividing throughout by a

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = \left(\frac{b}{2a}\right)^2 - \frac{c}{a}$$ i.e., completing the square.

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2}{4a^2} - \frac{c}{a}}$$ Taking the square root of both sides.

$$x = -\frac{b}{2a} \pm \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad \ldots 2$$

If we take $4a^2$ out of the square root sign, it becomes $2a$, and $2a$ is therefore the denominator of both terms.

Note that if b and c in I. have minus signs before them, then the terms b and $4ac$ in the formula 2 are positive.

Applying this to solve $3x^2 + 5x = 31{\cdot}25$, write it, $3x^2 + 5x - 31{\cdot}25 = 0$.

$$\text{Then } x = \frac{-5 \pm \sqrt{(5)^2 - 4[3 \times (-31{\cdot}25)]}}{2 \times 3}$$

Note $-12 \times (-31{\cdot}25) = +375$

$$x = \frac{-5 \pm \sqrt{25 + 375}}{6}$$

$$x = \frac{-5 \pm \sqrt{400}}{6} = \frac{-5 \pm 20}{6}$$

$x = +\frac{15}{6}$ or $-\frac{25}{6}$ as before.

Solve $2x^2 - 5x - 2{\cdot}8 = 0$.

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \times 2 \times (-2{\cdot}8)}}{2 \times 2}$$

$$x = \frac{5 \pm \sqrt{25 + 22{\cdot}4}}{4} = \frac{5 \pm 6{\cdot}88}{4}$$

$x = 2{\cdot}97$ or $-0{\cdot}47$ as before.

The student may use either the direct method of completing the square, or the quadratic formula. The formula, however, has to be remembered; but in completing the square, the solution is being done from first principles. Facility in solving these equations, by either method, may soon be acquired by practice.

Problems leading to Quadratics.

Example. The reciprocal of a number is equal to the number plus 1. Find the number.

Now the reciprocal of a number is $\frac{1}{\text{the number}}$, or one divided by the number.

Let x = the number, then $\frac{1}{x}$ is the reciprocal of x,

$\frac{1}{x} = x + 1$, by the condition stated in the problem ; cross multiply, then, $x(x + 1) = 1$

or $x^2 + x = 1$, here the co-efficient of x is 1

$x^2 + x + (\frac{1}{2})^2 = 1 + (\frac{1}{2})^2$, add $(\frac{1}{2})^2$ to each side.

$x + \frac{1}{2} = \pm\sqrt{1\frac{1}{4}} = \pm\sqrt{\frac{5}{4}}$

$$x = \pm\frac{\sqrt{5}}{2} - \frac{1}{2}, \text{ or } x = \pm\frac{2{\cdot}236}{2} - 0{\cdot}5$$

$$x = 0{\cdot}618 \text{ and } - 1{\cdot}618$$

Take the positive value, $x = 0{\cdot}618$, then $\frac{1}{0{\cdot}618} = 1{\cdot}618$, this fulfils the conditions of the problem. Ans $= 0{\cdot}618$.

Example. A certain number is such that its third part equals twice its square root. Find the number.

Let x = the number, $\frac{x}{3}$ is its third part, $2\sqrt{x}$ is twice its square root.

Then $\frac{x}{3} = 2\sqrt{x}$; this is an easy example.

Square both sides, $\frac{x^2}{9} = 4x$

By cross multiplying, $x^2 = 36x$

or $x = 36$. Ans.

Example. Two ships leave one port for another 168 miles away. One ship is 2 knots faster than the other, and starts 2 hours later than the slow ship. If they arrive in port at the same time, find the speeds of the ships.

Let x = speed of slow ship in knots.

$x + 2$ = speed of fast ship in knots.

In 2 hours the slow ship will go $2 \times x$ miles. When the fast ship starts the slow ship has $168 - 2x$ miles to go to port. Since they arrive together, the time taken by the fast ship to do 168 miles is the same as for the slow ship to do $168 - 2x$ mil

Now $\frac{\text{distance}}{\text{speed}}$ = time.

$$\therefore \frac{168}{x+2} = \frac{168-2x}{x}, \text{ cross multiply and we get :—}$$

$168\,x = (x+2)(168-2\,x)$, multiply out,

$168\,x = -2\,x^2 + 164\,x + 336$

$2\,x^2 + 168\,x - 164\,x = 336$, or, $2\,x^2 + 4\,x = 336$

$x^2 + 2\,x = 168$, after dividing by 2 throughout.

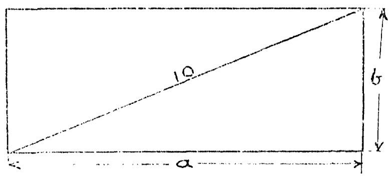
$x^2 + 2\,x + (\tfrac{2}{2})^2 = 168 + (\tfrac{2}{2})^2$

$x + 1 = \pm\sqrt{168+1} = \pm\sqrt{169} = \pm 13$

$x = \pm 13 - 1$, $x = +12$ or -14, the minus value cannot fit the problem, take the positive value, 12.

Slow ship does 12 knots
Fast ship does 14 knots. } Ans.

Example. The diagonal of a rectangle is 10 inches long, and its area is 48 sq. inches. Find the lengths of the sides.



$$\text{We have } a^2 + b^2 = 10^2$$
$$\text{and } a \times b = 48$$
$$\therefore 2\,ab = 96$$

Now if we add $2\,ab$ to $a^2 + b^2$, it becomes a complete square, and if we subtract $2\,ab$ from $a^2 + b^2$ it becomes a complete square.

We must add or subtract the value of $2\,ab$ from the right hand side to preserve the equality.

$a^2 + 2\,ab + b^2 = 10^2 + 96$
$a^2 - 2\,ab + b^2 = 10^2 - 96$ } Take the square roots of both sides of each equation.

then $a + b = 14$
and $a - b = 2$ } This is now a simultaneous equation. Add, and eliminate b.

$$2\,a = 16$$

$$a = 8,\ b = \tfrac{48}{8} = 6.$$

The sides of the rectangle are 8 inches and 6 inches. Ans.

TEST EXAMPLES V.

Problems leading to Quadratics.

1. The diagonal of a rectangular plate is to its length, as 17 is to 15, and its area is 30 square feet. Find its length and breadth.

Ans. $7\frac{1}{2}$ and 4 feet.

2. The half perimeter of a rectangular table is 100 inches. If a strip 5 inches wide is sawn off right round, the area left is $\frac{5}{8}$ths of the original area. Find the lengths of the sides before sawing.

Ans. 60 and 40 inches.

3. The sum of two numbers is 7, and the ratio of their squares is as 25 : 81. Find the numbers.

Ans. 2·5 and 4·5.

4. Two ships each steam a distance of 160 miles. The difference in their speeds is 2 miles per hour. One does the journey in 4 hours less than the other. Find the speeds of the vessels.

Ans. 10 and 8 miles per hour.

5. Divide 15 into two parts, such that three times the square of the lesser part, is less by 73 than the square of the greater part.

Ans. 4 and 11.

6. If, in the following formula, $u = 10$, $a = 6$, and $S = 125$, find t.

$$S = u\,t + \frac{at^2}{2}$$

Ans. $t = 5$.

CHAPTER V.

LOGARITHMS.

Logarithms are indices, used for the purpose of reducing the labour in the arithmetical operations of multiplication and division.

If $a^n = N$, then "n" is the power to which "a" must be raised to give the result N. "n" is the index or logarithm, "a" is the base, and "n" is the logarithm of N to base "a". Thus $2^3 = 8$, and we say that 3 is the logarithm of 8 to the base 2.

The logarithms used generally are those to base 10, because we use the decimal system of writing numbers.

Thus $10^2 = 100$, and 2 is the logarithm of 100 to base 10 also, $10^3 = 1000$ and 3 is the logarithm of 1000 to base 10.

It has already been shown that when two quantities are multiplied together, we add their indices, therefore

$10^2 \times 10^3 = 10^5 = 100000$, or 5 is the logarithm of the result or answer to base 10.

Given that the log. of 2 to base 10 is 0·301
and that the log. of 3 to base 10 is 0·4771,

find the logarithms of 4, 9 and 6.

Now by definition, $10^{\cdot 301} = 2$.
but $4 = 2 \times 2$ or 2^2, therefore $2^2 = 10^{\cdot 301 \times 2}$, because when we raise a power to a power we multiply the indices.

$\therefore$ 2^2 or $4 = 10^{\cdot 602}$, this is the same as squaring both sides, and log. of 4 to base 10 is 0·602. In the same way,

$= 3$, now $9 = 3^2$

$9 = 10^{\cdot 9542}$, therefore log. of 9 to base 10 is 0·9542

Now $6 = 2 \times 3 = 10^{\cdot 301} \times 10^{\cdot 4771} = 10^{\cdot 301 + \cdot 4771} = 10^{\cdot 7781}$, therefore log. of 6 to base 10 is 0·7781. The actual value of log. 6 given in the four figure log. tables is 0·7782, as being nearer to the correct value than 0·7781.

Logarithms are tabulated to many decimal places for calculations in which a higher degree of accuracy is needed.

For ordinary engineering calculations, the four figure log. tables, which give the value of the log. to four significant figures, are sufficiently accurate.

If then we are given a table of numbers with their corresponding logs., it is seen from the simple examples given that :—

1. Add the logs. of two numbers, and we have the log. of their product.
2. Subtract the log. of one number, from the log. of another number, and we have the log. of their quotient.
3. To raise a number to any given power, multiply the log. of the number by the index of the power, and we have the log. of the answer.

The log. of any number to base 10, may be got from the table of common logs. The same table is used to convert a given log. into its corresponding number.

Now a number such as 2405, may be written as $2{\cdot}405 \times 10^3$ or as $24{\cdot}05 \times 10^2$, and this form is sometimes useful as a compact method of writing a number of many figures.

Note that 0·2405 could be written as $2{\cdot}405 \times 10^{-1}$, since $= \frac{1}{10}$ and $2{\cdot}405 \times$ 0·2405 Also 0·02405 could written $2{\cdot}405 \times 10^{-2}$, since $2{\cdot}405 \times \frac{1}{100} = 0{\cdot}02405$.

To find the log. of a number from the tables. A small part of a logarithm table is given to illustrate the method.

	0	1	2	3	4	5	6	7	8	9	1 2 3 4	5	6 7 8 9
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2 4 6 8	10	12 14 16 [illegible]
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2 4 6 8	10	12 14 15 [illegible]
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2 4 6 7	9	11 13 15 [illegible]
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2 4 5 7	9	11 12 14 [illegible]

Find the log. of 2131. Look up the figures 21 on the left, then read horizontally along, and under the figure 3 which heads a column we read the number 3284. For the last figure in our number 2131, i.e., 1, we look under the column headed 1 on the right of the tables, and in the same horizontal line as 21 we find the figure 2. This is added to 3284, making it $(3284 + 2) = 3286$.

Now $10^1 = 10$, therefore by the definition of a logarithm, 1 is the log. of 10. It follows that the log. of any number less than 10, must be less than 1 ; also, 10 raised to a power greater than 1 but less than 2, must have a value between 10 and 100. This tells us that the log. of a number less than 10, but more than 1, is merely a decimal quantity ; and that the log. of a number more than 10 but less than 100, must be some quantity more than 1, but less than 2, or really 1 + some decimal number or numbers.

Now $10^3 = 1000$, and $10^4 = 10000$. Our number 2131 lies between 1000 and 10000, therefore the log. of 2131 must be 3 + some decimal quantity, the total value being less than 4. We see that the log. of 2131 must be 3·3286. Now the part

·3286 we read from the tables. Note that the tables do not show any decimal points, but all these groups of four figures taken from the tables are decimal quantities. By the methods shown above, it will be seen that the value of the whole number part of the log. depends upon whether it is the log. of a number in the units or in the tens, or in the hundreds, etc. The decimal part of a log. is always positive when we take it from the tables, the whole number part may be positive or negative.

Find log. 2·1. Look up 21 in our table, and read to the right under the 0 in the first column, the figures 3222. Now because 2·1 is less than 10 and more than 1, our log. is 0·3222. The log. of 21 is 1·3222, and the log. of 210 is 2·3222. The decimal part of a log. is called the "mantissa," and the whole number part is called the "characteristic" or "index" of the log. It is perhaps better to speak of the two parts as the "index" and the "decimal part."

Find log. of 2335. Look up 23 in the left hand column, read horizontally along, and under the figure 3 which heads a column, we read the number 3674. For the last figure in our number, i.e., 5, we look under the column headed 5 on the right hand side of the tables, and in the same horizontal line as 23, we find the figure 9. This we add to 3674 making 3683. The whole log. is 3·3683. The following table explains and illustrates the statements written above.

Number	May be Written	Log of the Number	Log as usually written
240500	$2·405 \times 10^5$	·3811 + 5	5·3811
2405	$2·405 \times 10^3$	·3811 + 3	3·3811
2·405	$2·405 \times 10^0$	·3811 + 0	0·3811
·2405	$2·405 \times 10^{-1}$	·3811 — 1	$\bar{1}·3811$
·02405	$2·405 \times 10^{-2}$	·3811 — 2	$\bar{2}·3811$
·002405	$2·405 \times 10^{-3}$	·3811 — 3	$\bar{3}·3811$

Note that the value of the whole number part, or index of the logarithm, depends on the position of the decimal point in the number. We see that :—

When we have 6 whole numbers before the decimal, the index of the log. is 5.

When we have 4 whole numbers before the decimal, the index of the log. is 3.

When we have 1 whole number before the decimal, the index of the log. is 0.

When we have no whole number before the decimal, the index of the log. is — 1.

When we have no whole number, and one 0 after the decimal, the index of the log. is — 2.

We see that the index, or characteristic, of the logarithm of a number greater than unity is always positive, and is one less than the number of digits in the number.

The index, or characteristic, of the log. of a number less than unity is always negative, and is one more than the number of noughts following immediately after the decimal point.

Given these Numbers	Find their Logs	Given these Logs	Find their Corresponding Numbers
Numbers.	Logs.	Logs.	Numbers.
3·516	0·546	$\bar{1}$·6308	0·4274
12·93	1·1116	0·7311	5·384
0·9123	$\bar{1}$·9601	$\bar{3}$·0080	0·001019
0·001523	$\bar{3}$·1826	0·9331	8·572
7651000	6·8838	3·8457	7010
0·7854	$\bar{1}$·8951	0·0613	1·152
0·000028	$\bar{5}$·4472	$\bar{2}$·0078	0·01018

Have a complete table of logs. before you to verify these results.

The method of taking the log. of a number from the tables has been explained. Now to convert a log. into its corresponding number :—Take $\bar{1}$·6308 in the third column above; the nearest group in the tables is ·6304, under the column headed 7. Reading horizontally to the left hand, we find 42 in the left hand column. The number corresponding to ·6304 is therefore 427. Now the difference between ·6308 and ·6304 is ·0004, but the tables show no decimal points, and this is merely written as 4. Reading to the right hand, in the same horizontal line, we notice that this 4 is in the column headed 4. The number corresponding to log. ·6308 is therefore 4274, and the index $\bar{1}$ tells us that it contains no whole numbers, and so the number must be 0·4274. Now find the log. of 0·4274 and prove your result.

From the same table above take log. $\bar{3}$·0080. Looking in the table of logs. we find no values between ·0043 and ·0086. Take ·0043, the number corresponding to this is 101. But the difference between ·0080 and ·0043 is 37. Looking for 37 in the right hand columns, we find it in the same horizontal line as ·0043 and in the column headed 9. The number is therefore 1019. The $\bar{3}$ tells us where to put the decimal. Log. $\bar{3}$·0080 corresponds to number 0·001019. Turn this into a log. again and prove your answer. Practice will soon give the student facility in the use of the tables.

Multiplication by Logs.

Add the logs. of the numbers, and the result is the log. of the answer.

Examples. 27·31 × 12·42.

Log. of 27·31 is 1·4364 }
Log. of 12·42 is 1·0941 } add

The log. of the answer is 2·5305
And 2·5305 is log. of 339·2. Ans.

Multiply 13·31 by 0·0531

Log. of 13·31 is 1·1242 }
Log. of 0·0531 is $\bar{2}$·7251 } add. Note $\bar{2} + 1 = \bar{1}$

Log. of the answer is $\bar{1}$·8493
And $\bar{1}$·8493 is log. of 0·7068.

Multiply 144 × 32·2 × 0·003152.

Log. of 144 is 2·1584
Log. of 32·2 is 1·5079 add. Note + 4 3 =
Log. of 0·003152 is $\bar{3}$·4986 + 1

Log. of the answer is 1·1649, and looking in the tables we find that 1·1649 is the log. of 14·62. Ans.

Note in the tables the nearest group to 1649 is 1644, corresponding to the number of 146, the difference is 5, but reading horizontally along we do not find a 5 in the right hand columns, but there is a 6 and this is nearer than 3. As the 6 is in the column headed 2, our last figure in the number is 2.

Division by Logs.

Subtract the log. of the divisor from the log. of the dividend, and the result is the log. of the answer. Divide 52·82 by 95·63,

or $\dfrac{52 \cdot 82}{95 \cdot 63}$

Log. of 52·82 is 1·7228 }
Log. of 95·63 is 1·9806 } subtract

The log. of the answer is $\bar{1}$·7422

The decimal parts of the logs. are subtracted in the usual way, but for the index, we use the algebraic method of subtracting, i.e., change the sign and add. Following the above example, 6 from 8 is 2 ; 0 from 2 is 2 ; 8 from 12 is 4 ; we have now 1 to add to the 9, and so 0 from 7 is 7 ; we have 1 to add to the index of the lower line, making it 2. Now change the sign of 2 from + to — and add. We get $-2 + 1 = \bar{1}$.

And from the tables we find that $\bar{1}$·7422 is the log. of 0·5524. Ans.

Examples. Divide 121·5 by 0·00312.

or $\dfrac{121\cdot5}{0\cdot00312}$ $\left.\begin{array}{l}\text{Log. of } 121\cdot5 \text{ is } 2\cdot0846 \\ \text{Log. of } 0\cdot00312 \text{ is } \bar{3}\cdot4942\end{array}\right\}$ subtract.

$$4\cdot5904$$

Subtracting, 2 from 6 is 4 ; 4 from 4 is 0 ; 9 from 18 is 9 ; 5 from 10 is 5 ; carrying + 1 over, + 1 and — 3 is — 2. Now change the sign of 2 from — to +, and add ; + 2 + 2 = + 4. Our answer is log. 4·5904, and 4·5904 is the log. of 38940. Ans.

If, in the above example in logs. the student has any difficulty with the characteristic of the log. of the answer, he may use the following method.

If $\dfrac{121\cdot5}{0\cdot00312}$ is multiplied by $\dfrac{1000}{1000}$, it becomes $\dfrac{121500}{3\cdot12}$ and the quotient is not altered.

$$\left.\begin{array}{l}\text{Log. of } 121500 \text{ is } 5\cdot0846 \\ \text{Log. of } \quad 3\cdot12 \text{ is } 0\cdot4942\end{array}\right\}\text{subtract}$$

$$4\cdot5904$$

And 4·5904 is the log. of 38940. Ans.

It may be helpful to anyone beginning to use logs., in such a case as the above, to add to the index or characteristic of both divisor and dividend a number sufficient to cause the negative index to disappear. This is in effect the same thing as multiplying both divisor and dividend by the same number, and therefore does not affect the quotient.

$$\frac{\frac{11}{14}\,(16\cdot5^2 - 13\cdot25^2) \times 5620}{2240}, \text{ simplify this first,}$$

$$\frac{13\cdot25)\,(16\cdot5 - 13\cdot25) \times 5620}{14 \times 2240}$$

$$\frac{11 \times 29\cdot75 \times 3\cdot25 \times 5620}{14 \times 2240}$$

Logs.		Logs.		
1·0414		1·1461	add	Put down the logs. of all numbers in the numerator and add. Put down the logs. of the numbers in the denominator in a separate column, and add. Finally subtract as shown to perform the division.
1·4735	add	3·3502		
·5119				
3·7497		4·4963		

$$\left.\begin{array}{l}6\cdot7765 \\ 4\cdot4963\end{array}\right\}\text{subtract}$$

2·2802, log. of 190·6. Ans.

In this example, note that if we evaluate ($16{\cdot}5^2 - 13{\cdot}25^2$), without taking the sum $\times$ the difference, then the log. of each quantity must be found, multiplied by 2, then brought back to a number before subtracting. This is more laborious than the method used in the solution.

To Evaluate Powers and Roots.

To raise a number to a power, multiply the log. of the number by the index of the power.

Thus Log. of a^2 = log. $a \times 2$

Log. of a^3 = log. $a \times 3$

$$\text{Log. of } \sqrt{a} \text{ or } a^{\frac{1}{2}} = \frac{\text{Log. } a}{2}$$

Examples.

Evaluate $\sqrt{137{\cdot}4}$, Log. $137{\cdot}4 = 2{\cdot}138$, divide this by 2

Then $\dfrac{2{\cdot}138}{2} = 1{\cdot}069$ = log. of Ans. = log. of $11{\cdot}72$. Ans.

Find the value of $(51{\cdot}34)^{\frac{2}{3}}$

Log. of Ans. = log. of $51{\cdot}34 \times \frac{2}{3} = 1{\cdot}7104 \times \frac{2}{3}$

$$\begin{array}{r} 1{\cdot}7104 \\ 2 \\ \hline 3)3{\cdot}4208 \\ \hline \end{array}$$

Log. of Ans. = $1{\cdot}1402$ = log. of $13{\cdot}81$. Ans.

Find the square root of $0{\cdot}443$. Now log. of $0{\cdot}443$ is $\bar{1}{\cdot}6464$

Log. of Ans. = $\dfrac{\bar{1}{\cdot}6464}{2} = \bar{1}{\cdot}8232$ = log. of $0{\cdot}6656$

Note that $\bar{1}{\cdot}6464$ is really $-1 + {\cdot}6464$, this is the same as $-2 + 1{\cdot}6464$, because -1 has been added to one part and $+1$ added to the other part. This is the same as adding 1 to the number and then subtracting 1 from it, the value is therefore not altered.

Dividing $-2 + 1{\cdot}6464$ by 2, we get $-1 + {\cdot}8232$, which is written as usual $\bar{1}{\cdot}8232$, and log. $\bar{1}{\cdot}8232$ corresponds to

Evaluate $\sqrt[3]{0{\cdot}8354}$, $\dfrac{\text{Log. of } 0{\cdot}8354}{3}$ is the log. of the answer.

Log. of 0·8354 is $\bar{1}{\cdot}9219$, now 3 will not go into $\bar{1}$, therefore add -2 to the index, and $+2$ to the decimal part. The log. then becomes $-3 + 2{\cdot}9219$, and dividing by 3, we get $-1 + {\cdot}9739$, which is written $\bar{1}{\cdot}9739$, and this is the log. of 0·9417.

Examples. Evaluate $(0{\cdot}6561)^{\frac{2}{3}}$

Log. of answer $=$ log. of $0{\cdot}6561 \times \frac{2}{3} = \bar{1}{\cdot}817 \times \frac{2}{3}$,

$$\begin{array}{r} \bar{1}{\cdot}817 \\ 2 \\ \hline 3)\bar{1}{\cdot}634 \\ \\ \bar{1}{\cdot}878 \end{array}$$

Note here in multiplying we have $+$ one to carry over from the decimal part. But $2 \times \bar{1} = \bar{2}$, and taking the $+1$ of carriage away we get $\bar{1}$, because $\bar{2} + 1$

Next divide by 3, using the method shown in previous examples, mentally.

$\bar{1}{\cdot}878$ is log. of 0·7551. Ans.

Find the fifth root of 0·05612, or $\sqrt[5]{0{\cdot}05612}$ or $(0{\cdot}05612)^{\frac{1}{5}}$

$$\text{Log. of fifth root} = \frac{\text{Log. of } 0{\cdot}05612}{5} = \frac{\bar{2}{\cdot}7492}{5}$$

The number to add mentally to the index to make it divisible by 5 is -3, but $+3$ must be added to the decimal part also.

Therefore $\dfrac{\bar{2}{\cdot}7492}{5} = \bar{1}{\cdot}7498 =$ log. of 0·562. Ans.

Evaluate $(\frac{1}{4})^{0{\cdot}4}$, this is $(0{\cdot}25)^{0{\cdot}4}$, now log. 0·25 is $\bar{1}{\cdot}3979$.

Log. of answer is $\bar{1}{\cdot}3979 \times 0{\cdot}4$, here it is best to express the whole log. as a minus quantity. Subtract 0·3979 from -1,

$$\begin{array}{r} -1{\cdot}0000 \\ {\cdot}3979 \end{array}$$

and we get $-0{\cdot}6021$

Now multiply $-0{\cdot}6021$ by 0·4,

$$\begin{array}{r} -0{\cdot}6021 \\ 0{\cdot}4 \\ \hline -0{\cdot}24084 \end{array}$$

Note, plus $\times$ minus is minus

and $-0{\cdot}24084$ is the same as $\bar{1}{\cdot}75916$, because, adding $+1$ to $-{\cdot}24084$ we get $+{\cdot}75916$, i.e., we have made the decimal part positive. To keep the value the same, however, we must subtract 1 from the index, and so our log. becomes $\bar{1}{\cdot}75916$, we cannot use the 6, $\bar{1}{\cdot}7592$ is the log. of 0·5742. Ans.

Example. Evaluate $S = \sqrt{\dfrac{C\,P}{f\left(2 + \dfrac{D^2}{d^2}\right)}}$ when $C = 27$, $P = 190$, $D = 81$, $d = 31$, $f = 1110$

$$S = \sqrt{\frac{27 \times 190 \times 81 \times 81}{1110\left(2 + \dfrac{81 \times 81}{31 \times 31}\right)}}$$

Solve the bracket $\left(2 + \dfrac{81^2}{31^2}\right)$ fir .

This becomes $(2 + 6{\cdot}826) = 8{\cdot}826$

$$\frac{\times\ 190 \times 81 \times 81}{1110 \times 8{\cdot}826}$$

log. 31	1·4914
	2
	2·9828
log. 81 =	1·9085
	2
	3·8170
	2·9828
	0·8342

and 0·8342 is log. of 6·826

Logs. for Numerator.		Logs. for Denominator.	
1·4314	add	3·0453	add
2·2788		0·9458	
1·9085			
1·9085		3·9911	
7·5272	subtract		
3·9911			

3)3·5361 Divide by 3 to extract the cube root.

1·1787 = log. of S. S = 15·09. Ans.

There are many important calculations in which we use logs. to the base 2·71828, a number so important in mathematics, that the letter e is generally used to denote it, just as π is used to denote 3·14159. Such logs. are called "Napierian" logs., after Napier who invented the logarithmic method of calculating; these logs. are also called hyperbolic logs., because we use them to calculate the area under a hyperbolic curve, i.e., a curve which may be expressed by $p\,v$ = constant.

To convert ordinary logs. to Napierian logs., multiply the ordinary log. by 2·30258 ; but for ordinary engineering purposes it is usual to multiply the ordinary log. by 2·3, as this is accurate enough.

Evaluate, $p = \frac{180}{12} [1 + \log_e 12] - 5$.

Note $\log_e$ is the symbol for logs. to base e, or Napierian logs.

$p = 15 [1 + \log. 12 \times 2{\cdot}3] - 5$. 1·0792 log. 12

$p = 15 [1 + (1{\cdot}0792 \times 2{\cdot}3)] - 5$

$p = 15 [1 + 2{\cdot}482] - 5$

$p = (15 \times 3{\cdot}482) - 5 = 52{\cdot}23 - 5$

$p = 47{\cdot}23$. Ans.

2·48216, we need take this to four figures only, say 2·482.

TEST EXAMPLES VI.

1. Multiply 3251 by 3·995. Ans. 12990.

2. Multiply 2·718 by 0·00371. Ans. 0·01009

3. Divide 0·392 by 0·895. Ans. 0·438.

4. Divide 0·0356 by 27·2. Ans. 0·001308.

5. Evaluate, $\sqrt[2]{0{\cdot}0813}$; $\sqrt[3]{1{\cdot}235}$; $(27{\cdot}91)^{\frac{2}{5}}$; $\sqrt[3]{0{\cdot}0593}$.
Ans. 0·2851 ; 1·073 ; 3·787 ; 0·3899.

6. Evaluate $\dfrac{\frac{11}{14} [(17{\cdot}5)^2 - (14{\cdot}75)^2] \times 5600}{2240} + \frac{223}{113}$.
Ans.

7. If $\dfrac{T_1}{T_2} = \Big(\dfrac{v_2}{v_1}$ v_1 when $T_1 = 855$, $T_2 = 505$, and $v_2 = 15{\cdot}2$. Ans. 4·075.

8. Evaluate, $\dfrac{215}{13{\cdot}5} [1 + \log_e 13{\cdot}5] - 4$. Ans. 53·32.

9. Evaluate, $\dfrac{100 [T + 1]^2}{S -}$, when $T = 13$, and $S = 165$.
Ans.

CHAPTER VI

VARIATION.

Variation is an extended form of the arithmetical theory of proportion. In dealing with problems, or in observing and collecting results in laboratory experiments, or in our ordinary daily work, it is the relation of one thing to another which constantly interests us. We apply the method called "Variation" to all cases where the value of one quantity depends on the value of some other quantity or quantities. For instance :—

The weight of a piece of iron depends upon its volume.

The circumference of a circle depends on its diameter.

If we have two pieces of iron, we know that the larger piece has the greater weight ; we say, therefore, that the weight of a piece of iron varies directly as its volume. The symbol $\propto$ denotes variation, and if we write "weight $\propto$ volume," we mean that the weight varies as the volume.

Direct Variation.

Two quantities are said to vary directly as each other, if the ratio of any two values of the one quantity, is equal to the ratio of the corresponding two values of the other quantity.

If we have two pieces of cast iron, of one and two cubic inches respectively, their weights are 0·26 and 0·52 pounds.

The ratio of the two weights is $\dfrac{0{\cdot}26}{0{\cdot}52} = \dfrac{1}{2}$

The ratios of volumes and weights are equal.

This may be expressed algebraically thus :—

$$\frac{V_1}{V_2} = \frac{W_1}{W_2}, \text{ or by transposing, } \frac{V_1}{W_1} = \frac{V_2}{W_2}$$

Now in the above simple example, any volume divided by its own weight gives the same result, i.e., a constant value.

We may therefore state the definition given above in a simpler form :—Two quantities are said to vary directly when their quotient is constant.

Example. The circumference of a circle varies directly as its diameter. When the diameter is 14 inches, the circumference is 44 inches, find the circumference when the diameter is 21 inches.

If C stands for circumference and d for diameter, we have,

$$C \propto d, \text{ or } \frac{C_1}{d_1} = \frac{C_2}{d_2} = \text{constant},$$

$$\frac{44}{14} = \frac{C_2}{21}, \text{ or by cross multiplying}$$

$$44 \times 21 = C_2 \times 14$$

$$C_2 = \frac{\overset{22}{\cancel{44}} \times \overset{3}{\cancel{21}}}{\underset{2}{\cancel{14}}} = 66 \text{ inches. Ans.}$$

If the value of one quantity depends directly on the value of other two quantities :—The volume of a cylinder, for instance, depends upon the square of its diameter and its length, if V = volume, d = diameter, l = length, then we have, $V \propto d^2 \times l$,

$$\therefore \frac{V_1}{d_1^2 \times l_1} = \frac{V_2}{d_2^2 \times l_2} = \text{constant.}$$

Inverse Variation.

If, as one quantity gets larger, another quantity which depends upon it gets smaller, the two quantities are said to "vary inversely." In the case of a perfect gas, it is known that if the temperature remains constant, the pressure varies inversely as the volume.

Let the pressure be 100 lb. per sq. inch when the volume is 10 cu. ins. Then when the pressure is 50 lb. per sq. inch, the volume is 20 cu. ins.

Here we find that the product of pressure and volume is constant, in the first case 100×10, and in the second 50×20, this constant value being 1000. We have, therefore, that two quantities vary inversely when their product is constant. If P varies inversely as Q we write it in symbols as $P \propto \frac{1}{Q}$, or $\frac{P}{\frac{1}{Q}}$ is constant, this is the same as $P \times Q$ is constant, or $P_1 Q_1 = P_2 Q_2$

Example. A ship steaming at 10 knots, covers a certain distance in 15 hours. If the ship had steamed at 12 knots, in how many hours would the same distance have been run ?

Now if the speed is greater the time taken must be less.

This is inverse variation, and we have $T \propto \dfrac{}{S}$

or $T_1 \times S_1 = T_2 \times S_2$ where T = time in hours.

and S = speed in knots.

or $15 \times 10 = T_2 \times 12$. $T_2 \dfrac{15 \times 10}{12} = 12\frac{1}{2}$ hours. Ans.

Example. The strength of a beam varies directly as the square of its depth, directly as its breadth, and inversely as its length. A beam is 20 ft. long, 10 inches deep and 5 inches broad. Find the depth of another beam 15 feet long, and 3 inches broad to have the same strength.

$$\left.\begin{array}{l}\text{Here, strength} \propto b \times d^2 \\ \text{and strength} \propto \dfrac{1}{l}\end{array}\right\} \begin{array}{l}\dfrac{\text{Strength}}{b \times d^2} = \text{constant.} \\ \text{Strength} \times l = \text{constant.}\end{array}$$

Combining the rules for direct and inverse variation we have

$$\frac{\text{Strength}_1 \times l_1}{b_1 \times d_1^2} = \frac{\text{Strength}_2 \times l_2}{b_2 \times d_2^2}, \text{ now Strength}_1 = \text{Strength}_2$$

$$\therefore \frac{l_1}{b_1 \; d_1^2} = \frac{l_2}{b_2 \; d_2^2}, \text{ or } d_2^2 = \frac{l_2 \; b_1 \; d_1^2}{l_1 \; b_2} \text{ by transposing.}$$

$$\therefore d_2^2 = \frac{\overset{5}{\cancel{15}} \times 5 \times (10)^2}{20 \times \cancel{3}} = \frac{2500}{20} = 125.$$

$$d_2 = \sqrt{125} = 11{\cdot}18 \text{ inches. Ans.}$$

In this problem, if the depth of the second beam was made 12 inches, how much stronger was it than the first beam ?

Calling S the strength, we have

$$\frac{S_1 \; l_1}{b_1 \; d_1^2} = \frac{S_2 \; l_2}{b_2 \; d_2^2}, \text{ or } \frac{S_1}{S_2} = \frac{b_1 \; d_1^2 \; l_2}{l_1 \; b_2 \; d_2^2} \text{ by transposing.}$$

$$\frac{S_1}{S_2} = \frac{\cancel{3} \times (10)^2 \times \overset{5}{\cancel{15}}}{\underset{4}{\cancel{20}} \times \cancel{3} \times (12)^2} = \frac{500}{576} \text{ or } \frac{1}{1{\cdot}152},$$ that is to say, that if the strength of the first beam is 1, the strength of the second one is now 1·152. The second beam is

$$\left(\frac{1{\cdot}152 - 1}{1}\right) \times 100, \text{ or } 15{\cdot}2 \text{ per cent. stronger.} \quad \text{Ans.}$$

Example. The absolute pressure of a perfect gas varies directly as its absolute temperature, and varies inversely as its volume.

Here we have $\dfrac{p}{T}$ is constant, and $p\ v$ = constant.

Combining we have $\dfrac{p\ v}{T}$ = constant, or $\dfrac{p_1 \ v_1}{T_1} = \dfrac{p_2 \ v_2}{T_2}$

If 6 cubic feet of a perfect gas at 100 lb. per sq. inch absolute and at absolute temperature 500° F. be compressed to 2 cubic feet the absolute temperature rising to 800° F., find the new absolute pressure.

$$\frac{p_1 \ v_1}{T_1} \qquad \frac{p_2 \ v_2}{T_2}$$

$$\frac{100 \times 6}{500} = \frac{p_2 \times 2}{800}, \text{ or } p_2 = \frac{100 \times 6 \times 800}{2 \times 500} = 480$$

The new absolute pressure is 480 lb. per sq. inch. Ans.

Note in the above example, we should have stated the pressure in pounds per square foot, because the volume is in cubic feet. As, however, the 144 would cancel, being common to both sides of the equation, we have omitted it.

Sometimes it is convenient to work out the numerical value of the constant. The volume of one pound of air at 14·7 pounds per sq. inch absolute, is 12·4 cubic feet (nearly), when its temperature is 32° F.

absolute temperature is $(460 + 32) = 492°$ F.

Then $\dfrac{p_1 v_1}{T_1}$ = cons, putting in the values :–

$$\frac{14\cdot7 \times 144 \times 12\cdot4}{492} = 53\cdot2 \text{ to the nearest decimal.}$$

Note.—If the volume is in cubic feet, we must express the pressure in pounds per sq. foot in this case.

The value of the constant then, for *One Pound* weight of air is 53·2.

Example.—One pound weight of air is at a temperature of 200° F., and its absolute pressure is 100 lb. per sq. inch. Find its volume.

$$\frac{p\ v}{T} = 53\cdot2, \qquad \frac{100 \times 144 \times v}{(200 + 460)} = 53\cdot2.$$

$$\frac{53\cdot2 \times 660}{144 \times 100} = 2\cdot43 \text{ cubic feet per pound. Ans.}$$

Note.—For absolute temperatures, add 460 to the temperature Fahr.

Example.—An air container of 21 cubic feet volume, contains air at 1500 lb. per sq. inch absolute and 100° F. Find the weight of air in the containing vessel.

$$\frac{p\ v}{T} = 53\cdot2, \text{ or } v = \frac{53\cdot2 \times T}{p}$$

Note again that v is the volume *per pound* of air.

$$v = \frac{53\cdot2 \times (100 + 460)}{1500 \times 144} = 0\cdot138 \text{ cu. foot per lb.}$$

$$\text{No. of lb. of air in vessel} = \frac{21}{0\cdot138} = 152\cdot17 \text{ lb. weight.}$$

Example.—If 10 men trim 300 tons of coal in 3 days of 8 hours, how many men will trim 600 tons in 4 days of 10 hours ?

Let m = men, T = tons, d = days, h = hours.

Consider how the number of days taken will vary with each of the other quantities.

The greater the number of men employed, the less will be the number of days taken to trim the coal.

$$\text{Therefore } d \propto \frac{1}{m}$$

The greater the number of tons to be trimmed the greater will be the number of days taken.

$$\text{Therefore } d \propto T.$$

The greater the number of hours worked per day, the less will be the number of days taken.

$$\text{Therefore } d \propto 1$$

Collecting, $d \propto \dfrac{}{m \times h}$ $\quad \dfrac{d \times m \times h}{T}$ $\quad$ constant

or m_1

$$10 \times 3 \times 8 \qquad m_2 \times 4 \times 10$$

$$300 \qquad\qquad 600$$

$$= \frac{8 \times 150}{10 \times 10} = 12 \text{ men. Ans.}$$

$$\text{Or, since } \frac{m\ d\ h}{T} = \text{constant,}$$

$$\text{we have } \frac{10 \times 3 \times 8}{300} = \tfrac{8}{10} \text{ as the constant.}$$

$$\therefore m = \frac{\text{Cons.} \times T}{d \times h} = \tfrac{8}{10} \times \frac{T}{d \times h}, \text{ or } m = \tfrac{8}{10} \frac{T}{d \times h}$$

a formula for any time and weight of coal,

$$m = \frac{8}{10} \times \frac{600}{4 \times 10} = 12 \text{ men as above. Ans.}$$

TEST EXAMPLES VII.

1. The working pressure of a boiler varies directly as the thickness of the shell, and inversely as the internal diameter. A boiler has a shell 1 inch thick, its internal diameter being 13 feet 4 inches and the working pressure 150 lb. per sq. inch. Find the working pressure of a boiler having a shell $1\frac{1}{2}$ inches thick and 15 feet diameter. Ans. 200 lb. per sq. inch.

2. Ten pounds of air at 200 lb. per sq. inch absolute and at 300° F., find the volume occupied in cubic feet. If this air cools down to 100° F., find its new pressure, if the volume remains constant.

Ans. 14·04 cu. feet, and 147·4 lb. per sq. inch absolute.

3. When the horse power is 1200 and the speed 10 knots, the pressure on the thrust is 62 lb. per sq. inch. Find the pressure on the thrust when the horse power is 1400 and the speed 9 knots. The pressure on the thrust varies directly as the horse power and inversely as the speed of the ship.

Ans. 80·37 lb. per sq. inch.

4. The stress in a turbine rotor due to centrifugal force is 2200 lb. per sq. inch, when running at 1700 revolutions per minute, the rotor being 4 feet 6 inches diameter. Find the stress in a rotor 10 feet diameter and made of the same material when running at 400 revolutions per minute. The stress varies as the square of the speed of a point on the rim of the rotor.

Ans. 601·7 lb. per sq. inch.

5. A shaft 6 inches diameter transmits 150 horse power at 70 revolutions per minute. What horse power can a shaft 4 inches diameter transmit when running at 190 revolutions per minute, if the stress in the second shaft is 80 per cent. of the stress in the first shaft ? Solve the problem from the formula,

$$\frac{\pi\ d^3}{16} \times \text{stress} = \frac{63000 \times \text{horse power}}{\text{revolutions}}$$

where d = diameter in inches. Ans. 96·5 horse power.

6. A wire 0·1 inch diameter and of a certain length, has a resistance of 3·2 ohms. What will be the diameter of a wire four times as long and made of similar material, that will have a resistance of 6·8 ohms ? The resistance in ohms varies directly as the length, and inversely as the area of the wire.

Ans. 0·1372 inch.

7. Ten men dig a trench 100 yards long in 5 days of 8 hours each. How many men will dig a trench 250 yards long, working 7 days of 10 hours each ? Ans. $14\frac{2}{7}$ men.

8. A flat plate of iron one foot square and $\frac{1}{8}$ inch thick weighs 5 lb. The specific gravity of iron is 7·7, and of lead 11·4. Find the weight of a flat plate of lead 15 inches diameter and $\frac{5}{8}$ inch thick. Ans. 45·43 lb.

CHAPTER VII.

BRITISH AND METRIC UNITS.

Fundamental Units of Mass, Length and Time.

All quantities with which we deal in Mechanics may be expressed in terms of Mass, or of Length, or of Time; or in some combination of these three.

To express the magnitude of anything which is to be measured whether this magnitude is to be in terms of length, area, volume, weight or time, it is necessary to have some fixed standard of comparison. Such a standard is called a Unit. From the three fundamental quantities stated above, many other units are derived.

Mass.

Mass we define as the quantity of matter in a given body or object. Now several properties possessed by a body of given mass may change, if heated or cooled, its volume will change; if it is hammered, or rolled, its form will change; if it is weighed by an accurate spring balance in different latitudes, we find that its weight is not constant; but the amount of matter in the body has not changed. Its mass, then, is the one invariable property of matter.

The weight of a body is a measure of the earth's attraction upon it, and this attraction is exerted from the centre of the earth. The nearer an object is to this centre of attraction, the greater is its weight. As the earth is a sphere slightly flattened at the poles, an object will be heavier at the poles than at the equator; a spring balance, however, must be used to detect this difference in weight, since if we use an ordinary balance, both the object and the "weight" used will the same change in attraction, and therefore in weight.

Mass, therefore, is not weight, but we can compare by weighing, and for this we need a standard of mass. Bodies of equal mass balance each other in the pans of an ordinary lever balance; and the weight of a body, for ordinary purposes, may be regarded as a measure of its mass.

In the British system, the legal unit of mass is the Imperial Standard Pound. This is the mass of a certain lump of platinum in the possession of the Board of Trade. The pound is subdivided into 16 equal parts called ounces.

In the Metric system the unit of mass is the Kilogram. This is the mass of a certain piece of platinum kept at Sévres, and it is heavier than the pound. A mass of one Kilogram is equal to the mass of one thousand cubic centimetres of water at a temperature of 4° Centigrade, and one gram is equal to the mass of one cubic centimetre of water at 4° Centigrade.

METRIC SYSTEM

10 Milligrams = 1 Centigram	10 Grams = 1 Dekagram
10 Centigrams = 1 Decigram	10 Dekagrams = 1 Hektogram
10 Decigrams = 1 Gram	10 Hektograms = 1 Kilogram

TO CONVERT

Metric to British.	British to Metric.
1 Gram = 15·43 Grains	1 Ounce = 28·35 Grams
1 Kilogram = 2·204 Pounds	1 Pound = 453·6 Grams

Length.

The legal standard unit of length in Britain is the yard, and this is the length of a certain bronze bar at a particular temperature. The unit of length used in engineering, however, is the "foot" which is one-third of the legal "yard." The foot is divided into 12 equal parts, each of which is called an "inch." There are 5280 feet in one land mile, and 6080 feet in one nautical mile. In the Metric system, the standard of length is the metre, and this is the length of a certain bronze bar kept at Sévres, measured when at a temperature of 4° Centigrade.

The metre is subdivided into 10 equal parts each of which is called a decimetre. The decimetre is divided into 10 equal parts, each of which is called a centimetre, and the tenth part of a centimetre is called a millimetre. For measurement of greater lengths, multiples of the metre are used.

10 Millimetres (mm.)	1 Centimetre (cm.)
10 Centimetres	1 Decimetre (dm.)
10 Decimetres	1 Metre (m.)

$\therefore$ 1 Metre = 10 Decimetres or 100 Centimetres or 1000 Millimetres.

10 Metres	**1 Dekametre**
100 Metres	**1 Hektometre**
1000 Metres	**1 Kilometre**

TO CONVERT

Metric to British.	British to Metric.
1 Centimetre = 0·3937 Inch.	1 Inch = 2·54 Centimetre
1 Metre = 39·37 Inches.	1 Foot = 0·305 Metre.
1 Kilometre = 0·621 Mile.	1 Mile = 1609 Metres or 1·609 Kilometre

1 Litre = 1000 Cubic Centimetres = 0·22 Gallon.

Time.

The earth, by virtue of its daily rotation about its axis, is the time-keeper. By reason of this rotation, the sun appears to make a daily transit of the heavens. When the sun is at its highest altitude, it is said to be over the meridian of the place of observation. The apparent solar day is the period of time between the sun's highest altitude on one day and its highest altitude on the next day. Now this period of time, the apparent solar day, varies from day to day throughout the year. As a unit must be an invariable standard, we cannot use an apparent solar day. If, however, we divide one solar year by the number of days in the year, we obtain what is called the mean solar day, and this is an invariable period of time. This mean solar day is divided up into 24 hours, the hour is divided up into 60 minutes and the minute is divided up into 60 seconds. The unit of time used in ordinary physical measurements is called the "mean solar second," and it is $\frac{1}{24} \times \frac{1}{60} \times \frac{1}{60} = \frac{1}{86400}$ part of a mean solar day. The clock keeps "mean solar time." The time, as given daily by the sun, is "apparent time."

Examples. Find a multiplier to convert pounds per sq. inch to kilogs. per sq. centimetre.

$$\text{lb. per sq. inch} = \frac{\text{lb.}}{2{\cdot}2} \text{ kilogs. per sq. inch.}$$

$$\text{One square inch} = \frac{1}{(0{\cdot}3937)^2} = 6{\cdot}46 \text{ sq. cm.}$$

$$\text{and } \therefore \frac{\text{lb.}}{2{\cdot}2 \times 6{\cdot}46} = 0{\cdot}07036 \times \text{lb.}$$

$\therefore$ Multiply lb. per sq. inch by 0·07036 and we have kilogs. per sq. cm. Ans.

Thus 14·7 × 0·07036 = 1·0343 kilogs. per sq. cm., or kilogs. per sq. cm. may be thought of as atmospheres per sq. inch (this is only an approximation).

Example. Find a multiplier to convert statute miles per hour into metres per second.

Let S = statute miles per hour, and m = metres per second.

Then $\dfrac{S \times 5280}{60 \times 60}$ = feet per second.

Also 1 metre = 39·37 inches or $\dfrac{39{\cdot}37}{12}$ feet.

$\therefore$ 1 foot $\dfrac{12}{39{\cdot}37}$ metres.

$$\frac{S \times 5280}{60 \times 60} \quad \frac{12}{39{\cdot}37} = m.$$

or, $0{\cdot}447\ S = m$.

The multiplier is 0·447. Ans.

Example. Convert 17·37 kilograms per sq. cm. to lb. per sq. inch.

2·54 centimetres = 1 inch
$\therefore$ 1 sq. inch = $(2{\cdot}54)^2$ sq. cm.
$\therefore$ pressure per sq. inch = $(2{\cdot}54)^2 \times 17{\cdot}37$ kilogs.
and 1 kilogram = 2·2 lb.
$\therefore$ pressure per sq. inch = $2{\cdot}2 \times (2{\cdot}54)^2 \times 17{\cdot}37$
= 246·8 lb. per sq. inch. Ans.

or, $\dfrac{17{\cdot}37}{0{\cdot}07036}$ = 246·8 lb. per sq. inch. Ans.

Example. Construct an expression to find the speed of a vessel :—

(a) In statute miles per hour (b) in kilometres per hour ; the speed of the vessel is such that a distance of K nautical miles is done in N minutes.

K nautical miles in N mins. = $\dfrac{K}{N}$ nautical miles per min.

K nautical miles in N mins. = $\dfrac{K\ 60}{N}$ nautical miles per hour = knots.

[Note that when we write "knots," we always mean nautical miles per hour.]

and $\dfrac{K \quad 60}{N} \times \dfrac{6080}{5280}$ = statute miles per hour.

$= 69{\cdot}09 \dfrac{K}{N}$ statute miles per hour. Ans. (*a*)

1 kilometre $= \dfrac{1000 \times 39{\cdot}37}{12}$ feet, $= \dfrac{1000 \times 39{\cdot}37}{5280 \times 12}$

= 0·621 statute mile.

. ·. K nautical miles in N mins. $= \dfrac{69{\cdot}09 \quad K}{0{\cdot}621 \quad N}$

. ·. K nautical miles in N mins. $= 111{\cdot}1 \dfrac{K}{N}$ kilometres per hour. Ans. (*b*).

Example. A liquid weighs 36 ozs. per quart. Find the weight of 1 litre of this liquid in grams.

1 quart of fresh water weighs 40 ozs.

. ·. this liquid is $\frac{36}{40}$ of the weight of water.

A litre of fresh water weighs 1000 grams.

. ·. 1 litre of this liquid weighs $\frac{36}{40} \times 1000 = 900$ grams. Ans.

Example. A tapered shaft is 880 millimetres long, and the taper is $\frac{3}{4}$ inch per foot. The diameter at the small end is 110 millimetres. Find the diameter of the large end in inches.

1 millimetre $= \dfrac{39{\cdot}37}{1000} = 0{\cdot}03937$ inch.

Length $= 880 \times 0{\cdot}03937$ inches, or $\dfrac{880 \times 0{\cdot}03937}{12}$ feet.

Difference in diameter $= \dfrac{880 \times 0{\cdot}03937}{12} \times \frac{3}{4}$ inches,

= 2·165 inches.

Small end is $110 \times 0{\cdot}03937 = 4{\cdot}3307$ inches diameter.

. ·. Diameter at large end $= 4{\cdot}3307 + 2{\cdot}165 = 6{\cdot}496$ inches. Ans.

CHAPTER VIII.

TRIGONOMETRY ; PROPERTIES OF THE CIRCLE.

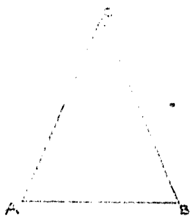
Trigonometry is the science which expresses the relation between the sides and the angles of triangles. Some of the important properties of the triangle are :—

Any two sides of a triangle are together greater than the third. The sum of the 3 angles of any triangle is equal to 2 right angles, or 180 degrees. Triangles upon the same base, or upon equal bases, and having the same vertical height, are equal in area.

The area of a triangle is equal to the product of its base and one-half of its vertical height.

A Right Angled Triangle is a triangle with one of its angles equal to 90 degrees. The hypotenuse of a right angled triangle is the side opposite to the right angle, and it is always the longest side. The square of the hypotenuse of a right angled triangle, is equal to the sum of the squares of the other two sides.

An Equilateral Triangle is a triangle with its three sides equal in length, and with its three angles equal in magnitude, these angles being each 60 degrees.

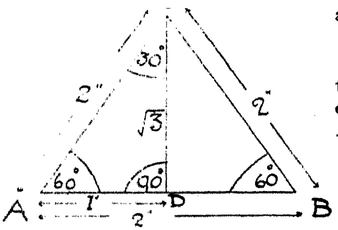


An Isosceles Triangle is a triangle with two of its sides equal, and with the angles opposite to these equal sides also equal.

Let A C B be an isosceles triangle.

Then A C = B C, and the angle at A must equal the angle at B. If the angle at C is 40°, then together the angles at A and B equal 180° — 40° = 140°

and A and B are each $\dfrac{140^\circ}{} = 70^\circ$.



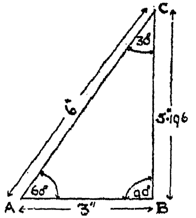
Let A B C be an equilateral triangle, having each of its sides equal to 2 inches. Let the base A B be bisected at D ; A D = 1 inch.

Then $(C\,D)^2 \quad 2^2 \quad = 4$ —
= 3.

$CD = \sqrt{3}.$

The line C D also bisects the angle A C B, making the angle A C D = 30 degrees. The angle C D A is 90°. In a right angled triangle therefore, if the angles are 30°, 60° and 90°, the sides always have the ratio 1 : 2 : $\sqrt{3}$.

Note that $\sqrt{3}$ is 1·732.



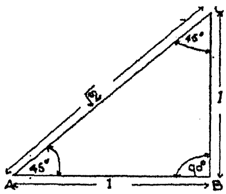
In the triangle A B C, let A B = 3 inches, the angles being 30°, 60° and 90°.

Then A C = 3 × 2 = 6 inches and B C = 3 × $\sqrt{3}$ = 3 × 1·732 = 5·196 ins.

If the length of B C had been given, then

$$A\ B = \frac{5{\cdot}196}{\sqrt{3}} = \frac{5{\cdot}196}{1{\cdot}732} = 3 \text{ inches.}$$

A C = 3 × 2 = 6 inches, as before.



In this triangle A B C, which has a right angle at B, the angles A and C being each 45 degrees, we have A B = B C. Then if A B = 1, B C = 1, and A C = $\sqrt{1^2 + 1^2} = \sqrt{2} = 1{\cdot}414$.

In a right angled triangle, having two angles of 45° each, if the short sides are each, say 5 inches long, then the hypotenuse is 5 × $\sqrt{2}$ = 5 × 1·414 = 7·07 inches long. If we are given that the hypotenuse is 7·07 inches, then the short sides are

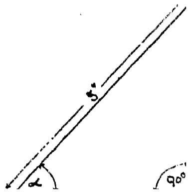
$$\text{each } \frac{7{\cdot}07}{1{\cdot}414} = 5 \text{ inches long.}$$

Similar Triangles are those in which the sides are in the same ratio, and in which the corresp angles are equal. Thus all right angled triangles having of 30° and 60°, will have their sides in the ratio of 1 : 2 : $\sqrt{3}$, and are said to be similar triangles. In the same way all right angled triangles having angles of 45° will have their sides in the ratio of 1 : 1 : $\sqrt{2}$.

The magnitude of an angle may be expressed in circular measure, i.e., in radians, or it may be stated in degrees, minutes and seconds. Sometimes in engineering we express the angle

in degrees and minutes. In many cases it is necessary only to express it to the nearest degree. There is, however, another method by which we may define the magnitude of a given angle.

Trigonometrical Ratios.



Draw a right angle A B C, make A B three inches, and B C four inches long, Then $(A\ C)^2 = 3^2 + 4^2$ or $A\ C = \sqrt{25} = 5$ inches.

Now measure the angle at A with a protractor; it is seen to be very nearly 53 degrees. This angle can now be expressed in terms of the sides of a right angled triangle, because in any right angled triangle having its sides in the ratio of 3 : 4 : 5, there will always be an angle of 53 degrees.

The angle at A (α), may be stated:

As a *ratio* of $\dfrac{B\ C}{A\ C}$ or called the *Sine*.

a *ratio* of $\dfrac{A\ B}{A\ C}$ called the *Cosine*.

As a *ratio* of $\dfrac{B\ C}{A\ B}$ or called the *Tangent*.

Three other ratios may be used to define the angle. They are the reciprocals of the Sine, Cosine and Tangent. Thus :—

$$\frac{A\ C}{B\ C} \text{ or } \frac{\quad}{\text{Sine } \alpha} = \text{Cosecant } \alpha$$

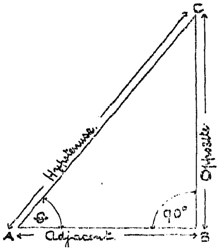
$$\frac{A\ C}{A\ B} \text{ or } = \frac{1}{\text{Cosine } \alpha} = \text{Secant } \alpha$$

$$\frac{A\ B}{B\ C} \text{ or } \frac{1}{\text{Tangent } \alpha} = \text{Cotangent } \alpha$$

We generally use only the Sine, Cosine and Tangent in our ordinary work, and it will be sufficient for the student to work with these ratios.

It is to the above definitions that we refer when we speak of the trigonometrical ratios of an angle.

The student will see that every angle will have its own value of Sine, Cosine and Tangent ; the particular values, $\frac{4}{5}$, $\frac{3}{5}$ and $\frac{4}{3}$ given above, are the approximate values of Sine, Cosine and Tangent of 53°.

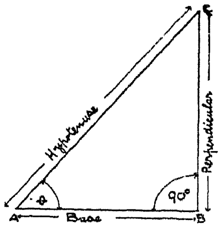


Take a right angled triangle A B C, calling the angle at A "theta," (θ), A C is the *hypotenuse*, C B is the side *opposite* the angle θ, and A B is the side *adjacent* to the angle θ.

Then, $\text{Sin. } \theta = \dfrac{\text{opposite side}}{\text{hypotenuse}} = \dfrac{BC}{AC}$

$\text{Cos. } \theta = \dfrac{\text{adjacent side}}{\text{hypotenuse}} = \dfrac{AB}{AC}$

$\text{Tan. } \theta = \dfrac{\text{opposite side}}{\text{adjacent side}} = \dfrac{BC}{AB}$

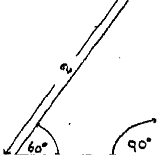


Instead of calling the sides "adjacent" and "opposite" as above, we may call them "base" and "perpendicular" as shown here. The ratios then are—

$\text{Sin. } \theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{BC}{AC}$

$\text{Cos. } \theta = \dfrac{\text{base}}{\text{hypotenuse}} = \dfrac{AB}{AC}$

$\text{Tan. } \theta = \dfrac{\text{perpendicular}}{\text{base}} = \dfrac{BC}{AB}$



To find the Sine, Cosine and Tangent of several important Angles.

The angles of 30°, 60° and 45° occur frequently in our work, and close attention should be given to the following explanations.

By the definitions already given :—

$$\text{Sin. } 60^\circ = \frac{\text{opp. side}}{\text{hypotenuse}} = \frac{\sqrt{3}}{2} = \frac{1{\cdot}732}{2}$$

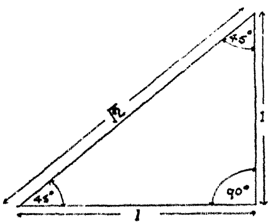
$$\text{Cos. } 60^\circ = \frac{\text{adj. side}}{\text{hypotenuse}} = \frac{1}{2} = 0{\cdot}5$$

$$\text{Tan. } 60^\circ = \frac{\text{opp. side}}{\text{adj. side}} = \frac{\sqrt{3}}{1} \quad 1{\cdot}732$$

$$\text{Sin. } 30^\circ = \frac{\text{opp. side}}{\text{hypotenuse}} = \frac{1}{} = 0{\cdot}5$$

$$\text{Cos. } 30^\circ \quad \frac{\text{adj. side}}{\text{hypotenuse}} = 0{\cdot}866$$

$$\text{Tan. } 30^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3} = 0{\cdot}5773$$



It is seen in this table, that the Sine of 60° has the same value as the Cosine of 30° ; and that the Cosine of 60° has the same value as the Sine of 30°. Now if the sum of any two angles is equal to 90°, the angles are said to be "complementary angles." Thus 30° is the complementary angle to 60°, and 20° is the complementary angle to 70°. We have the following rule :—The Sine of an angle is equal to the Cosine of its complementary angle.

$$\text{Sin. } 45^\circ = \frac{\text{opp. side}}{\text{hypotenuse}} \quad \frac{1}{\sqrt{2}} \quad \sqrt{2} \quad 0{\cdot}707$$

$$\text{Cos. } 45^\circ = \frac{\text{adj. side}}{\text{hypotenuse}} \quad 1 \quad 0{\cdot}707$$

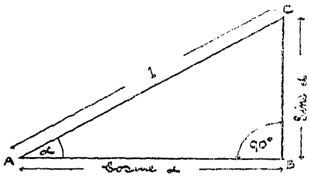
$$\frac{\text{opp. side}}{\text{adj. side}}$$

The Sin. and Cos. of 45° are each equal to 0·707, and the Tangent is unity, or 1.

The Sine and Cosine cannot have values greater than unity, or 1, but the Tangent may have any value.

Sines and Cosines are decimal quantities, ranging in value from 0 to 1. The value of the Tangent is less than unity up to 45°, above 45° the value is greater than unity. The value of the Tangent ranges from 0 at 0°, to infinity at 90°.

Relation between the Sine, Cosine and Tangent of an Angle.



A B C is a right angled triangle. Let the hypotenuse A C = 1.

$$\text{Then Sin. } \alpha = \frac{B\,C}{A\,C} = \frac{B\,C}{1} = B\,C.$$

$$\text{and Cos. } \alpha = \frac{A\,B}{A\,C} = \frac{A\,B}{1} = A\,B.$$

We may call B C the Sine and A B the Cosine.

$$\text{Now } (A\,C)^2 = (B\,C)^2 + (A\,B)^2$$

$$\text{or } 1^2 = \text{Sin.}^2\,\alpha + \text{Cos.}^2\,\alpha$$

$$\text{Also Tan } \alpha = \frac{B\,C}{A\,B} = \frac{\text{Sin. } \alpha}{\text{Cos. } \alpha}$$

Example. The Sine of an angle is 0·75, find the Cosine and the Tangent.

$$\text{Sin.}^2\,\alpha + \text{Cos.}^2\,\alpha = 1, \text{ or Cos.}^2\,\alpha = 1 - \text{Sin.}^2\,\alpha$$

$$\text{Cos.}^2\,\alpha = 1 - (\tfrac{3}{4})^2 = 1 - \tfrac{9}{16} = \tfrac{7}{16}$$

$$\text{Cos. } \alpha = \frac{\sqrt{7}}{4} = \frac{2{\cdot}646}{4} = 0{\cdot}6615.$$

$$\text{Tan. } \alpha = \frac{\text{Sin. } \alpha}{\text{Cos. } \alpha} = \frac{3}{\cancel{4}} \times \frac{\cancel{4}}{\sqrt{7}} = \frac{3}{\sqrt{7}}$$

$$= \frac{3 \times \sqrt{7}}{7} = 1{\cdot}134.$$

The Cos. is 0·6615; the Tan. is 1·134. Ans.

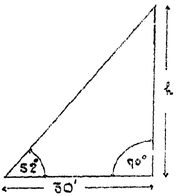
Sines, Cosines and Tangents for angles from 0° to 90° are tabulated and arranged in tables of degrees and minutes to an accuracy of 4 decimal places for ordinary purposes in engineering.

Note, $\text{Sin.}^2 \propto$ means $(\text{Sin.} \propto)^2$ and not $\text{Sin.} (\propto)^2$. That is, take the value of the Sine from the tables and square it. Thus, $\text{Sin.}^2\ 30° = (0{\cdot}5)^2 = 0{\cdot}25$.

How to Use Trigonometrical Tables.

Find the value of Sine 10° 12′. Looking in a table of Natural Sines we find 10 in the column headed "degrees"; in the same horizontal column under the heading 12′ we find the value of 1771. As Sines are decimal quantities, this is read as 0·1771. Find the Sine of 66° 33′. Find 66 in the column of degrees, and looking under the minutes column headed 30′ in the same horizontal line, we obtain the value 0·9171. Now the difference between 33′ and 30′ is 3′. Looking under the heading 3 in the "mean differences" column, and in the same horizontal line as 0·9171, we get the value 3. Adding 3 to 0·9171 we get 0·9174 which is the Sine of 66° 33′. Find the Cosine of 23° 46′. Looking in the table of Natural Cosines, we see that Cos. 23° 42′ is 0·9157, and in the difference column under the heading 4 in the same horizontal line, we find the number 5. In finding the value of the Cosine we subtract the difference, because as the angle gets greater, the value of the Cosine becomes less. Subtracting 5 from 0·9157 we get 0·9152 and this is the Cosine of 23° 46′. In some tables the angles are given in degrees and multiples of 6 minutes, say 6′, 12′, 18′, etc., up to 54′; in others the angles are given in degrees and multiples of 10 minutes. Suppose we find the value of the Sine of 66° 17′ from a table of the latter kind. We have Sine 66° 10′ = 0·9147, and as the difference is 7, in the column of mean difference under the heading 7 in the same horizontal line as 0·9147 we have the number 8; adding we get 0·9155 as Sin. 66° 17′. It sometimes happens that two different sets of tables will give a difference of 1 in the 4th decimal place. This is not of any importance in ordinary work, and makes only a very small difference in the answer. Sines and Cosines are generally printed separately, but as the Sine of an angle is the Cosine of its complement, one table will do for both. We then read the Sines from the top of the page and the Cosines from the bottom upwards. In such a table, for instance, the student should note that Sine 84° 10′, has the same value as Cosine 5° 50′, because 5° 50′ is 90° — (84° 10′). Tangents are taken from the tables in the same way as Sines, the difference being added.

Examples. **Find the height of a ship's funnel, if at a distance of 30 feet from the base of the funnel, the angle of elevation of the top of the funnel is 52°.**



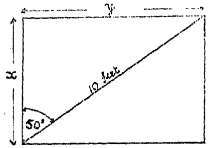
52° is here the angle of elevation.

$$\text{Tan. } 52^\circ = \frac{\ }{30}, \text{ or } h = 30$$

Now, from the tables, Tan. 52 [illegible] 1·2799

$\therefore h = 30 \times 1{\cdot}2799$, h [illegible] 97 feet. Ans.

Example. The diagonal of a r[illegible] feet long, and the short side of this rectangle ma[illegible] diagonal. Find the lengths of th[illegible]



Let the sides of the rectangle be x feet, and y feet.

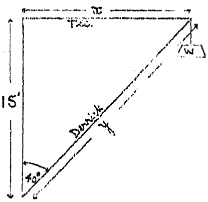
$$\text{Then Sin. } 50^\circ = \frac{y}{10}, \text{ or } y = 10 \times \text{Sin. } 50^\circ$$

Now Sin. $50^\circ = 0{\cdot}766$,

$\therefore y = 10 \times 0{\cdot}766 = 7{\cdot}66$ feet. Ans.

$$\text{Also Cos. } 50^\circ = \frac{x}{10}, \text{ or } x = 10 \text{ Cos. } 50^\circ$$

Now Cos. $50^\circ = 0{\cdot}6428$, $\therefore x = 10 \times 0{\cdot}6428$, [illegible] 6·428



Example. A derrick is inclined at 40° to the vertical. The tie is horizontal, and is fastened at a height of 15 feet above the foot of the derrick. Find the length of the tie, and of the derrick.

Let x = length of tie.

y = length of derrick.

$$\text{Tan. } 40^\circ = \frac{x}{15}, \text{ or } x = 15 \times \text{Tan. } 40^\circ$$

$$x = 15 \times 0{\cdot}8391 = 12{\cdot}5865 \text{ feet.}$$

15 15 15

$= 19{\cdot}58$ feet.

The Tie is 12·59 feet long, and the Derrick 19·58 feet. Ans.

tice, it is easy to multiply 15 by 0·8391 by arithmetic, but it is best to divide 15 by 0·766 by means of logs.

Log. 15	1·1761	subtract.
Log. 0·766	$\bar{1}$·8842	
	1·2919	

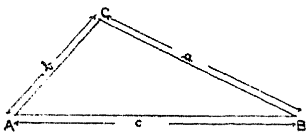
Note, the log. of Cos. 40° may be got from a table of logarithmic Cosines directly.

1·2919 is log. of 19·58 as before.

Area of a Triangle.

In the paragraphs which follow, the angle opposite the side a is wrtiten as A, and the angles opposite the sides b and c, are written as B and C respectively.

The area of a triangle is the product of its base and one-half its vertical height. In the case of a right angled triangle, if one side and one angle other than the right angle is known, then all the sides may be found, and the remaining sides determined. The area is then easily obtained. In the case of a triangle which does not contain an angle equal to 90 degrees, the following formulæ may be used.



If the lengths of the three sides are given, the following rule gives the area.

$$\text{Area} = \sqrt{s\ (s - a)\ (s - b)\ (s - c)}$$

$$\text{Where } s = \frac{b + c}{}, \text{ or half the sum of the three sides.}$$

Example. A triangle has sides, 8, 9 and 12 inches long. Find its area, $s = \dfrac{8 + 12}{} \quad \dfrac{9}{} = \dfrac{29}{2} = 14{\cdot}5.$

$$\text{Area} = \sqrt{14{\cdot}5} \qquad\qquad - 12)$$

$$= \sqrt{14{\cdot}5 \times 6{\cdot}5 \times 5{\cdot}5 \times 2{\cdot}5}$$

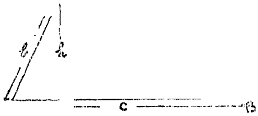
Log. of area = 1·5563

Area = 35·99 sq. inches. Ans.

Logs.
1·1614
0·8129
0·7404
0·3979

2)3·1126

1·5563



If two sides and the angle included between them is known, then,

$$\text{Area} = \frac{b \times c \times \text{Sin. A}}{2}, \text{ if } b, c \text{ and}$$

A are known.

$$\text{or } \frac{a \times c \times \text{Sin. B}}{2}, \text{ if } a, c \text{ and B are known}$$

$$\text{or } \frac{a \times b \times \text{Sin. C}}{2}, \text{ if } a, b \text{ and C are known.}$$

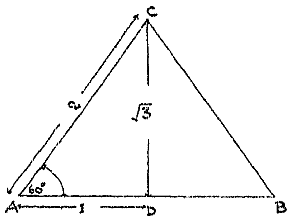
Note that $h = b$ Sin. A, or $h = a$ Sin. B and is the vertical height.

Example. Given $b = 11$ inches, $c = 14$ inches, $A = 50°$, find the area.

$$\text{Area} = \frac{11 \times 14 \times \text{Sin. } 50°}{2} = 77 \times 0{\cdot}766$$

$$= 58{\cdot}98 \text{ square inches. Ans.}$$

To find the area of an equilateral triangle.



The vertical height is D C

Now D C = A D $\times \sqrt{3}$, because A D = 1

but A D = $\frac{1}{2}$ side.

$\therefore$ height of an equilateral triangle

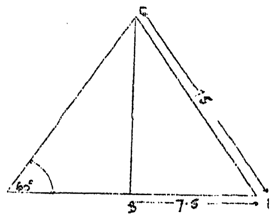
$$\frac{\text{Side} \times \sqrt{3}}{2}$$

$$\text{Area} = \text{Base} \times \tfrac{1}{2} \text{ Vertical Height}$$

$$= \text{Side} \times \frac{\text{Side} \times \sqrt{3}}{2} \times \frac{1}{2} = (\text{Side})^2 \times \frac{\sqrt{3}}{4}$$

$$= 0{\cdot}433\ S^2.$$

Example. An equilateral triangle has sides 15 inches long, find its area.



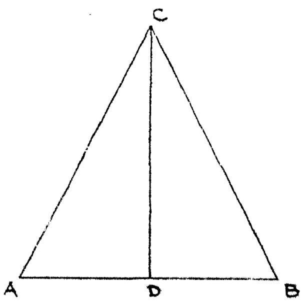
$$\text{Area} = 15 \times 15 \times \frac{\quad}{4} = 225 \times \frac{1·732}{4}$$

$$= 97·425 \text{ sq. inches.} \quad \text{Ans.}$$

otherwise :— $D\,C = \sqrt{15^2 - 7·5^2} = \sqrt{168·75}$

$$= 12·99 \text{ inches.}$$

$$\text{Area} = \frac{12·99 \times 15}{\quad} = 97·425 \text{ sq. inches, as above.}$$



To find the area of an isosceles triangle. If the three sides are known, then

$$D\,C = \sqrt{(B\,C)^2 - (D\,B)^2},$$

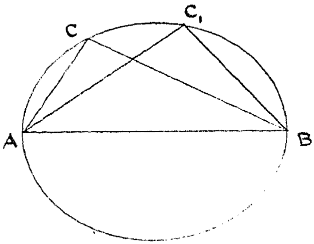
because perpendicular C D bisects A B.

The area is $\dfrac{A\,B \times D\,C}{2}$

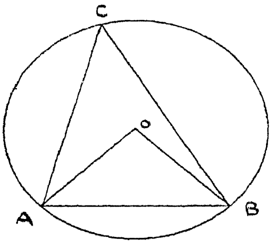
If A C, and B C are known, and either the angle C or the angle A is known, D C is easily determined. If A is given, D C = A C × Sin. A ; if C is given, then

$A = \dfrac{180° - C}{2}$, and D C may be found.

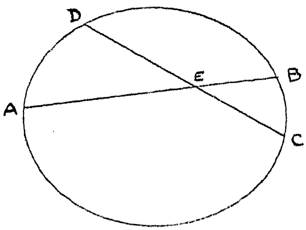
Properties of a Circle.



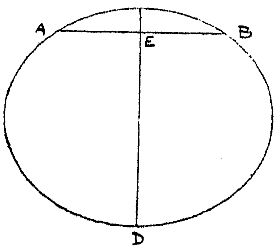
If any triangle is constructed with the diameter A B as its base, and its apex C or C_1, is on the circumference of the circle, then the angles C and C_1 are each right angles. This proposition is often stated as :—The angle in a semi-circle is a right angle.



If any triangle A B C, has its base on the chord A B, and its apex C anywhere on the circumference of the segment A C B ; and if another triangle be drawn on the chord A B but with its apex at O, the centre of the circle ; then the angle at C is half of the angle at O.



If any two chords A B and D C cut each other at the point E, then D E × E C = A E × E B.

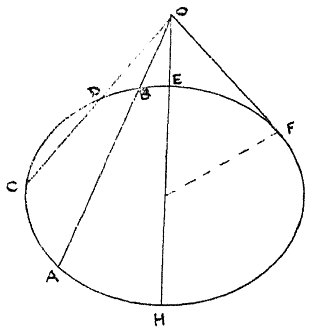


A special case of the above proposition occurs when one chord is a diameter of the circle, and the other chord cuts this diameter at right angles. We have

D E × E C = A E × E B

but A E = E B.

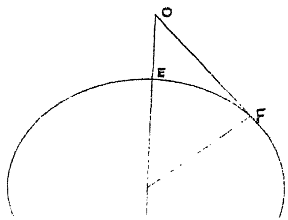
∴ D E × E C = $(A\ E)^2$.



If any two chords such as B A and D C be drawn from a point O outside the circle, then O C $\times$ O D = O A $\times$ O B.

When one chord is a diameter and the other becomes a line drawn as a tangent, from O to F, then

$$O H \times O E = (O F)^2$$



Example. A man on a ship's bridge is 50 feet above the water. Find the length of a line joining his eye to the horizon, if the earth is 8,000 miles diameter.

E H = 8,000 miles, O E = $\frac{50}{5280}$ = 0·00946 mile.

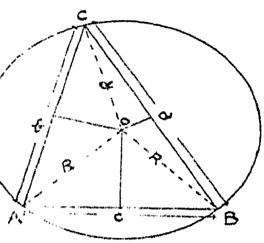
Then O H $\times$ O E = (O F)^2
O H = 8000 + 0·00946
= 8000·00946.

We may take O H as being very nearly 8,000 miles.

Then 8,000 $\times$ 0·00946 = (O F)^2

O F = $\sqrt{75 \cdot 68}$ = 8·75 miles. Ans.

Circumscribed Circle.



If a triangle be drawn inside a circle as shown, then if the sides *a*, *b* and *c* are bisected, perpendiculars erected from the middle points of *a*, *b* and *c* meet at the centre of the circle.

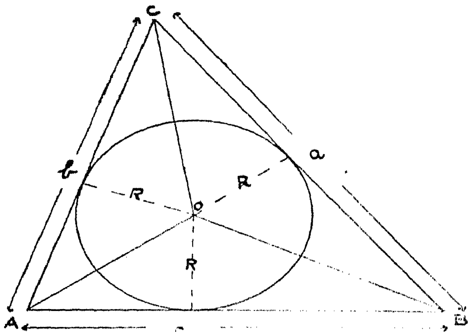
The Radius of the circumscribing circle is given by :—

$$R = \frac{a \times b \times c}{4 \times \text{area of triangle}}$$

or, which may be more convenient,

$$2R = \frac{\quad}{\text{Sin. A}}, \text{ or } \frac{\quad}{\text{Sin. B}}, \text{ or } \frac{\quad}{\text{Sin C}}$$

Inscribed Circle.



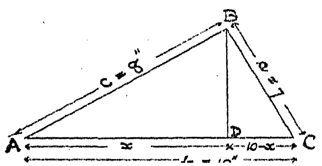
Lines joining the Centre O, of the circle to the corners of the triangle, bisect the angles A, B and C

$$\text{Then R} = \frac{\text{Area of the triangle} \times 2}{a + b + c}$$

Solution of Triangles.

In the previous paragraphs the trigonometry of the right angle triangle has been considered; we proceed to consider triangles which do not contain an angle of 90 degrees.

First Case.



Given three sides, to determine the angles.

In the triangle A B C, let $a = 7''$, $b = 10''$, $c = 8''$

From B drop the perpendicular B D, and let A D $= x$, then D C $= 10 - x$

$(\text{B D})^2 = 7^2 - (10 - x)^2$, and from the other side

$(\text{B D})^2 = 8^2 - x^2$

$\therefore\ 49 - (100 - 20\,x + x^2) = 64 - x^2$

$- 51 + 20\,x - x^2 = 64 - x^2$, cancel out x^2

and $20\,x = 115$, $x = 5''{\cdot}75$, D C $= 4''{\cdot}25$

$$\text{Cos. A} = \frac{5\cdot75}{8} = 0\cdot7187, \text{ and A} = 44^\circ\ 3' \text{ from the tables.}$$

$$\text{Cos. C} = \frac{4\cdot25}{7} = 0\cdot6071, \text{ and C} = 52^\circ\ 35' \text{ from the tables.}$$

$$\text{B} = 180^\circ - (44^\circ\ 3' + 52^\circ\ 35') = 83^\circ\ 22'.$$

Another method would be to find the area of the triangle by the formula, and divide by half the base to get the vertical height B D. Thus :—

$$s = \frac{8 + 7 + 10}{2} = 12\cdot5$$

$$\text{Area} = \sqrt{s\ (s - a)\ (s - b)\ (s - c)}$$
$$= \sqrt{12\cdot5 \times 5\cdot5 \times 2\cdot5 \times 4\cdot5}$$

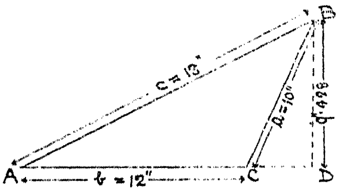
$$\text{Area} = 27\cdot81 \text{ sq. inches, B D} = \frac{\text{area}}{\tfrac{1}{2}\text{ base}}$$

$$\text{B D} = \frac{27\cdot81}{5} = 5''\cdot562$$

$$\text{Then Sin. A} = \frac{5\cdot562}{8} = 0\cdot6952, \text{ A} = 44^\circ\ 3', \text{ from the tables.}$$

$$\text{and Sin. C} = \frac{5\cdot562}{7} = 0\cdot7945, \text{ C} = 52^\circ\ 36', \text{as before.}$$

Example. A triangle has sides 10, 12 and 18 inches long. Find the angles.



$$\text{Area} = \sqrt{20 \times 10 \times 8 \times 2}$$
$$= 56\cdot57 \text{ sq. ins.}$$

$$\text{B D} = \frac{\text{area}}{\tfrac{1}{2}\text{ base}} = \frac{56\cdot57}{6}$$
$$= 9''\cdot428$$

$$\text{Sin. A} = \frac{9\cdot428}{18} = 0\cdot5237, \text{ A} = 31^\circ\ 35'. \quad \text{Ans.}$$

$\therefore$ Sin. of Angle B C D $= \dfrac{9 \cdot 428}{10} = 0 \cdot 9428$, Angle B C D $= 70° \; 32'$

$C = 180° - (70° \; 32') = 109° \; 28'$. Ans.

$B = 180° - (109° \; 28' + 31° \; 35') = 38° \; 57'$. Ans.

The following formulæ, which are adapted to logarithmic computation, may be used to solve the previous problems, or any case where three sides are given.

$$\text{Cos.} \frac{A}{2} = \sqrt{\frac{s\,(s - a)}{b\,c}}$$

$$\text{Cos.} \frac{B}{2} = \sqrt{\frac{s\,(s - b)}{}}$$

$$\text{Cos.} \frac{C}{2} = \sqrt{\frac{s\,(s - c)}{a\,b}}$$

s is the semisum of the three sides. As an aid to memory, the student should note that when we use the Cos. $\frac{A}{2}$ formula, the side "a" appears in the numerator of the fraction under the square root sign, but is not in the denominator. In the Cos. $\frac{B}{2}$ formula, the side "b" is in the numerator and not in the denominator. In the Cos. $\frac{C}{2}$ formula, c is in the numerator and not in the denominator.

Solving the first example again by formula, we have :—

$$s = \frac{8 + 7 + 10}{2} = 12 \cdot 5 \; ; \quad \text{Cos.} \frac{A}{2} = \sqrt{\frac{12 \cdot 5 \; (12 \cdot 5 - 7)}{10 \times 8}}$$

$$\therefore \frac{A}{2} = \sqrt{\frac{12 \cdot 5 \times 5 \cdot 5}{80}} = 0 \cdot 927$$

from the tables $\frac{A}{2} = 22° \; 2'$, multiply by 2

and $A = 44° \; 4'$. Ans.

$$\begin{array}{r} 1 \cdot 8373 \\ 2)\overline{1} \cdot 9342 \\ \hline \overline{1} \cdot 9671 \text{ log. of } 0 \cdot 927 \end{array}$$

We may work now for either angles B or C.

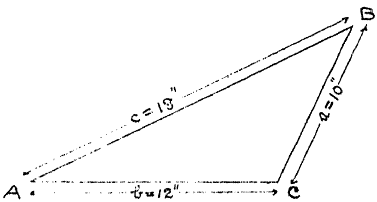
$$\text{Cos.}\ \frac{C}{2} = \sqrt{\frac{12{\cdot}5 \times 4{\cdot}5}{7 \times 10}} = 0{\cdot}8964$$

from the tables $\dfrac{C}{2} = 26^\circ\ 19'$, multiply by 2

$C = 52^\circ\ 38'$. Ans.

$$\begin{array}{r} 1{\cdot}0969 \\ 0{\cdot}6532 \\ \hline 1{\cdot}7501 \\ 1{\cdot}8451 \\ \hline 2)\overline{1}{\cdot}9050 \\ \hline \overline{1}{\cdot}9525 \end{array}$$
log. of 0·8964

B is found now by subtraction as before. The slight difference in the results is due to the fact that we are working to 4 places only. Note that Cos. $\dfrac{A}{2}$ which is 0·927, must not be multiplied by 2, as this does not give Cos. A. Find first, from the tables, the angle whose Cos. is 0·927, then multiply the angle so found by 2. Solving the second example again :—first put the letters on the sketch as shown.



$$s = 20$$

$$\text{Cos.}\ \frac{A}{2} = \sqrt{\frac{20\ (20 - 10)}{18 \times 12}}$$

$$= \sqrt{\frac{200}{216}}$$

$$\text{Cos.}\ \frac{A}{2} = 0{\cdot}962\ ;\ \frac{A}{2} = 15^\circ\ 50',\ A = 31^\circ\ 40'.\quad \text{Ans}$$

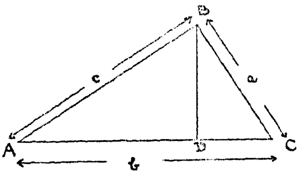
$$\frac{B}{2} = \sqrt{\frac{20 \times 8}{18 \times 10}} = \sqrt{\frac{8}{9}} = 0{\cdot}9426$$

$$\frac{B}{2} = 19^\circ\ 29',\ B = 38^\circ\ 58'.\quad \text{Ans.}$$

We would have to work to 7 places to get these angles exactly the same as before. In a case such as this, where one angle is greater than 90°, *always find the other angles by the formulæ,* and finally obtain the largest angle by subtraction. This is important.

Sine Rule.

This important rule states that the *sides* of a triangle are proportional to the *Sines* of their opposite angles.



Let A B C be any triangle.
Draw B D perpendicular to A C.
Then B D
the other side, B D = c Sin. A,
∴ a Sin. C = c Sin. A, or

$$\frac{a}{\text{Sin. A}} = \frac{c}{\text{Sin. C}}$$

By dropping a perpendicular from C to A B, it can be shown that

$$a \text{ Sin. B} = b \text{ Sin. A, or } \frac{\quad}{\text{Sin. A}} = \frac{\quad}{\text{Sin. B}}$$

$$\text{and therefore, } \frac{a}{\text{Sin. A}} = \frac{b}{\text{Sin. B}} = \frac{c}{\text{Sin. C}}$$

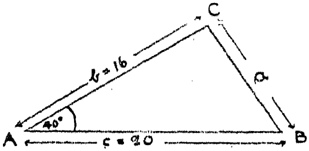
Second Case.

Given two sides and the angle included between them, to find the remaining side and angle.

Cosine Rule.

Given the sides b and c, and the angle A between them, then $a^2 = b^2 + c^2 - 2\ b \times c \times \text{Cos. A}$.

Example. Given $b = 16''$, $c = 20''$, A $= 40^\circ$; find a and the two remaining angles.



$$a^2 = b^2 + c^2 - 2\ b\ c \text{ Cos. A.}$$

$$a^2 = 16^2 + 20^2 - 2 \times 16 \times 20 \times \text{Cos. } 40^\circ.$$

$$a^2 = 256 + 400 - 640 \times 0{\cdot}766$$

$$a = \sqrt{656 - 490{\cdot}24} = \sqrt{165{\cdot}76} = 12''{\cdot}88. \quad \text{Ans.}$$

Now apply the Sine rule to obtain either C or B.

$$\frac{a}{\text{Sin. A}} = \frac{b}{\text{Sin. B}}\text{; or Sin. B} = \frac{b\ \text{Sin. A}}{a}$$

$$\text{Sin. B} = \frac{16 \times 0{\cdot}6428}{12{\cdot}88} = 0{\cdot}7984$$

from the tables B = 53° nearly. C is found by subtraction.

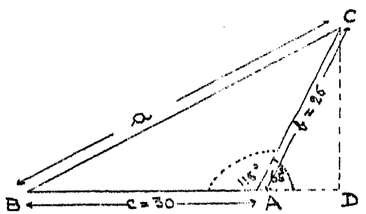
C = 180° — (40° + 53°) = 87°.

a = 12·88 ; B = 53° ; C = 87°. Ans.

It may be necessary to find the Sine, or the Cosine of an angle greater than 90°. For the Sine of such an angle the following rule holds :—The Sine of an angle is equal to the Sine of its supplement. The supplement of an angle A is (180° — A), so that if we need the Sine of 110° we look in the tables for Sine (180° — 110°), or Sin. 70°. Thus Sin. 160° is the same as Sin. 20°. All values of the Sine between 0° and 180° are said to be positive (i.e., +). The tables show all values of the Sine between 0° and 90° only.

To obtain the Cosine of 100°, we look in the tables for the Cosine of (180° — 100°), or Cos. 80°. We should note, however, that values of the Cosine are positive between 0° and 90°, and negative from 90° to 270°. Thus if the angle considered is greater than 90° and less than 270°, we say its Cosine is minus, or negative. The formula used in the last example :— $a^2 = b^2 + c^2 - 2\ b\ c$ Cos. A is perfectly general, but if A is greater than 90°, then its Cosine is negative, and $a^2 = b^2 + c^2 - 2\ b\ c$ (— Cos. A), which is the same as $a^2 = b^2 + c^2 + 2\ b\ c$ Cos. A. The student should note this formula carefully, remembering to use the minus sign before the third term in the equation when A is less than 90°, and to use the positive sign when A is greater than 90°. Angles greater than 180° do not occur in this part of the subject.

Example. In a triangle two sides are 25 and 30 feet long respectively, and the angle between them is 115°, find the third side and the remaining angles.



$a^2 = b^2 + c^2 + 2\ b\ c$ Cos. A.

$a^2 = 625 + 900 + 2 \times 30 \times 25$ Cos. 65°, note Cos. 65° = — Cos. 115°.

$a^2 = 1525 + 1500 \times 0{\cdot}4226 = 2158{\cdot}9.$

$a = \sqrt{2158{\cdot}9} = 46{\cdot}47$. Ans.

Apply the Sine rule

$$\frac{a}{\text{Sin. A}} \qquad \frac{b}{\text{Sin. B}} \quad \text{or Sin. B} = \quad b \text{ Sin. A}$$

$$\text{Sin. B} \quad \frac{25 \text{ Sin. } 115^\circ}{46{\cdot}47}, \text{ and Sine } 115^\circ = \text{Sine } 65^\circ.$$

$$\text{Sin. B} = \frac{25 \times 0{\cdot}9063}{46{\cdot}47} = 0{\cdot}4874$$

From the tables B = 29° 10′. c = 180° — (115° + 29° 10′) = 35° 50′.

The third side is 46·47 ft. ; B = 29° 10′ ; C = 35° 50′. Ans.

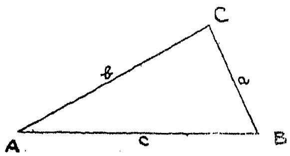
Note that this case may be solved by dropping a perpendicular from C on to B A produced. The angle C A D is 65°. Then C D and A D may be found : C D = 25 Sin. 65°, A D 25 Cos. 65°.

Then we have $\text{Tan. B} = \frac{\text{C D}}{\text{B D}}$ which determines angle B,

a is next found from $a \quad \frac{\text{C D}}{\text{Sin. B}}$, or $a = \frac{\text{B D}}{\text{Cos. B}}$, but there is probably just as much work in this solution as in the previous method.

Third Case.

Given two angles and one side, to find the remaining sides.



Let A and B and c be given. Then C = 180° — (A + B), next apply the Sine rule.

$$\text{Sin. A} \qquad \text{Sin. C}$$

which determines c

finally $\quad \text{Sin. A} \quad \frac{b}{\text{Sin. B}}$, which determines b.

If we are given A and C and c, we proceed exactly as in the above case.

Given two sides and the angle opposite one of them, to find the remaining side and angles.

Let b and c and B be given.

Then $\dfrac{\quad}{\text{Sin. B}} \qquad \dfrac{\quad}{\text{Sin. C}}$, which determines Sin. C

Then $A = 180° - (B + C)$, which determines A

Finally $\dfrac{b}{\text{Sin. B}} = \dfrac{a}{\text{Sin. A}}$, which determines a.

TEST EXAMPLES VIII.

1. An Isosceles triangle has its equal angles each 30°, and its base is 3 inches long. Find the lengths of the equal sides.

1·732 inches. Ans.

2. A regular hexagon has sides 6 inches long. Find the distance across the flats, and across the corners.

10·392 inches and 12 inches. Ans.

3. An equilateral triangle has a vertical height of 4 inches, find the length of its sides.

4·618 inches. Ans.

4. A right angled triangle has angles 90°, 50° and 40°. The side opposite the smallest angle is 7 inches long. Find the lengths of the other sides.

10·89 inches and 8·343. Ans.

5. Find the length of the diagonal of a square of 12 inches side; also the length of the diagonal of a cube of 12 inches side.

16·968 inches and 20·784 inches. Ans.

6. In a certain engine the crank is 3 feet long and the connecting rod is 12 feet long. Find the position of the piston when the crank has turned through 30 degrees from the top centre. Find the position of the crank when the piston is at half stroke; find also the position of the piston when the crank is at 90° to the line of stroke.

5·96 inches from start of stroke; 82° 49′ from top centre: 3·38 feet from start of stroke. Ans.

7. A man on look-out is 110 feet above the sea level. Find the length of the line joining his eye to the horizon, if the earth is 8,000 miles diameter.

12·9 miles. Ans.

8. A straight edge 12 inches long is laid horizontally in a furnace, at right angles to the longitudinal axis. From the centre of the straight edge to the lowest part of the furnace measures $\frac{3}{4}$ of an inch. Find the inside diameter of the furnace.

$48\frac{3}{4}$ inches diameter. Ans.

9. A furnace is 40 inches diameter inside. A 12 inch rule is laid inside as in question 8. Find the height of the two segments into which the rule divides the furnace.

Segments are 0·93 inch and 39·07 inches. Ans.

10. A bar 10 feet long is hinged at one end, and made to rotate at a uniform speed of 100 revolutions per minute. Find the linear velocity, in feet per second, of points on the rod at 3, 7 and 10 feet from the hinge. What property is common to all points on the rod?

31·41 feet per sec.; 73·29 ft. per sec.; 104·7 ft. per second. Every point on the rod has the same *angular* velocity. Ans.

11. Two ships are 31 miles apart, and they are steaming towards the same port. One ship travels at 10 knots and is $3\frac{1}{2}$ hours from port, the other ship travels at 7 knots and is 4 hours from port. Find the angle between the courses steered by the ships.

$57° 44'$. Ans.

12. Two ships leave the same port together, steering courses which diverge at an angle of 57° to each other. The speeds of the ships are 8 and 11 knots respectively, find how far the ships are apart after 3 hours steaming.

28·32 nautical miles. Ans.

13. Two trains are approaching a crossing. The lines upon which the trains are moving towards each other meet in an angle of 117 degrees. At a certain instant, one train, which moves at 40 miles per hour, is 30 miles from the crossing. At the same instant the other train, moving at 48 miles per hour is 20 miles from the crossing. How far apart are the trains one-quarter of an hour later?

24·68 miles. Ans.

14. A piece of plate, in the form of a triangle, has one side 2 feet long with which the other two sides make angles of 40° and 60°. Find the weight of the plate, which is 1 inch thick, if one cubic inch weighs 0·28 lb.

45·57 lb. Ans.

CHAPTER IX.

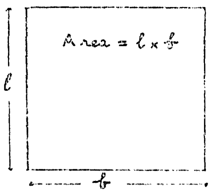
MENSURATION OF AREAS ; First Moment of an Area ; Second Moment of an Area.

To express the magnitude of an area or surface, two dimensions must be considered :—length and breadth.

It is not necessary, to use a new unit for area, since the area is expressed in terms of the fundamental unit Length. As both length and breadth are multiples of the same unit, i.e., feet or inches, the dimensions of an area are feet × feet = (feet)^2, or ins. × ins. = (ins.)^2, or, as we often write them, "square feet," or "square inches."

The "dimensions" of an area are said to be (length)^2.

Rectangle.

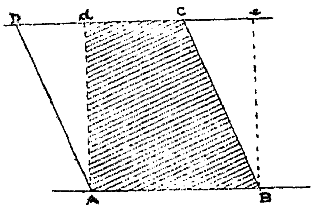


Area = length × breadth.

$$\therefore \text{length} = \frac{\text{area}}{\text{breadth}}$$

$$\text{or breadth} = \frac{\quad}{\text{length}}$$

Parallelogram.

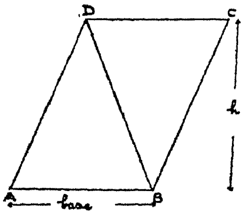


A B D C } D d = e
A B d e

A parallelogram is a figure having its opposite sides parallel. A B C D is such a figure. The figure A B *e d* is a rectangle. Now the shaded area is common to both rectangle and parallelogram. Also the triangle D *d* A = triangle C *e* B. The rectangle then, contains the common area and one triangle ; the parallelogram contains the common area and an equal triangle. Therefore the rectangle is equal in area to the parallelogram. All parallelograms upon the same base and between the same parallels are equal in area.

Area = base × vertical height.

Triangle.



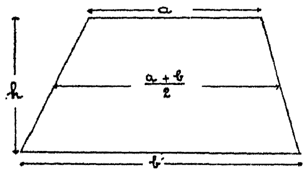
Every parallelogram may be divided by a diagonal into two equal triangles. Thus, D B divides A B C D into equal triangles, A D B and B D C. Therefore the area of each of these triangles is half that of the parallelogram.

Area of triangle =

base × vertical height

Calculations on the areas of triangles are given in the Chapter on Trigonometry.

Area of Trapezium.

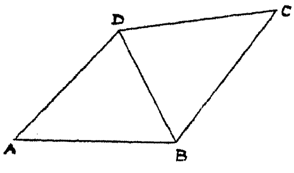


A Trapezium is a four-sided figure, having two of its sides parallel.

Area = mean width × vertical height.

$(a + b)$

Any Four-sided Figure.

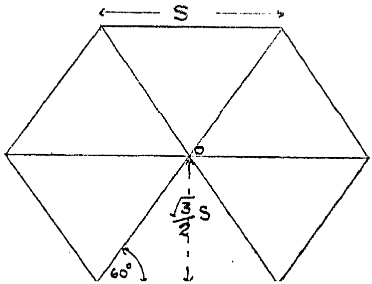


Any four-sided figure, such as A B C D, in which the sides may all be unequal, can be divided up into two triangles, the areas of which may be calculated separately by the rules already given. Adding these areas gives the area of the whole figure.

Hexagon.

The hexagon is made up of 6 equal triangles. Now as there are 360 degrees at the centre, the apex angle of each triangle is $\frac{360}{6} = 60°$, and as the angles at the bases of these triangles are equal, each angle must be 60°. The triangles are equilateral.

$$\text{Area of triangle } A\,O\,B = \frac{A\,B \times O\,C}{\quad} = A\,C \times O\,C$$



Now if S = length of side of hexagon, A C = ·

and O C = $\frac{\ }{2} \times \sqrt{3}$, already proved.

$\therefore$ Area of A O B $= \frac{S}{2} \times \sqrt{3} \times \frac{S}{2}$

$= \frac{\sqrt{3}}{4} S^2 = 0{\cdot}433\ S^2$

Area of hexagon $= \frac{\sqrt{3}}{4} S^2 \times 6$, since there are 6 triangles.

$$= \frac{3}{2} \times \sqrt{3} \times S^2$$

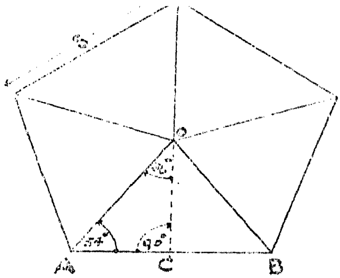
$$= \frac{1{\cdot}732}{2} \times 3 \times S^2$$

$$= 2{\cdot}598\ S^2$$

Pentagon.

The Pentagon is a five-sided figure having all its sides equal in length. The apex angle of each triangle is

$$\frac{360°}{5} = 72°. \quad \text{The angle A O C} = \frac{72°}{2} = 36°$$



$$\text{Now Tan. } 36^\circ = \frac{A\ C}{O\ C} \quad \therefore \quad O\ C = \frac{A\ C}{\text{Tan. } 36^\circ}$$

$$\text{Area of triangle A O B} = A\ C \times O\ C = \frac{A\ C \times A\ C}{\text{Tan. } 36^\circ}$$

$$\text{Area of Pentagon} = \frac{5 \times (A\ C)^2}{\text{Tan. } 36^\circ}, \text{ or if } A\ C = \frac{S}{2}$$

$$\text{Area} = \frac{5 \times \left(\frac{S}{2}\right)^2}{\text{Tan. } 36^\circ} = \frac{5}{4} \times \frac{S^2}{\text{Tan. } 36^\circ}$$

$$= \frac{5\ S^2}{4 \times 0{\cdot}7265}$$

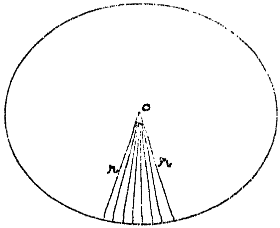
$$\text{Area} = 1{\cdot}721\ S^2.$$

The student should note that in each of these results we have S^2 which is (a length)^2, and therefore these formulæ express area. The co-efficients, 2·598 and 1·721, effect the *value*, but not the "dimensions," of these

The Circle.

The ratio of circumference to diameter in any circle is a constant quantity. This ratio is written π, and its value is 3·1416, but we often take it as $\frac{22}{7}$. The circumference of a circle is π D, or 2 π r.

Area.



We may regard the circular area as being made up of a great number of small triangles, their bases lying on the circumference, their apexes meeting in the centre.

Area of one such small triangle

$=$ length of its base $\times \frac{r}{2}$, because r is its vertical height.

Area of circle $=$ Sum of areas of all small triangles,

,, ,, $=$ Sum of $\left(\text{lengths of bases} \times \frac{r}{2}\right)$,

,, ,, $= \frac{r}{2} \times$ Sum of lengths of bases, since

$\frac{}{2}$ is a common multiplier.

But the sum of lengths of bases $=$ the whole circumference $= 2\,\pi\,r$

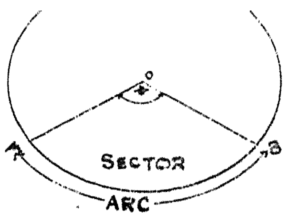
$\therefore$ Area of circle $= \frac{r}{2} \times 2\,\pi\,r = \pi\,r^2 = \frac{\pi\,D^2}{4}$

since $r = \frac{D}{2}$

The area, then, is given by either $\pi\,r^2$ or D^2

or $\frac{22}{7} \times \frac{1}{4}$ $\frac{11}{14}\,D^2$.

Area of Sector.



O A B is a sector of a circle. Now by the above method, the area of the sector will be equal to the sum of the areas of the small triangles which compose it.

Area $= \frac{r}{2} \times$ Sum of lengths of bases

$= \frac{r}{2} \times$ arc, or $\frac{\text{radius} \times \text{arc}}{2}$

Or we may, if θ is given in degrees, write :—

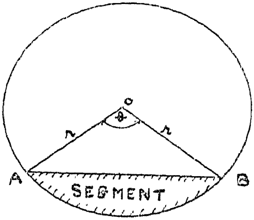
$$\text{area} \quad \frac{}{360^\circ} \times \pi\ r^2,$$

because the area is proportional to the angle at the centre.

Or if θ is given in radians, we may write,

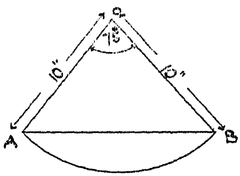
$$\text{Area} = \frac{\theta \text{ in radians}}{2\ \pi} \times \pi\ r^2 \qquad \theta \text{ in radians} \qquad r^2$$

Area of Segment.



To find the area, first determine the area of the sector, then find the area of the triangle and subtract.

Example. Find the area of the sector of a circle of 10 inches radius, the angle at the centre being 75°. Find also the area of the segment.



Area of sector O A C B

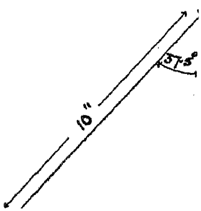
$$= \frac{75}{36\cancel{0}} \times \tfrac{22}{7} \times 1\cancel{0} \times 10$$

$= 65{\cdot}476$ sq. inches.

Area of Triangle O A B

$$= \frac{\text{height} \times \text{base}}{2}$$

$= 10 \text{ Cos. } 37^\circ{\cdot}5 \times 10 \text{ Sin. } 37^\circ{\cdot}5$

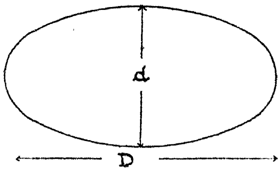


Area of Triangle $= 100 \times 0{\cdot}7934 \times 0{\cdot}6088 = 48{\cdot}302$ sq. inches.

Area of Segment $= 65{\cdot}476 - 48{\cdot}302 = 17{\cdot}174$ sq. inches.

Area of Sector is 65·476 sq. ins. Ans.
Area of Segment is 17·174 sq. ins. Ans.

Area of an Ellipse.



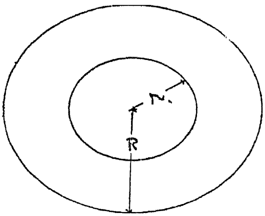
Area = (Major diameter × Minor diameter) × $\frac{\pi}{4}$

Area = $\frac{22}{7} \times \frac{1}{4} \times D \times d$
= $\frac{11}{14} \times D \times d$

If D = 16 inches, and d = 12 inches, Area = $16 \times 12 \times \frac{11}{14}$

Area = 150·857 square inches.

Area of an Annulus.

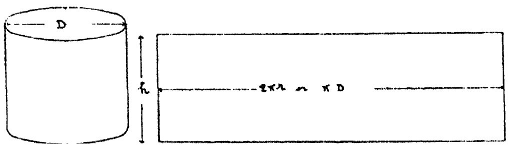


Area = $\pi R^2 - \pi r^2$
= $\pi (R^2 - r^2)$

This may be written as,

$$\frac{\pi}{4} (D^2 - d^2)$$

Curved Surface of a Cylinder.



The curved surface could be unrolled and laid out in the form of a rectangle, of length $2 \pi r$ or π D, and of height h, equal to the vertical height of the cylinder.

$$\text{Surface} = 2 \pi r h = \pi D h$$
$$= \frac{22}{7} \times D \times h.$$

The student should be able to put down these equations in terms either of r or D.

Area of Escape.

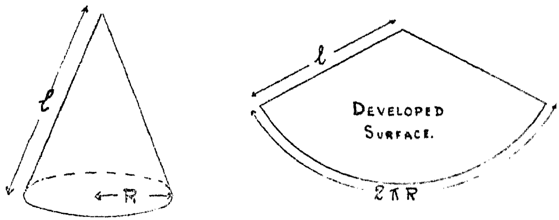
When a valve lifts, the area of escape is equal to the curved surface of a cylinder, the diameter of which is equal to the diameter of the valve, the height being equal to the lift of the valve.

Area of escape = π D × lift.

Find the lift to give an area of escape equal to the area of the valve.

$$D^2 \qquad D \times \text{lift} ; \text{ lift} = \frac{D}{4\,D} \text{ or lift} \qquad \text{Ans.}$$

Curved Surface of a Cone.



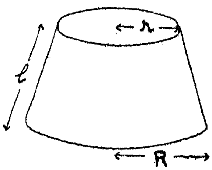
Let l equal the *slant* height of cone. A conical surface may be unrolled or developed into a sector of a circle.

Area $= \dfrac{\text{radius} \times \text{arc}}{}$, here the radius of the developed

sector $= l$, and the length of the arc $2\,\pi\,R$.

Area $= \dfrac{l \times 2\,\pi\,R}{}$ $\quad R\,l$, or $\dfrac{\pi\,D\,l}{}$

Frustum of a Cone.



Note here that l is the *slant height* of the

Area = mean circumference × slant height.

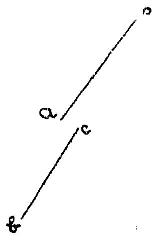
$= \dfrac{2\,\pi}{2}$ (R $\qquad$ (R +

(D + d)

We proceed now to give a brief account of an importan principle in mensuration, and these two last cases are again as examples.

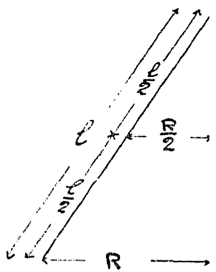
Theorem of Pappus.

If a line, straight or curved, be rotated about an axis in its own plane, it generates an area whose surface is equal to the length of the line multiplied by the distance its centre of gravity moves. When the centre of gravity of the line moves through a complete circle, the surface generated is called a "surface of revolution."



For instance, if the line *o a* be rotated about the axis *o o*, it will generate in space, the curved surface of a cone. If the line *c b* be rotated about the axis *o o*, it will generate the curved surface of a frustum of a cone. Now the centre of gravity of a straight line is always at the middle of its length, so we can find these surfaces easily.

Curved Surface of a Cone.



Let l be the slant height and R the radius of the base.

The C.G. of the line l is at $\frac{l}{2}$,

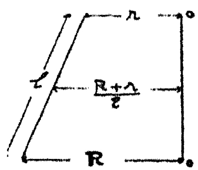
This C.G. is at $\frac{R}{2}$ from the axis *o o*,

because by similar triangles, $R \quad \frac{\frac{l}{2}}{\frac{R}{2}}$

Curved surface = length of line × distance C.G. moves in one turn.

$$\text{Curved surface} = l \times 2\,\pi \times \frac{R}{2} = \pi\, R\, l, \text{ or } \frac{\pi\, D\, l}{2}$$

Curved Surface of a Frustum.

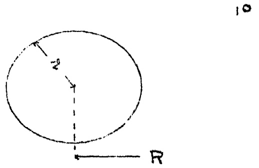


Here the C.G. of the line is at $\frac{R + r}{2}$ from *o o*, and l is the slant height of the frustum.

$$\text{Surface} = l \times 2\,\pi \left(\frac{R + r}{2}\right)$$
$$= \pi\, l\, (R + r)$$

$$\text{Surface} = \frac{\pi\, l}{2}(D + d), \text{ since } R = \frac{\quad}{2} \text{ and}$$

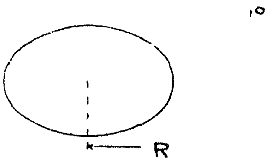
Curved Surface of an Anchor Ring.



Here the length of the line is the circumference of a circle, and its C.G. is at the centre.

Surface of ring $2 \pi r$
$= 4 \pi^2 r R.$

Curved Surface of an Elliptical Life Buoy.



The length of the perimeter of the ellipse must be measured by a piece of string, or thin wire ; call this l,

Then surface $= l \times 2 \pi R.$

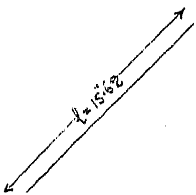
The C.G. of the perimeter here, is the geometrical centre of the ellipse.

The length of the perimeter may be found from the approximate

$$\text{formula} :- \pi \left(\frac{\text{Major Diam.} + \text{Minor Diam.}}{2}\right)$$

$$\text{or } \frac{\pi}{2}(D + d).$$

Examples. Find the curved surface of a cone, the base of which is 10 inches radius, the vertical height being 12 inches.



Slant height $= \sqrt{12^2 + 10^2}$

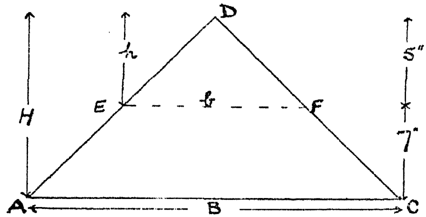
Curved

$= \pi R l = \frac{22}{7} \times 10 \times$

$= \frac{3436 \cdot 4}{7} = 490 \cdot 9$ sq

Ans.

Example. If the cone in the previous example is cut by a plane parallel to its base at a height of 7 inches above the base, find the curved surface of the frustum. Now the triangle D A C is similar to the triangle D E F, because the angle at the apex is common to both, and since E F is parallel to A C, the angle D E F = the angle D A C. The corresponding sides of these triangles have the same ratio.



$$\frac{\text{E F}}{\text{A C}} = \frac{\text{D E}}{\text{D A}} = \frac{h}{\text{H}}, \text{ or } \frac{h}{\text{H}} = \frac{b}{\text{B}}, \therefore b = \frac{h \cdot \text{B}}{\text{H}}$$

$$\frac{5 \times 20}{12} = 8\tfrac{1}{3} \text{ inches.}$$

$$\text{Also as } \frac{\text{D E}}{\text{D A}} = \frac{h}{\text{H}}, \text{ and D A} = 15{\cdot}62 \text{ inches,}$$

$$\therefore \text{D E} = \text{D A} \times \frac{h}{\text{H}} = \frac{15{\cdot}62 \times 5}{12} = 6{\cdot}5 \text{ inches (say)}$$

E A, the slant height of the frustum $= 15{\cdot}62 - 6{\cdot}5 = 9{\cdot}12$ inches.

Area = mean circumference $\times$ slant height

$$\text{Area} = \left(\frac{20 + 8\frac{1}{3}}{2}\right) \frac{22}{7} \times 9{\cdot}12 = \frac{85}{6} \times \frac{22}{7} \times 9{\cdot}12$$

$= 406{\cdot}06$ sq. inches. Ans.

By the same method as used above, if we are given the vertical height and the two diameters of a frustum of a cone, the height of the complete cone may readily be found.

The areas of similar figures vary as the squares of their corresponding dimensions. Thus, we compare the areas of circles by their (diam.)^2. In the case of the similar triangles in the last example the area of the small triangle is $\dfrac{b \times h}{2} = \dfrac{125}{6}$ sq. ins

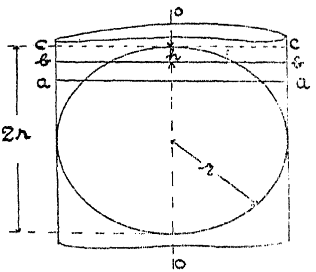
The area of the large triangle could be found as follows. Corresponding dimensions are h and H, or b and B.

$$\text{Area} = \frac{125}{6} \times \frac{H^2}{h^2} = \frac{125}{6} \times \frac{12^2}{5^2} = 5 \times 12 \times 2$$
$$= 120 \text{ sq. inches.}$$

Surface of a Sphere.

The surface of a sphere is equal to the curved surface of the circumscribing cylinder, i.e., the surface of the sphere is equal to the curved surface of a cylinder into which the sphere would just fit, the diameter of this cylinder being 2 r, and its vertical height 2 r.

Surface of sphere $= 2\ \pi\ r \times 2\ r = 4\ \pi\ r^2$.



Also if we draw two parallel lines a a and b b as shown, at right angles to o o, then the curved surface of the sphere between these two planes is exactly equal to the curved surface of the cylinder between the same planes; and the curved surface of the segment or cap of the sphere between planes b b and c c is equal to the area of the cylinder between these planes, or

Surface of segment $= 2\ \pi\ r \times h$.

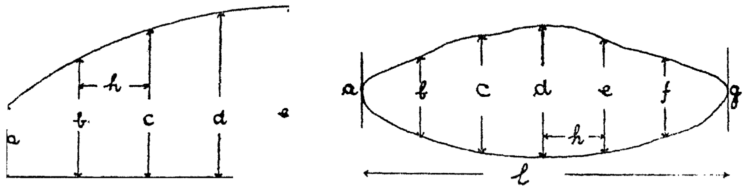
Examples. Find the curved surface of a sphere of 10 i radius.

Surface $= 2\ \pi\ r \times 2\ r = 4\ \pi\ r^2$ or, π D $\times$ D $= \pi$ D^2
$2 \times \frac{22}{7} \times 10 \times 2 \times 10 = \frac{22}{7} \times 20 \times 20 =$
sq. inches. Ans.

Find also the surface of a segment of this sphere if it is 3 inches high.

Surface of segment $= 2\pi r \times$ height $= 2 \times \frac{22}{7} \times 10 \times 3$ $= 188{\cdot}57$ sq. inches. Ans.

Area of Irregular Figures. Simpson's Rule.



Divide up the area into an *even* number of parts. This gives an *odd* number of ordinates. In the first figure the length l has been divided up into 4 parts, each equal to h, making the ordinates a, b, c, d, e, equidistant. These ordinates must be carefully measured. The area is given by :—

$$\frac{h}{3}[a + 4b + 2c + 4d + e].$$

This rule is sometimes stated as follows. Add to the sum of the first and last ordinates, four times the sum of the even ordinates and twice the sum of the odd ordinates. Multiply this result by one-third the common distance between the ordinates, and the product gives the area. If there are only three ordinates, a, b and c, then the area is $\frac{h}{3}[a + 4b + c]$, this is the simple form. Now $\frac{\text{area}}{\text{length}} =$ mean height, therefore looking at the first figure, up to the ordinate c, the mean height is

$$\frac{h}{3}[a + 4b + c] \div 2h = \frac{h}{3 \times 2h}[a + 4b + c]$$

$$= \frac{a + 4b + c}{6}$$

And this is the rule we use to get the mean width of a bunker.

Now Simpson's rule may be extended to any odd number ordinates. In the second figure, the area is given by :—

h

In this figure the value of a and g is 0. We must not, however, on that account begin at b and end at f, for in that case the end portions of the area would not be included. Let $a = 0$, $b = 2$, $c = 2\frac{1}{2}$, $d = 3$, $e = 3$, $f = 2\frac{1}{4}$, $g = 0$, all in inches, h being 2 inches. Find the area of the second figure.

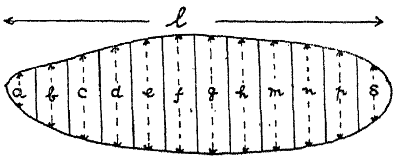
$$\text{Area} = \frac{\ }{3}\left[a + 4 \quad 2c + 4d + 2e + 4f + g\right]$$

$$= \tfrac{2}{3}\left[0 + (4 \times 2) + (2 \times 2\tfrac{1}{2}) + (4 \times 3) + (2 \times \quad + (4 \times 2\tfrac{1}{2}) + 0\right] = \tfrac{2}{3} \times 40 = 26\tfrac{2}{3} \text{ sq. inches.}$$

$$\text{Mean width} = \frac{\text{area}}{\text{length}} \quad \frac{26\frac{2}{3}}{6 \times 2} \quad 2{\cdot}2 \text{ inches.}$$

Note that the multipliers of the ordinates are, 1, 4, 2, 4, 2, 4, 1, in the above case. The first and last multipliers must always be unity, or one, no matter how many ordinates there are. To sum up then ; the ordinates must be carefully measured ; the ordinates must be equidistant ; there must be an odd number of ordinates.

Mid-Ordinate Rule.



Divide the area up into any number of parts by equidistant ordinates. In the figure shown. there are 11 ordinates, and 12 spaces. Now measure carefully, the mean width of each of the 12 spaces, as given by the dotted lines. Add the lengths of these dotted lines, divide the result by the number of these dotted lines, and we have the mean width of the area. Multiply this mean width by the total length l of the figure, and we have the area.

Or mean width =

$$\frac{a + b + c + d + e + f + g + h + m + n + p + s}{12}$$

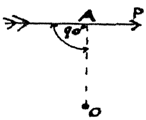
Area $= l \times$ mean width.

We assume here, that between each pair of ordinates the curve is a straight line, and if we take our ordinates very close to each other, this assumption is very nearly correct. The greater the number of ordinates taken, the nearer we approach to the true area. This is the method we use to get the mean pressure of the steam from the indicator diagram, but we always divide up the diagram into ten parts, so that we have 10 ordinates to measure, and it is easy to add up and then divide by ten.

Both Simpson's rule and the mid-ordinate rule give the area of irregular figures only approximately.

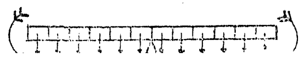
First Moments.

The first moment of a force about a point, is the product of the force and its perpendicular distance from the point. We generally state this quantity merely as the moment of a force about a point. It is important to remember that we mean a force multiplied by a distance; because we sometimes use a quantity which is called the second moment, in which a force, or perhaps an area or a volume is multiplied by the square of a distance.



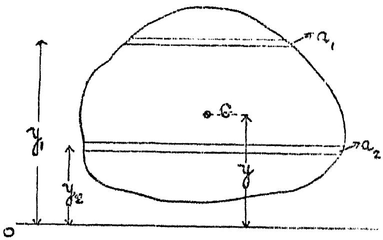
Let P be a force moving in the direction shown by the arrow. The perpendicular distance from the point O to the path of the force is O A, and the moment of the force P about the point O is said to be P $\times$ O A. If P is in pounds and O A in inches, the dimensions of this moment are expressed as inch-pounds.

Now we sometimes need the moment of an area, or of a volume about a point, or about a line. This brings us to the consideration of a certain position known as the centre of area or the Centroid, or as we rather loosely call it, the Centre of Gravity. As an area has no mass, it is not correct to call its Centroid by the name of Centre of Gravity; but the term is in very general use, and if we understand its meaning, there is no harm in using it. It is, of course, perfectly correct when dealing with a mass to speak of its Centre of Gravity.



If a uniform rod or bar is divided up into a number of equal divisions, as shown in the sketch, and supported on a fulcrum so that an equal number of divisions lie on each side, then the rod is found to balance about the fulcrum. Each of the small divisions has its moment about F, and the total moment to the right of the fulcrum tends to produce rotation in the direction indicated. The total moment of all the divisions to the left of F, tends to produce rotation in the reverse direction; but the rod remains at rest, so that the turning effect of the moments of all the divisions about the fulcrum is nothing. Further, if the whole mass of this rod was collected into one single heavy particle and placed at F, the effect would be the same, since this mass having then no leverage about F, would have no moment about F. The position at which this heavy particle must be placed is called the centre of gravity of the body.

The Centroid of an Area, or the centre of gravity of a mass, is that point in the area or in the mass through which, if a straight line is drawn in any direction, the sum of the first moments on one side of the line is equal to the sum of the first moments on the other side.



Take any area as shown, and divide it up into a great number of thin strips, of which only a_1 and a_2 are shown. Let the Centroid be at G, and let the whole area be A.

Taking first moments about *o o* we have :—

Moment of all strips about *o o* $= (a_1 \times y_1) + (a_2 \times y_2) +$ etc., the term "etc." including all moments not specially named, such as $a_3\, y_3 \;\ldots\ldots\; a_n\, y_n$.

But, by the principle of the centre of gravity, the effect of all these moments of elements of area about *o o*, must be the same

as if the whole area was concentrated at G, at a distance y from $o\ o$. Therefore we have :—

$$A \times y = a_1 y_1 + a_2 y_2 + \text{etc.}$$

$$\text{or } y = \frac{a_1 y_1 + a_2 y_2 + \text{etc.}}{A}$$

$$y = \frac{\text{sum of first moments about } o\ o}{\text{whole area}}$$

Note that since we take first moments about $o\ o$, y will be measured from $o\ o$. Also, since an area is expressed as (length)^2, say (inches)^2, the moment of an area must be area $\times$ distance $=$ (inches)^2 $\times$ inches $=$ (inches)^3.

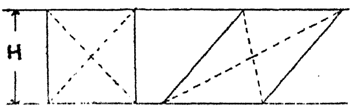
Now y is merely a length, and our equation is correct.

$$\text{Since, } \frac{\text{First moment}}{\text{Area}} \quad \frac{(\text{inches})^3}{(\text{inches})^2} = \text{inches.}$$

The line $o\ o$ might have been taken anywhere, but if it had been taken through the area, then the moments on one side would have had to be subtracted from the moments on the other side of the line.

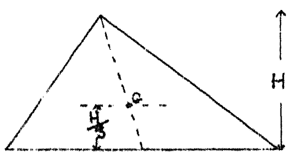
Centroids of Simple Figures. A few cases only are necessary.

Rectangle or Parallelogram.



Since both of these figures would balance about their diagonals, their centres of gravity must lie at the intersection of the diagonals. The C.G. is therefore at $\frac{H}{2}$ above the base.

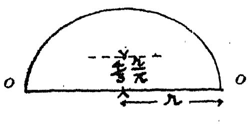
Triangles.



The centre of gravity of a triangle lies on a line joining the apex to the middle point of the base, at a vertical height of $\frac{H}{3}$ above the base.

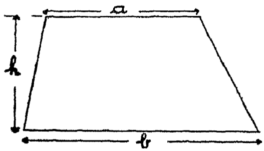
Circular Area. The C.G. lies at the centre of the circle.

Semicircular Area.



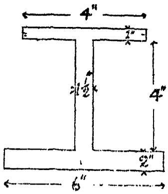
Let $o\ o$ be the diameter, then the C.G. is $\frac{4}{3} \times \frac{r}{\pi}$ from $o\ o$ (where r is the radius of the circle) and $\frac{4}{3}\frac{r}{\pi}$ $= 0{\cdot}424\ r$.

Trapezium.



The C.G. lies at a vertical height of $\frac{h}{3}\left[\frac{2\,a + b}{a + b}\right]$ above the base b.

a and b may have any values, a may be longer than b.



Examples. Find the position of the C.G. above the base in the figure shown.

Take moments about the base

Now C.G. above the base =

$$\frac{\text{Sum of moments of all areas about base}}{\text{total area}}$$

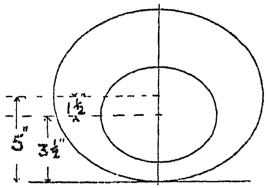
Now this whole area is made up of three small rectangles and their C's of G are at $6\frac{1}{2}$ inches, 4 inches and 1 inch respectively, above the base

∴ C.G. above base

$$= \frac{(4 \times 1 \times 6\frac{1}{2}) + (4 \times 1\frac{1}{2} \times 4) + (6 \times 2 \times 1)}{(4 \times 1) + (4 \times 1\frac{1}{2}) + (6 \times 2)} \qquad \frac{62}{22}$$

C.G. above base $= \frac{31}{11} = 2{\cdot}818$ inches above base.

Example. Find the C.G. of an eccentric sheave. The outside diameter is 10 inches, the hole is 5 inches diameter, the eccentricity being $1\frac{1}{2}$ inches.



Take moments about a tangent *o o*. Note that here the moment of the inner circular area (i.e., the hole), must be subtracted. Further, the total area is the difference between the areas of the two circles.

C.G. above *o o*

$$\frac{\text{Moment of large area} - \text{Moment of small area}}{\text{Area of large circle} - \text{Area of small circle}}$$

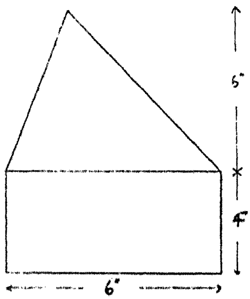
$$\frac{\pi\, 5^2 \times 5 - \pi\, (2\tfrac{1}{2})^2 \times 3\tfrac{1}{2}}{\pi\, 5^2 - \pi} \qquad \pi \;\; [5^3 \;\; - \qquad\qquad \times \;\; 3\tfrac{1}{2}]$$

$$\frac{125 - 21{\cdot}875}{25 \qquad 6{\cdot}25} \qquad \frac{103{\cdot}125}{18{\cdot}75} \qquad \frac{11}{2} \qquad 5{\cdot}5 \text{ inches.}$$

Ans.

C.G. is therefore 5·5 inches above *o o*, on a line joining the centres of the two circles, or 0·5 inch from the centre of the outer circle.

Example. Find the C.G. of the given figure above its base. Take moments about the base.



C.G. above base =

$$\frac{\text{Sum of moments about base}}{\text{Sum of areas}}$$

C.G. above base =

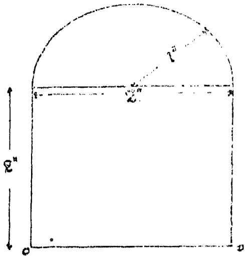
$$\frac{(6 \times 4 \times 2) + \frac{6 \times 6}{2} \times \left(4 + \right.}{4 \times 6 + \frac{6 \times 6}{2}}$$

C.G. above base =

$$\frac{48 + 108}{24 + 18} = \frac{156}{42} = \frac{26}{7}$$

C.G. above base $= 3\frac{5}{7}$ inches. Ans.

Example. A figure is made up of a square and a semicircle as shown, find the position of its C.G. Take moments about the base, then,



$$\frac{\text{Sum of moments about } o\ o}{\text{Sum of areas}} = \text{distance of C.G. from base.}$$

C.G. above base =

$$\frac{(2 \times 2 \times 1) + \frac{\pi \times r^2}{2} \times (2 + 0{\cdot}424\ r)}{4 + \frac{\pi\ r^2}{2}}, \text{ and } r \text{ is } 1$$

$$\text{C.G. above base} = \frac{4 + 3{\cdot}809}{4 + 1{\cdot}571} = 1{\cdot}4 \text{ inches. Ans.}$$

Second Moment of an Area. Definition.

The Second Moment of an area about an axis is the sum of all elements of area, each element being multiplied by the square of its distance from the given axis.

We have already shown that if an area be divided up into a great number of small elements, then the first moment of the whole area about an axis or a line, is equal to the sum of the first moments of all the elements of area about the same axis.

Now if each of these elements of area be multiplied by the *square* of its distance from the axis, the product is called the *second moment* of the element about the axis, because this time the element is multiplied by the square or second power of a distance. The sum of all these second moments of elements

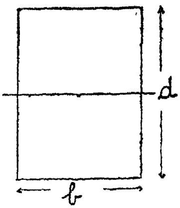
is equal to the second moment of the whole area about the axis considered. Sometimes this is written as the Geometrical Second Moment of the area. The question arises—Of what use in our work is this quantity? In many investigations in mechanics we deduce expressions which contain this very term, a term which is written as "the sum of all elements of area multiplied by the squares of their distances from some known axis." To this term has been given the name of "the second moment of the whole area" about the axis. Thus in evolving the expression for the resistance of a beam to bending, or of a shaft to torsion, such a term makes its appearance, hence its importance in mechanics. The second moment is often called the Moment of Inertia. Now this is not exactly correct, because an area has no mass, but this name is very generally used. In the case of bodies having mass, it is correct to speak of the second moment as the Moment of Inertia. In all cases it is important to state the position of the axis about which the second moment is required. In the examination for Second and First Class, there are no calculations given in which the second moment has to be worked out from first principles, but it is desirable that the student should have some notion of its meaning.

There is a certain position in the area at which if the whole area was concentrated, the effect of the second moment about an axis would not be altered. Such a position is called the Centre of Gyration, and the Radius of Gyration is the perpendicular distance of this centre from the axis considered. The student must not confuse this centre of gyration with the Centroid of the area. The two things are not the same. The centroid has an invariable position within the area, but the centre of gyration will change its position within the area for every new axis about which the second moment is taken. For one fixed axis, of course, the centre of gyration will have one position, and the radius of gyration one value. The symbol for the radius of gyration is k.

The same statements apply to moments of inertia of solids, but in the cases of bodies having mass, we are multiplying elements of mass, by a distance squared. In the case of a flywheel rotating about a fixed axis, i.e., its shaft, the radius of gyration is that distance from the axis of rotation at which, if the whole mass of the wheel was concentrated in one thin concentric ring, the second moment of the wheel would be the same, and the capacity of the wheel to store energy would not be altered, for it can be proved that the kinetic energy of a rotating body depends upon its Moment of Inertia, as well as upon the square of the speed of rotation. Further reference to the Second Moment or Moment of Inertia is made in the Chapters on Bending and Torsion.

Important Cases of Second Moments or Moments of Inertia, written (I.)

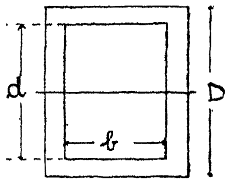
Rectangle about an Axis through its C.G.



$$\frac{b\ d^3}{12}, \text{ Radius of Gyration,}$$

$$k^2 = \frac{I}{\text{Area}} = \frac{b\ d^3}{12 \times b\ d} = \frac{d^3}{12}$$

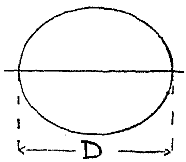
$$k = \frac{d}{\sqrt{12}}$$



I of Hollow Rectangular Area about an axis through the C.G.

$$\frac{B\ D^3 - b\ d^3}{12}$$

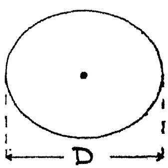
$$k = \sqrt{\frac{I}{B\ D -}}$$



I of Circular Area about a diameter in the plane of the area.

$$\frac{D^4}{64} \qquad = \sqrt{\frac{I}{\text{Area}}} = \frac{D}{4}$$

I of Circular Area about an axis through the centre at right angles to the plane of the area This is sometimes called the Polar Second Moment (I_p). Often denoted by J.



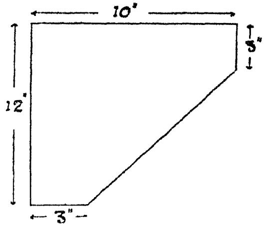
$$\frac{\pi\ D^4}{32}$$

$$\sqrt{\frac{I_p}{\text{Area}}} = \frac{D}{\sqrt{8}}$$

This is the radius of gyration we use for a spinning disc, such as a disc flywheel, and this result $\frac{R}{\sqrt{2}}$ is important. Some examples are given at the end of the Chapter.

$$I \text{ for an annulus about its diameter} = \frac{\pi}{64}[D^4 - d^4]$$

$$I \text{ for an annulus about its pole or centre} = \frac{\pi}{32}[D^4 - d^4]$$



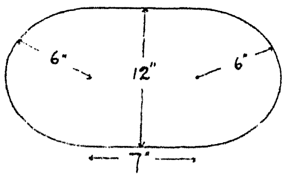
TEST EXAMPLES IX.

1. The beam knee shown is made of steel plate weighing 15 lb. per square foot. Find its weight in pounds.

9·22 lb. Ans.

2. The stuffing box for a 7 inch diameter piston rod is $9\frac{1}{2}$ inches diameter and $8\frac{3}{4}$ inches deep. What total length of $1\frac{1}{4}$ inch packing of square section will be required to pack the gland, allowing each turn to be 1 inch open.

7 turns. 14 feet $6\frac{1}{2}$ inches. Ans.



3. A floor plate is lightened by cutting out of it a hole of the given dimensions. Find the weight of plate cut out at 18·5 lb. per square foot.

25·32 lb. Ans.

4. An internal steam pipe is 8 inches inside diameter. The steam is admitted through rectangular slots 0·3 inch wide and 6 inches long. How many slots are needed to give $1\frac{1}{2}$ times the area of the pipe?

42 slots. Ans.

5. A paddle wheel is 18·75 feet in diameter from centre to centre of the floats. If the engines turn 32 revolutions per minute, and the slip is 15 per cent., find the speed of the vessel.

15·81 knots. Ans.

6. A propeller shaft is 18 inches diameter at the big end of the taper, and the taper is $\frac{3}{4}$ inch per foot. The length of the taper is 3 feet 10 inches. Find the diameter of the shaft at the small end.

$15\frac{1}{8}$ inches. Ans.

7. A crank pin is 15 inches diameter. There are 3 inches of liners between the two brasses. Find the length of the arc of contact between each half-brass and the pin. The crank pin is 16 inches long, find the rubbing area of the brasses if 7 per cent. is cut away to form oilways.

Each brass :—Arc, 20·55 inches ; Surface, 305·78 sq. ins. Ans.

8. A cylinder is 40 inches diameter. For every square inch of surface in the guide shoe, there are 4 square inches of piston area. The width of the shoe is 15 inches, find its length.

20·95 inches. Ans.

9. A safety valve is $3\frac{1}{2}$ inches diameter, and its bearing surface is quite flat. What lift has the valve when the area of escape is $\frac{1}{8}$ of the area of the valve ?

$\frac{7}{64}$ inch. Ans.

10. A conical buoy is 5 feet diameter and 8 feet vertical height, and the material of which it is made is 6 lb. per square foot. Find the total weight of the buoy.

513 lb. Ans.

11. Part of a funnel casing is shaped like a frustum of a cone. Its diameters are 8 feet and 7 feet, and its vertical height is 3 feet. Find the weight at 5 lb. per square foot.

358·4 lb. Ans.

12. Find the length of the side of the largest square which can be cut from a circular plate of 15 inches diameter.

10·605 inches. Ans.

13. A spherical buoy is 8 feet diameter and it floats at a draught of 5 feet. Find the total surface of the buoy and the area of the submerged portion.

201·14 and 125·7 sq. feet. Ans.

14. Find the area of the largest equilateral triangle which can be cut from a circular plate of 10 inches diameter.

32·49 sq. inches. Ans.

15. Jointing material 2·5 millimetres thick is wrapped round a drum 2 inches diameter. If the outside diameter of the roll is 9 inches, find the length of material on the roll.

51 feet 2·8 inches. Ans.

16. The steam space of a boiler at the water level is 14 feet 6 inches broad. The distance from the water level to the top of the boiler is 4 feet 11 inches. The vertical height from the water level to the boiler shell at the quarter breadths is 4 feet. Find the area of the top end plate in the steam space.

50·53 sq. feet. Ans.

17. A right angled triangle has its hypotenuse 10 inches long, its area being 25 square inches. What length must the hypotenuse of a similar triangle be if its area is 12 square inches?

6·928 inches. Ans.

18. If boiler tubes 7 feet long and 3 inches diameter give 22 square feet of heating surface per square foot of grate, what is the sectional area of the tubes per square foot of grate?

0·196 sq. foot. Ans.

19. Find the area of a sector of a circle, if the angle of the sector is 47°. Find also the area of the segment. The sector is 10 inches radius.

41·03 sq. inches. 4·46 sq. inches. Ans.

20. A boiler has 4 furnaces each $3\frac{1}{2}$ feet diameter and the grate is $5\frac{1}{2}$ feet long. If there are 74 tubes to each furnace, the tubes being 3 inches diameter and 7 feet long, find the heating surface of the tubes per square foot of grate.

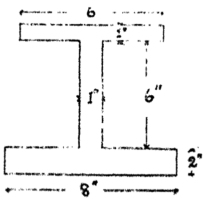
21·14 sq. feet. Ans.

21. An eccentric sheave is 25 inches outside diameter, and the hole is 14 inches diameter. The eccentricity is 3 inches. Find the position of the centre of gravity of the sheave.

1·37 inches from Centre of Sheave. Ans.

22. A trapezium has one of its parallel sides 10 inches long, the other being 15 inches long. The vertical height between these two sides is 8 inches. Find the height of the centre of gravity from the 15 inch side.

3·733 inches. Ans.

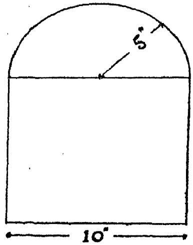


23. Find the height of the centre of gravity of the given section, from its base.

3·464 inches. Ans.

24. Find the surface of an anchor ring made of iron bar 2 inches diameter, the mean radius of the ring being 5 inches.

197·55 sq. inches. Ans.



25. A semicircular area is imposed upon the side of a square as shown. Find the centre of gravity of the figure so formed, above the base.

7·004 inches. Ans.

26. A thrust shaft is 12 inches diameter. Over the collars the diameter is 18 inches and there are 6 collars. If only 0·65 of the area of the collars is in contact with the shoes, find the total thrust transmitted when the pressure on the thrust shoe collars is 65 lb. per square inch.

35,852 lb. Ans.

27. A propeller blade is 7 feet long from the boss to the tip. At the boss the blade is $2\frac{1}{2}$ feet wide. The blade widths at equal distances apart from the boss are, $2\frac{3}{4}$ feet, $3\frac{1}{4}$ feet, $3\frac{1}{2}$ feet, 3 feet, $1\frac{3}{4}$ feet, 0. Find the area of the blade by Simpson's rule, and also by the mid ordinate rule.

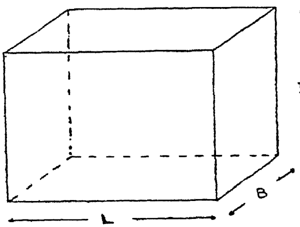
18·27 sq. feet by Simpson's Rule
18·08 sq. feet by mid ordinate } Ans.

CHAPTER X.

MENSURATION OF SOLIDS ; CENTRES OF GRAVITY.

Volume is expressed as area multiplied by thickness or depth in the simple case of a rectangular slab or plate. The dimensions of volume then are (inches)^2 $\times$ inches, or (inches)^3 ; if our sizes are in foot units, the volume will be (feet)^3 or as we write it, cubic feet.

Volume of a Rectangular Prism.



$$\text{Vol.} = \text{Area of base} \times \text{depth}$$
$$= L \times B \times D$$

$$\text{Note, } \frac{\text{Volume}}{D} = \text{Area of base.}$$

The lengths must be all in feet, or all in inches. This is very important.

Examples. A steel plate is $6\frac{1}{2}$ feet long, 3 feet broad, and $\frac{7}{8}$ inch thick. Find its volume in cubic feet. If one cubic foot of this steel weighs 485 lb., find the weight of the plate.

Volume $=$ Area $\times$ thickness (all in the same units).

Volume $= 6\frac{1}{2} \times 3 \times (\frac{7}{8} \times \frac{1}{12})$, note the 12 here, brings the thickness into foot units.

Volume $= \frac{13}{2} \times 3 \times \frac{7}{96} = 1{\cdot}422$ cu. feet (nearly). Ans.

Weight $= 1{\cdot}422 \times 485 = 689{\cdot}6$ lb. Ans.

Example. A plate of steel weighs 2,200 pounds. It measures 10 feet long and 6 feet broad. Find its thickness if a cubic foot of the material weighs 490 pounds.

Cubic feet of steel in the plate $=$ cubic feet.

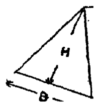
Now, area $\times$ thickness $= 4{\cdot}489$, but all units must be in feet.

$\therefore$ 10 $\times$ 6 $\times$ t (all in feet) $= 4{\cdot}489$ cubic feet.

$$\frac{4{\cdot}489}{10 \times 6} \text{ feet, or } t = \frac{4{\cdot}489 \times \cancel{12}}{\cancel{10} \times \cancel{6}} \text{ inches}$$

$t = 0{\cdot}8976$ inch. Ans.

Triangular Prism.



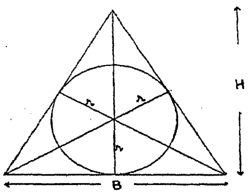
Volume = Area of end × length

$$= B \times \frac{H}{2} \times L$$

Example. Find the volume of a triangular prism, the end section of which is an equilateral triangle having sides 3 inches long, the total length being 12 inches.

Area of end = $(\text{side})^2 \times 0{\cdot}433 = 9 \times 0{\cdot}433 = 3{\cdot}897$ sq. inches.
Volume $= 3{\cdot}897 \times 12 = 46{\cdot}764$ cubic inches. Ans.

Example. Find the diameter of the largest bar which could be turned out of this prism.



The diameter of the inscribed circle is required.

$$r = \frac{\text{Area of triangle}}{\text{Semisum of the sides}}$$

$$\frac{3{\cdot}897}{\frac{(3 + 3 + 3)}{2}}$$

$$r = \frac{3{\cdot}897}{4{\cdot}5} = 0{\cdot}866 \text{ inch.}$$

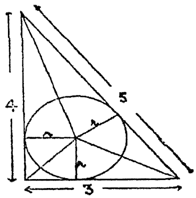
Diameter of largest bar $= 0{\cdot}866 \times 2 = 1{\cdot}732$ inches. Ans.

Or joining the corners to the middle points of the opposite sides, we have these lines intersecting at a common point which is the centre of the circle, and in this case

$$r = \frac{H}{3}, \text{ and } H = \sqrt{3} \times 1\tfrac{1}{2} = \sqrt{3} \times \tfrac{3}{2}$$

$$\text{and } r = \sqrt{3} \times \tfrac{3}{2} \times \tfrac{1}{3} = \frac{\sqrt{3}}{2} = 0{\cdot} \quad \text{inches as before.}$$

Example. Find the diameter of the largest bar which can be cut out of a triangular prism, whose end section is a right angled triangle having sides 3 inches, 4 inches and 5 inches long.



Angle of Triangle $= \dfrac{3 \times 4}{2} = 6$ sq. ins.

Area of Triangle = area of 3 small triangles

Area of Triangle

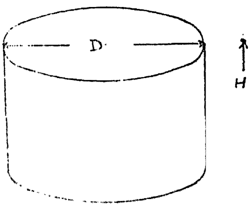
$= 6\,r$ sq. inches.

$= 6$ sq. inches.

$r = 1$ inch. Ans.

or, $\dfrac{\text{Area of triangle}}{\text{Semisum of sides}} = \frac{6}{6} = 1$ inch. Ans.

Volume of a Solid Cylinder.



Volume = Area of end $\times$ vertical height

$\times$ **H**, or $\frac{11}{14}$ $D^2 \times$ **H**.

Volume of a Hollow Cylinder.

Volume = Area of end $\times$ vertical height

$= \frac{11}{14}\,(D^2 - d^2) \times H.$

Example. A cast iron disc is 10 inches diameter and 5 inches thick. Find its weight if one cubic foot weighs 450 lb.

Volume = Area $\times$ thickness.

Volume $= \frac{11}{14} \times 10^2 \times 5$ cubic inches.

Volume $= \frac{11}{14} \times 100 \times \frac{5}{1728}$ cubic feet.

$$\text{Weight} = \frac{55 \times 100}{14 \times 1728} \times 450$$

$$\text{Weight} = 102{\cdot}3 \text{ lb.} \quad \text{Ans.}$$

Example. A hollow cylinder of cast iron is 12 inches long, its outer diameter being 6 inches. If the cylinder weighs 26·965 pounds, find its internal diameter. A cubic inch of cast iron weighs 0·26 lb.

$$\text{Volume} = \frac{26{\cdot}965}{0{\cdot}26} \text{ cubic inches.}$$

$$\text{Volume} = \tfrac{11}{14}\,[6^2 - d^2] \times 12$$

26·965

0·26

$$36 - d^2 = \frac{26{\cdot}965}{0{\cdot}26} \times \quad \times$$

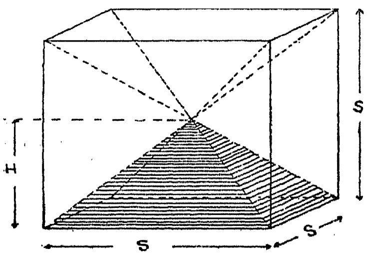
$$36 - d^2 = 11$$

$$d^2 = 36 - 11 = 25$$

$$d = 5 \text{ inches.}$$

∴ The internal diameter is 5 inches. Ans.

Volume of Pyramid having a square base.



If a cube is by its diagonals, it may considered as being up of six equal pyramids, the apeces meeting at the intersection of the diagonals, the base of each being a side of the cube. The volume of one pyramid is one-sixth of the volume of the cube.

$$\text{or Vol.} = S^3 \times \tfrac{1}{6} = S^2 \times \frac{S}{6}, \text{ now } H = \frac{S}{2}, \quad \text{or } S = 2\ H$$

$$\text{Vol.} = S^2 \times \quad 2\ H \qquad S^2\ H$$

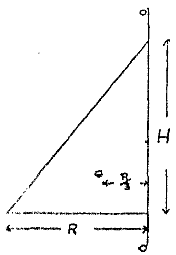
$$\text{Vol.} = \text{Area of base} \times \quad \text{vertical height}$$

For the volume of a pyramid on a triangular base, the same rule holds :—

$$\text{Vol.} = \text{Area of base} \times \quad \text{vertical height}$$

Theorem of Pappus for volumes. If an area, lying wholly on one side of an axis, be rotated about the axis in its own plane, it generates a solid, whose volume is equal to the product of the area and the distance its centre of gravity moves. If the area moves through a whole revolution, the solid is called a "solid of revolution." The theorems of Pappus are sometimes called "ring theorems."

Volume of a Cone.



If the triangle is swept about the axis *o o*, it will generate the volume of a cone. By the theorem, area × distance C.G. moves = volume.

Now the area of the triangle is $\frac{H \times R}{2}$, and the C.G. is at $\frac{R}{3}$ from the axis, and in one turn it moves a distance

R

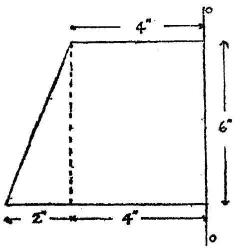
3

$$\text{Volume} = \frac{H \times R}{\cancel{2}} \times \cancel{2}\,\pi\,\frac{R}{3} = \pi\,R^2\,\frac{H}{3}$$

$$\text{Volume} = \text{Area of base} \times \quad \text{vertical height}$$

Frustum of a Cone.

Example. Find the volume of a frustum of a cone, the diameters being 8 inches and 12 inches and the vertical height being 6 inches.



If the figure shown be rotated about $o\ o$ it will generate the volume of a frustum. Regard the figure as made up of a rectangle and a triangle. The C.G. of the rectangle is at $\frac{4}{2}$ or 2 inches from $o\ o$, and the C.G. of the triangle is at $4 + \frac{2}{3} = 4\frac{2}{3}$ inches from $o\ o$.

Volume = Area × distance C.G. moves.

$$\text{Vol.} = 4 \times 6 \times 2\,\pi \times 2 + \frac{\quad \times 6}{\quad} \times 2\,\pi \quad 4\tfrac{2}{3}.$$

$$= 2\,\pi\,[\,(2 \times 6 \times 4) + 6 \times 4\tfrac{2}{3}] = 2\,\pi\,(48 + 28).$$

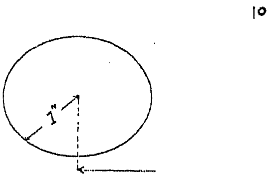
$$\text{Vol.} = 2 \times \tfrac{22}{7} \times 76 = 477{\cdot}7 \text{ cu. inches. Ans.}$$

If the student does the above work algebraically, calling d the smaller diameter, and D the large diameter, it is not difficult to prove that the volume is given by :—

$$\frac{\quad}{12}\,\text{H}\,[\text{D}^2 + \text{D}\,d + d^2], \text{ where H is the vertical height.}$$

Using this we have, Vol. $= \frac{22}{7} \times \frac{6}{12} \times [144 + (12 \times 8) + 64]$ $= 477{\cdot}7$ cu. inches.

Volume of Anchor Ring.



Vol. = Area × distance C.G. moves $= \pi\,r^2 \times 2\,\pi\,\text{R} = 2\,\pi^2\,r^2\,\text{R}.$

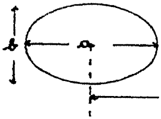
Example. Find the volume of an anchor ring made from a bar 2 inches diameter. The mean radius of the ring is 5 inches.

Volume = Area × distance C.G. moves.

$$= \pi \times 1^2 \times 2\,\pi \times 5 = \frac{22}{\quad} \times 2 \times \frac{22}{\quad} \times 5$$

Volume = 98·72 cubic inches. Ans.

Volume of an Elliptical Lifebuoy.



Volume = Area × distance C.G. moves.

$\times \quad \times 2\,\pi\,R.$

Example. A lifebuoy has a section $4\frac{1}{2}$ by $5\frac{1}{2}$ inches, its inner diameter being 15 inches. Find its weight if a cubic foot of cork weighs 15 lb.

$$\text{The mean radius is } 7{\cdot}5 + \frac{5\frac{1}{2}}{2} = 10\tfrac{1}{4} \text{ inches.}$$

$$\text{Volume} = \tfrac{22}{7} \times \tfrac{1}{4} \times \tfrac{9}{2} \times \tfrac{11}{2} \times 2 \times \tfrac{22}{7} \times \tfrac{41}{4} = 1252{\cdot}07 \text{ cu. ins.}$$

$$\text{Volume} = \frac{1252{\cdot}07}{1728} \text{ cu. feet.} \quad \text{Weight} = \frac{1252{\cdot}07}{1728} \times 15.$$

Weight = 10·86 lb. Ans.

Example. Find the weight of one turn of metallic packing for an eight inch piston rod. The section is as shown, and the material weighs 0·33 lb. per cubic inch.

Area of section

$\times \quad \times \tfrac{1}{2} = \tfrac{15}{16}$ sq. inches.

The C.G. is $4 + \frac{1}{3} \times \frac{3}{2} = 4\frac{1}{2}$ inches from *o o*.

$$\text{Volume} = \frac{15}{16} \times \cancel{2} \quad \times \frac{9}{\cancel{2}} = \frac{15}{16} \times \frac{22}{7} \times 9.$$

= 26·518 cubic inches.

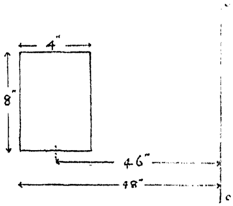
Weight = 26·518 × 0·33 = 8·75 lb. Ans.

Note that the volume of a cylinder or of a flywheel rim can be found by this method. Suppose the section of a flywheel rim is a rectangle 8 inches by 4 inches, and that the outside diameter is 8 feet.

The C.G. of the section is at 48 — 2 = 46 inches from o

Volume $= 8 \times 4 \times 2 \times \frac{22}{7} \times 46 \times \frac{1}{1728}$ cu. feet

$= 5 \cdot 354$ cubic feet. Ans.



Volume of a Sphere.

The sphere may be regarded as being made up of a great number of small pyramids, whose apeces all meet in the centre of the sphere, and whose bases lie on the surface of the sphere. Then the volume of one small pyramid :—

$$= \text{Area of its base} \times \frac{\text{height}}{3}$$

Volume of sphere = Sum of volumes of all small pyramids.

$$\text{Volume of sphere} = (\text{sum of areas of bases}) \times \frac{\text{height}}{3}$$

Note that the term $\frac{\text{height}}{3}$ is a common multiplier for all small pyramids, also the height is r, the radius of the sphere.

Now (sum of areas of bases) = Surface of whole sphere $= 4\,\pi\,r^2$,

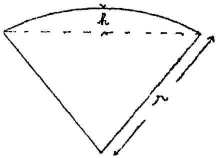
$$\therefore \text{ Volume of sphere} = 4\,\pi\,r^2 \times \frac{r}{3} = \frac{4}{3}\,\pi\,r^3,$$

or since $r = \frac{D}{2}$, $\quad$ Vol. $= \frac{4}{3}\,\pi \left\{\frac{D}{2}\right\}^3$ $\quad$ 6

and $\frac{\pi\,D^3}{6}$ is $\frac{2}{3}$ of the volume of the cylinder into which the sphere just fits.

Volume of Sector of Sphere.

A sector of a sphere is a cone having a spherical base. Its volume is easily found by the previous method. The sector is made up of a great number of small pyramids, and the sum of the areas of their bases is equal to the surface of the curved base of the sector.



Volume of sector $=$ curved surface $\times$

and if the segment is h high,

Vol. of sector $= 2\ \pi\ r\ \times\ h\ \times$ 3

$= \frac{2}{3}\ \pi\ r^2\ h.$

The student should remember that the curved surface of the base of the sector, is equal to the curved surface of the circumscribing cylinder between the same zones, as explained in Chapter IX.

Volume of Segment of Sphere.

This case is more difficult to prove, but it may readily be determined by subtracting from the volume of the sector, the volume of the flat based cone it contains. The formula is,

Vol. $= \frac{\pi}{6}\ h^2\ (3\ D - 2\ h)$, where D is the diameter of the sphere,

and h is the height of the segment.

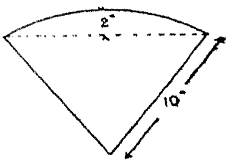
Volume of a Hollow Sphere is given by :

$$\text{Vol.} = \frac{\pi}{6}\ D^3 - \frac{\pi}{6}\ d^3 = \frac{\pi}{6}\ [D^3 - d^3]$$

or in terms of the radius $\frac{4}{3}\ \pi\ [R^3 - r^3]$

Example. Find the volume of a sphere of 10 inches radius.

Vol. $= \frac{4}{3}\ \pi\ r^3 = \frac{4}{3} \times \frac{22}{7} \times 1000 = 4190{\cdot}47$ cu. ins. Ans.



Find the volume of a sector of this sphere, if the height of the segment is 2 inches.

Area of curved base $= 2\ \pi\ r\ h = 2 \times 2 \times \frac{22}{7} \times 10 = \frac{880}{7}$ sq. inches.

Vol. of sector $= \frac{880}{7} \times \frac{10}{3} = 419{\cdot}04$ cubic ins. Ans.

Example. Find the volume of the segment of this sphere. The height of the segment is 2 inches.

$$\text{Vol.} = \frac{\pi}{6} \qquad\qquad = \frac{\;}{6} \times 4\ [3 \times 20 - 2 \times 2]$$

$$\text{Vol.} = \frac{88}{42} \times (60 - 4) = \frac{44}{\cancel{21}_{\,3}} \times \overset{8}{\cancel{56}} = \frac{352}{3}$$

Vol. $= 117\frac{1}{3}$ cubic inches. Ans.

Example. Find the volume of a hollow sphere 10 inches outer, and 8 inches inner diameter. Find its weight if one cubic inch weighs 0·26 lb.

$$\text{Volume} = \frac{\pi}{6}\ [10^3 - 8^3] = \tfrac{22}{42}\ [1000 - 512].$$

$$= \tfrac{22}{42} \times 488 = 255{\cdot}62 \text{ cu. ins.} \quad \text{Ans.}$$

$$\text{Weight} = 255{\cdot}62 \times 0{\cdot}26 = 66{\cdot}46 \text{ lb.} \quad \text{Ans.}$$

The student should notice that in all these cases of formulæ for volumes, the units must be $(\text{ins.})^3$ or $(\text{feet})^3$. Take the

formula, Volume $= \frac{\;}{6}\ h^2\ [3\ D - 2\ h]$; now D and h must either

be in feet or inches, say inches. Then in the bracket, when we subtract 2 h from 3 D, we get inches. Then multiplying inches by h^2, we get inches $\times$ $(\text{inches})^2 = (\text{inches})^3$. The constants do not affect the *order* of the unit, but only the *value* of the answer.

Example. A hollow sphere made of cast iron weighs 20 kilograms, and its shell is $\frac{3}{4}$ inch thick. Find the diameters of the sphere, if one cubic inch weighs 0·26 lb.

20 kg. $= 20 \times 2{\cdot}2 = 44$ lb.

$$\text{Volume of cast iron} = \frac{44}{0{\cdot}26} = 169{\cdot}2 \text{ cu. inches.}$$

Let r = outer radius of sphere, then $r - \frac{3}{4}$ = inner radius.

$$\text{Volume} = \tfrac{4}{3}\ \pi\ [r^3 - (r - \tfrac{3}{4})^3] = 169{\cdot}2.$$

Now the term $(r - \frac{3}{4})^3$ must be multiplied out,

$(r - \frac{3}{4})^2 = r^2 - \frac{3}{2}\, r + \frac{9}{16}$.

$(r - \frac{3}{4})^3 = (r^2 - \frac{3}{2}\, r + \frac{9}{16}) \times (r - \frac{3}{4})$, multiply out by the usual method.

$(r - \frac{3}{4})^3 = r^3 - \frac{9}{4}\, r^2 + \frac{27}{16}\, r - \frac{27}{64}$

$169{\cdot}2 = \frac{4}{3}\,\pi\,[r^3 - (r^3 - \frac{9}{4}\, r^2 + \frac{27}{16}\, r - \frac{27}{64})\,]$ take the inner brackets away and change all the signs in these brackets.

$169{\cdot}2 \qquad\qquad - r^3 + \frac{9}{4}\, r^2 - \frac{27}{16}\, r + \frac{27}{64}]$, cancel

then, $\dfrac{3}{4\,\pi} \times 169{\cdot}2 = \frac{9}{4}\, r^2 - \frac{27}{16}\, r + \frac{27}{64}$,

$$\frac{9}{4}\, r^2 - \frac{27}{16}\, r = \frac{3}{4\,\pi} \times 169{\cdot}2 - \frac{27}{64} = 39{\cdot}95$$

$$- \frac{3}{4}\, r = \frac{39{\cdot}95 \times 4}{\qquad}$$, by multiplying every term by $\frac{4}{9}$,

Completing the squares and solving as usual :-

$r - \frac{3}{8} = \pm$

$r = \pm\ 4{\cdot}23 + 0{\cdot}375$

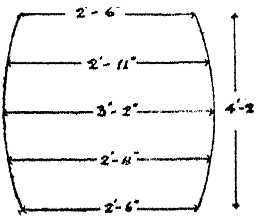
$r = 4{\cdot}605$, the minus value being impossible.

Outer diameter 9·21 ins. ; inner diameter 7·71 inches. Ans.

Volume of Irregular Solids.

If the sectional area of a solid varies, as in the case of a barrel or of a bunker, we apply Simpson's rule to find the volume. If a, a_1, a_2, a_3, a_4, etc., are the areas at equal distances h apart, then :—

$$\text{Volume} = \frac{h}{3}\,[a + 4a_1 + 2a_2 + 4a_3 + a_4]$$



An odd number of areas must be given.

Examples. A barrel is 2 feet 6 inches diameter at the ends, 3 feet 2 inches at mid-length, and 2 feet 11 inches at quarter length. It is 4 feet 2 inches long. Find its capacity in gallons.

h, **the common interval** **inches.**

Volume

$$\frac{50}{4 \times 3} \times \tfrac{11}{14}\,[30^2 + 35^2 \times 4 + 38^2 \times 2 + 35^2 \times 4 + 30^2]$$

cubic inches

$$\frac{550}{12 \times 14} \times \frac{[900 + 4900 + 2888 + 4900 + 900]}{1728} \text{ cu. ft.}$$

$$\frac{550 \times 14488}{12 \times 14 \times 1728} \times 6{\cdot}25 = 171{\cdot}5 \text{ gallons. Ans.}$$

Example. A'thwartship bunker is 48 feet long. The width at the top is 42 feet 4 inches, at $\frac{3}{4}$ height 41 feet 5 inches, at $\frac{1}{2}$ height 40 feet and at $\frac{1}{4}$ height 36 feet, and at bottom 25 feet. Find how many tons of coal the bunker will hold at 45 cubic feet per ton. The depth of the bunker is 26 feet.

Volume = mean area × length.

$$= \tfrac{26}{4} \times \tfrac{1}{3}\,[42\tfrac{1}{3} + 4 \times 41\tfrac{5}{12} + 40 \times 2 + 36 \times 4 + 25] \times 48.$$

$$= \tfrac{26}{12} \times 457 \times 48 \text{ cubic feet.}$$

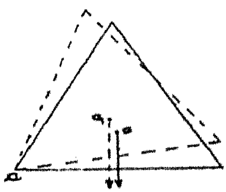
$$\text{Weight} = \frac{26 \times 457 \times 48}{12 \times 45} = 1056{\cdot}17 \text{ tons. Ans.}$$

Centre of Gravity.

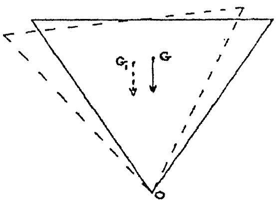
The Centre of Gravity of a solid is that position in the body through which we may regard the whole weight of the body as acting. It is the centre of mass; and if the whole mass of the body could be concentrated in one single heavy particle, this particle would lie at the centre of gravity of the body.

Properties of the Centre of Gravity.

If a body at rest is displaced through a small angle by an external force, and when released returns to its original position, it is said to be in *stable equilibrium*. If the cone as shown, when tilted about a point through a small angle and then released, at once returns to its original position, it is said to be in stable equilibrium. Another way of stating this is :—If when tilting a

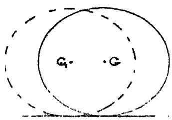


body slightly, its centre of gravity is raised, then when released it will return to its original position. Yet another statement is possible :—If when a body is tilted, a vertical line through its centre of gravity falls *inside* the point of contact, the body is stable and will return to its original position when released. The point of contact for the cone when tilted is *a* ; in tilting, G is raised to G_1 and a line through G_1 drawn vertically falls inside of *a*. This means that when we release the cone, it has a restoring moment about *a*, which brings it back to its original position.

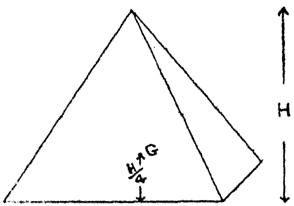


If now a cone is carefully balanced about its apex, and then displaced through a small angle, its centre of gravity is lowered and no recovery is possible. The cone falls completely over. We say that when standing in its original position it is in *unstable equilibrium*, because when displaced slightly it cannot return to its original position. Note that the body has, when displaced slightly, an over turning moment about *o*, the apex.

If a sphere is at rest upon a table, when we displace it slightly, it will stay in any position indifferently. Because all its radii are equal, no matter how we roll or displace it, its centre of gravity is always at the same vertical height above the table, and cannot be either raised or lowered by rolling the sphere. The sphere is said to be in *neutral equilibrium*.



Centres of Gravity.

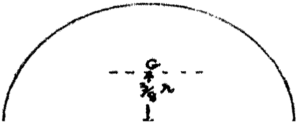


The centre of gravity of a cube is at the intersection of its diagonals.

The centre of gravity of a rectangle is at the intersection of its diagonals.

The centre of gravity of a cone or pyramid is at one quarter of the vertical height above the base, or

$$G \text{ is at } \frac{H}{4} \text{ above the base.}$$

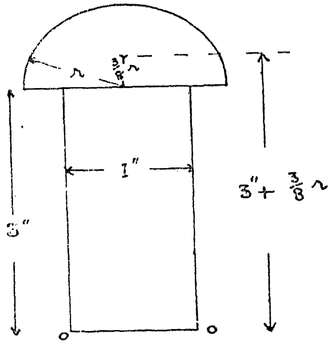


For a cylinder, the C.G. lies on the axis at the middle of the length.

For a hemisphere, G is at $\frac{3}{8}$ r from the diameter.

Example. A cone is 16 inches high and its base is 12 inches diameter, find the position of its C.G. The diameter is not necessary here.

C.G. is at $\dfrac{H}{4}$ or $\frac{16}{4}$ = 4 inches above the base. Ans.



Example. A rivet 1 inch in diameter has a hemispherical head $1\frac{1}{2}$ inches diameter. Find the point in its length about which the rivet will balance.

This means find the position of the C.G. Take moments about $o\ o$, the end of the rivet.

$$\text{C.G. above } o\ o = \frac{\text{Sum of moments of weights or volumes about } o\ o}{\text{Sum of weights or volumes}}$$

$$\text{C.G. above } o\ o = \frac{[\pi\,(\frac{1}{2})^2 \times 3 \times 1\frac{1}{2}] + [\frac{2}{3}\,\pi\,(\frac{3}{4})^3 \times (3 + \frac{3}{8} \times \frac{3}{4})]}{(\pi\,(\frac{1}{2})^2 \times 3) + \frac{2}{3}\,\pi\,(\frac{3}{4})^3}$$

$$\text{C.G. above } o\ o = \frac{(\frac{1}{4} \times 3 \times \frac{3}{2}) + (\frac{2}{3} \times \frac{27}{64} \times \frac{105}{32})}{(\frac{1}{4} \times 3) + (\frac{2}{3} \times} \qquad \frac{699}{352}$$

= 1·986 inches. Ans.

Note in this case that $\pi\,(\frac{1}{2})^2 \times 3$ is the volume of the cylindrical part of the rivet, and its C.G. is at $1\frac{1}{2}$ inches from $o\ o$. The volume of the head is $\frac{2}{3}\,\pi\,r^3$, and its C.G. from $o\ o$ is $(3 + \frac{3}{8}\,r)$ inches. The student is warned here to remember that he is now dealing with *volumes* and *not areas*. He must not, for instance, confuse the C.G. or centroid of a semicircular area which is at 0·424 r from its diameter, with the C.G. of a hemispherical solid, which is at $\frac{3}{8}\,r$ from its diameter.

Example. A fore peak bulkhead is 10 feet high. The semi-ordinates of width measured at regular intervals of $2\frac{1}{2}$ feet starting from the top are, 5·5, 5·1, 4·4, 3, and 0 feet respectively. Calculate its area, and the position of its centre of gravity from the top.

By Simpson's Rule :—

Semi-Ordinates	Simpson's Multipliers	Functions of Ordinates	Distances of Ordinates from top	Functions for Moments
5·5	1	5·5	0	0
5·1	4	20·4	$2\frac{1}{2} \times 1$	$2\frac{1}{2} \times 20{\cdot}4$
4·4	2	8·8	$2\frac{1}{2} \times 2$	$2\frac{1}{2} \times 17{\cdot}6$
3	4	12	$2\frac{1}{2} \times 3$	$2\frac{1}{2} \times 36$
0	1	0	$2\frac{1}{2} \times 4$	$2\frac{1}{2} \times 0$

Sum 7 Sum $= 2\frac{1}{2} \times 74$

Half area of bulkhead $\times\ 46{\cdot}7 = 38{\cdot}91$ sq. ft.

$\therefore$ Area $\times\ 2 = 77{\cdot}82$ sq. ft. Ans.

Moment of half area $\times\ 2{\cdot}5 \times 74$
3

$$\text{C.G} \quad \frac{\text{Moment of area}}{\text{Area}}$$

$$= \frac{2 \times 2{\cdot}5 \times 2{\cdot}5 \times 74 \times 3}{3 \times 2 \times 46{\cdot}7 \times 2{\cdot}5}$$

$= 3{\cdot}961$ feet from the top. Ans.

The student will find it a great help to tabulate as shown above, when moments are to be calculated by Simpson's rule.

The volume of similar figures vary as the cube of their corresponding dimensions.

Example. A ship 410 feet long has a wetted surface of 34,000 square feet and a displacement of 10,300 tons. A similar ship is 350 feet long. Find its wetted surface and displacement.

The area of similar figures vary as the square of their corresponding dimensions.

$\therefore$ Wetted surface of second ship $= 34{,}000 \times \dfrac{350^2}{410^2}$
$= 24{,}800$ sq. feet. Ans.

The displacement varies as the volume.

$\therefore$ Displacement of second ship $= 10{,}300 \times \dfrac{350^3}{410^3}$
$= 6{,}400$ tons. Ans.

TEST EXAMPLES X.

1. A thrust shaft is $12\frac{1}{2}$ inches diameter. The couplings are 25 inches diameter and $3\frac{1}{2}$ inches thick. Find the weight of the shaft if there are 6 collars 25 inches diameter and 2 inches thick, the overall length being 8 feet. One cubic foot of the material weighs 490 lb.

2·377 tons.

2. A circular oil cup is 2 inches diameter inside and the oil tube is $\frac{7}{16}$ inch diameter outside. When $\frac{1}{4}$ pint of oil is poured in, what is the depth of oil in the cup ?

2·89 inches. Ans.

3. The steam space of a boiler at the water level is 14 feet 6 inches broad, and the length is 16 feet. The distance from the centre of the water level to the top of the boiler is 4 feet 11 inches. The vertical height from the water level to the shell at the quarter breadths is 4 feet. Find the volume of the steam space in cubic feet.

808·75 cubic feet. Ans.

4. A brass liner for a propeller shaft is 16 inches diameter outside and 1 inch thick, its length being 12 feet. Find its weight if one cubic inch weighs 0·3 lb.

0·909 ton. Ans.

5. A hollow shaft is 20 feet long. Its outer diameter is 16 inches and its inner diameter 8 inches. The couplings are 32 inches diameter and 4 inches thick. Find the weight at 490 lb. to the cubic foot.

5·191 tons. Ans.

6. The steam spaces of the boilers contain 1,850 cubic feet of steam. The H.P. cylinder is 30 inches diameter, the stroke being 60 inches, cut-off taking place at 0·6 of the stroke. For how many strokes of the engine does the steam space contain steam ?

125·5 strokes. Ans.

7. A rectangular main bearing brass fits into a rectangular seat. The distance from the cap to the bottom of the recess is 13 inches and the width of the seat is 12 inches. The diameter of the shaft is 10 inches, and the overall length of the brass is $12\frac{1}{4}$ inches. The flanges are $\frac{5}{8}$ inch thick, 14 inches deep and 13 inches wide. Find the weight of the two half brasses, at 0·3 lb. to the cubic inch.

294·3 lb.

8. How many feet of wire $\frac{1}{16}$ inch in diameter can be made from a bar of copper 2 inches diameter and 4 inches long ?

$341\frac{1}{3}$ feet. Ans.

9. Find the weight of a circular anchor ring of 6 inches mean radius, the diameter of the bar from which it is made being 2 inches. One cubic inch weighs 0·28 lb.

33·18 lb. Ans.

10. A solid cast iron ball weighs 156 lb. Find its diameter if one cubic inch weighs 0·26 lb.

10·46 inches. Ans.

11. A hollow sphere is 6 inches external radius, and 5 inches internal radius. Find its weight if one cubic inch weighs 0·26 lb.

99·14 lb. Ans.

12. A flywheel rim is 10 feet external and 9 feet internal diameter, and 1 foot thick. The boss is 12 inches diameter, 12 inches long, the hole being 6 inches diameter. Find the weight of the wheel at 450 lb. to the cubic foot.

3·117 tons. Ans.

13. A frustum of a cone is 4 feet high. The radius of its base is $2\frac{1}{2}$ feet, the radius of the top being $1\frac{3}{4}$ feet. Find its volume in cubic feet.

57·35 cu. ft. Ans.

14. A cylindrical vessel 12 feet diameter and 8 feet long, floats in sea water at a draught of 5 feet with its axis horizontal. Find its weight if 35 cubic feet of sea water equal 1 ton.

10·199 tons. Ans.

15. A hollow sphere is 12 inches diameter outside. It is made of cast iron and weighs 99·15 lb. Find its internal diameter if a cubic inch of cast iron weighs 0·26 lb.

10 inches. Ans.

16. A rivet is $1\frac{1}{2}$ inches diameter and 6 inches long. The head is a hemisphere 2 inches diameter. Find the position of the centre of gravity.

3·557 inches from point of rivet. Ans.

17. A frustum of a cone has diameters 8 and 12 inches, its vertical height being 6 inches. How far is the centre of gravity above the base ?

2·61 inches above base. Ans.

18. A triangular prism 10 inches long is made of brass weighing 0·3 lb. per cubic inch. Find the weight of the largest cylinder which can be turned from the prism, (*a*) if the section of the prism is an equilateral triangle of side 5 inches long, (*b*) if the section is an isosceles triangle of sides 6 inches, 6 inches and 4 inches, (*c*) if the section is a triangle having sides 3 inches, 5 inches and 6 inches.

19·63 lb.; 18·85 lb.; 10·76 lb. Ans.

19. A conical buoy weighs 2 tons, and it floats in salt water with its apex upwards. The base of the buoy is 10 feet diameter and the draught is 1 foot. Find the diameter of the buoy at the water line.

8·86 feet. Ans.

20. A solid cone of cast iron weighs 500 lb., and its vertical height is 25 inches. Find the diameter of its base. One cubic foot weighs 450 lb.

17·124 ins. Ans.

21. A tank is 30 centimetres broad, 75 centimetres long and 1·4 metres in depth. Find its capacity in gallons and litres.

315 Litres, 69·3 Gallons. Ans.

22. A Diesel engine has a stroke of 1·27 metres, and the cylinder is 66 centimetres diameter. Find the volume swept out by the piston in cubic feet.

15·36 cu. ft. Ans

23. A hexagonal bar has sides of 26 centimetres, its length being 20 feet. Find its weight in kilograms and pounds if one cubic inch weighs 0·28 lb.

183 lb., 83·18 Kilos. Ans.

24. A cylindrical tank holds 75 gallons of oil. Its height is 4 metres, find its diameter in centimetres and inches.

32·94 centimetres, 12·97 inches. Ans.

25. The chord of the segment of a circle is 4 inches and the greatest height of the segment is 1 inch. Determine the radius of the circle of which this segment is part. Divide the segment into six equal parts by lines parallel to the chord, measure these lines and by Simpson's rule determine the position of the centre of gravity.

0·401 inch from the chord. Ans.

CHAPTER XI.

FORCES AND OTHER VECTOR QUANTITIES.

Scalar Quantities are those which have magnitude only. The mass of a body is a scalar quantity, because the mass of a body is completely specified when its magnitude in pounds is known. Other examples are volume, density and time; all these are scalar quantities.

There are, however, many quantities in Mechanics which cannot be so simply expressed. When we define the speed of an object, it may be necessary to know not only the velocity in feet per second, but also the direction in which the object moves. Thus two properties, one of magnitude and one of direction are necessary to completely specify the motion of the object.

If two properties are required to completely define a quantity then this is called a Vector quantity.

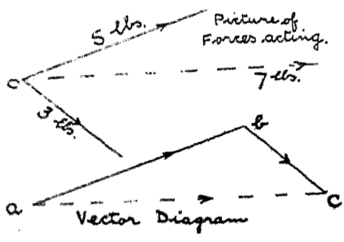
Vector Quantities.

Force is a Vector quantity, because we must know the magnitude of the force in pounds and the direction in which it acts; we also need to know the position at which the force acts. Displacement is also a vector quantity, because we must know the direction in which a body is displaced, as well as the distance through which it has been moved.

The term displacement, as used here, refers to the movement of a body from one place to another, and not to the displacement of a floating body such as a ship.

Velocity and Acceleration are further examples of vector quantities.

Diagrammatic Representation of Forces.

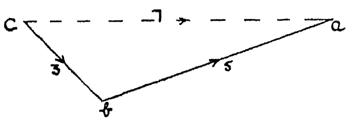


Let a force of 5 lb. act through the point *o*. If a line be drawn parallel to this force, the length of the line, drawn to some scale of force, will represent the magnitude of the force, and the arrow head will represent the direction in which this force acts. Draw the line *a b* parallel to the force of 5 lb., and denote the direction by the arrow head. This line *a b* is called a Vector. Let

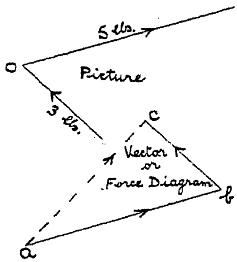
another force of 3 lb. act through o in the direction shown. From b draw another vector, parallel to the 3 lb. force, making $b\,c$ the correct length to the same scale as $a\,b$. Join $a\,c$ by a dotted line. Then the length of the line $a\,c$ to scale, is the magnitude of a single force, which acting alone through o, will have the same effect as the two original forces. The direction of this single force is given by the direction of the arrow head on the dotted line. Now draw through o a line parallel to $a\,c$, and of correct length to represent the magnitude of this single force. Suppose $a\,c$ to be 7 lb. This force is called the resultant force. The resultant force is that force, which, acting alone through a point, has the same effect as the system of forces acting through the same point, which it replaces.

Note that in drawing the vector diagram, the arrows go from a to b, and from b to c, as if proceeding continuously from a to c along a line bent at b. The forces may be taken in any order, but the direction of the arrows must be preserved. For instance, drawing the vector or force diagram again, draw the vector representing the 3 lb. force first, i.e., draw $c\,b$ parallel to the 3 lb. force. Now to draw the vector for the 5 lb. force :—

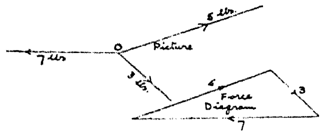
From which end of $c\,b$ shall it be drawn? It can only be drawn from b if we are to preserve the direction of the arrows continuously, since if we draw it from c, the arrows will be opposed in direction. Note that $a\,c$ is in the same direction as when the 5 lb. force was taken first in order.



Take now the same two forces acting at the point o, but suppose the 3 lb. force to act in the opposite direction as shown. Set out $a\,b$ parallel to the 5 lb. force and put the arrow on the vector in the same direction as it is shown in the picture. Then from b draw $b\,c$ parallel to the 3 lb. force, so that the arrows are continuous in direction. Join $a\,c$ with a dotted line. Then $a\,c$ is the magnitude and direction of the single force acting through o which would have the same effect as the two forces of 3 and 5 lb.

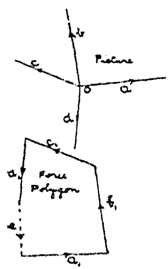


Now to return to the original case. The 7 lb. force, shown on the force diagram is the resultant of the 5 and 3 lb. forces. Further, if the force of 7 lb. be applied at the point o, but in the opposite direction to the arrow on the dotted line, it will



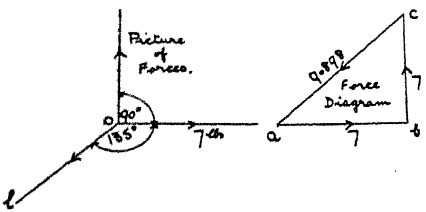
keep the point o in equilibrium, and will just counteract the effect of the 5 and 3 lb. forces. This force of 7 lb. is now called the Equilibrant. The picture now shows the point o in equilibrium under the action of three forces. This leads to the important statement :—If a point in a body is in equilibrium under the action of three forces, these forces may be represented by the three sides of a triangle, and the direction of the arrow heads will be continuous round the sides. This triangle is a closed figure. If we are given three forces acting at a point, and after drawing the force diagram we find that it is not a closed figure, then the point is not in equilibrium under the action of the three forces, and there will be a resultant force acting.

Polygon of Forces.



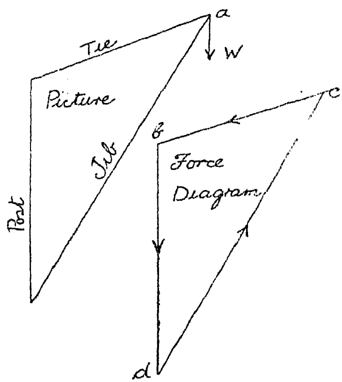
The law of vector addition is applicable to any number of forces acting through a point. Let forces a, b, c and d act through o. Draw a_1 parallel to a, putting the arrow on to show the direction ; to a_1 add b_1 parallel to b, being careful to draw it to the same scale ; draw c_1 and d_1 in the same manner, being careful to preserve the direction of the arrows. Now close the figure by the dotted line e, and e is the force to add to the system at the point o to keep the point in equilibrium. The length of e to scale denotes the magnitude of the force and its arrow the direction. Note if we reverse the direction of e, it becomes the resultant force, and if it acts alone, it will have the same effect as the forces a, b, c, d acting together. Graphical methods, i.e., actually drawing the diagrams to scale, give good results ; or the force e may be calculated.

Example. Three forces act at the point o, which is in equilibrium. The horizontal force is 7 lb. Find the magnitude of the other two.



Draw $a\ b$ parallel to 7 lb. force. Draw $b\ c$ parallel to vertical force. Through a draw $a\ c$ parallel to $o\ l$, then since the angle $c\ a\ b$ is 45°, $a\ b = b\ c = 7$ lb., and $a\ c = 7 \times 1{\cdot}414 = 9{\cdot}898$ lb. The other two forces are 7 lb. and 9·898 lb. Ans.

Forces in the Members of a Jib Crane.



Three forces meet at the point a,

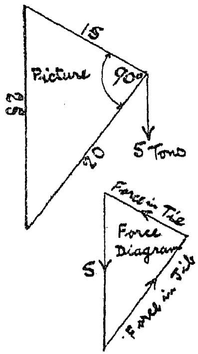
1. The downward weight W;
2. The pull in the Tie;
3. The thrust in the Jib;

therefore as the point a is in equilibrium, the forces may be represented by a triangle.

Draw $b\ d$ vertically downward to scale to represent the known weight W. Through b draw $b\ c$ parallel to the Tie. Through d draw $d\ c$ parallel to the Jib. Then to scale, $b\ c$ is the force in the Tie, and to scale $d\ c$ is the force in the Jib.

Note that in this case, the force diagram is similar to the picture of the crane. The tie is in tension, and the jib in compression.

Example. A derrick is held up by a topping lift or tie. From the foot of the derrick to where the tie is fastened to the mast is a distance of 25 feet, and the angle between the tie and the derrick is 90°. Find the forces in the tie and the derrick when a load of 5 tons is being lifted. The tie is 15 feet long.



The Jib is $\sqrt{25^2 - 15^2} = \sqrt{625 - 225}$ $= \sqrt{400} = 20$ feet long.

Draw the force diagram as shown. Now the two triangles representing the picture and the force or vector diagram are similar.

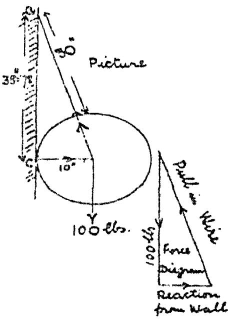
Force in Tie $= \quad \times 5 = 3$ tons. Ans.

Force in Jib $= \frac{20}{25} \times 5 = 4$ tons. Ans.

This is the same as writing Force in Tie

Force in Jib by similar Triangles.

and $\frac{20}{25} =$



Example. A sphere of 20 inches diameter weighs 100 lb. It is supported by a wire 30 inches long, and rests against the wall as shown. Find the pull in the string and the reaction on the wall.

Calculate first the length $a\ c$, this is found to be 38·72 inches.

Three forces meet at the centre of the sphere.

1. The downward weight of 100 lb.
2. The outward reaction from the wall.
3. The pull in the wire.

Draw the force diagram as shown.

$$\text{Then } \frac{\text{pull in wire}}{100} \quad \frac{40}{38{\cdot}72}, \text{ pull in wire} = \frac{4000}{38{\cdot}72} = 103{\cdot}3 \text{ lb.}$$

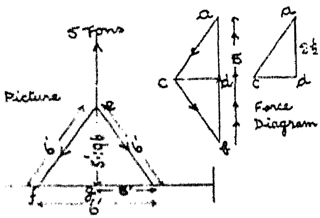
$$\text{and } \frac{\text{reaction from wall}}{100} \quad \frac{10}{38{\cdot}72}$$

$$\text{Reaction from wall} = \frac{1000}{38{\cdot}72} \quad 25{\cdot}82 \text{ lb.}$$

Pull in s 103·3 lb., Reaction from wall = 25·82 lb. Ans.

Example. A shaft weighing 5 tons is lifted by a chain sling, each arm of which is 6 feet long. The sling is fastened to the shaft at points 6 feet apart. Find the force in the arms of the sling.

Note that the pull in the single chain is upwards.



Draw $a\ b$ to represent 5 tons.

Through a draw $a\ c$ parallel to one arm of sling, and through b draw $b\ c$ parallel to the other arm.

Then $a\ c$ and $c\ b$ are equal, and $a\ c$ and $c\ b$ are the forces in the two arms respectively. $e\ g = \sqrt{6^2 - 3^2} = 5{\cdot}196$ feet.

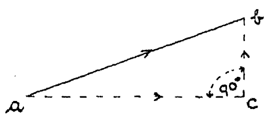
In the force diagram draw $c\ d$ horizontal, dividing $a\ b$ into two equal parts each of $2\frac{1}{2}$ tons, and drawing $a\ c\ d$ as a separate triangle we see that it is similar to $e\ f\ g$.

$$\text{Then } \frac{a\ c}{2\frac{1}{2}} = \frac{6}{5{\cdot}196} \quad \text{or } a\ c = \frac{6 \times 2\frac{1}{2}}{5{\cdot}196} \quad \frac{15}{5{\cdot}196}$$

$a\ c = 2{\cdot}886$ tons, and this is the force in each arm. Ans.

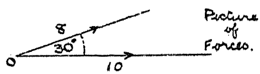
Note that this is more than the weight of the shaft. The length $c\ d$ represents the force needed to keep the ends of the chains apart.

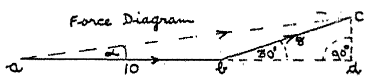
Resolution of Forces.



Let $a\ b$ be a force acting in the direction shown. Now this force, if applied to a body would tend to lift it vertically and to move it horizontally. Draw $a\ c$ horizontally and $c\ b$ vertically, then the angle $a\ c\ b$ is 90°. Then $a\ c$ and $c\ b$ are the two forces which could replace the force $a\ b$, and they are called the components of $a\ b$. Further, these forces $a\ c$ and $c\ b$, because they are at right angles to each other are called Rectangular Components. Note carefully the direction of the arrows. A force has no component at right angles to itself. For instance, a heavy force may be acting down upon a table, but it has no tendency to move the table horizontally, because a horizontal movement of the table would be at right angles to the line of action of the downward force.

Example. A force of 10 lb. acts horizontally through a point. Another force of 8 lb. acts through the same point as shown, the directions being given by the arrows. Find the magnitude and direction of the resultant force.





Draw $a\ b$ parallel to the 10 lb. force, and from b draw $b\ c$ parallel to the 8 lb. force. Then $a\ c$ is the resultant. Drop a perpendicular from c to $a\ b$ produced to d.

Now $b\ d = 8$ Cos. $30° = 8 \times 0{\cdot}866 = 6{\cdot}928.$

and $c\ d = 8$ Sin. $30° = 8 \times 0{\cdot}5 = 4.$

$\therefore$ $a\,d = (10 + 6{\cdot}928)$, and $c\,d = 4$.

$a\,c = \sqrt{(16{\cdot}928)^2 + 4^2}$

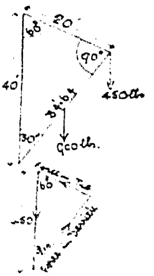
$= \sqrt{286{\cdot}7 + 16} = \sqrt{302{\cdot}7}$

$a\,c = 17{\cdot}4$ lb. Ans.

For the direction of $a\,c$,

$$\text{Sin } \alpha = \frac{d\,c}{a\,c} = \frac{4}{17{\cdot}4} = 0{\cdot}2299$$

and $\qquad$ $13^\circ\ 18'$ to the 10 lb. force.



Example. A derrick 34·64 feet long is supported by a tie 20 feet long, which makes a right angle with it. The weight of the derrick is 900 lb. Find the forces in both tie and derrick. (Note the given sides are 1 : 1·732, and this must be a triangle having sides 1 : 2 : $\sqrt{3}$).

The weight of the derrick may be taken as acting vertically downwards at the middle of its length.

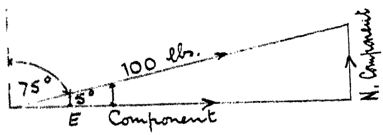
Of this 900 lb. we may regard half, or 450 lb. as acting downwards from the derrick head, as shown by the dotted line.

Draw the force diagram as before, but using the 450 lb. weight only.

Force in Tie $= \frac{450}{2} = 225$ lb. Ans.

e in Jib $= 225\sqrt{3} = 225 \times 1{\cdot}732 = 389{\cdot}7$ lb. Ans.

ple. A force of 100 lb. is acting in a direction of 75° East of North. Find its North and East components.

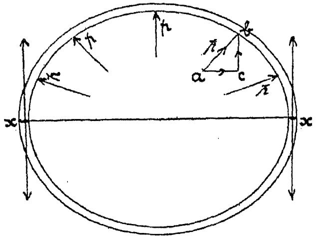


North Component $= 100 \times$ Sin. $15° = 100 \times 0{\cdot}2588$ $= 25{\cdot}88$ lb.

East Component $= 100 \times$ Cos. $15° = 100 \times 0{\cdot}9659$ $= 96{\cdot}59$ lb.

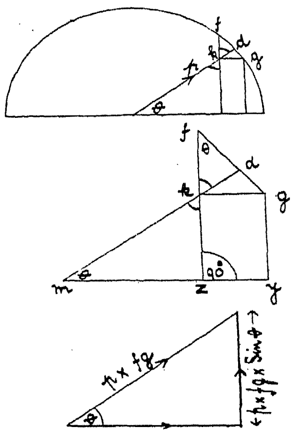
N. Component $= 25{\cdot}88$ lb., E. Component $= 96{\cdot}59$ lb.

Ans.



An example of this resolution of forces is its application to finding the strength of a thin hoop (such as a thin cylinder or a boiler shell) to resist rupture when exposed to internal fluid pressure.

The internal pressure acts radially all round the section, i.e., normally to the curved surface, for the radius is at right angles to the very short piece of the curve near it. Resolving one radial line of pressure (a b), only its vertical component b c is effective in tending to cause rupture as shown by the arrows at x and x.



Consider a very short arc of length f g.

Force acting radially on f g $= p \times f$ g.

Now if f g is an exceedingly small length, we may take it as being a straight line, and to make the angles more obvious we draw a great magnification of this line as shown. Draw k g parallel to the diameter.

Now triangle m k z is similar to triangle k f d, the angle k f d being equal to θ. Draw the force diagram, the radial force being $p \times f$ g, the resolved vertical component is $p \times f$ $g \times$ Sin. θ,

$$\text{but Sin. } \theta = \frac{k\ g}{f\ g}$$

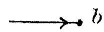
$$\therefore p \times f\,g \times \text{Sin. } \theta = p \times f\,g \times \frac{k\ g}{f\ g} = p \times k\ g, \text{ since } f\ g \text{ cancels.}$$

That is to say, the vertical force is the pressure multiplied by the projected length, since $k\ g$ is the projected length of the arc $f\ g$, and this will be true for any part of the circle, or for the whole semicircular arc. Therefore total force tending to cause rupture $= p \times$ diameter, the diameter representing the projected length of the semicircular arc.

Therefore no matter what shape we make the top of a piston, the total vertical force on the crosshead is the pressure multiplied by the cross sectional area of the cylinder.

Struts and Ties.

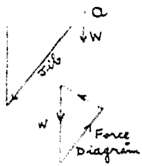
If a bar or rod is subjected to a compressive stress, the bar is said to be a strut. Now a strut pushes at its end joints.



Let $a\ b$ be a strut. Then if it is in compression it tends to push the joints or pins a and b further apart, and the arrows show that the direction of the force is *towards* each pin.

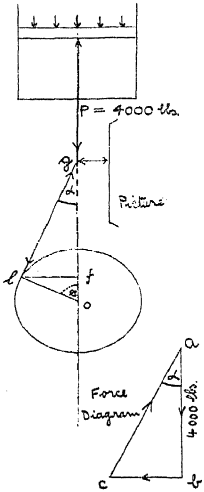
If a bar is in tension, it is called a tie. A tie pulls at its joints or pins.

If $c\ d$ is a bar in tension then it tends to pull c and d nearer together, and the arrows show the direction of the force pulling *away from* the pins.



Thus the problems solved, in the cases of jib cranes, the arrows show that the jib is a strut, and that the topping lift is a tie. This is a simple case and we know already by experience that the tie is in tension and the jib in compression. But if we did not know we may proceed as follows. At the pin a, the direction of *one* force (W) is known, for it is always downwards. Also for equilibrium, the arrows must go continuously round the force diagram. Transfer the arrows from the force diagram to the Jib and Tie, as nearly as possible to the point of pin a. We find that the arrow head on the Jib points towards a, the Jib is therefore a strut, and in compression. The arrow head on the Tie points away from a, and this is a Tie and in tension. It must be particularly noted that the arrows must be transferred from the force diagram to the corresponding bar or member of the crane, and placed as closely as possible to the pin or point for which the force diagram shows the forces. For instance, the force diagram here has been drawn for the three forces meeting at a, and it represents those forces only. Note that once we get the arrows on the bars close to a, we know how to place the arrows at the other end of the bars. The Jib here pushes at both its end pins, and the Tie pulls at both its end pins.

Steam Engine Mechanism.



The crosshead pin g is in equilibrium under the action of three forces :—

1. The compressive force in the piston rod pushing at the pin,

2. The compressive force in the connecting rod pushing at the pin.

3 The force acting outwards from the guide shoe.

A triangle will represent these forces. The magnitude of the force P is known, and its direction is known. Draw a vector $a\ b$ to represent P, the total force on the piston. From b draw $b\ c$, parallel to the force acting from the guide, and from a draw a line parallel to the connecting rod, because the force in this rod acts through the longitudinal axis of the rod. Transfer the arrows from the diagram to the picture, putting them close up to the point g. The connecting rod thrusts at the top end pin and also at the crank pin. The guide shoe is in compression, and so is the piston rod.

Draw $l\ f$ at right angles to the line of stroke. Then we have completed a triangle $g\ l\ f$ which is similar to the triangle $a\ c\ b$. It is usual in ordinary practice to make the connecting rod four times the length of the crank. Suppose that $o\ l$ is 2 feet long and $l\ g$ is 8 feet long, or calling $o\ l$ 1, then $l\ g$ is 4.

$$\text{Sin } \alpha = \frac{l\ f}{l\ g}, \qquad \text{Sin. } \theta = \frac{l}{o\ l}$$

$$\text{or Sin. } \alpha = \frac{l\ f}{4}, \qquad \text{Sin. } \theta = \frac{l\ f}{1}, \text{ it follows that Sin. } \alpha \text{ is}$$

$\frac{1}{4}$ of the Sine of θ, because in the above two fractions, $l\ f$ the numerator is the same, but the denominators are as 4 : 1.

For any position of the crank, if θ is known, find its Sine from the tables, divide the Sine by the ratio of rod to crank, in this case 4, and the result is the Sine of α. Suppose in the case above that $\theta = 50°$. Then Sin. $50° = 0{\cdot}766$,

$$\text{therefore Sin. } \alpha = \frac{0{\cdot}766}{} = 0{\cdot}1915 \text{ and from the tables}$$

$11°2'$. Let $P = 4{,}000$ lb.

In the force diagram :—

$a\ c$ represents the force in the connecting rod.
$c\ b$,, ,, on the guide.
$a\ b$,, ,, on the piston (P).

$$\text{Cos.}\ \alpha = \frac{a\ b}{a\ c} \quad \therefore\ a\ c = \frac{a\ b}{\text{Cos.}\ \alpha} = \frac{4000}{0{\cdot}9815}$$

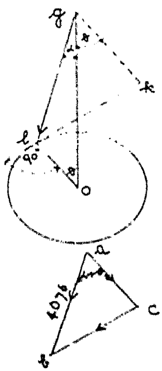
$= 4075$ lb. in rod. Ans.

$$c\ b = a\ b \times \text{Tan.}\ \alpha = 4000 \times 0{\cdot}195$$

$= 780$ lb. on guide. Ans.

When the piston is on either top or bottom centres, the force in the connecting rod is the same as the force on the piston; for all other positions between the dead centres, we see from the force diagram that the force in the connecting rod is greater than the force on the piston. This does not mean that more work is being done by the connecting rod than by the steam on the piston, for that is, of course, impossible. Consider how this force in the connecting rod is applied. During the first part of the down stoke, the force in the rod goes partly to put compression in the crank web, but mostly to turn the crank.

The force in the rod may be resolved into :—



1. A tangential force acting at right angles to the crank.

2. A force acting down the crank web, from the pin to the shaft centre.

For the position shown, draw $a\ b$ parallel to the rod. Resolving $a\ b$, draw $a\ c$ parallel to the crank, and $b\ c$ parallel to the tangent shown dotted at l.

Then $a\ c$ represents the compressive force in the crank, and $c\ b$ represents the actual force available to cause the turning moment about the centre of the shaft. If we draw $g\ k$ parallel to the crank, then angle $o\ g\ k$ = angle $g\ o\ l$ both marked θ, and as $a\ c$ and $a\ b$ are parallel to $g\ k$ and $g\ l$, angle $b\ a\ c = (\alpha + \theta)$. The two angles are known, or α may be found from θ as already shown.

Then $\text{Sin.}\ (\alpha + \theta) = \dfrac{b\ c}{a\ b}$, or $b\ c = a\ b \times \text{Sin.}\ (\alpha + \theta)$

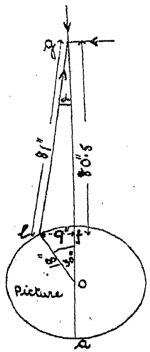
$(\alpha + \theta) = (50° + 11°2') = 61°2'$, and $b\ c = a\ b \times \text{Sin.}\ 61°2'$ $= 4076 \times 0{\cdot}8749$, or $b\ c = 3566$ lb.

And the turning moment on the shaft is $3566 \times 2 = 7132$ ft. lb.

We do not need the compression in the crank, but $a\ c$ can easily be found since $\text{Cos.}\ 61°\ 2' = \dfrac{a\ c}{a\ b}$

This condition of compression in the crank continues until the connecting rod is at right angles to the crank, then the whole force in the rod is turning the crank. After passing this position, most of the force in the rod turns the crank, and part goes to put the crank in tension, and the force in the rod may be resolved for any position of the crank. Be careful to note that the above force diagram has been drawn for the crank pin end of the rod. Another method of finding the twisting or turning moment on the shaft will be shown in the Chapter on Torsion.

Example. An engine has a cylinder 20 inches diameter, the crank is 18 inches long and the connecting rod is 81 inches long. Find the force in the connecting rod and on the guide when the crank is 30° from the top centre, and when the crank is at right angles to the line of stroke; the effective pressure on the piston for these positions being 200 lb. per sq. inch.



For 1st position, force on piston $= 20 \times 20 \times \frac{11}{14} \times 200 = 62{,}860$ lb., nearly.

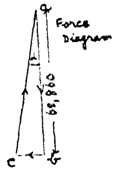
Draw $l\ f$ at right angles to $o\ g$. Then $l\ f = \frac{18}{2} = 9''$ since the angle is 30°.

$g\ f = \sqrt{81^2 - 9^2}$
$= \sqrt{6480} = 80{\cdot}5$ ins.

Draw the force diagram, making $a\ b = 62{,}860$ lb.

Draw $c\ b$ parallel to force on guide, and $a\ c$ parallel to rod.

Then triangle $g\ l\ f$ is similar to triangle $a\ c\ b$.



$$\frac{81}{80\cdot5} = \frac{a\ c}{62{,}860}$$

$$a\ c = \frac{81 \times 62{,}860}{80\cdot5}$$

$a\ c = 63{,}260$ lb. = force in connecting rod. **Ans.**

also, $\dfrac{9}{80\cdot5} = \dfrac{c\ b}{62{,}860}$, $c\ b = \dfrac{9 \times 62{,}860}{80\cdot5}$

$c\ b = 7028$ lb. on guide. Ans.

Or by trigonometry,

$$\text{Sin. } \alpha = \frac{\text{Sin } 30^\circ}{4\frac{1}{2}} = \frac{\frac{1}{2}}{\frac{9}{2}} = \tfrac{1}{9} = 0\cdot1111,$$

$\alpha = 6^\circ\ 23'$ from the tables.

Note here the ratio $\dfrac{\text{rod}}{\text{crank}} = \frac{81}{18} = 4\frac{1}{2}$

$$\text{Cos. } \alpha = \frac{a\ b}{a\ c},\quad a\ c = \frac{a\ b}{\text{Cos. } 6^\circ\ 23'} = \frac{62{,}860}{0\cdot9938}$$

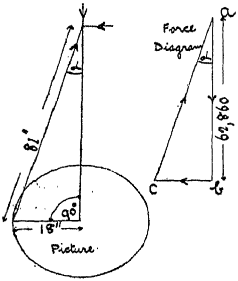
$= 63270$ lb. Ans.

$$\text{Tan. } \alpha = \frac{c\ b}{a\ b},\quad c\ b = a\ b \times \text{Tan.}$$

$= 62860 \times 0\cdot1119$

$c\ b = 7034$ lb. Ans.

Note the difference in the figures, 63270 and 63260 is only 10 lb. in more than 63 thousands, and this is exceedingly small. To get the answers the same we should have to use 7 figure logs. Also the difference in 7034 and 7028 is only 6 lb. and 6 in more than 7 thousands is a very small error.



For the 2nd position, piston load is the same $= 62{,}860$ lb.

Sine $\alpha = \frac{18}{81} = \frac{2}{9} = 0{\cdot}2222$,
$\alpha = 12° \ 50'$ from tables.

$a \ b$

$a \ c$

Cos. $12° \ 50'$

$0{\cdot}975$

$a \ c = 64480$ lb. Ans.

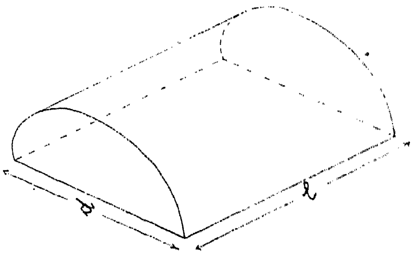
$c \ b = a \ b \times$ Tan. $12° \ 50' = 62860 \times 0{\cdot}2278$
$= 14320$ lb. Ans.

Note for this position, the force on the guide

force on rod

If the rod was only 4 cranks long, it would be, $\dfrac{\text{force on rod}}{4}$

Example. In the engine in the last question the crank pin was 11 inches diameter and 12 inches long. Find the maximum pressure on the oil between the pin and the brass.

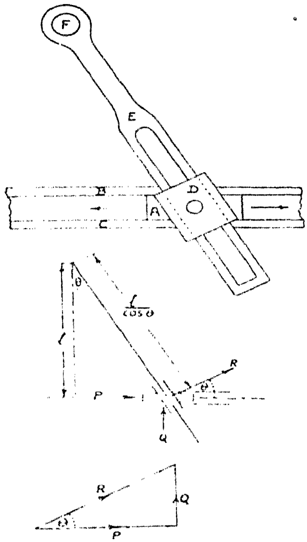


Now the projected area of the curved surface of the pin is a rectangle, its area being 11×12 square inches, i.e., $d \times l$

$$\text{Pressure per sq. inch on oil} = \frac{64480}{11 \times 12} = \frac{64480}{132}$$

$= 488{\cdot}5$ lb. per sq. inch. Ans.

***The Rapson Slide.**



A form of steering gear was devised by J. Rapson about 100 years ago, and its principle is shown by the sketch. A block A slides between parallel guides B and C and carries a pin which passes through another sliding block D which can move along the tiller E. F is the rudder stock. The direction of the pull in the tiller ropes is indicated by the arrows. The arrangement is known as the Rapson Slide and it has been revived in the electric-hydraulic steering gear.

To obtain an expression for the twisting moment exerted on the rudder stock :—

Let P = force exerted on the ram, and let l = the distance from the centre line of the rams to the centre of the rudder stock. Let the tiller be at any angle $\theta°$.

Now, the effect of P is to cause the block D to slide down the tiller, and there is a force Q called into action to prevent this. P and Q have a resultant R perpendicular to the centre line of the tiller.

$$\frac{\ }{\text{Cos.}\,\theta}\quad .\ \text{R acts at a distance of } \frac{l}{\text{Cos.}\,\theta} \text{ from the}$$

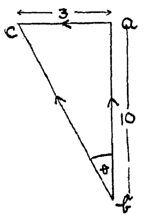
rudder stock.

$$\text{Twisting moment} = \frac{\ }{\text{Cos}\,\theta.}\quad \frac{l}{\text{Cos.}\,\theta}\quad \frac{\text{P}\,l}{\text{Cos.}^2\,\theta}$$

When $\theta = 0$, the twisting moment is P l ; but for any other value of θ, and assuming P to be constant, the twisting moment is greater than P l because Cos. θ and hence Cos.$^2\,\theta$ is less than 1. The greater the value of θ the greater is the twisting moment exerted.

Triangle of Velocities.

Example. A vessel steaming due North at 10 knots passes into a current running due West at 3 knots. Find the actual speed and course made by the vessel.

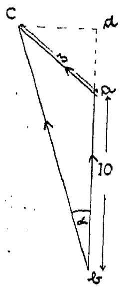


Let $b\ a$ be a vector representing the speed of the vessel through the water, and let $a\ c$ represent the magnitude and direction of the current. Then $b\ c$ represents the speed and direction of the vessel over the ground, in other words, $b\ c$ is the resultant velocity of the two velocities $b\ a$ and $a\ c$.

$$b\ c = \sqrt{10^2 + 3^2} = \sqrt{109}$$
$$= 10{\cdot}44 \text{ knots. Ans.}$$

The course made is found from Tan. $\theta = \frac{3}{10} = 0{\cdot}3$, $\theta = 16^\circ\ 42'$.

The course made is North 16° 42′ West. Ans.



Example. If the current changes in direction and runs at 3 knots in a North Westerly direction, find now the speed and course of the vessel.

The vector diagram is shown. Note angle $c\ a\ d$ is 45°.

$$a\ d = \frac{3}{\sqrt{2}} = 3 \times 0{\cdot}707 = 2{\cdot}121 \text{ nautical miles.}$$

$$c\ d = \frac{3}{\sqrt{2}} = 2{\cdot}121 \text{ nautical miles.}$$

$b\ c = \sqrt{(b\ d)^2 + (c\ d)^2}$ and $b\ d = 10 + 2{\cdot}121 = 12{\cdot}121$

$b\ c = \sqrt{(12{\cdot}121)^2 + (2{\cdot}121)^2}$

$b\ c =$

$b\ c = 12{\cdot}3$ knots nearly. Ans.

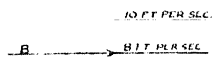
$b\ c$ is the resultant of $a\ c$ and $b\ a$.

$$\text{Tan. } \alpha = \frac{c\ d}{b\ d} \quad \frac{2{\cdot}121}{12{\cdot}121} = 0{\cdot}175$$

Course made is North 9° 56′ West.

Relative Velocity.

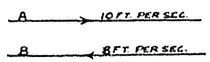
If two bodies are in motion the velocity of one body relative to the other body is the velocity that it appears to have when viewed from the other body. The velocity of a moving body compared to a fixed point is its **absolute velocity.** The speed of a moving body compared to the speed of another moving body is its relative velocity, sometimes called its apparent velocity.



Imagine two bodies, A moving at 10 feet per sec. and B at 8 feet per sec., to travel on parallel courses and in the same direction. If we were moving with B and viewing the motion of A it would appear to have a velocity of 10 — 8 = 2 ft. per sec. to the right. We consider B to be stationery, and A has a velocity of 2 feet per sec. to the right. The velocity of A relative to B is 2 feet per sec. to the right. The same result may be obtained in this manner. Apply a velocity of 8 feet per sec. to the left to each body. B is thus brought to rest and A has 10 — 8 = 2 feet per sec. velocity to the right.

If the velocity of B relative to A was required, then bring A to rest by applying a velocity of 10 feet per sec. to the left to each body.

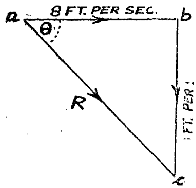
B then has 8 feet per sec. to right and 10 feet per sec. to left, resulting in a velocity of 2 feet per sec. to the left. The velocity of B relative to A is 2 feet per sec. to the left.



If A and B were moving on parallel courses but in opposite directions, and the velocity of A relative to B was required. Bring B to rest by applying a velocity of 8 feet per sec. to the right to each. A then has 10 + 8 = 18 feet per sec. to the right, and the velocity of A relative to B is 18 feet per sec. to the right. In a similar manner, the velocity of B relative to A is 18 feet per sec. to the left.

If A was moving due north at 10 feet per sec. and B due East at 8 feet per sec., and the velocity of B relative to A was required. Apply a velocity of 10 feet per sec. due South to each. A is thus brought to rest whilst B now has velocities of 8 feet per sec. due East and 10 feet per sec. due South. The resultant of these is the velocity of B relative to A.

Draw a velocity vector diagram for B.



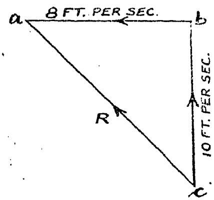
The resultant velocity of B is along the line a to c, and its value is $\sqrt{8^2 + 10^2} = 12{\cdot}81$ feet per sec.

$$\text{Tan. } \theta = \frac{10}{8}, \quad \theta = 51^\circ\, 20'.$$

The velocity of B relative to A is 12·81 feet per sec. at 51° 20′ South of East.

If the velocity of A relative to B is required, then bring B to rest by applying to each a velocity of 8 ft. per sec. due West, A now has velocities of 10 feet per sec. due North and 8 feet per sec. due West.

Draw a velocity vector diagram for A.

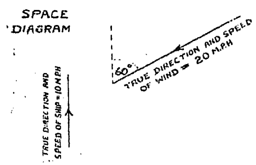


The resultant velocity of A is along the line c to a. Its value is $\sqrt{8^2 + 10^2}$ $= 12{\cdot}81$ feet per. sec.

$$\text{Tan } a = \frac{10}{8}, \quad a = 51^\circ\, 20'.$$

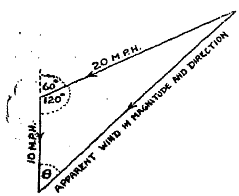
The velocity of A relative to B is 12·81 feet per sec. at 51° 20′ North of West.

Example. A ship is steaming due North at 10 miles per hour the wind is blowing from N.60°E. at 20 m.p.h. From what direction and with what velocity will the wind appear to blow to a man standing on the ship's deck ?



The velocity of the wind relative to the ship, considered stationary, is required. Bring the ship to rest by applying to each a velocity of 10 m.p.h. due South.

The wind now has velocities of 20 m.p.h. from N.60°E. and 10 m.p.h. due South. The resultant of these is the apparent velocity of the wind, or the velocity of the wind relative to the ship.



Draw the velocity vector diagram. By cosine rule :—

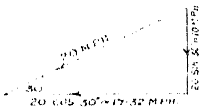
Apparent wind =

$$\sqrt{10^2 + 20^2 - 2 \times 10 \times 20 \text{ Cos. } 120^\circ}$$
$$= \sqrt{100 + 400 + 200} = \sqrt{700}$$
$$= 26{\cdot}46 \text{ m.p.h.}$$

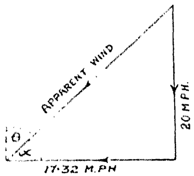
By sine rule :—

$$\frac{20}{\text{Sin. }\theta} = \frac{26{\cdot}46}{\text{Sin. }120^\circ},\ \text{Sin. }\theta = \frac{20}{26{\cdot}46}\,\text{Sin. }120^\circ,\ \theta = 40^\circ\ 53'$$

The apparent wind has a velocity of 26·46 m.p.h. from N.40° 53′E. To prove that this is correct, obtain the rectangular components of the true speed of the wind.



The ship has a speed of 10 m.p.h. due N. and the wind has a component of 10 m.p.h. due S., therefore the speed of the wind relative to the ship must be 20 m.p.h. due S. The wind has a component of 17·32 m.p.h. due W. and the ship has no velocity in the Westerly direction. Therefore the wind has a speed of 17·32 m.p.h. due W. relative to the ship. The resultant of 20 m.p.h. due S. and 17·32 m.p.h. due W. must be the speed of the wind relative to the ship.



$$\text{Apparent wind} = \sqrt{20^2 + (17{\cdot}32)^2} = \sqrt{400 + 300} = \sqrt{700} = 26{\cdot}46 \text{ m.p.h.}$$

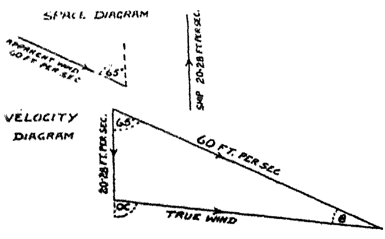
$$\text{Tan. } \propto = \frac{20}{17{\cdot}32},\ \propto = 49^\circ\ 7'$$

$$\theta = 90^\circ - 49^\circ\ 7' = 40^\circ\ 53'$$

The apparent wind has a velocity of 26·46 m.p.h. from N.40°53′E.

Example. A ship is steaming due N. at 12 knots and a man standing on the forecastle head feels a wind on the port bow at an angle of 65° to the direction of motion of the ship. The apparent wind velocity is 60 feet per sec. Find the true velocity of the wind and its direction.

$$1 \text{ Nautical mile per hour} = \frac{6080}{3600} = 1{\cdot}69 \text{ feet per sec. (this is a useful constant).}$$



$$12 \text{ knots} = 12 \times 1{\cdot}69 = 20{\cdot}28 \text{ feet per sec.}$$

Apply a velocity of 20·28 feet per sec. to both the ship and the true wind. The ship is now stationary, whilst the apparent wind of 60 ft. per sec. is the resultant of the true wind and a velocity of 20·28 feet per sec. due South.

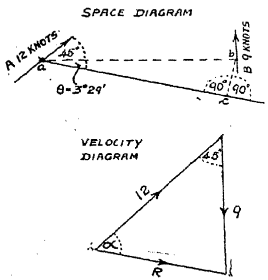
$$\text{True wind} = \sqrt{(20{\cdot}28)^2 + 60^2 - 2 \times 60 \times 20{\cdot}28 \text{ Cos. } 65^\circ}$$
$$= \sqrt{2982{\cdot}4} = 54{\cdot}6 \text{ feet per sec.}$$
$$= 32{\cdot}31 \text{ knots.}$$

$$\frac{20{\cdot}28}{\text{Sin. } \theta} = \frac{54{\cdot}6}{\text{Sin. } 65^\circ} \qquad \text{Sin. } \theta = \frac{20{\cdot}28 \text{ Sin. } 65^\circ}{54{\cdot}6} \qquad \theta = 19^\circ\ 40'$$

$$\alpha = 19^\circ\ 40' + 65^\circ = 84^\circ\ 40'$$

The true wind has a speed of 32·31 knots from N.84° 40′W. Ans.

Example. From a ship A, steaming N.E. at 12 knots, another ship B is observed 5 miles due E. and steaming N. at 9 knots. After what time will these ships be nearest together, and what is their nearest approach ?



Apply a velocity of 9 knots due South to each ship. B is thus fixed in position, and the resultant of 12 knots N.E. and 9 knots due S. is the velocity of A relative to B.

$$R = \sqrt{12^2 + 9^2 - 2 \times 12 \times 9 \text{ Cos. } 45^\circ}$$
$$= 8{\cdot}5 \text{ knots.}$$

$$\frac{9}{\text{Sin. } \alpha} = \frac{8{\cdot}5}{\text{Sin. } 45^\circ}$$

$$\text{Sin. } \alpha = \frac{9 \text{ Sin. } 45^\circ}{8{\cdot}5}$$

$$\alpha = 48^\circ\ 29'$$

Now transfer the relative velocity in magnitude and direction to the space diagram. $\theta = 48^\circ\ 29' - 45^\circ = 3^\circ\ 29'$

B has been fixed in position, and the velocity of A relative to B is along the line a to c. The nearest approach of the ships is $b\ c$, angle $a\ c\ b$ is 90°

$$b\ c = 5 \text{ Sin. } 3^\circ\ 29' = 0{\cdot}3035 \text{ mile.}$$

The distance gone at the relative velocity of 8·5 knots is $a\ c$.

$$a\ c = 5 \text{ Cos. } 3^\circ\ 29' = 4{\cdot}9905 \text{ miles.}$$

$$\therefore \text{ time} = \frac{4{\cdot}9905}{8{\cdot}5} = 0{\cdot}587 \text{ hour} = 35{\cdot}22 \text{ minutes.}$$

The nearest approach of the ships is 0·3035 mile after 35·22 minutes, and ship A passes astern of B. Ans.

TEST EXAMPLES XI.

1. Two forces meet at a point. One of 10 lb. acts due North, the other of 6 lb. acts 50° East of North. Find the magnitude and direction of the resultant force.

14·6 lb. acting 18° 22′ East of North. Ans.

2. Find the resultant of the above two forces if the 6 lb. force acts due East.

11·66 lb. acting 30° 58′ East of North. Ans.

3. A point is in equilibrium under the action of 3 forces. One of 8 lb. acts due South, another of 6 lb. acts due East. Find the magnitude and direction of the third force.

10 lb. acting 36° 52′ West of North. Ans.

4. A derrick 20 feet long is held up at an angle of 30° to the vertical by a topping lift which is horizontal. A load of $2\frac{1}{2}$ tons is hung from the derrick head. Find the forces in derrick and tie. Find the additional force in both derrick and tie, if the derrick weighs 1 ton.

1·443 tons in tie, 2·886 tons in derrick; 0·2886 ton in tie, 0·5774 ton in derrick. Ans.

5. A derrick 25 feet long is supported horizontally by a tie which makes an angle of 35° with the mast. The load carried at the derrick head is 4 tons. Find the force in derrick and tie. If the tie is shortened until it makes an angle of 90° with the derrick, find the position of the derrick after lifting up, and find the force in derrick and tie for the load of 4 tons.

4·883 tons in tie, 2·801 tons in derrick; 2·857 tons in tie, 2·801 tons in derrick. Ans.

6. In a jib crane, the post is 10 feet high, the jib is 15 feet long and the tie is 8 feet long. Find the forces in jib and tie when a load of 5 tons is carried.

4 tons in tie, 7·5 tons in jib. Ans.

7. A load is lifted out of an engine room by means of a wire stretched between two masts, 120 feet apart. The load hangs from the centre of this wire, which dips 45 feet below its points of support on the masts. Find the pull in the two parts of the wire when the load lifted is $2\frac{1}{2}$ tons.

2·08 tons in wire.

8. An engine has a stroke of 5 feet, and the connecting rod is 10 feet long. Find the forces in connecting rod and guide when the crank is 60° from the top centre, and when the crank is horizontal. The load on the piston is 80,000 lb.

81,960 lb. in rod, 17,740 lb. on guide: 82,620 lb. in rod, 20,655 lb. on guide. Ans.

9. A vessel steams at 11 knots due North, and enters a current running at 2·5 knots South East. Find the actual speed and course made by the ship. If the current changes and now runs at 2·5 knots North East, find the speed and course of the ship.

9·399 knots, 10° 49′ East of North ; 12·88 knots, 7° 53′ East of North. Ans.

10. A Diesel piston weighing 2,500 lb. is being lifted out of a cylinder by a lifting tackle inclined at 20° to the axis of the cylinder. Find the force on the tackle and the side thrust on the cylinder. 2,661 lb. pull, 910 lb. side thrust. Ans.

11. Two ships are bound for the same port on courses at 60° to each other. The speeds of the ships are 10 knots and 5 knots and they will arrive simultaneously. If they are 86·6 miles apart at a certain time, how far apart will they be after 3 hours steaming ? 60·62 miles. Ans.

12. A man, who can row in still water at 5 miles per hour, is to cross a river flowing at 2·5 m.p.h. and to reach the other bank exactly opposite the point he started from. The river is 0·75 mile wide. Find the course he should steer and the time he will take.

Up-stream at 60° to the river bank ; 10·392 minutes. Ans.

*13. A ship is fitted with electric hydraulic steering gear consisting of two hydraulic cylinders fitted with rams 10 inches diameter. The distance from the centre line of the rams to the centre of the rudder stock is 26 inches, and the fluid pressure in the cylinders is 1000 lb. per sq. inch, which may be taken as constant. Calculate the turning moment imposed on the rudder stock starting from mid-position, and when reaching the maximum helm position of 35°.

170,200 ft. lb. ; 253,600 ft. lb. Ans.

*14. A turbine rotor runs at 500 revs. per minute. The steam leaves the guide blades at a velocity of 250 feet per sec. and enters the moving blades at 20° to the plane of the wheel. The velocity of the steam relative to the blades is 140 feet per sec. What is the mean diameter of the rotor ?

4·739 feet. Ans.

*15. Two ships leave a port at the same time, the first steaming north-west at 15 knots and the second 30° south of west at 17 knots. What is the speed of the second relative to the first ? After what time will they be 100 nautical miles apart ? Also what will then be the bearing of the second ship relative to the first ?

Relative velocity is 19·54 knots ;
Time 5·118 hours ; Bearing 12° 8′ west of south.

CHAPTER XII.

LINEAR AND ANGULAR MOTION; VELOCITY AND ACCELERATION.

Motion is change of position. We often call a change of position a displacement.

Velocity is rate of change of position, or change of position with respect to time. In mechanics, velocity is generally stated in feet per second, but it may be expressed in miles per hour, or in knots. A point moves with uniform velocity when it passes through equal distances in equal periods of time. A body may, however, be increasing its velocity, and we say that an acceleration is acting upon it; if the speed is decreasing we say that a retardation is acting, or that the speed is being retarded.

Acceleration is rate of change of velocity, or change of velocity per unit time. Let a body be moving at the end of a certain second with a speed of 12 feet per second, and suppose that at the end of the next second its speed is 15 feet per second. The change in velocity is 3 feet per second, and the change has occurred in one second, and we say that the acceleration during this second has been 3 feet per second per second. The velocity has changed at the rate of 3 feet per second during one second and this has been already defined as the acceleration. If this acceleration remains constant, then at the end of another second the speed will be 18 feet per second. We may now state acceleration to be the addition made to the velocity every second. Now it can be shown that the acceleration of a body is a measure of the force acting upon it, hence its importance. In describing the performance of motor cars, makers often give the time taken to achieve a certain speed starting from rest. This is obviously a measure of the accelerating power of the engine, and depends upon the ratio of power to weight of car; it is sometimes called the power ratio. Again, the time in which a car can be brought to rest from a high speed is a measure of the retarding or braking capacity of the brakes. On any railway where there are many stations with only a few miles between them, the acceleration of a train is of great importance; if the acceleration is high, then the train can do most of the distance at its highest speed. The train must be retarded also at a high rate, but this is always easy to achieve.

Dimensions of Velocity.

Speed, if uniform is a measure of $\dfrac{\text{distance}}{\text{time}}$. When the speed

varies, then the average speed is a measure of $\dfrac{\text{distance}}{\text{time}}$; in both cases the dimensions are the same, and may be expressed as $\dfrac{L}{T}$ or $L\,T^{-1}$. Velocity is generally written as "feet per second" in mechanics.

Dimensions of Acceleration.

Acceleration is a measure of $\dfrac{\text{change of velocity}}{\text{time to change}}$, and its dimensions are those of $\dfrac{\text{velocity}}{\text{time}}$ or $\dfrac{L}{T} \times \dfrac{1}{T} = \dfrac{L}{T^2}$ or $L\;T^{-2}$. It is usual to express acceleration in feet per (second)^2, but other units such as miles per (hour)^2 could be used.

Formulæ for Linear Motion.

Let v = final velocity in feet per second.
,, u = initial velocity in feet per second.
,, a = acceleration in feet per (sec.)^2
,, t = time in seconds.
,, s = space passed over in feet.

Then,
$$v = u \pm a\,t \quad \ldots \quad \ldots \quad \ldots \quad (1)$$
$$s = u\,t \pm \frac{a\,t^2}{2} \quad \ldots \quad \ldots \quad \ldots \quad (2)$$
$$v^2 = u^2 \pm 2\,a\,s \quad \ldots \quad \ldots \quad (3)$$

Let us suppose that, when we first observe a body moving under some acceleration, its speed is u feet per second. Now since the acceleration is the addition to the velocity every second, then :—

After 1 second $v = u + a$
,, 2 seconds $v = u + a + a$, or $v = u + 2 \times a$
,, 3 seconds $v = u + 3 \times a$
,, t ,, $v = u + a \times t$ or $v = u + a\,t$ ... (1)

If the body is being retarded the minus sign would be used.

The space or distance passed over is found by multiplying the average velocity by the time taken, or :—

s = (average velocity) $\times$ time.

$$\left(\frac{u}{2} + \frac{v}{2}\right) \times t = \left(\frac{u}{2}\right.$$

Note both u and v in the brackets are divided by 2.

But $v = u + a\,t$, (already proved), $\therefore$ substituting for v, we get

$$\left[\frac{u}{2} + \tfrac{1}{2}(u + a\,t)\right] \times t \qquad \left[u\right.$$

Or $s \quad u\,t + \frac{a\,t^2}{}$ ... ... (2)

The third formula in terms of v^2 is derived from (1) and (2).

We have that $v \quad u + a\,t$, square both sides,

$$v^2 = u^2 + 2\,u\,a\,t + a^2\,t^2,$$

This may be written, $v^2 = u^2 + 2\,a\left[u\,t + \frac{a\,t^2}{2}\right]$,

Note that if we multiply the bracket terms by $2a$, we get the original expression.

But $u\,t + \frac{a\,t^2}{2} \quad s$ (already proved),

$$\therefore\ v^2 = u^2 + 2\,a\,s \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

These are the *three fundamental formulæ* which the student must remember.

Note carefully, that if the body starts from rest, then u, the initial velocity, becomes nothing (0), and these expressions become, $v = 0 + a\,t$, or $v = a\,t$

$$\times\ t) \qquad \frac{a\,t^2}{2}$$

$$v^2 = (0)^2 \qquad \text{or} \qquad 2\,a\,s.$$

It is therefore sufficient for the student to remember the three fundamental forms ; when $u = 0$, the step shown above is easily performed mentally after a little practice.

The acceleration written "a," is in perfectly general form in the above equations, and it may have any value. When, however, a body falls from a height freely, it falls under the

acceleration of gravity, and the value of this particular acceleration is 32·2 feet per $(\sec)^2$. The value varies slightly, depending upon the latitude, in London it is 32·2 feet per $(\sec.)^2$; its symbol is "g." For motion of this kind, that is motion under the acceleration of gravity, we write g for a in the above equations.

Examples. A stone falls from rest. Find its velocity after 4 seconds, and the distance it has fallen.

$v = u + a\,t$, and in this case $u = 0$, and $a = g$, $\therefore$ we have $v = g\,t$, $v = 32{\cdot}2 \times 4 = 128{\cdot}8$ feet per second. Ans.

$s = u\,t + \dfrac{a\,t^2}{2}$, here again $u = 0$, $a = g$, $\therefore$ we have

$$\frac{g\,t^2}{2},\ s = \frac{32{\cdot}2}{2} \times 4^2 = 257{\cdot}6 \text{ feet. Ans.}$$

The average velocity over the period of 4 seconds would be :-

$$\frac{\text{initial velocity} + \text{final velocity}}{2} = \frac{0 + 128{\cdot}8}{2}$$

$= 64{\cdot}4$ ft. per second.

and the distance travelled $=$ average velocity $\times$ time
,, ,, $= 64{\cdot}4 \times 4 = 257{\cdot}6$ feet as above.

Example. With what initial velocity must a stone be projected upwards if it is to reach a height of 120 feet, and how long will it take to reach this height.

In this case we choose an equation connecting velocity and distance :—

$v^2 = u^2 - 2\,a\,s$, now v, the final velocity, $= 0$, $a = g$, and we use the minus sign because g is retarding the upward motion.

$\therefore\ 0^2 = u^2 - 2\,g\,s$, or $u^2 = 2\,g\,s$,

$u^2 = 2 \times 32{\cdot}2 \times 120$ or $u = \sqrt{64{\cdot}4 \times 120}$

$u = 87{\cdot}9$ feet per second. Ans.

Also, average velocity $\times$ time $=$ distance

$$\left(\frac{87{\cdot}9 + 0}{2}\right) \times t = 120$$

$$t = \frac{120}{43{\cdot}95} = 2{\cdot}73 \text{ seconds. Ans.}$$

Note the time could have been found from the equation connecting time, distance and acceleration :—

Thus $t^2 = \dfrac{120}{16 \cdot 1}$, and $t = 2 \cdot 73$ seconds as before.

Note further, that if the stone fell from rest through a height of 120 feet, its velocity at the end of 2·73 seconds would be 87·9 feet per second, $v = g\,t$, $v = 32 \cdot 2 \times 2 \cdot 73 = 87 \cdot 9$ feet per second.

Example. A body is moving with an initial velocity of 5 feet per second. Find its velocity at the end of 10 seconds if the acceleration is 2 feet per (second)^2, and the distance travelled in that time.

$v = u + a\,t$, $v = 5 + 2 \times 10 = 25$ feet per second. Ans.

$s = u\,t + \dfrac{a\,t^2}{2}$, $s = (5 \times 10) + \dfrac{2 \times 10^2}{2} = 150$ feet. Ans.

Example. A body is thrown upwards with an initial velocity of 200 feet per second. How long will it be before its velocity is reduced to 100 feet per second ?

$v = u - a\,t$, $v = 100$, $u = 200$, $a = g$.

$100 = 200 - 32 \cdot 2\,t$, or $32 \cdot 2\,t = 100$

$t = \dfrac{100}{32 \cdot 2} = 3 \cdot 106$ seconds. Ans.

Example. A body moves with initial velocity of 10 feet per second, and after 3 seconds its velocity is 25 feet per second, find the acceleration and the distance passed through in these 3 seconds.

Acceleration $= \dfrac{\text{change in velocity}}{\text{time to change}}$, by definition

$25 - 10$

$= \frac{15}{3} = 5$ feet per (sec.)^2. Ans.

Distance $=$ average velocity $\times$ time

$= \left(\dfrac{10 + 25}{2}\right) \times 3 = 17 \cdot 5 \times 3 = 52 \cdot 5$ ft. Ans.

or by formulæ, $v = u + a\ t$

$$25 = 10 + a \times 3,\ 3\ a = 25 - 10$$

$$a = 5 \text{ feet per (sec.)}^2$$

$$s = u\ t + \frac{a\ t^2}{2},\ s = (10 \times 3) + \tfrac{5}{2} \times 3^2 = 30 + \tfrac{45}{2}$$

$$s = 52{\cdot}5 \text{ feet, as before.}$$

Example. A body is projected upwards with a velocity of 250 feet per second. Find the greatest height to which it will rise. Find also its velocity 3 seconds after projection, also after it has risen 60 feet from the ground.

$$v^2 = u^2 - 2\ a\ s,\ u = 250,\ a = g,\ v = 0$$

$$0 = 250^2 - 2 \times 32{\cdot}2\ s,\ s = \frac{250 \times 250}{64{\cdot}4}$$

$\therefore$ greatest height $= 970{\cdot}3$ feet. Ans.

$$\text{Time to reach greatest height} = \frac{250}{32{\cdot}2} = 7{\cdot}76 \text{ secs.}$$

$$v = u - g\ t,\ v = 250 - 32{\cdot}2 \times 3$$

$v = 250 - 96{\cdot}6 = 153{\cdot}4$ feet per sec. Ans.

or velocity 3 secs. after projection $= 153{\cdot}4$ feet per sec.

$$v^2 = u^2 - 2\ g\ s, \text{ here } s = 60 \text{ feet}$$

$$v^2 = 250^2 - 2 \times 32{\cdot}2 \times 60,\ v^2 = 62{,}500 - 3{,}864$$

$$v^2 = 58{,}636,\ v = 242{\cdot}2 \text{ feet per sec.}$$

Suppose we want to know where the body is 10 seconds after leaving the ground, then :—

$$s = u\ t - \frac{g\ t^2}{2},\ s = 250 \times 10 - \frac{32{\cdot}2}{2} \times 10^2$$

$s = 2{,}500 - 1{,}610 = 890$ feet above the ground. The body is now in its downward course, having fallen $970{\cdot}3 - 890 = 80{\cdot}3$ feet from its highest position. The body reached its highest position $7{\cdot}76$ secs. after projection.

Example. Convert an acceleration of 40 miles per hour per minute to feet per (sec).^{2}

40 miles per hour per minute = 40 × 5,280 feet per hour per minute.

$$\frac{40 \times 5280}{60} \text{ feet per minute per minute.}$$

$$\frac{40 \times 5280}{60 \times 60} \text{ feet per second per minute.}$$

$$\frac{40 \times 5280}{60 \times 60 \times 60} \text{ feet per second per second.}$$

$$= \frac{88}{90} \text{ or } 0{\cdot}977 \text{ feet per (sec.)}^2. \quad \text{Ans.}$$

Example. A vessel starting from rest and moving under a constant acceleration, has a speed of 12 knots at the end of 8 minutes. Find the acceleration, the speed at the end of 3 minutes, the distance moved through in the 5th minute, and the total distance covered in the 8 minutes.

$$\text{One knot} = \frac{6080}{60 \times 60} = 1{\cdot}69 \text{ feet per sec.}$$

$$12 \text{ knots} = 12 \times 1{\cdot}69 = 20{\cdot}28 \text{ feet per sec.}$$

$$v = a\,t, \therefore a = \frac{v}{t} = \frac{20{\cdot}28}{8 \times 60} = 0{\cdot}04225 \text{ feet per (sec.)}^2 \quad \text{Ans.}$$

Or, acceleration $= \dfrac{\text{change in velocity}}{\text{time to change}}$, and the change is from 0 to 20·28 feet per sec.

$$a = \frac{20{\cdot}28}{8 \times 60} = 0{\cdot}04225 \text{ ft. per (sec.)}^2\text{, as above.}$$

Speed at end of 3 minutes, $v = a\,t$, $v = 0{\cdot}04225 \times 3 \times 60$

$$v = 7{\cdot}605 \text{ feet per sec., or } \frac{7{\cdot}605 \times 60 \times 60}{6080} \text{ knots.}$$

$$\frac{7{\cdot}605}{1{\cdot}69} = 4{\cdot}5 \text{ knots.} \quad \text{Ans.}$$

The distance moved through in the 5th minute, is the difference in the distances moved in 5 minutes and in 4 minutes from rest.

$$s = \frac{a\,t^2}{2}, \text{ in 5 minutes, } s = \frac{0{\cdot}04225}{2} \times (300)^2 = 1901{\cdot}2 \text{ ft.}$$

$$\text{in 4 minutes, } s = \frac{0{\cdot}04225}{2} \times (240)^2 = 1216{\cdot}8 \text{ ft.}$$

$$\text{In the 5th minute, distance moved} = \frac{1901{\cdot}2 \qquad 1216{\cdot}8}{6080}$$

$$= 0{\cdot}1125 \text{ nautical mile. Ans.}$$

Total distance covered = average speed × time

$$= \left(\frac{0 + 12}{2}\right) \times \qquad = 0{\cdot}8 \text{ nautical mile. Ans.}$$

Flight of Projectiles.

If air friction is neglected then the path of the projectile is a parabola. The vertical component of the initial velocity is u Sin. θ, and this has a negative acceleration of g feet per sec.^{2} due to the force of gravity.

The horizontal component, u Cos. θ remains unchanged whilst the projectile is in the air. At all instants the projectile is moving with a horizontal velocity of u Cos. θ feet per sec.

After t seconds the vertical velocity will be u Sin. $\theta - g\,t$, and when this becomes zero, the projectile has attained its maximum height.

$$u \text{ Sin. } \theta - g\,t = 0, \therefore t = \frac{u \text{ Sin. } \theta}{g} \text{ where } t \text{ is the time}$$

to attain maximum height.

The value of t might be obtained in this manner :

$$\text{accel.} = \frac{\text{change of velocity}}{\text{time to change}} \quad \therefore \text{ time} = \frac{\text{change of velocity}}{\text{accel.}}$$

When the projectile attains its maximum height its velocity in the vertical direction is 0, and the change of velocity has been u Sin. θ. Also, the acceleration is g.

$$\therefore \text{ time} = \frac{u \text{ Sin. } \theta}{g}$$

$$\text{Average vertical velocity} = \frac{u \text{ Sin. } \theta + 0}{2} = \frac{u \text{ Sin. } \theta}{2}$$

$$\therefore h = \text{Average velocity} \times \text{time}$$

$$\frac{u \text{ Sin. } \theta}{2} \times \frac{u \text{ Sin. } \theta}{g} \quad \frac{u^2 \text{ Sin.}^2 \theta}{2\, g}$$

The total time in the air is twice the time for the projectile to reach its maximum height,

$$\therefore \text{ time to hit the horizontal plane} = \frac{2\, u \text{ Sin. } \theta}{g}$$

$\therefore$ Range (d) on horizontal plane

$$= u \text{ Cos. } \theta \times \frac{2\, u \text{ Sin. } \theta}{g} = \frac{2\, u^2 \text{ Sin. } \theta \text{ Cos. } \theta}{g}$$

$d = \dfrac{2\ u^2}{g}$ Sin. θ. Cos. θ. This has its greatest value when

Sin. θ = Cos. θ, which is true only for 45°. The maximum range is therefore attained when the elevation of the gun is 45°.

$$\therefore \text{ Maximum range} = \frac{2\ u^2}{g} \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{u^2}{g}$$

Example. A projectile is fired from a gun with an initial velocity of 1,800 feet per sec., at 20° to the horizontal. Find the greatest height attained by the projectile and the range on the horizontal plane.

Vertical component of initial velocity = 1,800 Sin. 20° = 615·6 ft. per sec.

Horizontal component of initial velocity $= 1{,}800$ Cos. 20° $= 1691{\cdot}46$ ft. per sec.

$$\text{Time to attain maximum height} = \frac{615{\cdot}6}{32{\cdot}2} = 19{\cdot}12 \text{ seconds.}$$

$$\text{Average vertical velocity} = \begin{matrix}615{\cdot}6\\ \end{matrix} = 307{\cdot}8 \text{ ft. per sec.}$$

$\therefore$ Maximum height $= 307{\cdot}8 \times 19{\cdot}12 = 5885$ feet. Ans. (1).

Time to hit $= 19{\cdot}12 \times 2 = 38{\cdot}24$ secs.

$\therefore$ Range $= 38{\cdot}24 \times 1691{\cdot}46 = 64{,}680$ feet. Ans. (2).

$$\text{The maximum possible range would be } \frac{1800^2}{32{\cdot}2} = 100{,}600 \text{ feet}$$

and this would be when the gun was elevated at 45°.

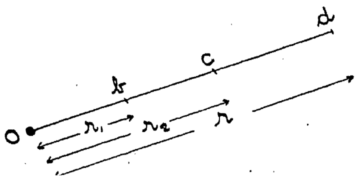
Angular Velocity and Acceleration.

The definition of a radian has been given in Chapter I., and angular velocity is written as "radians per second," the symbol being ω. Angular acceleration is written as radians per (sec.)^2 and the symbol is ϕ.

The methods used in problems on angular motion are exactly the same as those used for linear or straight line motion. In problems of rotation it is always best to work in circular measure or radians, and the student should remember how to convert revolutions per minute to radians per second. For instance, 100 revolutions per minute is $100 \times 2\,\pi = 200\,\pi$ radians per minute.

$$\begin{matrix}200\\ \\ 60\end{matrix} = \tfrac{10}{3}\,\pi \text{ radians per second} =$$

Remember that there are $2\,\pi$ radians in one circle.



Let $o\,d$ be a bar hinged at o. Now in making one complete turn round o, every part of the bar moves through 360° or $2\,\pi$ radians in the same time, therefore the angular velocity of every part of the bar is the same; but the points b, c and d although

moving with the same *angular* velocity, and through the same *angular* distance, have not the same *linear* velocity along their circular paths. The point d will travel through a greater distance and move with a greater speed along its path than the points c and b, and we can see that the linear speed of a point will vary as its distance from the centre of rotation. To find the linear speed of a point along its circular path, we multiply the angular velocity by the distance of the point from the centre of rotation. Thus, if the bar $o\,d$ rotates at radians ω per second, and if r_1, r_2, and r are distances in feet,

then, Linear speed of d along its path $= \omega \times r$ feet per sec.

,, ,, c ,, ,, $= \omega \times r_2$ feet per sec.

,, ,, b ,, ,, $= \omega \times r_1$ feet per sec.

Also, to convert linear into angular velocity, we divide the linear speed in feet per second, by the distance at which the point rotates from the centre, this distance being in feet. The result will be radians per second.

We have then, linear velocity, $v = \omega\, r$

$$\text{and angular velocity, } \omega = \frac{v}{r} \text{ or } \frac{\text{linear velocity in feet per sec.}}{\text{radius in feet}}$$

In the linear formulæ, s is a distance moved in a given time. In the case of angular motion, the angular distance turned through, as a number of radians, is denoted by θ, and we have

$$\text{that } \theta = \frac{s}{r} \text{ radians.}$$

and linear acceleration $=$ angular acceleration $\times\ r$,
or, $a = \phi \times r$.

Example. A wheel turns at 100 revolutions per minute. After 5 seconds its speed is 160 revolutions per minute. Find the angular acceleration and the number of revolutions turned in the 5 seconds.

Change in speed $= 160 - 100$ revs. per min. $= 60$ revs. per min.

$$\text{and 60 revs. per min.} = \frac{60 \times 2\,\pi}{60} \text{ radians per sec.}$$

$$= 2\,\pi \text{ radians per sec.}$$

$$\text{Acceleration} = \frac{\text{change in velocity}}{\text{time to change}} = \frac{2\,\pi}{5} \text{ radians per (sec.)}^2$$

The angular acceleration is $\dfrac{2\,\pi}{5} = 1{\cdot}257$ radians per (sec.)^2 Ans.

No. of revs. turned $=$ average speed $\times$ time.

No. of revs. $= \left(\dfrac{100 + 160}{2}\right) \times \frac{5}{60} = 130 \times \frac{5}{60} = 10{\cdot}83$ revs. Ans.

Note as we take the average speed in revs. per minute, we must take the time in minutes to be consistent.

Example. A wheel, mounted upon an axle, is made to rotate by means of a cord wrapped round the axle. A weight is hung on the cord, and starting from rest, is observed to fall 4 feet in 10 seconds. Find the angular acceleration produced. The axle is $1\frac{1}{2}$ inches diameter.

As the cord is wrapped round the spindle and does not slip, then when the weight falls 4 feet, a point on the circumference of the spindle turns through a linear distance of 4 feet.

No. of radians turned in 10 secs. $= \dfrac{4 \times 12}{\frac{3}{4}}$, since the radius of the axle is $\frac{3}{4}$ inch. $\theta = \frac{48}{3} \times 4 = 64$ radians.

Also, $s = \dfrac{a\,t^2}{2}$ linear, but putting in the angular notation, we get, $\theta = \dfrac{\phi\,t^2}{2}$ angular, $64 = \dfrac{\phi \times 10^2}{2}$

or $\phi = \frac{64}{50}$ radians per (sec.)^2

The angular acceleration is $\frac{64}{50}$ or $1{\cdot}28$ radians per (sec.)^2 Ans.

TEST EXAMPLES XII.

1. A body falls from rest through a distance of 600 feet. Find its velocity at the end of the fall, and the time taken.
6·103 seconds. 196·5 feet per sec. Ans.

2. A stone is thrown up with an initial velocity of 220 feet per second. Find the greatest distance to which it will rise, and the total time from leaving the ground until it falls to the ground again. 751·5 feet. 13·66 seconds. Ans.

3. A stone falls from rest for 6 seconds. Find its final velocity and the space fallen through.
193·2 feet per sec. 579·6 feet. Ans.

4. A stone is thrown up with an initial velocity of 250 feet per second, after how many seconds will its velocity be 120 feet per second, and after how many seconds from the start will the stone be 50 feet above the ground, on its return from the highest position. 4·036 seconds. 15·32 seconds. Ans.

5. A body moving with initial velocity of 20 feet per second, is acted upon by a retardation of 2 feet per second per second. When will the body come to rest, and what distance will it have covered in coming to rest? 10 secs. 100 feet. Ans.

6. A balloon is ascending with a constant velocity of 35 feet per second. A stone is thrown out and reaches the ground in 15 seconds. How high was the balloon when the stone was thrown out? 3,097 feet above ground. Ans.

7. A vessel starting from rest under constant acceleration travels 1 nautical mile in 12 minutes. Find her speed in knots at the end of that time, and what has been the acceleration.
10 knots; 0·02349 ft. sec.^{2} Ans.

8. The range of a projectile upon a horizontal plane was 70,000 feet, and the maximum height attained by the projectile was 6,000 feet. Find the elevation of the gun, and the initial velocity of the shell. 18° 56′ and 1,916 feet per sec. Ans.

9. A wheel turning at 400 revolutions per minute, has its speed reduced to 150 revolutions per minute in 8 seconds. Find the angular retardation, and the number of revolutions turned in this time. How long will it take to bring the wheel to rest if the same retardation is acting?
3·273 radians per sec.^{2}. $36\frac{2}{3}$ revs. 4·8 secs. Ans.

10. When the above wheel is turning at 400 revolutions per minute, find the linear velocity of points distant 2 and 3 feet from the centre of the shaft. 83·8 and 125·7 feet per sec. Ans.

CHAPTER XIII.

FORCE, WORK AND POWER.

Newton's Three Laws of Motion.

First Law.—Every body continues in its state of rest or of uniform motion in a straight line, unless acted upon by an external force.

Second Law.—Rate of change of momentum is proportional to the impressed force, and takes place in the direction of the impressed force.

Third Law.—To every action there is a reaction, equal in magnitude but opposite in direction.

Mass is the quantity of matter in a body, and it is the one invariable property of matter. The temperature of a body may change, or its apparent weight may change, but the quantity of matter contained in the body remains constant.

Weight is a measure of the earth's attraction. Since the earth is not a perfect sphere, then a mass placed at the poles is nearer to the earth's centre, or centre of attraction, than when at the equator, and its weight would be greater. Again, imagine a piece of iron suspended from a spring balance. The balance registers the earth's attraction on the iron, and we call that the weight of the iron. If now we place a magnet below the iron then the balance will record a further amount, and the apparent weight of the iron has changed. The mass, or quantity of matter, of the iron has not changed, but its apparent weight has changed. Mass and weight are therefore not the same thing. The common British unit of mass is one pound, and this is the unit of mass in the F.P.S. (foot, pound, second) system. But we generally find it more convenient to use a unit of mass of 32·2 lb., and this unit may be referred to as the "gravitational or engineers' unit of mass."

In the C.G.S. (centimetre, gram, second) system the unit of mass is the gram. This is about $\dfrac{1}{453 \cdot 6}$ lb., because 1 kilogram (1000 grams) = 2·2046 lb.

Inertia is that property of matter by which it resists change of motion.

Momentum may be defined as the quantity of motion possessed by a body, and it is expressed as the product of mass and velocity.

The unit of momentum is the momentum possessed by a mass of 1 lb. moving at a velocity of 1 foot per second.

The second law of motion states that force is proportional to the rate of change of momentum, that is, force is proportional to the rate of change of (velocity $\times$ mass). The mass is constant, therefore ;

Force is proportional to (rate of change of velocity) $\times$ mass

$$. \text{ Force } \quad \frac{\text{change of velocity}}{\text{time to change}} \times \text{mass}$$

$$\therefore \text{ Force } \quad ,, \quad ,, \quad \text{acceleration} \times \text{mass}$$

and Force $=$ acceleration $\times$ mass $\times$ a constant.

In order to obviate the use of a constant we say that the unit of force is that force which will give an acceleration of 1 foot per sec. per sec. to a mass of 1 pound. This unit of force is the absolute unit and is termed the **poundal.** Its value is

$$\frac{\ }{32\cdot2} \text{ lb., or about } \tfrac{1}{2} \text{ ounce.}$$

Force (poundals) $=$ acceleration (feet per sec.^{2}) $\times$ mass (lb.) This is the F.P.S. (foot, pound, second) system. Engineers, however, prefer to use the pound as the unit of force. If a mass of 1 lb. is allowed to fall freely, then there is a force of 1 lb. acting upon it due to the earth's attraction, and the acceleration is g feet per sec.^{2}, or 32·2 feet per sec.^{2} This force of 1 lb. is the gravitational unit of force.

If, therefore, the statement Force $=$ acceleration $\times$ mass is to be true, then the unit of mass must be g lb. and

$$\text{Force (lb.)} = \text{acceleration (feet per sec.}^2 \times \frac{\text{mass (lb).}}{\ }$$

This is the gravitational, or engineers', system and is generally adopted. Similarly, in the C.G.S. (centimetre, gram, second) system, if a mass of 1 gram is allowed to fall freely then there is a force of 1 gram acting upon it due to the earth's attraction, and the acceleration will be g feet per sec.^{2}

Now g is 32·2 feet per sec.^{2} or $32\cdot2 \times 12 \times 2\cdot54 = 981$ centimetres per sec.^{2}

$$\text{Force} = \text{acceleration} \times \text{mass}.$$

$$\therefore \text{Force (grams)} = \text{acceleration (cms. per sec.}^2) \times \frac{\text{mass (grams)}}{981}$$

We see, therefore, that to give an acceleration of 1 centimetre per sec.^{2} to a mass which weighs 1 gram, the force required would be $\frac{1}{981}$ gram. This unit of force, which is the absolute unit of force, is called the **dyne.** Hence 981 dynes = 1 gram, this being the same relation between absolute and gravitational units of force as 32·2 poundals is to 1 pound.

It has been shown that

$$\text{Force (lb.)} = \frac{\text{Change of velocity}}{\text{time}} \times \frac{\text{mass (lb.)}}{g}$$

$$\therefore \text{Force} \times \text{time} = \text{change of velocity} \times \text{mass}.$$

$$\text{Force} \times \text{time} = \text{change of momentum}.$$

Now, force × time during which the force acts is the Impulse of the force :

$$\therefore \text{Impulse} = \text{change of momentum}.$$

Force may therefore be defined as :

1. That which gives acceleration to mass.
2. That which produces or tends to produce change of motion.
3. Time rate of change of momentum, or the change of momentum per unit time.

Example. What force acting on a body of 20 lb. mass will increase its velocity from 15 ft. per sec. to 28 ft. per sec., in 10 seconds ?

$$\text{Acceleration} = \frac{\text{change of velocity}}{\text{time to change}} = \frac{28-15}{10} = 1{\cdot}3 \text{ ft. per (sec)}^2.$$

$$\text{Force} = a \times \frac{W}{g} = \frac{1{\cdot}3 \times 20}{32{\cdot}2} = 0{\cdot}807 \text{ lb. Ans.}$$

or, force × time = change of momentum.

$$\therefore \text{force} \times \text{time} = \frac{W}{g}(V_2 - V_1)$$

$$\therefore \text{Force} \times 10 = \frac{20}{32{\cdot}2}(28 - 15)$$

$$\therefore \text{Force} = \frac{20 \times 13}{32{\cdot}2 \times 10} = 0{\cdot}807 \text{ lb.} \quad \text{Ans. as before.}$$

Example. In a certain engine the piston, rod and crosshead weigh 150 lb. At a certain instant the accelerating force acting on the piston is 300 lb. Find the acceleration of the piston.

$$\text{Force} = a \times \frac{W}{g} \qquad \therefore \text{Acceleration} = \frac{\text{Force} \times g}{W}$$

$$\therefore \text{Acceleration} = \frac{300 \times 32{\cdot}2}{150} = 64{\cdot}4 \text{ ft. per (sec.)}^2. \quad \text{Ans.}$$

Example. A bullet weighing 2 ozs. has a velocity of 1000 ft. per sec. What is its momentum ? If the bullet is arrested in $\frac{1}{100}$ of a second, find the average force exerted.

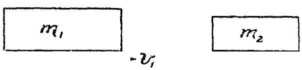
$$\text{Momentum} = \frac{W\ v}{g} = \frac{2 \times 1000}{16 \times 32{\cdot}2} = 3{\cdot}88 \text{ lb. secs. units. Ans.}$$

Force $\times$ time $=$ change of momentum. Since the bullet is stopped, the change of momentum must be 3·88

$$\therefore \text{Force} \times \tfrac{1}{100} = 3{\cdot}88 \qquad \therefore \text{Force} = 3{\cdot}88 \times 100$$

$$\therefore \text{Force exerted} = 388 \text{ lb.} \quad \text{Ans.}$$

Note that the shorter the interval of time to arrest the motion, the greater will be the force exerted.



If two bodies, m_1 moving with velocity v_1, and m_2 moving with velocity v_2, collide and then move on together, the force exerted by m_1 on m_2 must be exactly equal to the force exerted by m_2 on m_1, and the time during which the force acts is the same for each. The impulse of m_1 on m_2 is the same as the impulse of m_2 on m_1 and the change of momentum of each must be the same. m_1 loses momentum equal to the gain of momentum by m_2, the algebraic sum of momentum remaining unchanged. This is the **Law of the Conservation of Momentum.** No momentum is lost due to impact, but some energy is dissipated in the form of heat.

This loss of energy is investigated later. Since momentum is conservative, then $m_1 v_1 + m_2 v_2 = (m_1 + m_2)v$ where v is the velocity with which the two masses move on together.

$$v = \quad m_1 \qquad m_2$$

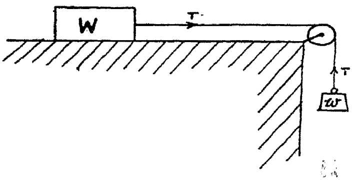
When a shot is fired from a gun, the force of the explosion gives an equal impulse to the shot in one direction and to the gun in the opposite direction. The momentum given to the shot in one direction equals the momentum given to the gun in the opposite direction, and the nett change of momentum is zero.

Example. A bullet weighing 1 ounce is fired into a block of wood weighing 18 lb. which is free to move. The bullet remains embedded and causes the wood to start moving at 5 ft. per sec. What was the velocity of the bullet before it entered the wood ?

Momentum of bullet before impact = Momentum of wood and bullet after impact, $\frac{\frac{1}{16} \times v}{g} = \frac{18\frac{1}{16} \times 5}{g}$

$\times\ 5$

$= 18\frac{1}{16} \times 5 \times 16 = 1445$ ft. per sec. Ans.



Let a weight W, on a smooth horizontal table, be connected by a light cord passing over a light pulley to another weight w which hangs vertically.

Neglecting friction :—

Accelerating force $= w$

Mass accelerated $= w + W$

Now acceleration $= \frac{\text{accelerating force}}{\text{mass}}$

$\therefore$ a (the acceleration of the whole system) $= \frac{w\,g}{(w + W)}$ ft. per

The tension T is the same in all parts of the cord.

Consider W. The accelerating force is T and the acceleration is $\frac{w\,g}{(w + W)}$ $\frac{w\,g}{(w + W)}$ $\frac{T\ g}{W}$ $.\ T = \frac{w\,W}{(w + W)}$

If we consider w, then the accelerating force acting on w is $(w - T)$ and the acceleration is $\frac{w\,g}{(w + W)}$

$$\frac{w\,g}{(w + W)} = \frac{(w - T)\,g}{w} \quad \therefore \; w - T = \frac{w^2}{w + W}$$

$$\therefore \; T = w - \frac{w^2}{(w + W)} = \frac{w^2 + w\,W - w^2}{(w + W)}$$

$$\therefore \; T = \frac{w\,W}{(w + W)} \text{ as before.}$$

If the table is not smooth, and there is a friction force of F opposing the motion of W. Accelerating force $= (w - F)$

Mass accelerated $= (w + W)$

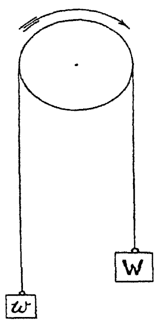
$$\therefore \; \text{Acceleration} = \frac{(w - F)\,g}{(w + W)} \text{ feet per sec.}^2$$

Consider W. The accelerating force is $(T - F)$

$$\frac{(w - F)\,g}{(w + W)} = \frac{(T - F)\,g}{W} \quad \therefore \; T - F = \frac{W\,(w - F)}{(w + W)}$$

$$T = F + \frac{W\,(w - F)}{(w + W)}$$

$$T = \frac{W\,w + w\,F}{(w + W)} = \frac{w\,(W + F)}{(w + W)}$$



Now take the case of two unequal weights W and w lb. connected by a light flexible cord over a pulley whose mass may be neglected. Neglecting friction :—

The unbalanced load $(W - w)$ is an accelerating force, total mass accelerated $= (W + w)$, then,

$$\text{acceleration} = \frac{(W - w)\,g}{(W + w)} \text{ ft. per sec.}^2$$

Consider w.

Pull in cord $=$ load w $+$ force producing acceleration of w

$$= w + \frac{w}{g} \times \frac{(W - w)\,g}{(W + w)}$$

$$w \quad \frac{+ w^2 + w\,W - w^2}{(W + w)} \quad \frac{2\,w\,W}{(W +} \text{ lb.}$$

Now consider W.

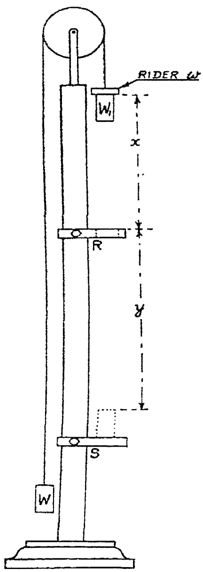
Pull in cord = load W — force producing acceleration of W

$$= W - \frac{W}{g} \times \frac{(W - w)\,g}{(W + w)}$$

$$= \frac{W^2 + W\,w - W^2 + W\,w}{(W + w)} = \frac{2\;W\;w}{(W + w)} \text{ lb.}$$

as before.

Atwood's Machine. This machine is used to determine the value of g and to prove the truth of the laws of motion. It consists of a pillar about 8 feet high, on which a ring R and a stopper S can slide up and down, and be fixed in any desired positions. Two equal weights W and W_1 are attached to the ends of a light flexible cord passing over a perfectly balanced aluminium pulley mounted on agate bearings, so that the inertia of the pulley and the friction of the bearings is so small as to be negligible. The weights W and W_1 are cylindrical and can pass freely through ring R. The rider weight w is slotted and, being larger in diameter than the ring, will not pass through it. To use the machine, fix ring R and stopper S at some distance y apart. Hold W_1 at x above the ring and place the rider in position. Now release the weights and the system moves with uniform acceleration through distance x until the rider is intercepted, and then with uniform velocity through distance y until brought to rest by the stopper. Whilst the rider is in position the accelerating force is w, and the total mass accelerated $= W + W_1 + w = (2W + w)$ since $W_1 = W$.



Therefore, acceleration $a = \dfrac{w\,g}{(2W + w)}$ ft. per sec.^{2}

Now, since $v^2 = 2\,a \times$ distance fallen, the velocity of the system when the rider is intercepted $= \sqrt{\frac{2\,w\,g}{(2W + w)} \times x}$ feet per sec.

Observe the time taken to move over the distance y, let this be t seconds. The velocity during the distance y is uniform because there are now equal weights at each end of the cord.

Therefore $y = \frac{2\ w\ g\ x}{(2\ W + w)} \times$

All the quantities in this expression have been directly measured except g and this may be calculated.

$$\frac{2\,w\,g\,x\,t^2}{(2W + w)} \quad . \quad g = \frac{y^2\,(2\ W + w)}{2\,w\,x\,t^2} \text{ ft. per sec.}^2$$

Work.

When a force is exerted through a distance, work is said to be done. The unit of work is the "foot pound" and is a measure of the work done when a force of one pound is exerted through a distance of one foot.

Energy is capacity to do work.

Potential Energy is energy stored in a body by virtue of its position, as of a weight which could give out energy by falling through a distance, such as a weight driving a clock. Water, stored in a high tower, is capable of doing work by falling through a distance. Water flowing over a high fall gives up potential energy, which is often used to generate electricity by means of water turbines. Energy stored in a spring is also potential energy. The work stored in a body by lifting it to a certain height, is equal to the work done to place it in that position. In all cases this is equal to the weight of the body multiplied by the vertical distance its centre of gravity is raised.

Kinetic Energy.

All bodies in motion possess energy. The energy of the wind is used to drive sailing ships; or windmills for grinding or pumping. The energy in a small bullet, by virtue of high velocity, may be used to puncture holes in bodies. A ball or fly press, may be used to cut holes in sheet metal, by the energy stored in its moving weights. This energy, due to motion, is called kinetic energy. Kinetic energy stored in the flywheel of a shearing machine during the idle stroke, is partly given up during the

working stroke. In exactly the same way, the energy developed during the working stroke of a gas or oil engine is partly stored in the flywheel, and given out during the idle strokes. The wheel, by virtue of its rotation, is a storehouse of energy.

Whether energy is Potential or Kinetic, its units are the same namely, foot pounds units, or units or work. Kinetic Energy is written K.E. We may also express K.E. in inch lb., or inch tons, but we must always state the units in which we give the answer.

Kinetic Energy = force × distance.

We have shown that force $= \dfrac{W \times v}{g \times t}$

and distance = mean velocity × time

$$= \left(\frac{o + v}{2}\right) \times t, \text{ where } v \text{ is final velocity}$$

K.E. = force × distance

$$\frac{W \times v}{g \times t} \quad \frac{v}{2} \times t = \frac{W}{2\,g} \qquad t \text{ cancels.}$$

v is velocity in feet per second, as usual.

This formula represents foot pounds of work stored in a body of weight W moving with velocity v feet per second.

Remembering that a velocity is a measure of $\dfrac{L}{T}$

and that an acceleration is a measure of $\dfrac{L}{T^2}$

lb. ×

we have, dimensions of K.E. =

$$\text{dimensions of K.E.} = \frac{\text{lb.} \times L^2 \times T^2}{L \times T^2} = \text{lb.} \times L, \text{ or lb.} \times \text{ft.}$$

proving that the formula gives ft. lb. units. Note that the constant 2 does not affect the *kind* of unit. It does, of course, affect the value, and must always be used.

Conservation of Energy.

Energy, like matter, can neither be created nor destroyed. Coal burned on a grate, generating steam in a boiler, and this steam doing work in an engine, is an example of the transformtion of energy ; but we did not create the energy originally held in latent form in the coal. Nor do we destroy the energy. The energy after we have done using it still exists, but perhaps in a form such that we cannot again avail ourselves of it. Energy may be changed into different forms, but these various forms are all manifestations of the same thing. Energy may be dissipated, or reduced to a state or form in which it is no longer available for our purposes in engineering, but it is still in existence. The statement that energy can neither be created nor destroyed, is called the law of the conservation of energy.

Example. A pump lifts 30 cubic feet of salt water per minute to a height of 25 feet. What work is done per hour ?

Work per min. $= 30 \times 64 \times 25$ ft. lb.

Work per hour $= 30 \times 64 \times 25 \times 60 = 2{,}880{,}000$ ft. lb. of work. Ans.

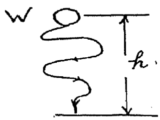
Example. A man and his load weigh 200 lb. The man ascends a ladder 20 feet long, inclined at 30° to the wall. Find the work done in ascending to the top.

The work done is always equal to the weight $\times$ the vertical distance through which it moves.

Vertical height of the ladder $= 20 \times \dfrac{\sqrt{3}}{2} = 17{\cdot}32$ feet.

Work done $= 200 \times 17{\cdot}32 = 3464$ ft. lb. of work. Ans.

Note, whether a weight falls vertically or along the zig zag path shown, the work done is only W $\times$ h.



Example. A weight of 4 tons falls 5 feet on to a bloom of steel and compresses it $\frac{1}{4}$ of an inch. Find the average force sustained by the bloom.

The weight falls 5 feet $+ \frac{1}{4}$ inch $= 60\frac{1}{4}$ inches.

Work done by weight $= 60\frac{1}{4} \times 4 = 241$ inch tons of work. All this work is absorbed in crushing the bloom, and by the law of the conservation of energy we have :—

Work done by falling weight $=$ Work done on bloom

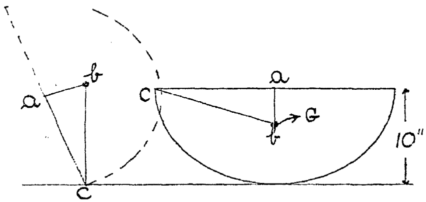
$241 =$ average force on bloom $\times \frac{1}{4}$ inch.

because the average force on bloom is exerted through $\frac{1}{4}$ inch.

Average Force $= 241 \times 4 = 964$ tons. Ans.

Example. A hemisphere rests with its curved surface on a table. It is 10 inches radius and weighs 30 lb. Find the work done in turning it over.

Once we tilt the hemisphere until the C.G. is just vertically over the point of contact, c, it will fall completely over.



The work done is the weight $\times$ the *vertical* distance the C.G. is raised. Let G be the centre of gravity.

Vertical distance C.G. is raised $= c\,b - (10 - a\,b)$

Now $a\,b = \frac{3}{8} \times 10 = 3{\cdot}75$, $\because$ C.G. of a hemisphere is $\frac{3}{8}\,r$ from the diameter. $c\,b = \sqrt{10 + (3{\cdot}75)^2} = 10{\cdot}68$ inches.

C.G. is raised $10{\cdot}68 - (10 - \frac{30}{8}) = 4{\cdot}43$ inches.

Work done $= 30 \times 4{\cdot}43 = 132{\cdot}9$ inch lb. of work. Ans.

Example. A car weighing $1\frac{1}{2}$ tons is travelling at 30 miles per hour. Find its kinetic energy.

30 miles per hour is 44 feet per second.

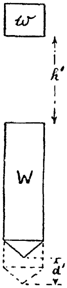
$$\text{K.E.} = \frac{W\ v^2}{2\ g} = \frac{3}{2} \times \frac{(44)^2}{2 \times 32{\cdot}2} = 45{\cdot}1$$

K.E. $= 45{\cdot}1$ ft. tons of work. Ans.

To express this in ft. lb., multiply $45{\cdot}1$ by 2240,

K.E. $= 45{\cdot}1 \times 2240 = 101{,}000$ ft. lb. of work. Ans.

***Pile Driver.**



The tup w (lb.) falls through a distance h feet on to the pile W (lb.) and drives it a distance of d feet into the ground against an average resistance of F (lb.)

Velocity of tup when it strikes the pile $= \sqrt{2\,g\,h}$

By the law of the conservation of momentum :—

$$\frac{w \times \sqrt{2\,g\,h}}{g} + 0 = \frac{(w + \mathrm{W})}{g} \times v,$$ where v is the velocity of tup and pile immediately after impact.

$$\therefore\ v = \frac{w \times \sqrt{2\,g\,h}}{(w + \mathrm{W})} \text{ ft. per sec.}$$

$$\text{Kinetic energy of tup and pile} = \frac{(w + \mathrm{W})}{2\,g} \times \frac{w^2 \times 2\,g\,h}{(w + \mathrm{W})^2}$$

$$= \frac{w^2\ h}{(w + \mathrm{W})}$$

Now this energy is dissipated in overcoming the average resistance through the distance d.

$$\therefore\ \frac{w^2\ h}{(w + \mathrm{W})} = \mathrm{F} \times d.$$

Example. The tup of a pile driver weighs 500 lb. and the pile weighs 600 lb. The tup falls 12 feet before striking the head of the pile. If the pile is driven 6 inches into the ground, find the average resistance of the ground.

$$\mathrm{F} = \frac{w^2\ h}{(w + \mathrm{W})\ d} = \frac{500^2 \times 12 \times 2}{1100 \times 1} = 5454 \text{ lb. or } 2{\cdot}435 \text{ tons.}$$

Ans.

Note. The velocity of the tup after falling 12 feet $= \sqrt{2\,g \times 12}$ ft. per sec

$$\text{K.E. of tup} = \frac{500 \times 2\,g \times 12}{2\,g} = 500 \times 12 = 6000 \text{ ft. lb.,}$$

i.e., the loss of potential energy of the tup.

By the law of conservation of momentum :—

Velocity of pile and tup immediately after impact

$$\frac{500 \sqrt{2\ g \times 12}}{1100} \text{ ft. per sec.}$$

$$\therefore \text{ K.E. of pile and tup } = \frac{1100 \times 500^2 \times 2\ g \times 12}{2\ g \times 1100^2}$$

$$\frac{500^2 \times 12}{1100} = 2727 \text{ ft. lb.}$$

Loss of energy due to impact $= 6000 - 2727 = 3273$ ft. lb. and this energy re-appears principally in the form of heat energy.

Rotation.

The formula $\dfrac{W\ v}{2\ g}$, may be expressed in another form for dealing with rotating bodies.

Let N = revolutions per minute.

R = radius of gyration in feet.

$$\text{Then } v = \frac{2\ \pi\ R\ N}{60} \text{ feet per second}$$

$$\text{and } v^2 = \frac{4\ \pi^2\ R^2\ N^2}{3600}$$

$$\text{K.E.} = \frac{W}{2\ g} \qquad \frac{W\ 4\ \pi^2\ R^2}{2 \times 32{\cdot}2 \times 3600} \qquad \frac{W\ R^2\ N^2}{5870}$$

Note that 5870 is not the exact figure for the denominator, but it is quite near enough for four-figure accuracy.

$$\text{K.E.} = \frac{W\ R^2}{5870} = 0{\cdot}00017\ W\ R^2\ N^2$$

Either of these forms may be used. These formulæ give the result in ft. lb. if W is expressed in pounds, or ft. tons, if W is expressed in tons. Care must be taken to have R in feet and N in revolutions per minute.

Example. Find the kinetic energy stored in a flywheel weighing 2 tons, its radius of gyration being 3 feet, and its speed 100 revolutions per minute.

$$\text{K.E.} = 0{\cdot}00017 \times 2 \times 3 \times 3 \times 100 \times 100 = 30{\cdot}6 \text{ ft. tons.}$$

K.E. $= 30{\cdot}6$ ft. tons, or $30{\cdot}6 \times 2240$ 3,544 ft. lb. of energy. Ans.

In the case of a flywheel of ordinary proportions where the rim is of moderate thickness, the radius of gyration may be taken as the mean radius ; but for a rotating disc, i.e., a solid disc flywheel, we cannot assume this to be true.

Example. A solid disc of cast iron is 3 feet diameter, 6 inches thick, the material weighing 450 lb. per cubic foot, and it rotates at 200 revolutions per minute. Find the radius of gyration of the wheel, and its kinetic energy.

It has been shown that the radius of gyration of a circular disc about its shaft or polar axis is $\dfrac{R}{\sqrt{2}}$, R being the radius of the disc.

$$\text{Radius of gyration} = \frac{1\frac{1}{2}}{\sqrt{2}} = 1{\cdot}0605 \text{ feet. Ans.}$$

$$\text{Wt. of wheel} = \text{area} \times \text{thickness} \times 450 = \pi R^2 \times \tfrac{1}{2} \times 450 \text{ lb.}$$

$$= \tfrac{22}{7} \times \tfrac{3}{2} \times \tfrac{3}{2} \times 450 \times \tfrac{1}{2} = 1591 \text{ lb. (approx).}$$

K.E. $= 0{\cdot}00017\ W\ R^2\ N^2$, note R $=$ radius of gyration in feet in this formula.

$$\text{K.E.} = 0{\cdot}00017 \times 1591 \times (1{\cdot}0605)^2 \times 200 \times 200.$$

$$\text{K.E.} = 12{,}160 \text{ ft. lb. of energy. Ans.}$$

Power is rate of doing work. If a weight of one pound be lifted 4 feet, then the work done is 4 foot lb. ; whether the weight is lifted in one second or in one hour, the work done is the same. The power required is by no means the same. The shorter the time taken to do the work, the greater is the power applied.

The Horse Power is the unit of power. One horse power is equal to 33,000 ft. lb. of work per minute, or 550 ft. lb. of work per second. We may also state it as 1,980,000 ft. lb. of work per hour.

Example. A pump lifts 200 tons of water per hour to a height f 20 feet. Find the horse power exerted.

$$\text{Work per minute} = \frac{200 \times 2240 \times 20}{60} \text{ ft. lb.}$$

$$\text{Horse Power} = \frac{\text{work per minute}}{33{,}000} \qquad \frac{400 \times 2240}{6 \times 33{,}000} = 4{\cdot}52 \text{ H.P.}$$

Ans.

If this work is done with a loss of 40 per cent., find the horse power of the pump.

This means that the efficiency of the pump is 60 per cent.

$4{\cdot}52$ represents $\frac{60}{100}$ of the H.P. of the pump.

$\frac{4{\cdot}52}{60}$ represents $\frac{1}{100}$ of the H.P. of the pump.

$\frac{100 \times 4{\cdot}52}{60}$ represents $\frac{100}{100}$ or the actual H.P. developed by the pump.

Or, in one step, H.P. of pump $= 4{\cdot}52 \times \qquad = 7{\cdot}53$. Ans.

It is important that the student should remember that horse power is a measure of the work done per unit time.

In reading over this Chapter, the student should remember that momentum is force multiplied by time, and that K.E. is force multiplied by distance. We generally use the momentum formula to find the force if the time of application of the force is given, and the K.E. formula to find the force if the distance is given.

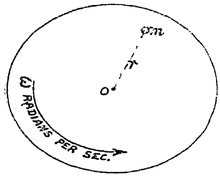
Thus :—

$$\text{Force} = \frac{\text{Momentum}}{\text{Time force is applied}}$$

$$\text{Force} = \frac{\text{Kinetic Energy}}{\text{Distance moved by force}}$$

*Angular Momentum

The angular momentum of a rotating body, sometimes referred to as its *moment of momentum,* is the first moment of the momentum possessed by the rotating body.



Let a body rotate around the fixed point o with constant angular velocity ω radians per sec. Consider an element of mass of the body m at a radius of r.

The linear velocity of m is v, and $v = \omega\, r$. Momentum of $m = m\, v = m\, \omega\, r$. 1st moment of the momentum of m

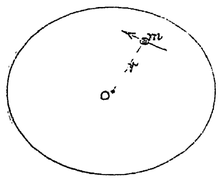
$\therefore$ the 1st moment of the momentum of all the elements of mass which constitute the rotating body = Summation of $m\, \omega\, r^2$.

Now, since ω is constant, then the angular momentum = $\omega \times$ summation of $m\, r^2$. But the summation of $m\, r^2$ is the summation of each element of mass, each element being multiplied by the square of its distance from o.

This has already been defined as the 2nd moment, or moment of inertia about o. It is a polar 2nd moment, and may be denoted by I_p or by J.

$\therefore$ Angular Momentum $= \omega\, I_p$ in absolute units, or

$$= \frac{\omega\, I_p}{g} \text{ in gravitational units.}$$



Consider, now, the element of mass m to be acted upon by a force F, which always acts at right angles to r.

The turning moment of F about o $= F \times r$.

Force = mass $\times$ acceleration.

$\therefore F = m \times a$, a being the linear acceleration of m.

Now linear acceleration (a) = angular acceleration $\times\, r$

$\therefore F = m \times \phi \times r$.

Turning moment about $o = F \times r = m \times \phi \times r \times r = m\, \phi\, r^2$

$\therefore$ total turning moment about o = Summation of $m\, \phi\, r^2$

$= \phi \times$ summation of $m\, r^2$,

since all elements of mass must have the same angular acceleration.

$\therefore$ Turning Moment (M) $= \phi\, I_p$ in absolute units, or

$= \dfrac{\phi\, I_p}{g}$ in gravitational units. This is important.

$$\text{Again, } M = \frac{I_p}{g} \times \phi = \frac{I_p}{g} \times \frac{\text{Change of angular velocity}}{\text{time}}$$

$\therefore$ Turning moment $\times$ time $= \dfrac{\;}{g} \times \omega$, if the final angular velocity is ω radians per sec., and the initial velocity was zero.

$\dfrac{\;}{g} \times \omega =$ angular momentum

$\therefore$ Turning moment $\times$ time $=$ change of angular momentum.

Example. An engine is running at 240 revolutions per minute. Steam is suddenly shut off, and the load removed from the engine at the same instant. The engine turns 300 revolutions before stopping. If the flywheel weighs 2,000 lb. and its radius of gyration is 3 feet, find the frictional resisting moment of the engine, assuming it to be constant at all speeds.

240 revs. per min. $=$ 4 revs. per sec. $= 8\,\pi$ radians per sec.

Average revs. in coming to rest $= \dfrac{240}{\;} =$ 120 revs. per min.

$$\text{Time to come to rest} = \frac{300}{120} \times 60 = 150 \text{ secs.}$$

$$\text{Angular acceleration} = \frac{\text{Change of angular velocity}}{\text{time}}$$

$= \dfrac{\;}{150}$ radians per sec. per sec.

I_p of wheel $= 2{,}000 \times 3^2$ lb. feet^2 units.

$$M = \frac{\phi\, I_p}{\;} \quad \frac{8\,\pi \times 2000 \times 9}{150 \times 32{\cdot}2} = 93{\cdot}64 \text{ ft. lb.} \quad \text{Ans.}$$

Example. In order to turn a flywheel in its bearings it is found necessary to exert a couple of 350 ft. lb. The weight of the flywheel is 5 tons and its radius of gyration is 3·5 feet. If the driving power is cut off when the wheel is running at 120 revs. per min., find how long the wheel will take to come to rest.

120 revs. per min. = 2 revs. per sec. = 4 π radians per sec.

I_p of wheel $= 5 \times 2240 \times (3{\cdot}5)^2$ lb. feet^2 units.

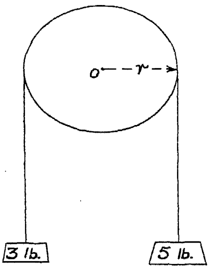
Turning moment $\times$ time = Change of angular momentum

$$350 \times \text{time} = \frac{4\pi \times 5 \times 2240 \times (3{\cdot}5)^2}{32{\cdot}2}$$

$$\text{Time} = \frac{4\pi \times 5 \times 2240 \times (3{\cdot}5)^2}{32{\cdot}2 \times 350} = 153 \text{ secs.} = 2 \text{ minutes } 33 \text{ secs.}$$ Ans.

Example. Two weights of 3 and 5 lb. are fastened to the ends of a light cord hanging over a frictionless pulley. If the pulley is a circular disc and weighs 12 ounces, find the time for either weight to move 10 feet from rest.

Let the radius of the pulley be r feet.



The radius of gyration of a circular disc is $\frac{}{\sqrt{2}}$ feet.

$\therefore$ I_p of pulley about o

$$= \frac{12}{16} \times \left(\frac{r}{\sqrt{2}}\right)^2 \quad 8 \quad \text{lb. ft.}^2 \text{ units.}$$

I of the weights with respect to o
$= 3r^2 + 5r^2 = 8r^2$ lb. feet^2

$\therefore$ total I_p of the system about o

$$= \frac{3r^2}{8} + 8r^2 = 8\tfrac{3}{8}r^2 \text{ lb. feet}^2.$$

Turning moment about $o = (5 - 3)\ r = 2\ r$ ft. lb.

$$M = \frac{\phi\ I_p}{g}, \quad \phi = \frac{M g}{I_p} = \frac{2\ r \times g}{8\frac{3}{8}\ r^2} = \frac{16\ g}{67\ r} \text{ radians per sec.}$$

Linear acceleration (a) = Angular acceleration $(\phi) \times r$

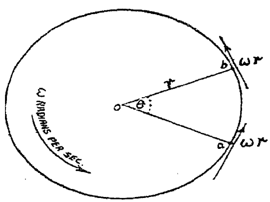
$$\therefore a = \frac{16\ g}{67\ r} \times r = \frac{16\ g}{67} \text{ feet per sec. per sec.}$$

Now $s = \frac{1}{2}\ a\ t^2$

$$\therefore t = \sqrt{\frac{2\ s}{a}} = \sqrt{\frac{2 \times 10 \times 67}{16\ g}} = 1{\cdot}613 \text{ secs.} \quad \text{Ans.}$$

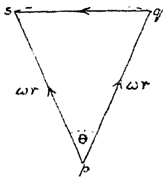
Centripetal Acceleration and Centrifugal Force.

It is a matter of common experience, that if a weight at the end of a string be whirled round in a circle, a pull is transmitted along the string to the hand. This outward pull is called the centrifugal force. Now imagine the weight to be still moving round on its circular path without the string. A force exactly equal to the centrifugal force, but acting in the opposite direction must be applied to the weight to keep it on its circular path. This force is called the centripetal force, and it acts radially in towards the centre of rotation. If this force was not applied, the weight would move off in a straight line.



Imagine a body moving on a circular path of radius r feet about a fixed centre o, and with constant angular velocity ω radians per second. Its constant linear velocity $v = \omega\ r$ feet per sec. and the direction of this velocity is always tangential to the circular path. The direction of motion therefore changes from instant to instant. Now velocity is a vector quantity and has magnitude and direction. If either of these properties change there must be a force acting to produce this change. Here the magnitude is constant, but the direction changes.

Let the body move from a to b through a small angle θ radians in a small interval of time t seconds.



Draw the vector $p\ q$ to represent the linear velocity at a in magnitude and direction, and the vector $p\ s$ to represent the velocity at b. The angle between these vectors is θ. Join $q\ s$, then $q\ s$ represents the change of velocity. It is that velocity which must be vectorially added to $p\ q$ in order to give the resultant velocity $p\ s$.

Since θ is a small angle the arc and the chord have the same length, therefore the change of velocity $q\ s = \omega\ r\ \theta$.

$$\text{Acceleration} = \frac{\text{change of velocity}}{\text{time to change}} \quad \frac{\omega\ r\ \theta}{t} = \omega\ r \times \frac{\ }{t}$$

but the constant angular velocity $\times$ time $=$ angle passed through

$$\times\ t = \theta \text{ and } \omega = \frac{\ }{t}$$

$$\therefore \text{ Acceleration} = \omega\ r \times \frac{\ }{t} = \omega\ r \times \omega = \omega^2\ r \text{ feet per sec.}^2$$

$$\text{Also, } v = \omega\ r, \ \therefore\ v^2 = \omega^2\ r^2 \text{ and } \omega^2 =$$

$$\therefore \text{ Acceleration} = \omega^2\ r = \frac{v^2}{r^2} \times r = \frac{v^2}{r} \text{ ft. per sec.}^2$$

θ was considered to be a small angle, and angle $p\ q\ s$ is nearly 90°. When θ is indefinitely small the angle $p\ q\ s$ is 90°. The change of velocity, and hence the acceleration is at 90° to the vector $p\ q$, that is, it is along the radius of the circular path, and is directed towards o. This is the Centripetal Acceleration.

and its value is $\omega^2\ r$ or $\dfrac{\ }{r}$ feet per second per second.

Now, force $=$ mass $\times$ acceleration,

$$\therefore \text{ Centripetal force} = \frac{W}{g} \times \omega^2\ r, \text{ or } \frac{W}{g} \times \frac{v^2}{r} \text{ lb.}$$

This is the force which acts on the body towards o, in order to produce the change of direction. The reaction of the centripetal force is the Centrifugal Force, and its value is $W \omega^2 r$ or

$$\frac{W \quad v^2}{g} \text{ lb.}$$

The formula C.F. $= \dfrac{W}{g \quad r}$ be expressed in another form which is often convenient to use, let N = revolutions per minute, then :—

$$\text{C.F.} = \frac{W \quad v^2}{g \quad r} = \frac{W}{g \quad r} \times \left(\frac{2}{60}\right) = 0{\cdot}00034 \ W \ r \ N^2$$

or $\dfrac{W \ r}{2935}$

Care must be taken to have r in feet. The centrifugal force will be in lb. if W is expressed in lb., and in tons if W is expressed in tons.

Stress Due to Centrifugal Force.

It has been shown that in a cylindrical vessel subject to an internal pressure ;—

$$P = \frac{2 \ T \ S}{D}, \text{ where } P = \text{pressure, } T = \text{thickness, } D = \text{diameter.}$$

Consider a revolving hoop such as the rim of a flywheel. Let the section of the rim be one foot square. The weight of one foot length of the circumference will equal the weight of a cubic foot of the material, and the centrifugal force it exerts radially from the centre will $= W \quad v^2$

Now this force may be regarded as the internal pressure per sq. foot and $\therefore \frac{W}{r\,g} \qquad \frac{2\,T\,S}{D}$ (all in foot units).

$D = 2\,r$, and $T = 1 \quad \therefore S = \quad W\ v^2$

If s = stress per square inch ; $144\,s = S$; and if w = weight per cubic inch $1728\,w = W$.

$$\therefore 144\,s = \frac{1728\ w\ v^2}{g}$$

$$\therefore s = \frac{12\,w\,v^2}{g}$$

If speed = N revs. per minute, $v^2 = \left(\frac{2\ \pi\ r\ N}{60}\right)^2$ and

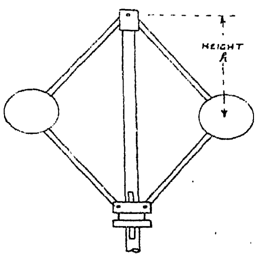
putting in this value for v^2 and 32·2 for g, we get :—

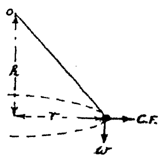
$$w\ r^2\ N^2$$

$$245$$

Governors.

The principle of the simple governor is that of a conical pendulum. Consider a ball of w lb. suspended from o by a link whose weight may be neglected, rotating in a circular path around a vertical axis. The centrifugal force set up by w causes it to swing outwards, thus increasing the radius r and decreasing the height h.



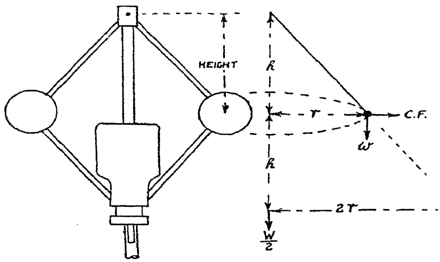


By moments about *o* (neglecting friction) :—

$$w \times r = \text{C.F.} \times h \quad \therefore h = \frac{w \times r}{\text{C.F.}} = \frac{w \times r \times 2935}{\times r \times \text{N}^2}$$

$$\therefore h = \frac{2935}{\text{N}^2}$$

Hence, in this type of governor, the height varies inversely as the speed² and is independent of the weight of the balls.



* In the Porter governor, a central weight is connected by links to the balls, the height of this governor for a given speed, and the change of height for a given change of speed, can be pre-determined by the central load W and the weights of the balls. The sketch shows such a governor where all the links are of the same length. Taking moments about *o*, neglecting the weight of the links and the friction of the sleeve, etc. :—

$$\text{.F.} \times h = w \times r + \frac{\text{W}}{2} \times 2\,r$$

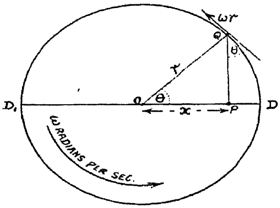
$$\therefore h = \frac{w\,r + }{\text{C.F.}} \qquad \frac{r\,(w + \text{W}) \times 2935}{w \times r \times \text{N}^2}$$

$$\therefore h = (1 \qquad 2935$$

* Simple Harmonic Motion.

Up to the present uniform acceleration only has been considered, but there are forms of motion where the acceleration is not uniform, and simple harmonic motion is one of these.

Simple harmonic motion is a to and fro motion in a straight line, that is, a reciprocating motion, and its important characteristic is that the acceleration is everywhere proportional to the distance from mid-travel. Clearly, then, this is a case of variable acceleration. Such a motion may be generated by the projection upon a diameter, of a point moving in a circular path about a fixed point with constant angular velocity.



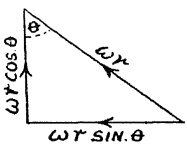
Imagine point Q moving in a circular path about the fixed centre O, with constant angular velocity ω radians per second and with radius r feet Let P be the projection of the point Q upon the diameter DD_1. As Q travels around its circular path, P travels to and fro along the diameter DD_1, and it will be shown that the acceleration of P is always directly proportional to its distance, or displacement, from O.

The **amplitude** of the motion is the maximum displacement from mid-travel, the amplitude is therefore r feet.

The **periodic time** is the time taken for P to move from D to D_1, and back again to D. It is the same time as Q takes to complete its circular path.

Consider OQ at any angle θ to the diameter DD_1, and let the displacement of P from O be x feet.

The constant linear velocity of Q is $\omega\, r$ feet per sec., and at every instant is tangential to the circular path.

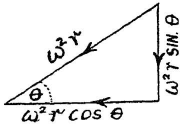


Resolve $\omega\, r$ into components parallel to and perpendicular to $D\,D_1$. Then $\omega\, r$ sin. θ is the velocity of P.

Velocity of P $= \omega\, r$ sin. θ. This must be greatest when sin. θ is greatest because $\omega\, r$ is constant. Now Sin. $90° = 1$ and this is the maximum value of the sine.

$\therefore$ Maximum velocity of P $= \omega\, r$ and it occurs when $\theta = 90°$, that is, at mid-travel.

$$\text{Velocity of P} = \omega\, r \sin.\ \theta = \omega\, r \times \frac{Q\,P}{r} = \omega \times Q\,P$$



The acceleration of Q is $\omega^2 r$, and it is directed towards O. (See page 177).

Resolve $\omega^2 r$ into components parallel to and perpendicular to D D_1. Then $\omega^2 r \cos. \theta$ is the acceleration of P and it is directed towards O.

$$\text{Acceleration of P} = \omega^2 r \cos. \theta = \omega^2 r \times \frac{}{r} = \omega^2 x.$$

$\therefore$ Acceleration of P $= \omega^2 \times$ displacement, that is, the acceleration of P is proportional to the displacement from mid-travel, since ω^2 is constant. This defines simple harmonic motion, and therefore P moves to and fro along D D_1, with S.H.M.

Acceleration of P $= \omega^2 r \cos. \theta$. This is a maximum when $\cos. \theta$ is a maximum. Cos. $0° = 1$, therefore maximum acceleration of P $= \omega^2 r$, and this occurs at the ends of the travel.

$$\text{Acceleration} = \omega^2 \times \text{displacement} \quad \therefore \omega = \sqrt{\frac{\text{acceleration}}{\text{displacement}}}$$

Periodic time :—The time taken for Q to complete the circular path is $\frac{2\pi}{}$ seconds.

$$\therefore \text{Periodic time} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}} \text{ seconds.}$$

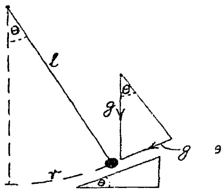
Also, since $\omega = \sqrt{\frac{\text{acceleration}}{\text{displacement}}}$, then when the displacement from mid-travel is one foot, $\omega = \sqrt{\text{acceleration}}$, or, the acceleration is ω^2 when the displacement is 1 foot.

The **Simple Pendulum** is regarded as being a bob weight hung from a weightless string. One complete oscillation is made up of two swings, a swing being described from the highest position on one side to the highest position on the other side.

It may be shown that if the bob swings through a small displacement on either side of its central position then it swings with simple harmonic motion.

Let l = length from the centre of suspension to the centre of the pendulum bob in feet.

Let r = displacement of the bob from mid-position in feet and T = periodic time of one complete oscillation in seconds.



Imagine the bob is on a small inclined plane, the angle of the plane being θ. The acceleration of the bob down the incline is the component of g parallel to the incline.

Acceleration of bob $= g \sin \theta = g\ \theta$ (radian) for a small angle θ.

Acceleration of bob $= g\ \theta$

$$= g \times \frac{\quad}{l} = \frac{\quad}{l} \times$$

Now since g and l are constants, the acceleration is proportional to r, or proportional to the displacement.

This defines simple harmonic motion, and the bob swings with S.H.M.

$$\text{Time} \quad \sqrt{\frac{\text{displacement}}{\text{acceleration}}} \quad \pi \sqrt{\frac{r \times}{g \times r}} = 2\,\pi \sqrt{\frac{l}{\quad}}$$

TEST EXAMPLES XIII.

1. A force of 10 lb. acts on a body of 400 lb. Find the acceleration produced, and the velocity of the body after 12 seconds.

Accel. 0·805 ft. per sec.^{2}; Vel. 9·66 ft. per sec. Ans.

2. A force acting on a body weighing 200 lb. for 10 seconds, changes the velocity of the body from 10 to 22 feet per second. Find the force. 7·452 lb. Ans.

3. A hammer weighing 7 lb. moving with a velocity of 30 feet per second, strikes an object which arrests its motion in $\frac{1}{20}$ of a second. Find the average force of the blow.

130·4 lb. Ans.

4. Through what distance must a force of 8 lb. act on a body of 22 lb. weight, to change its velocity from 12 to 28 feet per second. 27·34 feet. Ans.

5. A cone is solid and weighs 25 lb. It is 20 inches vertical height. Find the work done in turning it over. The base is 20 inches diameter, and the cone is resting on its base.

12·85 ft. lb. Ans.

6. A chain weighs 50 lb. per fathom. If 100 fathoms of this chain hang from a pulley, find the work done in heaving up 80 fathoms of the chain.

1,440,000 ft. lb. Ans.

7. A cage weighing 1,500 lb. is being lowered by a wire rope. Find the tension in the wire (*a*) when the speed is uniform (*b*) when the cage is increasing its downward speed at a rate of 6 feet per (second)^2, (*c*) when the speed is being retarded at 6 feet per (second)^2.

(*a*) 1500 lb.; (*b*) 1220·5 lb.; (*c*) 1779·5 lb. Ans.

8. A chain weighing 10 lb. per foot hangs over a frictionless pulley. The length of the chain hanging below the centre of the pulley on one side is 30 feet, and on the other side 10 feet of chain hang below the centre. Find the work done in pulling the chain down until the lengths on each side of the pulley are equal. 1,000 ft. lb. Ans.

9. A flywheel rim is 10 feet external diameter and 8 feet internal diameter, the width of the rim being 1 foot, and the material weighs 450 lb. per cubic foot. Find the K.E. at 100

revs. per minute. How much work is given out by the wheel when its speed is reduced to 80 revolutions per minute.

438,200 ft. lb. at 100 revs. Work given out 157,700 ft. lb. Ans.

10. A solid disc of cast iron, 4 feet diameter, 6 inches thick, rotates at 80 revolutions per minute. Find the K.E. stored.

6,152 ft. lb. Ans.

11. If in question 9 the shaft is 8 inches diameter and the change of speed takes place in 5 seconds, find the force on the key.

10,035 lb. Ans.

12. Find the pull on a turbine blade due to centrifugal force when rotating at 2,000 revolutions per min., the blade weighing 0·4 lb., and its distance from the rotor centre being 15 inches.

680 lb. Ans.

13. A pulley weighing 50 lb., has its centre of gravity $\frac{1}{8}$ of an inch from its geometrical centre, find the additional load on the bearings when running at 1,000 revs. per minute.

177·1 lb. Ans.

*14. A bullet weighing $1\frac{1}{2}$ ounces leaves the muzzle of a gun at a velocity of 1600 feet per second and is fired into a freely suspended block of wood which weighs 10 lb., and remains embedded in the wood. Find (*a*) the kinetic energy stored in the bullet before impact (*b*) the velocity at which the wood starts to move, and (*c*) the kinetic energy lost in the impact.

3,727 ft. lb. ; 14·86 ft. per sec. ; 3,692·4 ft. lb. Ans.

15. Find the height of a simple conical pendulum governor when rotating at speeds of 50, 75 and 100 revs. per minute respectively, neglecting friction, and the weights of the connecting arms and sleeve.

1 ft. 2·1 ins. ; 6·26 ins. ; 3·52 ins. Ans.

*16. The links of a Porter governor are all the same length. The central weight is 30 lb., and the two balls each weigh 3 lb. Find the difference in height when the speed is increased from 200 to 250 revs. per minute ; neglecting the weights of the links, and friction.

3·487 ins. Ans.

*17. The stroke of an engine piston is 4 feet and the speed of the crank pin is 12·28 radians per second. Find the piston speed and the acceleration of the piston, as the crank passes the points 30°, 60°, 90°, 120° and 150° respectively, neglecting the angularity of the connecting rod (that is, assuming the piston moves with simple harmonic motion).

Velocity : 12·28, 21·27, 24·56, 21·27, 12·28 ft. per sec. Ans.

Accel. : 261·2, 150·75, 0, — 150·75, — 261·2 ft. per (sec).^{2} Ans.

*18. The reciprocating parts of an engine weigh 310 lb., the stroke is 2 ft. 3 ins., and the speed is 120 revs. per minute. Find the accelerating force at the beginning of the stroke.

1,710 lb. Ans.

19. Find the time of a complete oscillation for a simple pendulum 3 feet long.

1·919 seconds. Ans.

20. Find the length of a simple pendulum to make twice as many beats per minute as the one in the previous example.

9 inches. Ans.

CHAPTER XIV.

FRICTION FORCE.

Friction Force.

When a body rests upon a horizontal plane, it exerts a force equal to its weight upon the plane; and since the direction of this force is vertically downwards, it must be at right angles to the plane. If a force is said to be "normal" to a plane, we mean that it acts at right angles to the plane. This downward force has no component at right angles to its own direction, i.e., no component acting horizontally. When we try, by means of a horizontal force to slide the body along the plane, we do, however, find a resisting force acting. It is a matter of experience that this force always acts so as to resist motion; it is called the friction force. It is also known that some materials offer a greater resistance to sliding than others, therefore the friction force appears to be a property of the materials in contact, and to depend upon the condition of the sliding surfaces. Applying a horizontal force then to the body—if this force is too small to make sliding occur, it is less than the friction force, and no movement takes place. Increasing the force until the body is just about to move, i.e., until the applied force is just equal to the resisting friction force, we say that we have reached the *limiting value* of the friction force. Once the body moves, the force required to keep it moving without accelerating, is rather less than the force needed to start it moving; or we may say that the friction of rest is greater than the friction of motion.

The Laws of Friction.

The laws set down here refer to solids moving over plane dry surfaces.

1. The friction force depends upon the nature of the surfaces.
2. It depends upon the condition of the surfaces.
3. It is directly proportional to the normal pressure between the surfaces.
4. It is independent of the area of the surfaces in contact.
5. It is approximately independent of the velocity of rubbing at low and moderate speeds. It is slightly greater just before movement occurs than during motion.
6. It acts in the opposite direction to the force causing motion.

Co-efficient of Friction. Angle of Friction.

Since it is found experimentally that the friction force varies directly as the normal pressure between the surfaces, then

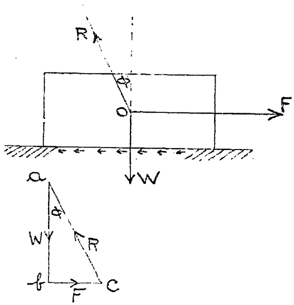
Friction Force $\propto$ Normal Pressure.

$$\therefore \frac{\text{Friction Force}}{\text{Normal Pressure between the surfaces}} = \text{constant.}$$

This constant is called the co-efficient of friction, and it is generally written μ.

$$\therefore \mu = \frac{\text{Friction Force}}{\text{Normal Pressure}} = \frac{\text{Force to move body}}{\text{Weight of body}}$$

when on a horizontal plane, and when the force is applied horizontally, from which Friction Force $= \mu \times W$.



In the diagram, let F be the friction force, and W the weight of the body. F and W have a resultant force. Draw a vector $a\ b$ to represent W, from b draw $b\ c$ parallel to, and representing F. The resultant is $a\ c$. Through o draw R parallel to $a\ c$ in the top figure.

$$\text{Now Tan. } \phi = \frac{F}{W}$$

$$= \frac{\text{Friction Force}}{\text{Normal Pressure}}$$

The angle ϕ is called the friction angle. It is the angle which the reaction R makes with the normal when motion is about to occur, and the tangent of the friction angle is equal to the co-efficient of friction, or Tan. $\phi = \mu$.

Example. An object weighs 20 lb. and rests on a horizontal plane. If the co-efficient of friction is 0·25, find the horizontal force necessary to move the object, and the work done in moving it 10 feet along the plane.

Friction force $= \mu$ W $= 0{\cdot}25 \times 20 = 5$ lb. Ans.

Work done $= 5 \times 10 = 50$ ft. lb. Ans.

Example. A turbine rotor weighs 2 tons, and it runs on a shaft 8 inches diameter. If the co-efficient of friction is 0·05, find the horse power lost in friction when the shaft runs at 2,000 revolutions per minute.

$$\text{Friction Force} = \text{normal pressure} \times \mu$$
$$= 2 \times 2240 \times 0{\cdot}05 \text{ lb.}$$
$$\text{Work per min.} = \text{Force} \times \text{feet moved per minute.}$$
$$\text{Work per min.} = 2 \times 2240 \times 0{\cdot}05 \times \tfrac{22}{7} \times \tfrac{8}{12} \times 2000 \text{ ft. lb.}$$

$$\text{Horse Power} = \frac{\text{Work per minute in ft. lb.}}{33000}$$

Horse Power lost in Friction

$$\frac{2 \times 2240 \times 0{\cdot}05 \times 22 \times 8 \times 2000}{7 \times 33000 \times 12}$$

Horse Power lost in Friction

$$= \frac{4480 \times 0{\cdot}05 \times 2 \times 8 \times 2}{7 \times 3 \times 12} = 28{\cdot}42. \quad \text{Ans.}$$

Example. Find the speed at which a motor car will start to skid when travelling around a horizontal circular track of 40 yards radius. The co-efficient of friction between the tyres and the ground is 0·7.

Let W = weight of car in lb., and v its linear velocity in feet per sec.

$$\text{Then centrifugal force} = \frac{W\,v^2}{r\,g} = \frac{W\,v^2}{40 \times 3 \times 32{\cdot}2} \text{ lb.}$$

and this force tends to cause the car to skid. Resisting this there is frictional force $= \mu\,W = 0{\cdot}7\,W$.

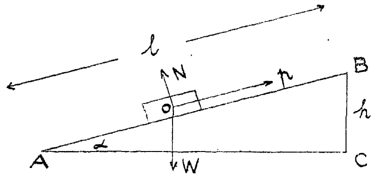
$$\text{When the car is about to skid, } \frac{W\,v^2}{40 \times 3 \times 32{\cdot}2} = 0{\cdot}7\,W.$$

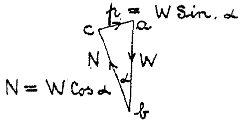
$v^2 = 40 \times 3 \times 32{\cdot}2 \times 0{\cdot}7$ and $v = 52{\cdot}01$ feet per sec., or 35·46 miles per hour. Ans.

Inclined Plane. Without Friction.

Let p be the force, which acting parallel to the plane, keeps the block of weight W in equilibrium.

Now, work done in hauliug the block a distance $l = p \times l$ but this work is also equal to W h.





$$\therefore \; p \times l = W\,h,$$

$$\text{or } p = W\frac{h}{l} = W \text{ Sin. } \alpha$$

or by the triangle of forces: three forces in equilibrium act at o, the downward weight W, the force p, and N, the reaction normal to the plane. Draw a vector $a\ b$ to represent W. From b, draw $b\ c$ parallel to N. From a draw $a\ c$ parallel to p. Then triangle $a\ b\ c$ is similar to A B C. Note that here we have resolved W parallel to the plane; and the component of W at right angles to the plane has no effect in causing motion. The component down the plane is $a\ c$, and we must apply an equal and opposite force p to keep the body in equilibrium.

We have, $p = W \text{ Sin. } \alpha$

$N = W \text{ Cos. } \alpha$

Taking Friction into Account.

From the above force diagram, the normal force between the block and the plane is W Cos. α, and therefore the friction force to be overcome before sliding begins is μ W Cos. α.

Total Force to move body up plane $= W \text{ Sin. } \alpha + \mu\, W \text{ Cos. } \alpha$, or Total Force $=$ force due to incline $+$ force due to friction.

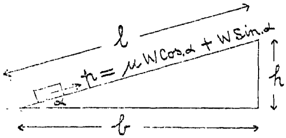
Also, the force to draw body down the plane $= \mu\, W \text{ Cos. } \alpha - W \text{ Sin. } \alpha$, and when the inclination of the plane is such that the body is just on the point of moving down itself without assistance, then $\mu\, W \text{ Cos. } \alpha - W \text{ Sin. } \alpha = o$

$$\text{or } \mu\, W \text{ Cos. } \alpha = W \text{ Sin. } \alpha$$

$$\mu = \frac{W \text{ Sin. } \alpha}{W \text{ Cos. } \alpha} = \text{Tan. } \alpha,$$

but μ is the co-efficient of friction, and we have shown that μ is equal to the tangent of the friction angle; therefore in the case where the body just slides down unaided, the angle of the plane (α) is equal to the friction angle (ϕ). The angle of a plane upon which a body will just begin to slide unaided is often called "the angle of repose" and the tangent of the angle of repose is equal to the co-efficient of friction.

Work done on an Inclined Plane.



From the equation proved, $p = \text{W Sin.} \propto + \mu \text{ W}$

Multiply every term by l, then $p \times l = \text{W Sin.} \propto \times l + \mu \text{W Cos.} \propto \times l$ but $p \times l$ = work done on plane

$$\text{and Sin.} \propto = \frac{h}{l}, \text{ and Cos.} \propto = \frac{b}{l}$$

$$\therefore \text{ Work done} = \text{W} \frac{\ }{l} \times l + \mu \text{ W} \frac{\ }{l} \times l, \text{ cancel } l$$

$$\text{Work done} = \text{W } h + \mu \text{ W } b.$$

or work done = work done in lifting W a vertical height h + the work done against friction in hauling W through a distance equal to the base of the plane.

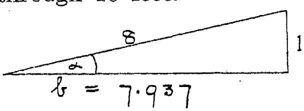
This is a very useful rule, and is important.

Note that when we say that an incline rises 1 in 5, we mean that it rises 1 foot vertically for every 5 feet measured along the incline, and not along the base of the plane.

Example. An inclined plane rises 1 in 10. Find the force to move a body weighing 25 lb. up the plane neglecting friction, and the work done in drawing it a distance of 8 feet along the plane.

$$p = \text{W Sin.} \propto, \text{ and Sin.} \propto = \tfrac{1}{10}$$
$$p = 25 \times \tfrac{1}{10} = 2{\cdot}5 \text{ lb. Ans.}$$
$$\text{Work done} = 2{\cdot}5 \times 8 = 20 \text{ ft. lb. Ans.}$$

Example. An inclined plane has an inclination of 1 in 8, and the co-efficient of friction is 0·2. Find the force to move a body of 50 lb. weight up the plane, and the work done in moving it through 10 feet.



$$b = \sqrt{8^2 - 1^2} = \sqrt{63} = 7{\cdot}937$$

$$\text{Sin.} \propto = \tfrac{1}{8}, \text{ Cos.} \propto = \frac{7{\cdot}937}{8}$$

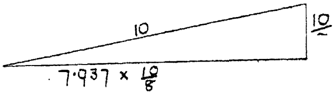
$$\text{Force} = \text{W Sin.} \propto + \mu \text{ W Cos.}$$

$$= 50 \times \tfrac{1}{8} + 0{\cdot}2 \times 50 \times \frac{7{\cdot}937}{\ }$$

$$= 6{\cdot}25 + 9{\cdot}921 = 16{\cdot}171 \text{ lb. Ans.}$$

Work $=$ force $\times$ distance $= 16{\cdot}171 \times 10 = 161{\cdot}71$ ft. lb.
Ans.

or Work $= \mathrm{W}\,h + \mu\,\mathrm{W}\,b$.



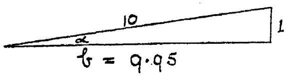
$$= 50 \times \tfrac{10}{8} + 0{\cdot}2 \times 50 \times \frac{7{\cdot}937}{8} \times 10$$

$= 62{\cdot}5 + 99{\cdot}21 = 161{\cdot}71$ ft. lb. as above.

Example. A plane has an inclination of 1 in 10. Find the force needed to draw a weight of 50 lb. down the incline, the co-efficient of friction being 0·18.

$$b = \sqrt{10^2 - 1^2} = \sqrt{99} = 9{\cdot}95$$

Force to draw body down $= \mu$ W Cos. α — W Sin. α ,



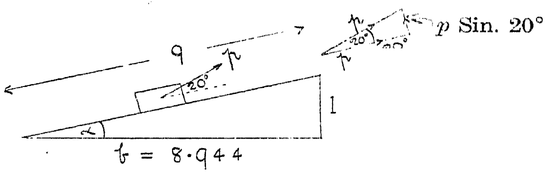
$$= 0{\cdot}18 \times 50 \times \frac{9{\cdot}95}{10} - 50 \times \tfrac{1}{10}$$

$= 8{\cdot}955 - 5 = 3{\cdot}955$ lb. Ans.

In this question the angle of the plane is less than the friction angle, and therefore the body cannot move down unaided; but if the angle of the plane was greater than the friction angle, a force would have to be supplied to prevent the body from moving down. This force $=$ W Sin. $\alpha - \mu$ W Cos. α .

In the examples considered before, the force applied has been parallel to the plane. In the following two examples, the force is taken as acting at angle θ to the plane.

Example. A plane rises 1 in 9. Find the force to move a body of 25 lb. up the plane, if the force makes an angle of 20° with the plane, the co-efficient of friction being 0·16.



$$b = \sqrt{81 - 1} = 8{\cdot}944.$$

Now p, inclined at 20° to the plane, may be resolved into a component parallel to the plane, and a component at right angles to the plane. The component, p Sin. 20°, reduces the normal pressure between the plane and the body.

Normal Press. between plane and body $= W \text{Cos.} \propto - p \text{ Sin.} 20°$
Friction Force parallel to plane $= \mu\ (W \text{ Cos.} \propto - p \text{ Sin.} 20°)$
Total Parallel Force $= W \text{ Sin.} \propto + \mu\ (W \text{ Cos.} \propto - p \text{ Sin.} 20°)$
But, Parallel Force $= p \text{ Cos.} 20°$.

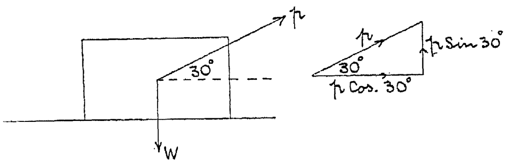
$$\therefore p \text{ Cos. } 20° = W \text{ Sin.} \propto + \mu\ (W \text{ Cos.} \propto - p \text{ Sin. } 20°)$$

$$0{\cdot}9397\ p = \tfrac{25}{9} + 0{\cdot}16 \left(25 \times \frac{8{\cdot}944}{9} - p \times 0{\cdot}342\right)$$

$$0{\cdot}9397\ p + 0{\cdot}05472\ p = 2{\cdot}7777 + 3{\cdot}975$$

$$\frac{6{\cdot}7527}{0{\cdot}99442} = 6{\cdot}79 \text{ lb. Ans.}$$

Example. A body weighing 30 lb. rests on a horizontal plane. Find what force inclined at 30° to the body will be required to cause motion, the co-efficient of friction being 0·2.



Resolving p into components at right angles to each other, we have :—

Vertical Component of $p = p$ Sin. 30°.
Horizontal ,, ,, $= p$ Cos. 30°.

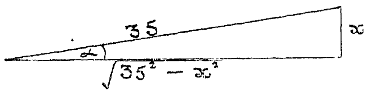
Now the normal pressure between the body and the plane is W before the force p acts. When p is applied, the normal pressure between the surfaces is W — p Sin. 30°. The horizontal component p Cos. 30° is the force available to cause motion.

Force to draw body along plane $= \mu \times$ normal pressure between surfaces.

$$p \text{ Cos. } 30° = \mu\ (W - p \text{ Sin. } 30°)$$
$$p \times 0{\cdot}866 = 0{\cdot}2\ (30 - p \times 0{\cdot}5)$$
$$0{\cdot}866\ p = 6 - 0{\cdot}1\ p$$
$$0{\cdot}966\ p = 6,\ p = 6{\cdot}21 \text{ lb. Ans.}$$

In cases of the inclined or of the horizontal plane, the force is supposed to be acting parallel to the plane, unless the question definitely states otherwise.

Example. A wagon weighing 8 tons is drawn up an incline 35 feet long by a force of 3200 lb. Find the height of the incline if the friction on the level is equal to 15 per cent. of the load.



Total force $=$ μ W Cos. α $+$ W Sin. α

Now friction force $= \mu$ W $= \frac{15}{100} \times 8 \times 2240$.

$$3200 = \tfrac{15}{100} \times 8 \times 2240 \times \text{Cos. } \alpha + 8 \times 2240 \times \text{Sin. } \alpha$$

$$3200 = 8 \times 2240 \, [0{\cdot}15 \text{ Cos. } \alpha + \text{Sin. } \alpha\,]$$

$$\frac{3200}{8 \times 2240} = 0{\cdot}15 \text{ Cos. } \alpha + \text{Sin. } \alpha \text{, put in values of Cos. and Sin.}$$

$$\tfrac{5}{28} = \frac{0{\cdot}15\sqrt{35^2 - x^2}}{35} + \frac{x}{35}\text{; multiply by 35,}$$

$$= 0{\cdot}15\sqrt{35^2 - x^2} + x$$

or $\frac{25}{4} - x = 0{\cdot}15 \qquad - x^2$, square both sides.

$$\frac{625}{16} \quad \frac{25\,x}{2} + x^2 = 0{\cdot}0225\,(35^2 - x^2)\text{, simplifying,}$$

$x^2 - 12{\cdot}22\,x = - 11{\cdot}24$, the solution is

$x \quad 6{\cdot}11 = \pm\, 5{\cdot}108$, $x = 1{\cdot}002$. Ans.

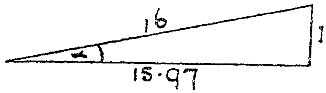
In this problem, however, as the angle of the plane is very small, an approximate method may be used.

Force on plane = friction force on level + force due to incline

$$3200 = \frac{8 \times 2240}{100} \times 15 + 8 \times 2240 \times \frac{x}{35}$$

From which $512\,x = 512$, and $x = 1$ foot, the difference in these two results is negligible. It should be pointed out that the friction force upon the incline is less than the friction force on the level, being the friction force on the level multiplied by the Cosine of the angle of the plane. For inclines where the slope is small the latter method may be used.

Example. A force of 6,000 lb. is required to draw a truck up an inclined plane of 1 in 16. The friction on the level is 10 per cent. of the load. Find the weight of the truck.



$$\text{Base} = \sqrt{16^2 - 1^2} = 15{\cdot}97$$

$$6000 = \mu \, W \text{ Cos. } \propto + W \text{ Sin. } \propto$$

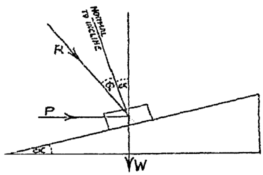
$$6000 = \frac{W}{10} \text{ Cos. } \propto + W \text{ Sin. } \propto$$

$$6000 = \frac{W}{10} \times \frac{15{\cdot}97}{16} + \frac{W}{16}$$

$$6000 = 0{\cdot}0998 \, W + 0{\cdot}0625 \, W = 0{\cdot}1623 \, W$$

$$W = \frac{6000}{0{\cdot}1623} = 36980 \text{ lb. Ans.}$$

***Horizontal Force to move a body up an Incline.**



At the instant the body starts to move up the incline, R the resultant of P and W makes an angle ϕ to the normal to the incline.

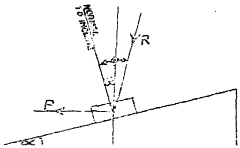
$\text{Tan } \phi = \mu$ (co-efficient of friction)

Draw the vector diagram.

$$\frac{\quad}{W} = \text{Tan } (\phi + \propto)$$

$$P = W \tan (\phi + \propto)$$

***Horizontal Force to pull a body down an Incline.**



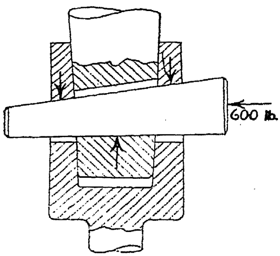
At the instant the body starts to move down the incline the resultant of P and W makes an angle ϕ to the normal to the incline.

Draw the vector diagram.

$$= \text{Tan}$$
$$W$$

$$P = W \tan(\phi - \propto)$$

Example. A cotter, tapered on one edge only, has a taper of 1 in 8. It is driven into a cottered joint with a force of 600 lb. If the co-efficient of friction between the surfaces is 0·2, find the force with which the two parts of the joint are drawn together, and the force to withdraw the cotter.



$\text{Tan} \propto = \frac{1}{8}$, $\propto = 7^\circ\ 7'$ (angle of cotter)

$\text{Tan.} \phi = 0{\cdot}2$, $\phi = 11^\circ\ 19'$ (friction angle)

Adding, $\phi + \propto = 18^\circ\ 26'$

$$P = W \tan(\propto + \phi) + \mu W$$
$$600 = W [\tan 18^\circ\ 26' + 0{\cdot}2]$$
$$600 = W [0{\cdot}3333 + 0{\cdot}2]$$
$$W = \frac{600}{0{\cdot}5333} = 1126 \text{ lb.} \quad \text{Ans.}$$

Note that on the edge of the cotter not tapered, the force to cause sliding is W × co-efficient of friction.

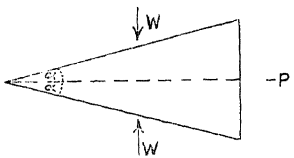
$$- \propto = 11^\circ\ 19' - 7^\circ\ 7' = 4^\circ\ 12'$$

Force to withdraw cotter $= W \tan(\phi - \alpha) + \mu W$

,, ,, ,, $= W [\tan 4° 12' + 0{\cdot}2]$

,, ,, ,, $= 1126 [0{\cdot}0735 + 0{\cdot}2]$

$= 307{\cdot}8$ lb. Ans.

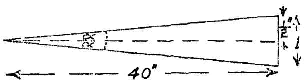


A wedge, tapered on both faces, may be regarded as two inclined planes back to back. The weight to be lifted must (in effect) be caused to move up the incline by a horizontal force.

$$P = W \tan(\phi + \alpha) + W \tan(\phi + \alpha) = 2 W \tan(\phi + \alpha)$$

Note that α is one half of the angle of the wedge.

Example. A propeller wedge, 40 inches long and 1 inch thick at the butt end, is used to start a propeller off the shaft. The estimated force of the blow on the end of the wedge is 1·5 tons. If the co-efficient of friction is 0·2, find the force tending to wedge the propeller off the shaft.



$$\text{Tan } \alpha = \frac{\text{[illegible]}}{40} = 0{\cdot}0125,$$

$$\text{Tan } \phi = 0{\cdot}2, \qquad \phi = 11° 19'$$

$$+ \alpha = 12° 2'$$

$$P = 2 W \tan(\phi +$$

$$W = \frac{P}{2 \tan 12°2'} = \frac{1{\cdot}5}{2 \times 0{\cdot}2132} = 3{\cdot}52 \text{ tons. Ans.}$$

The student should note that $\tan(\phi + \alpha)$ is *not* the same as $\tan \phi + \tan \alpha$.

$\text{Tan}(\phi - \alpha)$ is *not* the same as $\tan \phi - \tan \alpha$.

TEST EXAMPLES XIV.

1. A truck weighing 5 tons is drawn up an incline 30 yards long and 10 feet high. The friction on the level is 15 lb. per ton. If this work is done in 30 seconds, find the horse power exerted. H.P. is 7·196. Ans.

2. The weight of a truck is 5 tons. It is drawn up an incline 12 feet long and one foot high. The tractive force on the level is 12 lb. per ton. Find the work done. 11,919 ft. lb. Ans.

3. An incline slopes at 16 degrees to the horizontal. Find the force to draw a load of 5 tons up the incline if the co-efficient of friction is 0·2. Would the load remain at rest if unsupported ? What must be the angle of the plane if the load is to remain just at rest when unsupported ? 11° 19′. 2·339 tons. Ans.

4. A flywheel weighs 10 tons, its radius of gyration is 3 feet, and it is turning at 200 revolutions per minute. The shaft is 8 inches diameter, and the co-efficient of friction is 0·03. Find the number of revolutions made by the wheel before coming to rest, and the time taken to come to rest. 976 revs. 9·76 mins. Ans.

5. The work done in hauling a load of 10 lb. up a certain inclined plane a vertical distance of 2 feet is 60 ft. lb. The co-efficient of friction is 0·2, find the inclination of the plane. 5° 43′. Ans.

6. Find the force to haul a truck weighing 10 tons down an inclined plane of 1 in 20, if the co-efficient of friction is 0·18. 1·297 tons. Ans

7. An engine exerts a pull of 3 tons on a train of 120 tons weight. The frictional resistance is 10 lb. per ton. How long will it take the train to attain a speed of 30 miles per hour, starting from rest, up an incline of 1 in 80 ? 2 mins. 50·5 secs. Ans.

8. A body of 400 lb. is just about to move up an inclined plane when a force of 142 lb. is applied to it ; and is just about to move down when supported by a force of 25 lb. Find the angle of the plane and the co-efficient of friction . 12° 3′ and 0·15. Ans.

9. A body of 400 lb. moves up an inclined plane when a force of 152 lb. is applied to it, and a force of 5 lb. is required to move it down. Find the angle of the plane and the co-efficient of friction. 10° 35′ and 0·2. Ans.

CHAPTER XV.

MOMENTS : THE SIMPLE MACHINES.

Moment of a Force.

The moment of a force about a point is the product of the force and the perpendicular distance from the point to the line of action of the force.

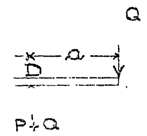
Thus the moment of the force P about the point o is $P \times O\,B$, or $P \times$ arm or leverage.

If any number of forces act upon a rigid body, and if the body remains in equilibrium, then it is apparent that the sum of the moments of the forces tending to turn the body in a clockwise direction, must exactly equal the sum of the moments tending to turn the body in an anti-clockwise direction. Since the body remains in equilibrium, the total effect of all the turning moments about any one point is zero. Also since the body is in equilibrium, the effect of all the vertical forces acting on it must be zero, and the effect of all the horizontal forces acting must be zero. Stating these three fundamental conditions for equilibrium briefly we have :—

The algebraic sum of all the vertical forces must be zero.

The algebraic sum of all the horizontal forces must be zero.

The algebraic sum of all the moments about a point must be zero.



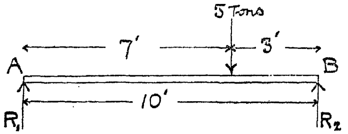
A B is a lever having its fulcrum at D.

Force Q is applied at A, and force P at B.

The moments of these forces about the fulcrum will act in opposite directions, and if the lever is to be in equilibrium then

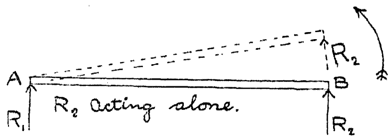
$P \times b = Q \times a$, or $P = Q \times \frac{a}{b}$, and we see that a small force P at a large leverage b may be made to balance a larger force Q at a small leverage a. Since the total upward force must equal the total downward force, it follows that the upward

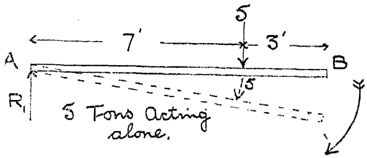
reaction at D must be $P + Q$. There are no horizontal forces acting here. The forces P and Q are parallel forces and they act in the same vertical plane.



Consider a beam 10 feet long, supported at A and B, carrying a load of 5 tons at 7 feet from A. We have to determine the reactions or supporting forces at the ends.

Now both R_1 and R_2 are unknown, but if we take moments round A we only have to deal with the force R_2 and its moment about A, since R_1 acting through the point A has no leverage and therefore no moment about A. Taking moments round A :—the only moment tending to turn, or rotate, the beam in an anti-clockwise direction about A is the moment $R_2 \times 10$, and if this moment was acting alone, the beam would rotate about A.





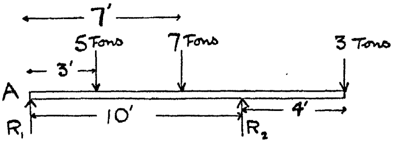
The only moment tending to turn, or rotate, the beam about A in a clockwise direction is the moment 7×5 ft. tons. This moment acts so as to cause rotation in the opposite direction to the moment $R_2 \times 10$, and as the beam is in eqilibrium about the point A under the action of these moments, then $R_2 \times 10 = 5 \times 7$, from which $R_2 = \frac{35}{10} = 3{\cdot}5$ tons.

Now $R_1 + R_2 = 5$, because the sum of the upward forces must equal the sum of the downward forces.

$$\therefore R_1 = 5 - R_2 = 5 - 3{\cdot}5 = 1{\cdot}5 \text{ tons.}$$

The method adopted here is often called the Principle of Moments. We sometimes call a clockwise moment a right hand moment, and an anti-clockwise moment a left hand moment.

Example. A sketch is shown of a beam having three loads. Determine the two reactions R_1 and R_2.



Take moments round A. Then for equilibrium :—

Right hand moments = left hand moments.

$(5 \times 3) + (7 \times 7) + (3 \times 14) = R_2 \times 10$. Observe that moments of *all* forces must be taken about A.

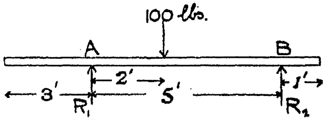
$15 + 49 + 42 = R_2 \times 10$ from which

$R_2 = \frac{106}{10} = 10\cdot6$ tons.

$R_1 = (5 + 7 + 3) - 10\cdot6 = 15 - 10\cdot6 = 4\cdot4$ tons. Ans.

When solving a problem by the Principle of Moments, the point about which the moments of the forces are taken *must* be stated. It is *not* sufficient to make the statement " Right hand moments = left hand moments."

Example. A uniform bar is 9 feet long and weighs 5 lb. per foot of length. It is supported as shown, and carries a load of 100 lb. at 2 feet from A. Find the two reactions.



Since the bar is uniform, its centre of gravity is at its centre of length, and the whole weight of the bar may be taken as being concentrated there. The weight of the bar then is always taken as acting at the centre of gravity.

The whole weight of the bar is $9 \times 5 = 45$ lb.

Take moments round A.

Left hand moments = right hand moments.
$R_2 \times 5 = 100 \times 2 + 45 \times (4\cdot5 - 3)$

Note the C.G. is at 4·5 feet from the left hand, and this is $4\cdot5 - 3$ or 1·5 feet from A.

$$R_2 \times 5 = 200 + 45 \times 1\cdot5$$

$$R_2 = 267\cdot5 \qquad = 53\cdot5 \text{ lb. Ans.}$$

Note that a moment is in feet × lb. units, and therefore a moment divided by a distance must be lb. Thus :—

$$\frac{\text{Moment}}{\text{Distance}} \qquad \frac{\text{foot lb.}}{\text{feet}} \qquad \frac{\text{length} \times \text{lb.}}{\text{length}}$$

R_1 = total downward weight — R_2

$R_1 = (100 + 45) - 53{\cdot}5 = 91{\cdot}5$ lb. Ans.

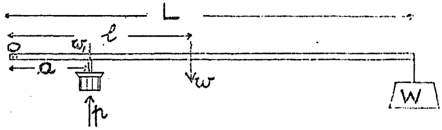
The student may check these results by taking moments round B, as follows :—

$R_1 \times 5 = (100 \times 3) + 45 \times (4{\cdot}5 - 1)$

$R_1 \times 5 = 300 + 157{\cdot}5,\ R_1 = \dfrac{457{\cdot}5}{5} = 91{\cdot}5$ lb.

The Lever Safety Valve.

The lever is pivoted or hinged at the fulcrum o. The heavy load W, the weight of the lever w, and the weight of the valve and spindle w_1, keep the lever down on top of the valve.



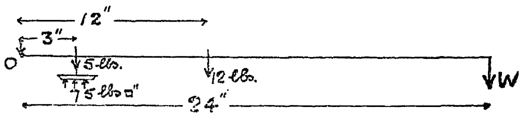
Let A be the area of the valve, p the steam pressure in lb. per square inch. Note that w, the weight of the lever, acts at l from the fulcrum. Take moments round o, then :—

Left hand moments = right hand moments.

$(p \times A) \times a = (W \times L) + (w \times l) + (w_1 \times a)$

Now in this equation, any *one* term may be unknown. We may be asked to find p, A, a, W, L, w or w_1, but only one of these.

Example. A safety valve is 3 inches diameter, and the boiler pressure is 75 lb. per square inch. The uniform lever weighs 12 lb., the valve and spindle weigh 5 lb. If the lever is 24 inches long, and the valve 3 inches from the fulcrum, find the load needed to hang from the end of the lever.



Area of valve $= 3 \times 3 \times \frac{11}{14} = 7{\cdot}07$ sq. inches.

Take moments round o, then :—

Right hand moments = left hand moments.

$$(W \times 24) + (12 \times 12) + (5 \times 3) = 7{\cdot}07 \times 75 \times 3$$
$$24\ W = 1590{\cdot}75 - 144 - 15 = 1431{\cdot}75$$

$$W = \frac{1431{\cdot}75}{24} = 59{\cdot}65 \text{ lb.} \quad \text{Ans.}$$

To find the shearing force on the fulcrum pin, we have,

Total downward force = total upward force.

Force on fulcrum $+ 5 + 12 + 59{\cdot}65$ = area of valve $\times$ steam pressure.

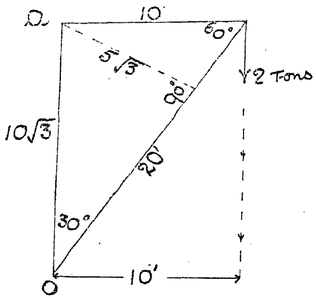
Force on fulcrum $= 7{\cdot}07 \times 75 - (5 + 12 + 59{\cdot}65)$
Force on fulcrum $= 530{\cdot}25 - 76{\cdot}65$
Force on fulcrum $= 453{\cdot}6$ lb. Ans.

Note this force is downwards, the only upward force being the total steam load on the valve.

Application of Method of Moments to Simple Frames.

A simple frame is a structure made up of members or bars, pin jointed at their ends. For instance, a derrick used for lifting weights is hinged on the mast by means of a double eye, and this is called a pin joint ; at the head of the derrick, the tie is fastened to an eye or ring on the derrick, and to an eye on the mast at its other end. We may say that all the three joints have the same freedom of movement as if they were pin jointed. The method of taking moments about a point, may be used to determine the forces acting in the members or bars of framed structures. We proceed to give some simple cases of the application of this method.

Example. A derrick 20 feet long makes an angle of 30° with the mast, the tie from the head of the derrick being horizontal. Find the forces acting in the derrick and the tie when a load of 2 tons is hung from the derrick head.



Consider the point o. The force acting in the derrick has no moment about o, because it has no leverage about o. The load of 2 tons has a moment about o; the line of action of this force is distant from o a perpendicular distance of 10 feet. Also, the force in the tie has a moment about o; the line of action of this force being distant from o a perpendicular distance of $10\ \sqrt{3}$ feet.

Taking moments about o, we have :—

$$2 \times 10 = \text{force in tie} \times 10\ \sqrt{3}$$

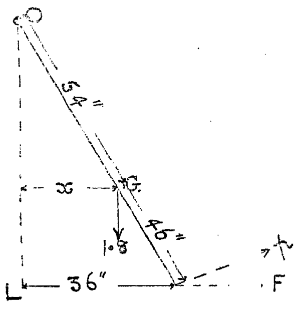
$$\text{Force in tie} = \frac{20}{10\ \sqrt{3}} = 1{\cdot}154 \text{ tons. Ans.}$$

Consider next the point a. The force in the tie has no moment about a. The force of 2 tons has a moment about a, equal to 2×10. The line of action of the force in the derrick is distant $5\ \sqrt{3}$ feet from a. The student must remember that the moment of a force about a point is the product of the force and its *perpendicular* distance from that point.

Taking moments about a, we have :—

$$2 \times 10 = \text{force in derrick} \times 5\ \sqrt{3}$$

$$\text{Force in derrick} = \frac{2 \times 10}{5\ \sqrt{3}} = 2{\cdot}309 \text{ tons. Ans.}$$



Example. A connecting rod 100 inches long weighs 1·8 tons, its centre of gravity is 46 inches from the lower end, and the rod hangs vertically. Find the least force, also the horizontal force applied at the lower end of the rod to pull it 3 feet from the vertical.

Let the rod hang from the point o. The weight of 1·8 tons has its line of action a perpendicular distance x from the point o.

By similar triangles, $\frac{}{54} =$ $\qquad x = 19{\cdot}44$ inches.

The least force p will be at 90° to the rod.

Taking moments about o, we have :—

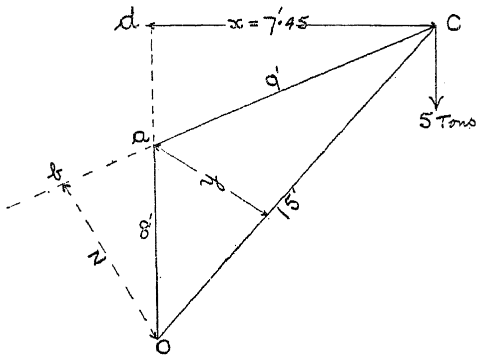
$19{\cdot}44 \times 1{\cdot}8 = p \times 100$; $p = 0{\cdot}3499$ ton. Ans.

The leverage of the horizontal force F will be the distance o L, and o L $= \sqrt{100^2 - 36^2} = 93{\cdot}28$ inches.

Taking moments about o, we have :—

$19{\cdot}44 \times 1{\cdot}8 = \text{F} \times 93{\cdot}28$; $\text{F} = 0{\cdot}3752$ ton. Ans.

Example. A derrick 15 feet long is supported by a tie 9 feet long, the tie being fastened to the mast 8 feet above the foot of the derrick, find the forces in the tie and derrick, when a load of 5 tons is lifted.



The line of action of the force in the tie is along $b\ c$, and the perpendicular distance of the line $b\ c$ from the point o is z. The line of action of the load of 5 tons is a perpendicular distance x from the point o.

$$\text{Vertical height of triangle} = \frac{\text{area}}{\frac{1}{2}\text{ base } a\ o}, \text{ above base}$$

$$x = \frac{\text{area of triangle } o\ a\ c}{}$$

$$\sqrt{s\,(s - a)\,(s - b)\,(s - c)}$$

$$= \tfrac{32}{2} = 16 \; ; \; x = \sqrt{16\,(16-15)\,(16-9)\,(16-8)}$$

$$\frac{29\cdot93}{4} = 7\cdot48 \text{ feet.}$$

Also triangles $c\ d\ a$ and $a\ b\ o$ are similar.

$$\therefore \frac{7\cdot48}{\quad} = \frac{z}{}, \text{ and } z = 6\cdot65 \text{ feet, nearly.}$$

Take moments about o,

$$5 \times x = \text{force in tie} \times z$$

$$5 \times 7\cdot48 = \text{force in tie} \times 6\cdot65$$

$$\text{Force in tie} = \frac{5 \times 7\cdot48}{\quad} = 5\cdot624 \text{ tons.} \quad \text{Ans.}$$

The force in the derrick acts about a at a perpendicular distance y, and y is readily found by dividing the area of the triangle $o\ a\ c$ by 7·5.

$$y = \frac{\text{Area of triangle } o\ a\ c}{\frac{1}{2} \text{ base } o\ c} = \frac{29\cdot93}{7\cdot5} = 3\cdot99 \text{ feet.}$$

Take moments about a,

$$5 \times x = \text{force in derrick} \times y$$

$$5 \times 7\cdot48 = \text{force in derrick} \times 3\cdot99$$

$$\text{Force in derrick} = \frac{5 \times 7\cdot48}{3\cdot99} = 9\cdot373 \text{ tons.} \quad \text{Ans.}$$

Examples of this kind have already been solved by the method of the triangle of forces in the Chapter on Vectors, and that method should generally be used in such problems. The solution by the method of moments is given as a further illustration of the principle of moments.

The Simple Machines.

A machine is a mechanism, or an apparatus, for transforming work, or for modifying force. The force which a man unaided has at his command is not great, but he may be capable of exerting this force through a great distance, thus performing work. The same amount of work could be done by exerting a greater force through a smaller distance. To take in work, supplied to it

by a comparatively small force moving through a great distance and to deliver this work in the form of a large force moving through a small distance, is the function of all lifting machines.

The Velocity Ratio of a Machine, written V.R., is the ratio of the distance moved by the applied force or effort, to the distance moved by the weight or load lifted, or :—

$$\text{V.R.} = \frac{\text{Distance moved by effort in a given time}}{\text{Distance moved by load in the same time}} \quad . \quad . \quad (1)$$

By the law of the conservation of energy, the machine should give out work equal to the work supplied to it. Some of the work given is dissipated in overcoming the friction of the moving parts of the machine, and cannot be recovered. We have therefore :—

Work put in = work given out + work lost in friction.

The Mechanical Advantage of a machine, written M.A., is the ratio of the load lifted to the effort applied, or :—

$$\text{M.A.} = \frac{\text{Load lifted}}{\text{Effort applied}} \qquad (2)$$

The Efficiency of a machine is the ratio of the work given out by the machine, to the work put into it.

$$\text{Efficiency} = \frac{\text{Work given out}}{\text{Work put in}}$$

$$\text{Efficiency} = \frac{\text{Load} \times \text{distance moved by load}}{\text{Effort} \times \text{distance moved by effort}}$$

$$\text{Efficiency} = \text{M.A.} \times \frac{}{\text{V.R.}} = \frac{\text{M.A.}}{\text{V.R.}} \text{ from (2) and (1)}$$

$$\text{From (2) we have, Effort} = \frac{\text{Load lifted}}{\text{M.A.}}$$

Note.—If there was no friction, V.R. would equal M.A., and the effort would be $\frac{\text{Load lifted}}{\text{V.R.}}$, or for a given effort P, the load lifted would be P × V.R. Hence the "load lost" due to friction is Theoretical load — Actual load = **P × V.R. — W.**

Also, since Efficiency $= \dfrac{\text{Mechanical advantage}}{\text{Velocity ratio}}$, we have

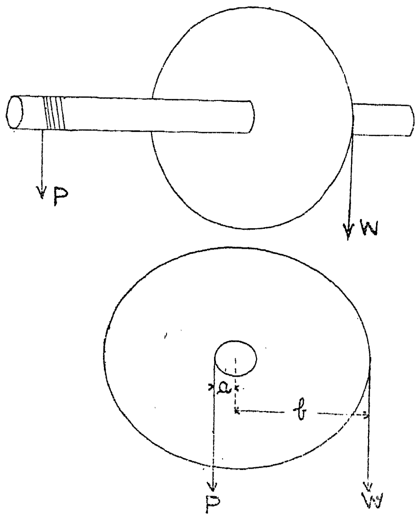
$$\text{Efficiency} = \frac{\text{Load lifted}}{\text{Effort}} \times \frac{1}{\text{Velocity ratio}},$$ and by

transposing we may write :—

$$\text{Load lifted} = \text{Effort} \times \text{Efficiency} \times \text{Velocity ratio.}$$

Under the heading **Simple Machines** we generally classify the lever, the wheel and axle, lifting tackle or pulley blocks, the compound winch, the screw and the inclined plane. The lever and the inclined plane have already been explained.

Wheel and Axle.



This apparatus is made up of a cylinder of small diameter and a wheel or pulley of larger diameter rigidly fastened togethe as shown.

Taking moments about the centre :—

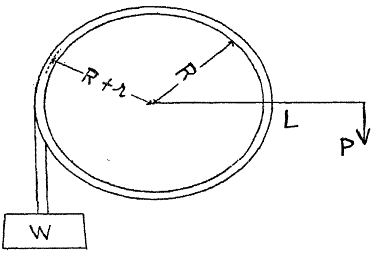
$$P \times a = W \times b$$

$$P = \frac{W\ b}{a}, \text{ or } W = \frac{P\ a}{b}$$

$$\text{V.R.} = \frac{}{2\ \pi\ a} \qquad \frac{b}{a}$$

An example of the wheel and axle is the hand ash hoist, the cranked handle taking the place of the pulley. Another example is the hand capstan.

If L is the length of the handle and r the radius of the rope, then by the principle of work :—



$$\text{or } W = \frac{P\ L}{(R + r)}$$

$$\text{and V.R.} = \frac{L}{(R + r)}$$

The same result is obtained by taking moments about the centre.

Example. In a machine of the wheel and axle type, the handle is 15 inches long, the barrel or drum is 6 inches diameter, and the rope is 1 inch diameter. Find the load which can be lifted by a force of 50 lb. applied to the handle (a) neglecting friction ; (b) if the efficiency is 88 per cent.

$$\text{V.R.} = \frac{}{R + r} \qquad \frac{15}{3 + \frac{1}{2}} = \frac{30}{7}$$

(a), Load lifted $=$ Effort $\times$ V.R.

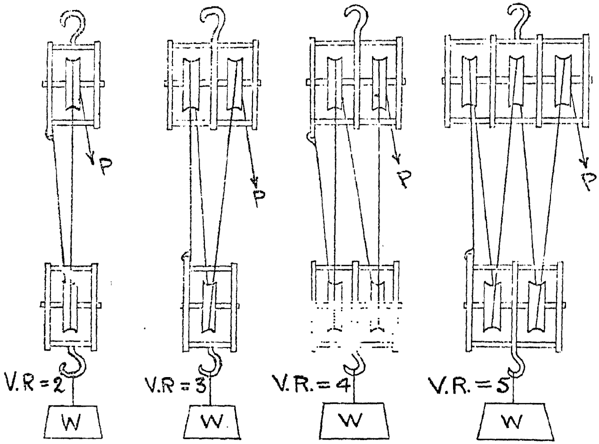
$= 50 \times \frac{30}{7} = 214\frac{2}{7}$ lb. Ans.

(b), Load lifted $=$ Effort $\times$ V.R. $\times$ Efficiency

$= 50 \times \frac{30}{7} \times \frac{88}{100} = 188\frac{4}{7}$ lb. Ans.

Rope Blocks.

The sketches show the usual arrangements for rope blocks. Note that if there are the same number of pulleys in the top and bottom blocks, the end of the rope is made fast to the top block; if there is one sheave more in the top than in the bottom block, the end of the rope is fastened to the bottom block.



In any of the arrangements shown, if the lower block is raised one inch, then each of the ropes fastened to the lower block must be shortened one inch; let there be n ropes supporting the lower block, then when the lower block is raised one inch each rope is shortened one inch, and the fall end must be moved through $n \times 1$ inches.

$$\text{V.R.} = \frac{\text{Distance moved by Effort}}{\text{Distance moved by Load}} = \frac{n \times 1}{1} = n$$

Or V.R. = number of ropes supporting the lower block.

= total number of pulleys in the arrangement.

Assuming the friction of each pulley to be the same :—

Let E = overall efficiency, and e = efficiency of each sheave,

then, $E = e_1 \times e_2 \times e_3 \times$ etc., $\therefore E = e^n$ $\qquad e =$

where n is the number of pulleys.

Example. A pair of rope pulley blocks has 3 pulleys in each block. An effort of 80 lb. raises a load of 360 lb. hanging on the lifting hook. The weight of each block is 20 lb. Neglecting the weight of the rope, find (a) the overall efficiency, (b) efficiency of each pulley, and (c) the total load on the top hook.

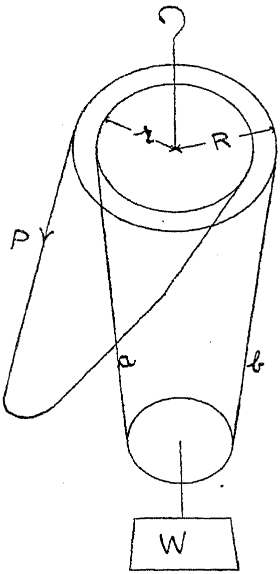
$$\text{V.R.} = \text{number of pulleys} = 6, \quad \text{M.A.} = \frac{W}{P} = \frac{360}{80} = 4{\cdot}5$$

$$\text{Overall efficiency} = \frac{\text{M.A.}}{\text{V.R.}} = \frac{4{\cdot}5}{6} = 0{\cdot}75 \text{ or } 75\% \quad \text{Ans. } (a)$$

$$\text{Efficiency of each pulley} = \sqrt[6]{0{\cdot}75} = 0{\cdot}9532 \text{ or } 95{\cdot}32\% \quad \text{Ans. } (b)$$

$$\begin{aligned}\text{Total load on the top hook} &= \text{load} + \text{effort} + \text{weight of gear}\\ &= 360 + 80 + 20 + 20\\ &= 480 \text{ lb.} \quad \text{Ans. } (c)\end{aligned}$$

Differential Pulley Block.



The top block consists of two pulleys of slightly different diameters, cast in one piece.

Let the top pulleys make one complete revolution. The chain b will move up a distance $2\pi R$, and if the chain a did not move, the lower block would rise a distance

of · During the same revolution the chain a moves down a distance of $2\pi r$, and if chain b did not move, the lower block would be

lowered a distance But as

b moves up, a moves down, and the block actually rises a distance

of · $2\pi R$ $2\pi r$ or $(R - r)$

$$\text{V.R.} = \frac{\text{Distance moved by Effort}}{\text{Distance moved by Load}} \qquad \frac{2\ \pi\ \text{R}}{\pi\ (\text{R} - r)}$$

$$\text{V.R.} = \frac{\text{R}}{(\text{R} - r)} \qquad \frac{2\ \text{D}}{(\text{D} - d)}$$

$$\text{or V.R.} = \frac{2 \times \text{No. of teeth in large wheel}}{\text{Diff. in No. of teeth in large and small wheels}}$$

The loss by friction is always great in these blocks, amounting to more than 50 per cent., so that the efficiency is always less than 50 per cent. In any lifting tackle, if the efficiency is less than 50 per cent., the block cannot overhaul or "run back" if left with the load suspended.

Suppose that in a certain set of blocks, V.R. = 10, that the efficiency is 0·48, and that the effort is 20 lb., then

Useful Effort = 20 × 0·48 = 9·6 lb.

Effort wasted = 10·4 lb.

Load lifted = 9·6 × 10 = 96 lb.

Now let the Effort be removed, the load being left suspended from the block. The Load of 96 lb. is equivalent to 9·6 lb. at the Effort end of the machine, and this is not capable of overcoming the friction, since the force needed to overcome friction is 10·4 lb., and the block cannot run back.

Example. In a Weston pulley block the diameters of the top sheaves are 10 and 9 inches respectively. The efficiency is 0·45 ; find the load which can be lifted by an effort of 40 lb.

$$\text{V.R.} = \frac{2\ \text{R}}{(\text{R} - r)} \qquad \frac{2 \times 5}{5 -} = \frac{10}{\frac{1}{2}} = 20$$

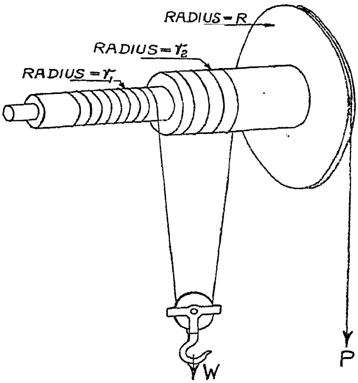
Load lifted theoretically = 20 × 40 = 800 lb.

Load actually lifted = 20 × 40 × 0·45 = 360 lb. Ans.

M.A. = 20 × 0·45 = 9.

Wheel and Differential Axle.

Sometimes called the Chinese Windlass. Although a heavy weight can be lifted by a comparatively small effort, this machine is of little use in practice on account of the great length of rope that would be required to lift the weight through an appreciable distance.



It consists of two cylinders having unequal diameters, and a wheel or pulley rigidly fastened together on a common axle. A crank handle may take the place of the wheel.

Let the machine be turned one complete revolution, the effort P will move a distance of 2 π R, and by the same reasoning as in the differential pulley block the lifting hook will rise a distance of

$$\text{V.R.} = \frac{\text{Distance moved by Effort}}{\text{Distance moved by Load}} \qquad \text{R}$$

2 R 2 D

or

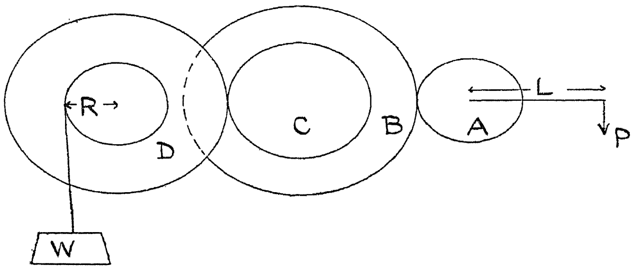
$\cdot\, d_1$

Compound Winch.

The compound winch, a lifting machine in which a great velocity ratio may be obtained, is a developed form of the wheel and axle.

A, B, C and D are gear wheels. The drum is rigidly secured to the wheel D, and wheels B and C are rigidly fastened together.

Let wheel D turn one revolution, then the drum also turns one revolution, and W moves a distance 2 π (R + r).



R = radius of drum.
= radius of rope.
= length of handle.

= weight lifted.
= effort or force applied.

For one turn of D,

$$\text{Revs. of C} = \frac{\text{Number of teeth in D}}{\text{Number of teeth in C}}$$

$$\text{Revs. of A} = \text{Revs. of C} \times \frac{\text{Number of teeth in B}}{\text{Number of teeth in A}}$$

Revs. turned by A for one turn by D

$$= \frac{\text{Teeth in D}}{\text{Teeth in C}} \times \frac{\text{Teeth in B}}{\text{Teeth in A}}$$

$$\text{Distance moved by P} = 2\,\pi\,\text{L} \times \frac{\text{Teeth in D}}{\text{Teeth in C}} \times \frac{\text{Teeth in B}}{\text{Teeth in A}}$$

$$\text{V.R.} = \frac{\text{Distance moved by P}}{\text{Distance moved by W}}$$

$$= \frac{2\,\pi\,\text{L}}{2\,\pi\,(\text{R} + r)} \times \frac{\text{Teeth in D}}{\text{Teeth in C}} \times \frac{\text{Teeth in B}}{\text{Teeth in A}}$$

$$\text{V.R.} = \frac{}{(\text{R} + r)} \times \frac{\text{Teeth in D}}{\text{Teeth in C}} \times \frac{\text{Teeth in B}}{\text{Teeth in A}}$$

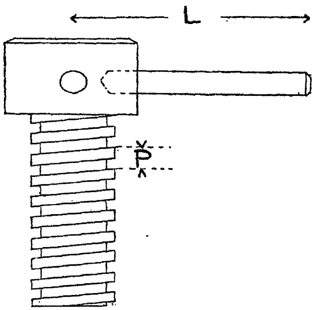
Example. In the compound winch shown in sketch, A has 12 teeth, B has 48 teeth, C has 36 teeth and D has 110 teeth. The drum is 9 inches diameter, the rope 1 inch diameter and the handle is 18 inches long. If the force at the handle is 50 lb., find what weight can be lifted, if the efficiency is 80 per cent.

$$\text{V.R.} = \frac{18}{(4\frac{1}{2} + \frac{1}{2})} \qquad \times \tfrac{48}{12} = 44$$

Weight lifted theoretically $= 44 \times 50 = 2200$ lb.

Weight actually lifted $= 44 \times 50 \times \frac{80}{100} = 1760$ lb.

The Screw.



A very useful form of lifting machine is the screw jack. The pitch of a screw is the axial distance which the screw advances through a fixed nut, in one revolution. For one complete revolution of the handle, the load will be lifted a distance equal to the pitch.

$$\text{V.R.} = \frac{\text{Distance moved by Effort}}{\text{Distance moved by Load}} \qquad \text{Pitch of thread}$$

Example. A screw jack, in which the pitch of the thread is $\frac{1}{4}$ inch, or 4 threads per inch, is worked by a handle 20 inches long. If the efficiency is 35 per cent., find the load which can be lifted by a force of 60 lb. at the handle.

$$\text{V.R.} = \quad \times 20 \quad = 4 \times 40$$

$$\text{V.R.} = 502{\cdot}8, \text{ load lifted } = 502{\cdot}8 \times \tfrac{35}{100} \times 60$$

Load lifted $= 10{,}559$ lb. or $4{\cdot}714$ tons. Ans.

By performing a number of experiments on any of these simple machines, the efforts required to lift different loads can be obtained, and from these data, graphs representing the effort, efficiency, and load lost due to friction, with respect to the load, can be plotted on squared paper.

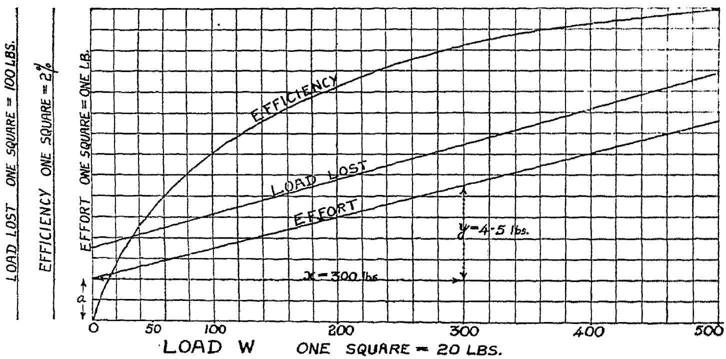
Let the data taken from an experiment on a screw jack whose velocity ratio is 176, be as follows :—

Load W (lb.)	0	50	100	200	300	400	500
Effort P (lb.)	2	2·75	3·5	5·1	6·5	8	9·5

The mechanical advantage $\left(\frac{W}{P}\right)$, efficiency $\left(\frac{M.A.}{V.R.}\right)$, and load lost (P × V.R. — W), should now be worked out for each case and tabulated, thus :—

Load W lb.	0	50	100	200	300	400	500
Effort P lb.	2	2·75	3·5	5·1	6·5	8	9·5
M.A. ...	0	18·2	28·6	39·2	46·1	50	52·6
Efficiency %	0	10·33	16·24	22·28	26·22	28·41	29·9
Load lost lb.	352	434	516	697·6	844	1008	1172

With load W as a base, plot the curves to suitable scales :-



It will be observed that the curve joining the plotted values of effort and load is a straight line. In some cases the plotted points may not lie exactly in line, a straight line, however, should be drawn as near as possible through these points; the difference between this straight line and the given values of the efforts are probably due to errors of observation during the experiment.

We can now evolve an expression to connect P and W. Choose any two points on this curve and construct y and x as shown, y is the increase of effort for a corresponding increase of load

represented by x. The ratio $\dfrac{y}{\quad}$ is the same for any part of this curve, because, being a straight line, its slope is constant. Let this ratio $\dfrac{y}{x}$ be represented by b. Also, let an effort of a lb. be required to cause movement in the machine when there is no load. If the load be increased from 0 to W lb., the effort must be increased from a lb. to b W lb., and therefore the total effort required to lift W will be $a + b$ W lb. Therefore, $P = a + b\,W$. This expression is called the **Linear Law** of the machine. a and b will, of course, have different values for every different machine.

In the above example, $a = 2$ and $b = \dfrac{y}{x} = \dfrac{4 \cdot 5}{300} = 0 \cdot 015$, hence the linear law of this screw jack is $P = 2 + 0 \cdot 015\,W$.

If we wish to lift a load of 350 lb. with this screw jack, the effort required would be, $P = 2 + 0 \cdot 015 \times 350 = 2 + 5 \cdot 25 = 7 \cdot 25$ lb.

The mechanical advantage when lifting this load is $\dfrac{W}{P} = \dfrac{350}{7 \cdot 25} = 48 \cdot 3$. The efficiency is $\dfrac{48 \cdot 3}{176} \times 100 = 27 \cdot 4\%$, and the load lost due to friction is $7 \cdot 25 \times 176 - 350 = 926$ lb.

If two definite values of efforts are known for two different loads lifted on a machine, the values of a and b in the linear law equation for that machine can be calculated by a simultaneous equation, as in the following example.

With a differential pulley block, an effort of 13·5 lb. just lifts a load of 100 lb. and an effort of 43·5 lb. just lifts a load of 400 lb. Find the probable effort required to lift 500 lb.

$$= a$$
$$= a +$$

$$30 = 300$$

$$\therefore b = \frac{30}{300} = 0 \cdot 1$$

$13 \cdot 5 = a + 0 \cdot 1 \times 100 \quad \therefore a = 13 \cdot 5 - 10 = 3 \cdot 5$

The law is $P = 3 \cdot 5 + 0 \cdot 1\,W$

When $W = 500$, $P = 3 \cdot 5 + 0 \cdot 1 \times 500 = 53 \cdot 5$ lb. Ans.

TEST EXAMPLES XV.

1. A uniform bar weighs 10 lb. and is 8 inches long. It is supported by a fulcrum which is not at the centre of length of the bar, and maintained in equilibrium by a load of 3 lb. hung from one end. Find the position of the fulcrum.

3·077 inches from 3 lb. weight. Ans.

2. A horizontal bar is 48 inches long. A weight of 4 lb. is hung from one end, and a weight of 7 lb. from the other end, the bar itself weighing 20 lb. Find the position of the fulcrum to give equilibrium.

21·68 inches from 7 lb. end. Ans.

3. A connecting rod 6 feet long lies horizontally with one end on the ground and the other end on the table of a weighing machine. In this position the scale reads 575 lb. The rod is now reversed so that its other end lies on the weighing machine, and the scale now reads 300 lb. Find the total weight of the rod, and the position of the centre of gravity.

Weight 875 lb. 2·057 feet from heavy end. Ans.

4. A lever safety valve is $3\frac{1}{4}$ inches diameter. From the end of the lever to the valve is 20 inches, and from the fulcrum to the valve is 4 inches. The valve and spindle weigh 3 lb., the lever weighs 10 lb., and its centre of gravity is 8 inches from the fulcrum. Find the weight to hang from the end of the lever if the valve is to lift at 50 lb. per sq. inch.

65·34 lb. Ans.

5. In a lever safety valve of 3 inches diameter, the lever weighs 12 lb., is 28 inches long and its centre of gravity is 10 inches from the fulcrum. When 56 lb. is hung from the end of the lever, the valve blows at 50 lb. per sq. inch. Find the distance from the valve to the fulcrum, the valve and spindle weigh 4 lb.

4·829 inches. Ans.

6. A pump lever on the main engines is 9 feet long, the rocking shaft being at 3 feet from the pump end. When the force applied at the pump end is 2 tons, find the total force on the rocking shaft bearing.

3 tons. Ans.

7. A beam 24 feet long has one support at its left hand end, its other support being 8 feet from the right hand end. From the right hand end a load of 5 tons is hung, and the beam carries a uniform load of half a ton per foot. Find the load on each support.

0·5 ton. 16·5 tons. Ans.

8. A body weighing w lb., is weighed in a false balance. It is found to weigh w_1 lb. in one scale and w_2 lb. in the other scale. Show that its true weight w is $\sqrt{w_1 \times w_2}$. In such a balance a body weighs 50 lb. in one scale and 45 lb. in the other scale, find its true weight. 47·43 lb. Ans.

9. A square plate of 2 feet side, has a hole 8 inches diameter drilled in it on a diagonal, the centre of the hole being 8 inches from a corner of the plate. Find the position of the centre of gravity of the plate.

0·85 inch from centre of square. Ans.

10. A triangular plate has sides 12, 15 and 18 inches A square hole of 4 inch side is cut out, the side of the hole being parallel to the 18 inch side, and the centre of the hole being 3 inches from this side Find the distance the centre of gravity of the plate is from the 18 inch side.

3·373 inches. Ans.

11. A beam of I section has a top flange 6 inches wide, a bottom flange 8 inches wide and a total depth of 10 inches, the thickness throughout for web and flanges being half an inch. Find the position of the centre of gravity.

4·587 inches above base. Ans.

12. A casting for an oil engine piston is 8 inches diameter and 20 inches total length. The flat end is one inch thick, the trunk being 0·75 inch thick. The bosses for the top end pin are 4 inches diameter and 2 inches long, their centres being at 8 inches from the closed end. Find the position of the centre of gravity.

9·02 inches from closed end. Ans.

13. A set of rope blocks has 3 sheaves at the top and 2 sheaves at the bottom. Find the weight lifted when a pull of 230 lb. is applied, the efficiency being 62 per cent. Find also the load which can be lifted theoretically.

713 lb. ; 1150 lb. Ans.

14. In a differential pulley block the pulleys are 10 inches and 9 inches diameter respectively. If the efficiency is 35 per cent., find the pull needed to lift 3 tons.

$\frac{3}{7}$ ton or 960 lb. Ans.

15. In the ratchet wheel of a hand turning gear there are 80 teeth, the worm wheel having 128 teeth. The ratchet moves over 2 teeth for each pull of the lever. Find the time to turn the engines one half of a revolution, if the lever makes 20 per minute.

128 minutes.

16. In a single purchase winch there are 15 teeth in the wheel on the handle shaft, and 82 teeth in the wheel on the barrel shaft. The barrel is 10 inches diameter, and the handle is 20 inches long. Find the load which can be lifted by a force of 60 lb. at the handle, the lifting rope being $1\frac{1}{2}$ inches diameter. Friction is to be neglected.

1140·6 lb. Ans.

17. In a double purchase winch the driving pinions have 15 and 18 teeth respectively, the driven wheels having 40 and 65 teeth respectively. The barrel is 9 inches diameter, the rope 2 inches diameter, and the handle is 24 inches long. Find the load which can be lifted by a force of 100 lb. at the handle neglecting friction.

4202 lb. Ans.

18. A cylinder cover weighing 2 tons is lifted by a screw tackle. The screw is 2 inches diameter and its pitch is $\frac{3}{8}$ inch. The toggle bar is 30 inches long. Find the force needed to lift the cover if the efficiency is 36 per cent.

24·75 lb. Ans.

19. A screw jack lifts a load of 5 tons. The pitch of the thread is $\frac{1}{4}$ inch, the handle is 25 inches long and the force applied is 60 lb. Find the efficiency, and state the mechanical advantage.

0·296 ; 186·6. Ans.

20. The valve for a lever safety valve has an area of 8 sq. inches. The lever weighs 6 lb. and its centre of gravity is 7·5 inches from the centre line of the valve. The weight is hung at 21 inches from the centre line of the valve. When there is no weight on the lever, the valve lifts at 3 lb. per sq. inch, and when there is 62 lb. on the lever, the valve lifts at 65 lb. per sq. inch. Find the weight of the valve, and the distance from valve to fulcrum.

3 lb. ; 3 inches. Ans.

21. A lever safety valve has an area of 8 sq. inches. From fulcrum to the valve is $3\frac{1}{4}$ inches, and the C.G. of the lever is 10 inches from the fulcrum. The valve and spindle weigh 8 lb. and the lever weighs 9 lb. The lever is fitted with a moveable weight of 60 lb. Find the distance the weight has to be moved for a difference of 10 lb. pressure per sq. inch.

$4\frac{1}{3}$ inches. Ans.

22. In a wheel and differential axle, the diameter of the big axle is $7\frac{1}{2}$ inches and that of the small axle 5 inches, and the diameter of the lifting rope is $\frac{1}{2}$ inch. An effort of 25 lb. is applied to the end of the handle which is 16 inches long and the load lifted is 300 lb. Find the velocity ratio, mechanical advantage, efficiency, and load lost due to friction.

25·6 ; 12 ; 46·8% ; 340 lb. Ans.

23. In a Weston differential pulley block the top sheaves were 9 and 10 inches diameter respectively. The following data were taken during an experiment with this machine :—

Load W lb.	25	50	75	100	200	300	400	500
Effort P lb.	7·25	9·5	11·8	14	22·9	32	41	50

On a base of load, plot the curves of effort, efficiency and load lost due to friction. Express the relation between P and W and calculate the probable effort that would be required to lift a load of 240 lb.

$P = 5 + 0{\cdot}09\ W$; 26·6 lb. Ans.

24. With a double purchase crab winch, an effort of 20 lb. just lifts a load of 210 lb. and an effort of 80 lb. just lifts a load of 1200 lb. Express the relation between effort and load, and find the probable effort required to lift a load of 1000 lb.

$P = 7{\cdot}27 + 0{\cdot}0606\ W$; 67·87 lb. Ans.

CHAPTER XVI.

STRENGTH OF MATERIALS.

Stress.

When a body is subjected to the action of external loads, applied to it anywhere, internal forces called stresses are induced in the material of the body. Stress is the resisting force induced in the material by the action of the external loads. This internal force or stress is measured in pounds per square inch, or in tons per square inch.

$$\text{Stress} = \frac{\text{Load in lb.}}{\text{Area in square inches}} = \text{lb. per square inch.}$$

A stress is always of this character, and its dimensions may be represented by

$$\frac{\text{Load}}{\text{Area}} \qquad \frac{\text{lb.}}{(\text{inches})^2}$$

Strain is the change of shape, or distortion of the material caused by the load, and it is expressed per inch of length.

$$\textbf{Strain} = \frac{\text{Total change of length in inches}}{\text{Original length in inches}}$$

$$= \text{Extension or contraction per inch of length.}$$

of number, since it is the ratio of two things Its dimensions are

$$\frac{\text{inches}}{\text{inches}} = \text{a pure number.}$$

There are three kinds of simple stress which may be induced in a body.

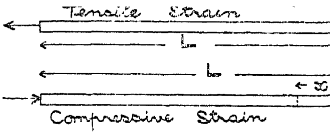
1. **Tensile Stress,** due to a direct pull.

2. **Compressive Stress,** due to a direct push.

3. **Shearing Stress,** due to transverse loading. A shear stress is sometimes called a sliding stress, or a tangential stress.

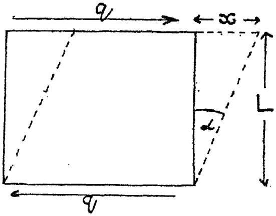
Both tensile and compressive stresses are caused by bending action, as well as by direct loading.

Shearing stresses are induced by transverse forces, such as those acting on the rivets in a riveted joint, or in a shaft transmitting torque ; and the material, when it fails, fails by transverse cutting as in the case of plates being sheared or punched.



Direct Strain =

in each of these cases.



Shear Strain = $\frac{\quad}{\mathrm{L}}$, and this measures the *angle* of distortion.

In the case of shearing strain, there is no change in length in the direction of the stress, but merely a change of position, and the strain is measured by the angle of distortion, $\frac{\quad}{\mathrm{L}}$ being the tangent of the angle α.

When a body is acted upon by a tensile stress alone, or by a compressive stress alone, or by a shear stress alone, it is said to be in a state of *simple stress.*

The Elastic Law.

All materials used in engineering are elastic, although the elastic strains may not be visible to the eye. When the load causing the change of shape is removed, the material returns to its original size and shape ; but there is a limit to this power of recovery.

The Elastic Limit.

The elastic limit defines the range within which the elastic law operates. Beyond a definite stress intensity, fixed by the inherent quality of the material, the power of elastic recovery practically disappears, and the material is said to have taken a "permanent set." For iron and steel the elastic limit occurs at about $\frac{2}{3}$ of the breaking stress.

Modulus of Elasticity.

It is found experimentally, that within the elastic limit, the stress is proportional to the strain; this statement is called Hooke's Law. It is the same as saying that equal additions to the load on a bar, will cause equal additions to its length.

We may write : $\dfrac{\text{Stress}}{\text{Strain}}$ = a constant; the symbol for this constant is E, in the case of direct loading; it is sometimes called Young's modulus, but it is generally called the Modulus of Elasticity. Now each material has its own elastic property; that is, for a given stress, different materials will exhibit different strains, therefore every material will have its own value for E, the Modulus of Elasticity.

We have : $\dfrac{\text{Stress}}{\text{Strain}} = \text{E}$; it has been shown that Strain is a pure number, therefore the dimensions of E are those of

$$\frac{\text{Stress}}{\text{A pure number}} = \text{a stress}$$

and E should always be expressed as a stress in pounds per square inch, or as tons per square inch.

Values of E.

Mild steel, 30,000,000 lb. per sq. inch.
Wrought Iron, 28,000,000 lb. per sq. inch.
Cast Iron, 18,000,000 lb. per sq. inch.

These are average values, and are approximate; for instance, E is often given as 13,000 tons per square inch for mild steel which differs only by a small amount from 30,000,000 lb. per square inch.

Beyond the elastic limit, the strain is no longer proportional to the stress, and the previous statements refer only to stress and strain within the elastic limit. In the case of ductile materials such as mild steel and wrought iron, after the elastic limit is passed, the material is in a plastic condition, and if the load is increased the specimen rapidly elongates in the direction of the applied load, the stretch which takes place being easily seen. At the same time the specimen is reduced in thickness or diameter, finally breaking at the section where this reduction of sectional area is most marked, that is, it breaks where the section has the smallest area.

In the case of cast iron, which is a brittle material having no ductility, when a specimen is broken in tension it exhibits practically no elongation or contraction of area.

Ultimate or Breaking Stress is found by dividing the load causing fracture by the original area of the bar. Ultimate or

$$\text{breaking stress} = \frac{\text{Load causing fracture}}{\text{Original area of bar}}$$

The breaking stress of mild steel in tension is from 28 to 32 tons per square inch for ordinary steels used in boilers and ships' hulls; for wrought iron, from 22 to 25 tons per square inch. The ultimate stress of mild steel and wrought iron in compression may be taken as about the same as in tension. The ultimate stress of mild steel in shear is about 23 to 24 tons per square inch. Cast iron for ordinary purposes may have a tensile strength or stress of from 8 to 10 tons per square inch, but special cast iron may have a tensile stress of upwards of 16 tons per square inch. Cast iron is very strong in compression, its ultimate strength being from 40 to 55 tons per square inch.

The ductility of the material is measured by the elongation of the bar up to the point of breaking, and also by the reduction of area. Before testing a specimen in tension, its diameter is carefully measured, and in the case of an eight inch specimen pop marks eight inches apart are lightly punched on the bar. After the bar is tested to destruction, the severed parts are held together and the distance between the original pop marks is measured. Then we have:—

$$\frac{\text{Final length} - \text{original length}}{\text{Original length}} \times 100 = \text{percentage elongation}$$

About 25 per cent. is a common value for mild steel, and about 20 per cent. for wrought iron.

$$\frac{\text{Original area} - \text{final area}}{\text{Original area}} \times 100 = \text{percentage reduction of area,}$$

and this may be from 50 per cent. to 60 per cent. for mild steel, and from 30 per cent. to 40 per cent. for wrought iron.

Working Stress.

The stress at which a material works must be well within the elastic limit. This is arranged by adopting a factor of safety; thus if it be decided to work steel at $\frac{1}{6}$ of its breaking stress, the factor of safety is said to be six. The factor of safety depends

upon the conditions under which the material works. In cases where the stress is applied slowly, and in which the material is not exposed to shock, a low factor such as four or five may be adopted; but if the stress is suddenly applied, or if the character of the stress is suddenly and often reversed, a much higher factor must be used; that is, the material must be worked at a much lower stress. Piston rods and connecting rods are alternately in tension and compression, and are therefore subject to complete reversal of stress, and the working stress allowed is low. In the case of a boiler shell, where the stress is applied gradually and maintained almost constant and is always of one kind, i.e., tensile, a high working stress is allowed.

$$\textbf{Factor of Safety} = \frac{\text{Breaking stress}}{\text{Working stress allowed}}$$

Example. A load of 110 lb. is hung from a steel wire 0·125 inch diameter and 20 feet long. The wire stretches 0·072 inch. Find the stress in the wire, the strain and the modulus of elasticity.

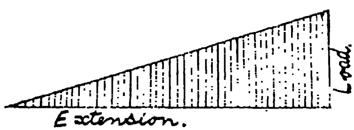
$$\text{Stress} = \frac{\text{Load}}{\text{Area}} \quad \frac{110 \times 14}{(0{\cdot}125)^2 \times 11} = 8960 \text{ lb. per sq. inch. Ans.}$$

$$\text{Strain} = \frac{\text{Extension}}{\text{Orig. length}} = \frac{0{\cdot}072 \text{ inch}}{20 \times 12 \text{ ins.}} = 0{\cdot}0003. \quad \text{Ans.}$$

$$\text{Modulus E} = \frac{\text{Stress}}{\text{Strain}} = \frac{8960}{0{\cdot}0003} = 29{,}870{,}000 \text{ lb. per sq. inch. Ans.}$$

Work done in Straining a Bar.

Since within the elastic limit the extension is proportional to the load, if the loads and the corresponding extensions are plotted on a diagram, a line drawn through the points will be



a straight line. The work done will be the area of the triangle shown shaded. Note carefully that the work done is *not* the

final load multiplied by the whole extension, because the bar has not been subject to the whole final load during its extension.

Work done $= \dfrac{\text{Load} \times \text{Extension}}{}$, this work is sometimes

called Resilience. Since the loading of the bar takes place within the elastic limit, the work stored in the bar will be given out when the load is removed. It would be difficult to use this stored energy, because it represents a large force moving through an exceedingly small distance, and in engineering we mostly deal with forces of moderate amount moving through fairly great distances.

$$\text{Work done, or Resilience} = \text{load} \times \text{extension}$$

Now, load $=$ stress $\times$ sectional area,

and extension $=$ strain $\times$ length $= \dfrac{\text{stress}}{\text{E}} \times$ length.

$$\therefore \text{Resilience} = \frac{\text{stress} \times \text{sectional area} \times \text{stress} \times \text{length}}{2 \times \text{E}}$$

$$\frac{(\text{stress})^2}{2\,\text{E}} \times \text{sectional area} \times \text{length}$$

$$= \frac{(\text{stress})^2 \times \text{volume}}{2\,\text{E}}$$

If stress and E are both expressed in lb. per sq. inch, and the volume in cubic ins., then the resilience is expressed in inch lb.

If stress and E are both in tons per sq. inch, and the volume in cubic inches, then the resilience is in inch tons.

Example. A steel bar 10 feet long and one inch diameter is stressed in tension to 6 tons per square inch. Find the load on the bar, the total extension and the work done in straining the bar. E $=$ 13,000 tons per square inch.

Load $=$ area $\times$ stress $= 1^2 \times \frac{11}{14} \times 6 = 4{\cdot}714$ tons. Ans.

$$\text{Strain} = \frac{\text{Stress}}{\text{Modulus}} \quad \frac{6 \text{ tons per sq. inch}}{13{,}000 \text{ tons per sq. inch}}$$

$$\text{Extension} = \text{Original length} \times \text{strain}$$

$$= 10 \times 12 \times$$

Ans.

$$\text{Work done} = \frac{\text{Load} \times \text{extension}}{2} = \frac{4{\cdot}714}{2} \times \frac{720}{13000}$$

$$= 0{\cdot}1306 \text{ inch ton of work}$$

or $0{\cdot}1306 \times 2240 = 292{\cdot}5$ inch lb. of work. Ans.

Example. The stress allowed on piston rods at the bottom of the thread is 5,000 lb. per square inch. Find the smallest diameter for a piston rod for a cylinder 27 inches diameter, the steam pressure being 200 lb. per square inch. If the ultimate strength of the material is 28 tons per square inch, what factor of safety has been allowed ?

Area at bottom of thread $\times$ safe stress
$= \text{area of cylinder} \times \text{steam pressure.}$

$$d^2 \times \tfrac{11}{14} \times 5000 = 27 \times 27 \times \tfrac{11}{14} \times 200$$

$$\frac{27 \times 27 \times 200}{5000} = 29{\cdot}16$$

$$d = \quad = 5{\cdot}399, \text{ say } 5{\cdot}4 \text{ inches. Ans.}$$

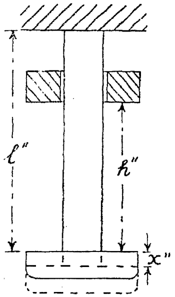
$$\text{Factor of safety} = \frac{28 \times 2240}{5000} = 12{\cdot}54. \text{ Ans.}$$

Note that 5,000 lb. per sq. inch is the tensile stress at the smallest diameter. The diameter of the body of the rod would be about 7 inches, and the stress in the body of the rod would be only

$$\frac{27 \times 27 \times 200}{7 \times 7} = 2975 \text{ lb. per sq. inch.}$$

A fairly low stress is allowed because the stresses are reversed in character every stroke. Piston rods are often made from steel of 35 tons ultimate strength, and then a stress of more than 5,000 lb. may be allowed at the bottom of the thread.

***Effect of Impact** produced by a falling weight.



The sketch represents a bar of material l inches long and A sq. inches sectional area, arranged with a stopper at the lower end, and being rigidly held at the upper end. A sliding weight W lb. may be allowed to fall through h inches on to the stopper, causing a maximum extension of x inches. The impact is thus wholly taken up by stretching the bar.

Let p be the maximum stress induced.

The work done by the falling weight = work stored in the strained material (i.e., the resilience).

$$\therefore \mathrm{W}\,(h + x) = \frac{p^2}{2\ \mathrm{E}} \times \mathrm{A} \times l$$

but, x = strain $\times$ length $= \dfrac{p}{\mathrm{E}} \times l$

$$\therefore \mathrm{W}\left(h + \frac{p\,l}{\mathrm{E}}\right) = \frac{\mathrm{A}l \times p^2}{2\ \mathrm{E}}$$

$$\mathrm{W}\,h + \frac{\mathrm{W}\,l \times p}{\mathrm{E}} \qquad \frac{\mathrm{A}\,l \times p^2}{2\ \mathrm{E}}$$

$$\frac{\mathrm{A}\,l}{2\ \mathrm{E}} \times p^2 - \frac{\mathrm{W}\,l}{\mathrm{E}} \qquad - \mathrm{W}\,h = 0,$$ a quadratic equation

where p is the unknown quantity.

Solving by the quadratic formula :—

$$p = \frac{\dfrac{\mathrm{W}\,l}{\mathrm{E}} \quad \mathrm{E}^2 \quad 2\ \mathrm{E}}{\dfrac{2\ \mathrm{A}\,l}{2\ \mathrm{E}}} \qquad \frac{\dfrac{\mathrm{W}\,l}{\mathrm{E}} \pm \sqrt{\dfrac{\mathrm{W}^2\,l^2}{\mathrm{E}^2} + \quad \mathrm{E}^2}}{\mathrm{A}\ l}$$

$$\frac{\mathrm{W}\,l \pm 1}{\dfrac{\mathrm{A}\,l}{\mathrm{E}}} \qquad \frac{\mathrm{W}}{\mathrm{A}l}$$

disregarding the negative sign.

Now, if $h = 0$, then $2\,W\,h\,A\,l\,E = 0$,

$$\text{and } p = \frac{W\,l + \sqrt{W^2\,l^2}}{A\,l} = \frac{2\,W\,l}{A\,l} = \frac{2\,W}{A}$$

If $h = 0$, then the load is suddenly applied, and we find that the maximum stress induced is $\frac{2\,W}{A}$, that is twice the stress intensity which would be induced by the same load W slowly applied.

If W falls through a distance h before impact on the material, then the maximum stress is given by,

$$p = \frac{W\,l + \sqrt{W^2\,l^2 + 2\,W\,h\,A\,l\,E}}{A\,l}$$

Example. Find the intensity of stress and the extension produced in a bar 12 feet long, and 2 square inches cross section, by the sudden application of a tensile load of 12 tons. What suddenly applied load would cause an extension of 0·036 inch? E = 12,000 tons per sq. inch.

$$p = \frac{2\,W}{A} = \frac{2 \times 12}{2} = 12 \text{ tons per sq. inch. Ans.}$$

$$\text{Strain} = \frac{\text{stress}}{E},\ \text{extension} = \frac{p}{E} \times l = \frac{12 \times 12 \times 12}{12000}$$

$= 0{\cdot}144$ inch. Ans.

$$\frac{p \times l}{E} = 0{\cdot}036,\ \text{but } p = \frac{2\,W}{A} \qquad \frac{2\,W \times l}{A \times E} = 0{\cdot}036$$

$$\therefore W = \frac{0{\cdot}036 \times 2 \times 12000}{2 \times 12 \times 12} = 3 \text{ tons. Ans.}$$

Example. A load of 280 lb. falls through $\frac{1}{4}$ inch on to a stop at the lower end of a vertical bar 10 feet long and 1 square inch cross section. Find the maximum stress induced.

E = 12000 tons per sq. inch.

$$p = \frac{(280 \times 120) + \sqrt{(280 \times 120)^2 + (2 \times 280 \times 0{\cdot}25 \times 1 \times 120 \times 12000 \times 2240)}}{1 \times 120}$$

$$= \frac{(280 \times 120) + \sqrt{(280 \times 120)^2 + \left(\frac{140 \times 120^2 \times 12000 \times 2240}{120}\right)}}{120}$$

$$= \frac{280 \times \cancel{120} + \cancel{120}\sqrt{280^2 + 140 \times 100 \times 2240}}{\quad}$$

$$= 280 + \sqrt{280^2 + 280 \times 50 \times 280 \times 8}$$

$$= 280 + 280\sqrt{1} \qquad = 280 + 280 \times 20{\cdot}03$$

$$= 280 + 5608 = 5880 \text{ lb. per sq. inch.} \quad \text{Ans.}$$

Strength of Boiler Stays.

The stays in a boiler are in tension and they are therefore Ties. For steel stays a maximum working stress of 9,000 lb. per square inch is allowed, and steel stays must not be welded. For wrought iron stays a maximum of 7,000 lb. per square inch is allowed as the working stress, but if wrought iron stays have been worked or forged in the fire then only 5,000 lb. per square inch is allowed. If the spacing of combustion chamber stays is 8 inches, then each stay supports an area of 8 × 8 or 64 square inches, and the load carried by the stay will be 64 × boiler pressure; but the load on the stay is also its area × working stress, therefore :—

Area of stay × Working Stress = Surface upheld × Boiler Pressure.

Example. Combustion chamber stays of steel are 1·375 inches diameter at the bottom of the thread. The pitch of the stays is 8 inches, find the working pressure of the boiler if the stress on the material of the stays is to be 8,600 lb. per square inch.

Area of stay × Stress = Surface upheld × Boiler Pressure.

$$\tfrac{11}{8} \times \tfrac{11}{8} \times \tfrac{11}{14} \times 8{,}600 = 8 \times 8 \times \text{Boiler Pressure.}$$

$$\text{Boiler Pressure} = \tfrac{11}{8} \times \quad \times \tfrac{11}{14} \times \frac{8600}{8 \times 8}$$

$$= 199{\cdot}6 \text{ lb. per square inch.} \quad \text{Ans.}$$

Note that the factor of safety allowed in this case is $\dfrac{28 \times 2240}{8600}$ = 7·29, assuming an ultimate strength of 28 tons per square inch in tension.

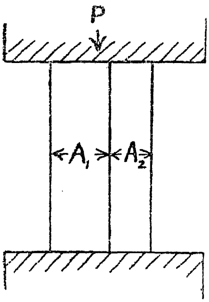
It is seen from the equation used in this example, that for the same stress on the stay, the working boiler pressure must vary as the area of the stay, or as the square of the diameter of the stay. If in this example the stays were reduced by corrosion to 1·125 inches diameter, we could find the new working pressure on the boiler by using the formula again, or we may write :—

$$\text{Reduced pressure} = 199{\cdot}6 \times \frac{(1{\cdot}125)^2}{(1{\cdot}375)^2} = 133{\cdot}5 \text{ lb. per sq. inch.}$$

this assumes that the stress on the stays is unaltered, but we can work the stays at a stress of 9,000 lb. per square inch, and the boiler pressure would then be :—

$$133{\cdot}5 \times \qquad = 139{\cdot}8 \text{ lb. per square inch.}$$

***Stress in Compound Bars.**



Consider two bars of different material jointly subjected to a load P. Let the area of the cross sections be A_1 and A_2; the stresses S_1 and S_2 and their moduli of elasticity be E_1 and E_2

The sum of the loads in the bars = P

$$\therefore A_1 S_1 + A_2 \qquad P \quad . \quad . \quad (1)$$

$$\text{Strain} = \frac{\text{Stress}}{\text{E}}$$

$\therefore$ The strains are $\frac{S_1}{E}$ and

But each bar has the same change of length, and their strains must be equal.

$$S_1 \qquad \frac{\ \ 2}{E_2} \therefore S_1 =$$

Putting this value of S_1 into equation (1) :—

$$= S_2 \quad \frac{E_1 A_1 + E_2 A_2}{E_2} \Big)$$

$$\therefore\ S_2 = \frac{P\ E_2}{E_1\ A_1 + E_2\ A_2} \quad \ldots \qquad \ldots \qquad \ldots \qquad (2)$$

$$\text{Similarly } S_1 = \frac{P\ E_1}{E_1\ A_1 + E_2\ A_2}$$

Example. A bar of copper of 1·5 sq. inches cross sectional area, and a bar of steel 0·75 sq. inch sectional area, are jointly subjected to a load of 10 tons in the direction of their lengths Find the stress in each bar and the strain. E for copper = 18×10^6 lb. per sq. inch and E for steel = 30×10^6 lb. per sq. inch.

$$\text{Stress in copper} = \frac{10 \times 2240 \times 18 \times 10^6}{18 \times 10^6 \times 1{\cdot}5 + 30 \times 10^6 \times 0{\cdot}75}$$

$$\frac{10 \times 2240 \times 18}{18 \times 1{\cdot}5 + 30 \times 0{\cdot}75}$$

$= 8{,}145$ lb. per sq. inch. Ans.

Stress in steel $\times\ 0{\cdot}75 + 8145 \times 1{\cdot}5 = 10 \times 2240$

$\therefore$ Stress in steel $= 13{,}480$ lb. per sq. inch.

The strain in each bar is the same,

$$\text{Strain} = \frac{13{,}480}{30 \times 10^6} \text{ or } \frac{8145}{18 \times 10^6}$$

$= 0{\cdot}00045$. Ans.

Stress due to Restricted Expansion.

Most materials expand when their temperatures rise. The expansion per unit length per degree rise of temperature is called the Co-efficient of Linear Expension.

Extension = Co-efficient (K) $\times$ length (L) $\times$ rise of temperature (T)

$= K\ L\ T.$

Now suppose a bar is prevented from expanding. The result is the same as allowing it to expand while being heated, and then compressing it an amount equal to the expansion.

$$\text{The strain} = \frac{\text{compression}}{\text{length}} \quad \frac{\text{K L T}}{\text{L}} = \text{K T}.$$

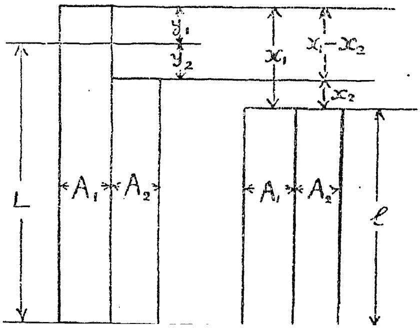
$$\text{But strain} = \frac{\text{stress}}{\text{E}} \qquad \text{stress} = \text{K T E}.$$

Example. A bar of cast iron is fitted between two plates which cannot move. The co-efficient of linear expansion for cast iron is 0·000006 per deg. Fah., and the modulus of elasticity is 18,000,000 lb. per sq. inch. Find the stress and strain in the bar when the temperature rises 200 Fah. deg.

Strain = K T = 0·000006 × 200 = 0·0012. Ans.

Stress = K T E = 0·000006 × 200 × 18,000,000 = 21,600 lb. per sq. inch. Ans.

***Stress due to Unequal Expansion.**



Consider two bars of different material joined together throughout their lengths. Suppose one bar to have a greater co-efficient of expansion than the other. When the bars are heated one tends to expand more than the other, but being joined together they must both expand by the same amount.

The bar which would have the greater expansion if free, is prevented from expanding its full amount by the pull in the second bar; while the second bar is extended more than it would have done if free.

Consider two bars the original length of each being l; the areas of section A_1 and A_2; co-efficients of linear expansion

K_1 and K_2, and their moduli of elasticity E_1 and E_2. Let the stresses set up be S_1 and S_2 and the rise of temperature T.

If free, the first bar would extend $K_1 \, T \, l = x_1$, and the second $K_2 \, T \, l = x_2$.

The difference of these, $x_1 - x_2 = T \, l \, (K_1 - K_2)$ ... (1)

But the lengths of the bars must be equal since they are secured together. The first bar is shortened by an amount y_1 and the second bar is extended by an amount y_2. The *sum* of these two amounts is equal to the *difference* the two bars would have expanded if free :—

$$\therefore x_1 - x_2 = y_1 + \qquad (2)$$

Let new length $= L$.

$$\frac{\text{Extension}}{\text{Length}} = \text{strain} = \frac{\text{Stress}}{E}$$

$$\therefore \text{extension} = \frac{\text{stress} \times \text{length}}{E}$$

$$\therefore y_1 = \frac{S_1 \, L}{E_1}, \text{ and } y_2 = \frac{S_2 \, L}{E_2}$$

Putting these values and the values in equation (1) into equation (2) :—

$$T \, l \, (K_1 - K_2) = \frac{S_1 \, L}{E_{\cdot}} + \frac{S_2 \, L}{E}$$

But l is very nearly equal to L, since the amount a bar extends due to increased temperature is only a very small proportion of its length. We can write therefore :—

$$\ _1 - K_2) = \qquad \frac{S_2}{E_2} \qquad \text{... (3).}$$

In words this expression may be stated :—The difference in expansion per unit length is equal to the sum of the strains.

If no external load is applied the load in each bar is the same. Therefore :—

$$\ _1 = A_2 \, S_2 \text{ or } S_1 = {}^{A_2} \quad S_2 \qquad (4).$$

Putting this value in (3) :-

$$T\ (K_1 - K_2) = \frac{A_2\ S}{A_1\ E} \qquad\qquad E_2$$

$$\overline{A_1\ E_1\ E_2}$$

$$\therefore\ S_2 = \frac{A_1\ E_1\ E_2\ T\ (K_1 - K_2)}{A_2\ E_2 + A_1\ E_1} \qquad (5).$$

This last expression is rather cumbersome, and the student is recommended to use equations 3 and 4.

Example. A steel bar 1 inch diameter is sheathed with a brass tube $\frac{3}{8}$ inch thick. The two are firmly secured at the ends. The modulus of elasticity for steel is 13,000 tons per sq. inch and its co-efficient of linear expansion is 0·0000067 per deg Fah. The modulus of elasticity for brass is 5,000 tons per sq. inch, and its co-efficient of linear expansion is 0·0000096 per deg. Fah. Find the stress in each material when the rise of temperature is 150 degrees Fah. If the original length was 20 inches, what is the length after heating ?

Load on steel = Load on brass.

Area of steel $\times$ stress in steel = Area of brass $\times$ stress in brass.

$$\therefore \frac{\ }{4} 1^2 \times S_s = \frac{\ }{4}$$

$$\therefore S_s = 2\tfrac{3}{4} \times \tfrac{3}{4} \times S$$

$$T\ (K_1 - K_2) = \frac{S_1}{E} +$$

$$150\ (0{\cdot}0000096 - 0{\cdot}0000067) = \frac{16\ S_b}{13,000} \qquad 5000$$

$$= S_b \left(\frac{33 \times 5,000 + 16 \times 13,000}{13,000 \times 16 \times 5,000} \right.$$

$$\left. \frac{33 \times 5 + 16 \times 13}{13 \times 16 \times 5,000} \right)$$

$$= S_b \left(\frac{373}{13 \times 16 \times 5,000} \right)$$

$$\therefore S_b = \frac{150 \times 0{\cdot}0000029 \times 13 \times 16 \times 5{,}000}{373}$$

$$= 1{\cdot}213 \text{ tons per sq. inch. Ans.}$$

$$S_s = \tfrac{33}{16} S_b = \frac{33 \times 1{\cdot}213}{16}$$

$$= 2{\cdot}502 \text{ tons per square inch. Ans.}$$

The final length of, say, the steel bar is the original length, plus the extension due to the stress, plus the expansion due to the rise in temperature.

$$\therefore \text{Final length} = 20 + \frac{20 \times 2{\cdot}502}{13{,}000} + 20 \times 0{\cdot}0000067 \times 150$$

$$= 20 + 0{\cdot}003848 + 0{\cdot}0201$$

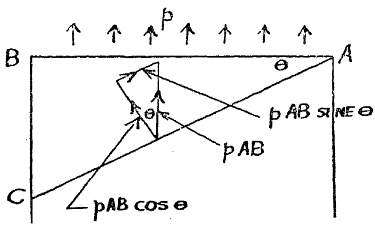
$$= 20{\cdot}023948 \text{ inches. Ans.}$$

Or, taking the brass, the final length is the original length minus the compression due to the stress and plus the extension due to the increase of temperature.

$$\therefore \text{Final length} = 20 - \frac{1{\cdot}213 \times 20}{5000} + 20 \times 0{\cdot}0000096 \times 150$$

$$= 20{\cdot}023948 \text{ inches. Ans. (as before).}$$

***Stresses on an oblique section of a bar subjected to direct stress.**



Consider a bar subject to a tensile stress p. Let the bar be of unit thickness. Then the load in the direction of p is p A B. This load is equivalent to two loads on the face A C. One is perpendicular or normal to A C, and one tangential to A C. The normal load is p A B cos. θ and the normal stress

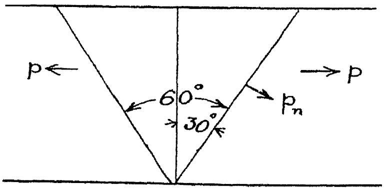
$$= \frac{\text{load}}{\text{area}} = \frac{p \text{ A B} \cos \theta}{\text{A C}}, \text{ but } \frac{\text{A B}}{\text{A C}} = \cos.\ \theta.$$

$\therefore$ Normal stress $p_n = p \cos.^2 \theta.$

The tangential load $= p$ A B sine θ, and the tangential or shear stress $q = p \dfrac{\text{A B}}{\text{A C}}$ sine $\theta = p \cos. \theta$ sine θ.

The direct stress p is equivalent to a direct stress on A C of $p \cos.^2 \theta$, and a shear stress of $p \cos. \theta$ sine θ. The shear stress will be greatest when cos. θ sine θ has its greatest value, and this occurs when $\theta = 45°$. The greatest shear stress is therefore $p \cos. 45°$ sine $45° = \dfrac{p}{2}$, and acts on all planes at 45° to the direction of p.

Example. Two plates 10 ins. wide and ½ inch thick are joined by a vee weld. The angle of the weld is 60°. The plates have a tensile load of 25 tons. Find the stress in the plates and the normal and shear stress at the junction of the plate and the deposited metal.



$$\text{Stress in plate } p = \frac{25}{\frac{1}{2} \times 10} = 5 \text{ tons per sq. inch.}$$

Normal stress, $p_n = 5 \cos.^2 30° = 3\frac{3}{4}$ tons per sq. inch.

Shear stress, $q = 5$ sine $30° \cos. 30° = 2{\cdot}165$ tons per sq. in.

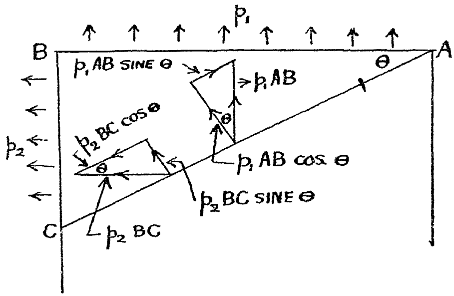
Consider now a bar with two tensile stresses p_1 and p_2 acting at right angles to each other. The stress p_1 produces a load in its own direction of p_1 A B, and the stress p_2 produces a load in its direction of p_2 B C. The components of these two forces normal to the plane A C are p_1 A B cos. θ and p_2 B C sine θ, and both produce a tension on A C. This tension is

$$\frac{p_1 \text{ A B} \cos. \theta}{\text{A C}} + \frac{p_2 \text{ B C sine } \theta}{\text{A C}}$$

$$p_n = p_1 \cos.^2 \theta + p_2 \text{ sine}^2 \theta.$$

$$\text{If } \theta = 45°, \; p_n = p_1 \cos.^2 45° + p_2 \text{ sine}^2 45°$$

$$= \frac{p_1}{2} + \frac{p_2}{2} = \frac{p_1 + p_2}{2}$$



The forces tangential to A C are p_1 A B sine θ and p_2 B C cos. θ, and these two act in opposite directions. The net force is therefore the difference of the two, p_1 A B sine θ — p_2 B C cos. θ.

$$\text{and the shear stress } q = \frac{p_1 \text{ A B sine } \theta}{\text{A C}} - \frac{p_2 \text{ B C cos. } \theta}{\text{A C}}$$

$$= p_1 \text{ sine } \theta \cos. \theta - p_2 \text{ sine } \theta \cos. \theta$$

$$= (p_1 - p_2) \text{ sine } \theta \cos. \theta$$

The value of θ which gives this expression its maximum value is $\theta = 45°$, $\therefore q$ (max.) $= (p_1 - p_2)$ sine 45° cos. 45°.

Example. An air vessel 4 feet diam. is made of plates 1 inch thick welded together, the seams being 45°. The air pressure is 400 lb. per sq. in. Find the stress on the longitudinal section and the circumferential section of the vessel. Find also the normal and shear stress on the welded seams.

Stress on longitudinal section =

$$\frac{\text{P D}}{2\text{T}} \quad \frac{400 \times 48}{2 \times 1} = 9{,}600 \text{ lb. per sq. inch.}$$

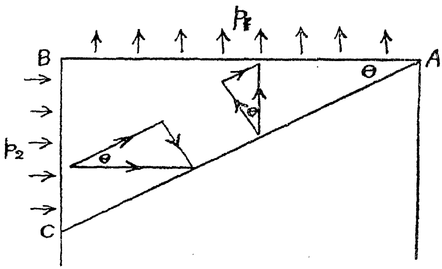
Stress on circumferential section = = 4,800 lb. per sq. inch.

Since seams are at 45°, normal stress =

$$\frac{p_1 + p_2}{2} = \frac{9{,}600 + 4{,}800}{2} = 7{,}200 \text{ lb. per sq. inch.}$$

$$\text{Shear stress} = \frac{p_1 - p_2}{2} = \frac{9{,}600 - 4{,}800}{2} = 2{,}400 \text{ lb. per sq. inch.}$$

In the case considered with two stresses, both p_1 and p_2 were tensile stress, and the normal stress p_n was a tensile stress. If both p_1 and p_2 were compressive stresses, the normal stress would be a compressive stress. Stresses which are of the same kind, all tensile or all compressive are called like stresses. Tensile stresses together with compressive stresses are called unlike stresses.



Consider the case when p_1 is tensile and p_2 compressive. The components of the forces p_1 A B and p_2 B C normal to A C oppose each other and the net normal force on A C is p_1 A B cos. θ — p_2 B C sine θ. The normal stress p_n is then $p_1 \cos.^2 \theta - p_2 \text{ sine}^2 \theta$. Also the components tangential to A C are in the same direction, and the total shear stress is p_1 sine θ cos. θ + p_2 sine θ cos. $\theta = (p_1 + p_2)$ sine θ cos. θ. These results are just what we would obtain by writing the compressive stress as $-\ p_2$, and this method holds generally.

It has been shown that a single tensile or compressive stress on one plane produces another tensile or compressive stress together with a shear stress on planes not parallel to the first. Similarly a normal stress and a shear stress are together equivalent to a single simple stress acting in another direction. Planes on which a single direct stress acts are called planes of principal stress, and planes upon which both direct and shear stresses act are called planes of complex stress.

TEST EXAMPLES XVI.

1. A mild steel specimen 2 inches long, 0·2 sq. inch cross section, was tested in a testing machine and the following results obtained by an extensometer :

Load in tons	0·5	1	1·5	2
Extension in ins.	0·00038	0·00075	0·00112	0·0015

2·5	3	3·5	3·6	3·7
0·0019	0·00225	0·00263	0·00271	0·0028

3·8	3·9	4·0	4·1	4·16
0·00285	0·00295	0·00305	0·0032	0·0034

At a slight increase above the load of 4·16 tons, the specimen stretched about $\frac{1}{16}$ inch without further increase in the load.

Plot a curve on a base of extension, and with reference to the curve state briefly what you observe about the limit of proportionality. Give the approximate elastic limit, yield point, and modulus of elasticity in tons per sq. inch.

19, 20·8 and 13160 tons per sq. inch respectively. Ans.

2. A specimen of mild steel, 0·7 inch diameter, and 8 inches between the gauge points was tested to destruction in tension. When the load on the bar was 0·77 ton the elongation was 0·00128 inch, and when the load was 2·31 tons, the elongation was 0·00384 inch. The yield point was reached when the load was 6·93 tons, and the specimen broke when the load was 11·55 tons, the total elongation being 2·25 inches, and the diameter at the place of fracture was 0·48 inch. Calculate the ultimate strength and the stress at yield point in tons per square inch, find also the percentage elongation and the percentage contraction of area, and find the value of E in lb. per square inch.

30 tons square inch ; 18 tons square inch ; 28·12 per cent. ; 53 per cent. ; 28,000,000 lb. square inch. Ans.

3. Allowing a stress of 5,000 lb. per square inch at the bottom of the thread, calculate the least diameter of a piston rod for a cylinder 24 inches diameter, the steam pressure being 220 lb. per square inch. 5·034 inches. Ans.

4. A piston 27 inches diameter has a piston rod 7 inches diameter, the diameter at the bottom of the thread being 5·4 inches. The steam pressure is 200 lb. per square inch, the pressure on the intermediate gauge being 52 lb. per square inch, and the pressure drop between the cylinders is 3 lb. per square inch. Find the compressive stress in the body of the rod, and the greatest tensile stress in the rod, when the pressures are as stated.

2,158 lb. sq. inch ; 3,625 lb. sq. inch. Ans.

5. A cylinder 28 inches diameter inside the joint of the cover is supplied with steam at 200 lb. per square inch. How many studs, each $1\frac{3}{8}$ ins. diameter, are necessary to hold the cover, if the stress on the material of the studs must not exceed 5,000 lb. per square inch ?

Note.—The sectional area of Whitworth screw bolts at the bottom of the thread in square inches is given by :—

$$\text{Area} = \frac{\text{Number of 8ths in diameter} \times (\text{number of 8ths} - 1)}{100}$$

23 studs. Ans.

6. A Diesel cylinder is 66 centimetres diameter, and the cover is held down by 34 studs 1·5 inches diameter. Find, using the rule for area given in Question 5, the stress in the studs when the pressure in the cylinder is 35 kilograms per square centimetre.

5,868 lb. per square inch. Ans.

7. Find the diameter of stays for a boiler if the working pressure is 165 lb. per square inch, the pitch of the stays 8 inches, and the stress allowed 9,000 lb. per square inch. Of what material are these stays made ? What is the factor of safety if the breaking strength is 29 tons per square inch ?

1·222 inches ; 7·217. Ans.

8. The main stays in the steam space of a boiler are 3 inches diameter. Find the working pressure of the boiler if a stress of 9,000 lb. per square inch is allowed in the stays, their pitch being 18 inches.

196·4 lb. per square inch. Ans.

9. Find the diameter of the top end bolts for an engine having a cylinder 42 inches diameter, working at 65 lb. per square inch (*a*) if there are two bolts ; (*b*) if there are four bolts. In each case the stress is not to exceed 6,000 lb. per square inch.

3·09 inches ; 2·185 inches. Ans.

10. A cotter stay is 3 inches diameter, and is swelled to $3\frac{3}{4}$ inches where the cotter passes through. The cotter is $\frac{7}{8}$ inch thick, find its depth assuming that the stress in the cotter is the same as the stress in the smallest part of the stay. Allow $1\frac{7}{8}$ for double shear.

4·31 inches. Ans.

11. The web of a built up crank shaft is to be bored out for shrinking on a shaft 12 inches diameter. Find the size of the hole to be bored in the web so that after shrinking the stress in the material is not more than 12 tons per square inch. The modulus of elasticity is 30,000,000 lb. per square inch.

11·9892 inches. Ans.

12. A stay $2\frac{1}{2}$ inches diameter has a swelled end to allow for a cotter passing through. Find the approximate diameter of the swelled part, if the strength is to be the same throughout, and the thickness of the cotter is $\frac{5}{8}$ inch.

2·929 inches. Ans.

13. A combustion chamber plate is $\frac{9}{16}$ inch thick, and the boiler pressure is to be 170 lb. per square inch. Find the pitch of the stays.

Note :—Pressure $= \dfrac{100\ (T + 1)^2}{S \quad 6}$, where T is thickness in sixteenths of an inch and S is surface upheld in square inches.

8·052 inches. Ans.

14. A piston of 52 inches diameter, has a piston rod 8 inches diameter and a tail rod 6 inches diameter, this being the diameter at the bottom of the thread. The steam pressure is 60 lb. per square inch by gauge and the back pressure is 14 lb. per square inch by gauge, find the stress in lb. per square inch in the small part of the rod.

3,362 lb. square inch. Ans.

15. A stay tube is $3\frac{1}{2}$ inches diameter outside and $2\frac{3}{4}$ inches diameter inside. The depth of the thread is 0·06 inch. The area supported by the stay is 97 square inches and the boiler pressure is 165 lb. per square inch. Find the stress in the stay tube at the bottom of the thread.

5,276 lb. square inch. Ans.

16. A bar $1\frac{1}{2}$ inches diameter is stressed to 10,000 lb. per square inch. The bar is 12 feet long, and E = 30,000,000 lb. per square inch. Find the work done on the bar.

$424\frac{2}{7}$ inch lb. Ans.

CHAPTER XVII.

THE THEORY OF BENDING.

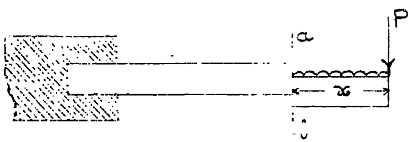
A Beam is a rigid bar set, in most cases, horizontally, and supported at one or more points. When the beam has only one support, it is called a *cantilever* ; when it has two supports, one at each end, it is called a *simple beam* ; when it has more than two supports, it is called a *continuous beam.*

A concentrated load is a load applied at a *point* on the beam.

A uniform load is a load uniformly spread, or distributed over the whole length of the beam. Thus if the beam is l feet long, and it carries a uniform load of w lb. per foot run, the total load carried is $w \times l$ lb. On x feet of such a beam, the load carried is $w \times x$ lb.

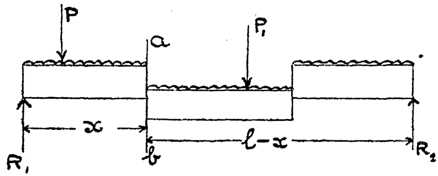
The Reactions are the vertical forces acting upward at the supports, and the sum of these upward reactions must equal the sum of the downward forces or loads acting on the beam. It has already been shown, in the Chapter on Moments, how to find the reactions for beams having two supports placed in any position.

Vertical Shear.



A beam is subjected to shearing force in a vertical direction. In the cantilever shown, let the outer portion, x feet long, carry a uniform load of w lb. per foot run, and a concentrated load P at its extreme end. Then a total force of $w\,x + P$, acts downwards on the right of the section $a\,b$, and an equal force, but of opposite direction, i.e., upwards, must act on the left of the section $a\,b$, causing a sliding or shearing force across the section.

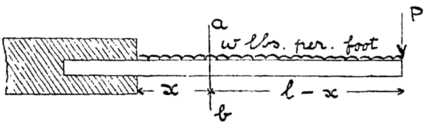
In a similar manner, let the simply supported beam shown be loaded with concentrated loads P and P_1 and a uniform load of w lb. per foot run. Then on a section $a\,b$, at x feet from R_1 there is an upward force R_1 acting, and a downward force of $P + w\,x$ acting. The vertical shear on the section $a\,b$ must be $R_1 - (P + w\,x)$ acting on the left of $a\,b$, and there must be an equal and opposite force acting on the right of $a\,b$,



Now at this section $a\ b$, the shearing force can have only *one* value which must be the same whether calculated from R_1 or from R_2, therefore :—

The Shearing Force on any section is the algebraic sum of all the forces on *one* side of the section, and this force may be reckoned from either end of the beam. Reckoning from R_2, the shearing force at $a\ b$ is $R_2 - (l - x)\ w - P_1$ and this must be the same as $R_1 - (w\ x + P)$, which is the shearing force reckoned from R_1.

Bending Moment.



It is evident that the force P and that the part of the uniform load on $(l - x)$ feet, must have a bending moment about $a\ b$.

The moment of P about $a\ b$ is $P\ (l - x)$.

On $(l - x)$ feet the load is $w\ (l - x)$ lb., and this uniform load acts at its Centre of Gravity at $\dfrac{(l - x)}{2}$ feet from $a\ b$.

The total Bending Moment at x feet from the support, written M_x is :—

$$M_x = P\ (l - x) + w\ (l - x) \times \frac{(l - x)}{2}$$

$$\text{or } M_x = P\ (l - x) + \frac{w}{2}\ (l - x)^2.$$

These considerations follow directly from the definition of the moment of a force about a point.

The Bending Moment on this cantilever will be greatest at the support, because the loads acting have their greatest leverage

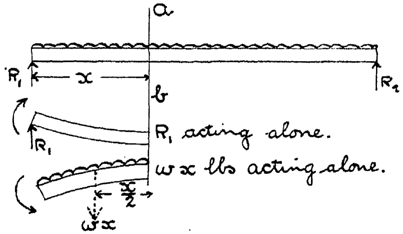
about the support; stating this in another form, the maximum value of the Bending Moment occurs when $x = o$; thus :—

when $x = o$, $\quad M = P(l - o) + \dfrac{w}{2}(l - o)^2$

$M = P\,l + \dfrac{w\ l^2}{2}$, or we may take moments directly about the support :—

M support $= P \times l + w\,l \times \dfrac{l}{2} = P\,l + \dfrac{w\ l^2}{2}$ as before.

Consider next a beam, simply supported at its ends, carrying a uniform load of w lb. per foot run. The reactions must first be found, but for the present demonstration, let them be written as R_1 and R_2. Taking moments about a section $a\ b$ at x feet from R_1 we have :—



Moment of R_1 about $a\ b = R_1 \times x$, this tends to cause "sagging" of the beam, as shown.

Moment of distributed load on x feet about $a\ b = w\ x \times \dfrac{x}{2}$

since on x feet we have $w\ x$ lb., and this load acts at its centre of gravity at $\dfrac{x}{2}$ feet from $a\ b$. This tends to cause "hogging" of the beam as shown. Since these two moments tend to cause rotation about $a\ b$ in opposite directions, the nett Bending Moment about $a\ b$ is :—

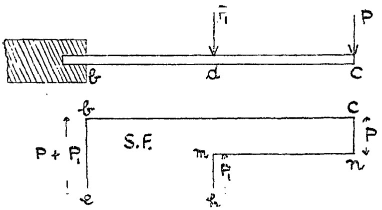
$M = R_1\,x - w\,x \times \dfrac{x}{2} = R_1\,x - \dfrac{w\ x^2}{2}$, and as the

Bending Moment at $a\ b$ can have only *one* value, whether calculated from R_1 or from R_2, we have the following definition :—

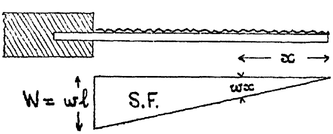
The Bending Moment at any section, is the algebraic sum of all the moments on *one* side of the section, and may be found by taking moments about the section from *either* end of the beam. In the subject of beams we are concerned mostly with stresses caused by the Bending Moment, the stresses caused by the Shearing Force being negligible in ordinary cases.

Diagrams of Shearing Force and Bending Moment.

The shearing forces and bending moments acting on a beam may be shown graphically in the form of diagrams. In the cantilever, loaded with concentrated loads P and P_1, the vertical shear at the free end of the beam is P lb. Neglecting the weight of the beam, there is no change in the shearing force between loads P and P_1; the shearing force from c to d being P lb. Between d and b the shearing force is $P + P_1$ lb.

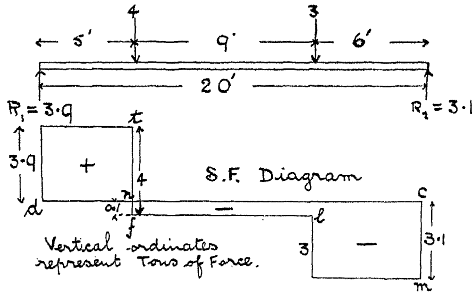


Draw $b\ c$, the base line of the shearing force diagram. From c set down a distance $c\ n$ representing P to some convenient scale; draw $n\ m$ parallel to $b\ c$. From m set down $m\ h$ representing P_1; from h draw $h\ e$ parallel to $b\ c$. The diagram is a picture which shows at a glance the manner in which the shearing force varies along the beam. Consider next a cantilever loaded with a uniform load of w lb. per foot. At the free end the downward force is zero, and therefore the vertical shear is zero. At one foot from the free end, the load acting downwards is w lb., at two feet from the free end, the load acting downwards is 2 w, and at x feet the load is $w\ x$. The load at any point is proportional to the distance from the free end, therefore the shearing force diagram is a triangle as shown, its maximum ordinate being $w\ l$, or W, the total load on the cantilever.

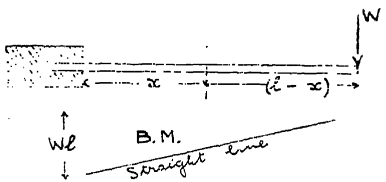


Take next a beam simply supported, having loads of 4 and 3 tons disposed as shown. To find the reaction R_2 take moments round R_1. Then $R_2 \times 20 = (5 \times 4) + (3 \times 14)$, from which $R_2 = 3{\cdot}1$ tons, $R_1 = 4 + 3 - 3{\cdot}1 = 3{\cdot}9$ tons.

From d, on the base line of shearing force, set up a vertical ordinate to represent 3·9 tons. This force is here taken as positive (+). Neglecting the weight of the beam, there is no change in the shearing force between R_1 and the load of 4 tons.

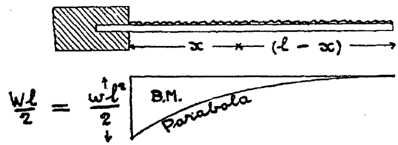


From t, distance 5 feet from R_1 measure down an ordinate of 4 tons; this cuts through the base line. As 3·9 tons has been taken as positive and measured upwards from the base, then 4 tons acting in the opposite direction to the 3·9 tons must be measured downwards from t. From t to f is 4 tons in the scale of shearing force, $n\ f$ being $3{\cdot}9 - 4 = -0{\cdot}1$ ton on the force scale below the base line. The remainder of the diagram is completed as shown, for at l we have $-0{\cdot}1 - 3 = -3{\cdot}1$ tons of shearing force, and this checks back from m to c as the reaction R_2. Note that the diagram shows the algebraic sum of all the forces (including the reactions) acting on the beam. The diagram may be started from either end, and the minus sign used on the diagram is not generally put down in the answer. The diagrams are often sketched roughly, and not drawn to scale.



Because the Bending Moment (B.M.), in the case of a cantilever having one load at the free end, is proportional to the distance from the free end, the B.M. diagram is a straight line as shown; or we may write :—At x feet from the support, $M_x = W\ (l - x)$; this is called an equation of the first order, because it contains the first power of x. This equation may be represented by a straight line.

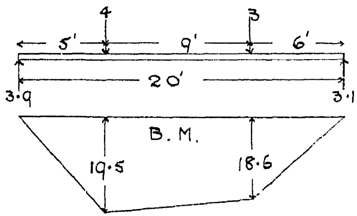
In the case of a cantilever carrying a uniform load of w lb. per foot,



$$M_x = w\,(l - x) \times \frac{(l - x)}{2} \quad \text{or } M_x = \frac{w}{2}\,(l - x)^2,$$

this is an equation of the second order, because it contains the square or second power of x. This equation is represented by a curve called a parabola. If it is desired to plot this curve, various values of x may be taken, and the ordinates measured down from the base line.

The B.M. at the support is $w\,l \times \frac{l}{2} = \frac{w\ l^2}{2}$, or calling $w\ l$, W the total load, this becomes $\frac{W\,l}{2}$. In beams carrying only concentrated loads, the B.M. diagram is always bounded by straight lines; but if the load is uniformly spread, the diagram is bounded by a parabolic curve.



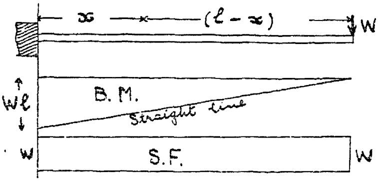
Take the beam shown in a previous paragraph, the B.M. at 5 feet from $R_1 = 3{\cdot}9 \times 5 = 19{\cdot}5$ foot tons.

B.M. at 6 feet from $R_2 = 6 \times 3{\cdot}1 = 18{\cdot}6$ foot tons.

Since there is no uniform load, the B.M. diagram is bounded by straight lines.

BENDING MOMENTS AND SHEARING FORCES.

Cantilever, with one load W at the free end.

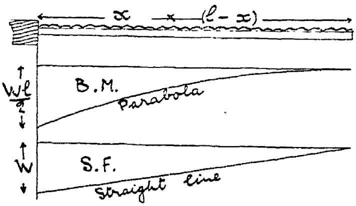


$M_x = W(l - x)$, M is greatest at the support.

M at support $= W\ l$.

M is zero at the free end, and varies from zero at the free end to the maximum value W l at the support. The shearing force is W over the whole length of the beam.

Cantilever, with a uniform load of w lb. per foot run.
Total load $W = w\ l$



Load on $(l - x)$ feet $= w\ (l - x)$ and this acts at $\dfrac{(l - x)}{}$ feet.

$$= w\ (l - x) \times \frac{(l - x)}{2}$$

$$= \frac{w}{2}\ (l - x)^2$$

this equation may be represented by a curve called a parabola.

$$M \text{ at support } = \frac{w}{2}\ (l - o)^2 = \frac{w\ l^2}{2}, \text{ or since } W = w\ l$$

$$M \text{ at support } = \frac{W\ l}{2}$$

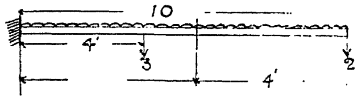
As the Bending Moment is mostly needed where it is greatest, i.e., at the support, we may take moments directly about the support. The total load W acts at $\dfrac{l}{2}$, the position of the C.G. from the support, and $\therefore$ M at support $= W \times \dfrac{l}{2} = \dfrac{W\ l}{2}$ as before.

The shearing force is *o* or zero at the free end, and varies directly as the distance from the free end; the shearing force at the support is therefore $w\ l$ lb. or W lb.

The student should remember that a uniform or distributed load always acts through its centre of gravity. This is true for the whole distributed load, and for any small portion of it.

Example. A Cantilever 10 feet long carries a uniform load of 0·5 ton per foot, a concentrated load of 2 tons at the free end, and a concentrated load of 3 tons at 4 feet from the support. Calculate the B.M. and S.F. at a point 6 feet from the support, and at the support.

Taking Moments about a point 6 feet from the support :—



Moment of the 2 ton load $= 2 \times 4$ foot tons; on 4 feet the amount of the uniform load is $0·5 \times 4 = 2$ tons and this acts at its C.G. at 2 feet from the point considered.

$\therefore M_6 = (0·5 \times 4 \times 2) + (2 \times 4) = 12$ ft. tons of moment. Ans.

$S.F._6 = 2 + 4 \times 0·5 = 4$ tons. Ans.

Note that the B.M. is the sum of the individual moments acting on *one side* of the point considered, and that the shearing force is the sum of the individual forces on *one side* of the point considered.

$M_{\text{support}} = (2 \times 10) + (4 \times 3) + (0·5 \times 10 \times \frac{10}{2})$
$= 20 + 12 + 25 = 57$ ft. tons of moment. Ans.

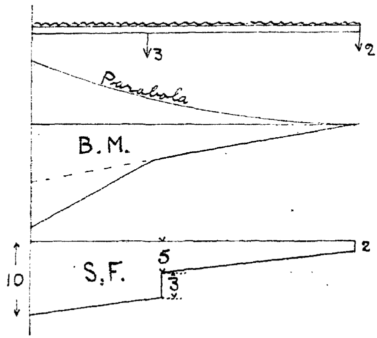
$S.F._{\text{support}} = 2 + 3 + 0·5 \times 10 = 10$ tons. Ans.

Units.

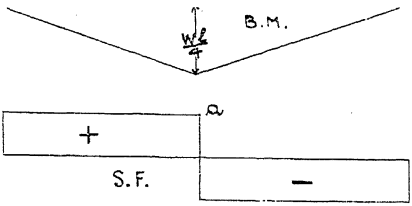
The units of a moment are force $\times$ distance, and are therefore the same as those of "work." An answer should always be expressed in the correct units. Work is often written as "foot lb. of work," and moment as "foot lb. of moment"; this distinguishes one from the other. Moment may be expressed as "foot lb.," or as "inch lb." or as "foot tons," or as "inch tons," depending upon which is the most convenient form for the problem. Shearing force is expressed as tons, or as lb. of force, its units being merely those of a load or force.

The diagrams of B.M. and S.F. are shown, though these are not necessary for the solution of the problem. The B.M. diagram

for the uniform load is drawn first above the base line, and for the concentrated loads it is drawn below the base line. The shearing force diagram needs no explanation.



Beam Supported at Both Ends, carrying a central load



Since the load W is at the centre of the beam, each reaction will be $\frac{W}{2}$.

Taking moments about a point at x from R_1 we have :-

$M_x = \frac{W}{2} \times x$ and this will be greatest when $x = \frac{l}{2}$, i.e., at the centre of the beam.

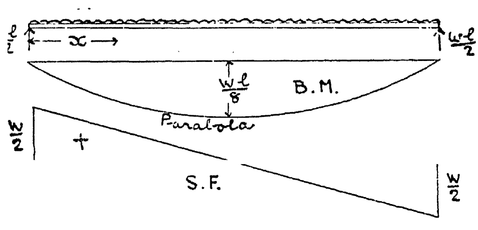
$$\text{M centre} = \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4}$$

This result may be obtained by taking moments directly about the centre of the beam. Note that W has no moment about its own point of application. The B.M. diagram is bounded by straight lines as shown. For the shearing force diagram, starting from the left end, set up an ordinate to equal $\frac{W}{2}$, this represents the shearing force anywhere between R_1 and the load W. From a, measure an ordinate to equal W downwards, cutting through the base line. Then if we call R_1 or $\frac{W}{2}$ positive, W acting in the opposite direction must be negative, and therefore

$$\frac{W}{2} - W = -\frac{W}{2}$$

this latter value being shown below the base line. As stated before the minus sign is not used in the answer.

Beam Supported at Both Ends, carrying a load of w lb. per foot run.



Each reaction is $\frac{w\ l}{2}$

$$M_x = \frac{w\ l}{2} \times x - w\ x \times \frac{x}{2}$$

$$\frac{w\,l\,x}{2} - \frac{w\,x^2}{2}, \text{ this is greatest when } x = \frac{l}{2}$$

$$\text{M centre} = \frac{w\ l}{2} \times \frac{l}{2} - \frac{w}{2} \times \Big(\qquad \frac{w\ l^2}{8} \text{ or } \frac{W\,l}{8}$$

We could proceed as follows, instead of in the previous manner.

$\frac{Wl}{2}$

Calling each reaction $\frac{W}{2}$, then taking moments :-

$$M_x = \frac{W}{2}\,x - \frac{x}{l}\,W \times \frac{x}{2}$$

$$\frac{W\ x}{2} \qquad \frac{W\ x^2}{2\,l} \qquad x = \cdot$$

$$\text{M centre} = \frac{W\,l}{4} - \frac{W\,l}{8} = \frac{W\,l}{8}$$

Note that on x feet we have $\frac{-}{l} \times W$ of the total load, and

this acts at $\frac{}{2}$ from the point considered.

The result $\frac{W\,l}{8}$ may be got by taking moments about the centre.

$$\text{M centre} = \frac{W}{2} \times \frac{l}{2} - \frac{W}{2} \times \frac{l}{4} = \frac{W\,l}{8}$$

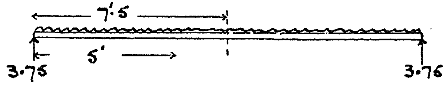
The shearing force at $x = \frac{w\ l}{2} - w\,x,$

or $\dfrac{W}{2} - \dfrac{x}{l}W$, and at the centre this is :—

$$\text{S.F.}_{\text{ centre}} = \frac{W}{2} - \frac{l}{2} \times \frac{W}{l} = o.$$ The shearing force is

a maximum at the ends, where its value is $\dfrac{W}{2}$

Example. A beam 15 feet long, supported at its ends, carries a uniform load of 0·5 ton per foot run. Find the B.M. at the centre and at 5 feet from either support. Find the shearing force at the ends and at 5 feet from either end.



Total load on beam $= \frac{15}{2} = 7{\cdot}5$ tons.

R_1 and R_2 each equal $\dfrac{7{\cdot}5}{} = 3{\cdot}75$ tons.

$$M = 3{\cdot}75 \times 7{\cdot}5 - 7{\cdot}5 \times \tfrac{1}{2} \times \frac{7{\cdot}5}{}$$

$= 28{\cdot}125 - 14{\cdot}0625 = 14{\cdot}0625$ ft. tons of moment. Ans.

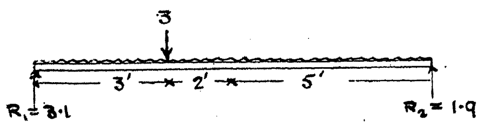
or $M_{\text{centre}} = \dfrac{W\,l}{} = \dfrac{7{\cdot}5 \times 15}{} = 14{\cdot}0625$ ft. tons as before.

$M_5 = 3{\cdot}75 \times 5 - 5 \times \frac{1}{2} \times \frac{5}{2} = 12{\cdot}5$ ft. tons of moment. Ans.

Shearing force at supports $= 3{\cdot}75$ tons. Ans.

Shearing force at $5' = 3{\cdot}75 - 5 \times \frac{1}{2} = 1{\cdot}25$ tons. Ans.

Example. A beam, 10 feet long, supported at its ends, carries a uniform load of 2 tons, and a concentrated load of 3 tons at 3 feet from the left support. Find the bending moment at the centre and at 3 feet from the left support. Find also the shearing force at each end of the beam and at the centre.



Take moments round R_1 then :—

$R_2 \times 10 = 3 \times 3 + 2 \times 5$; from which $R_2 = 1{\cdot}9$ tons.

$R_1 = 3 + 2 - 1{\cdot}9 = 3{\cdot}1$ tons.

$M_{\text{centre}} = 1{\cdot}9 \times 5 - 1 \times 2{\cdot}5 = 7$ foot tons of moment. Ans.

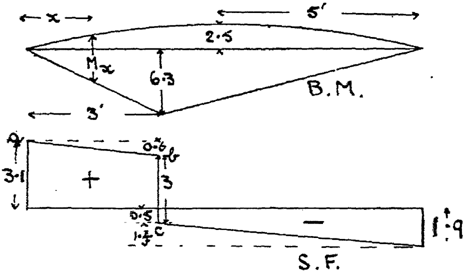
Or, from the other end, $M_{\text{centre}} = 3{\cdot}1 \times 5 - 3 \times 2 - 1 \times 2{\cdot}5$ $= 7$ ft. tons as before.

$_{\text{at load}} = 3{\cdot}1 \times 3 \qquad \times 2 \times \frac{3}{2} = 8{\cdot}4$ foot tons. Ans.

S.F. $= 3{\cdot}1 - 3 - 1 = -0{\cdot}9$ ton. Ans.

$\text{S.F.}_{\text{ends}} = 3{\cdot}1$ tons at R_1 and $1{\cdot}9$ tons at R_2. Ans.

It is convenient to draw the B.M. diagram in two parts. Above the base line, draw the B.M. curve for the uniform load. The maximum ordinate for this curve is $\dfrac{W\,l}{8} = \dfrac{2 \times 10}{8} = 2{\cdot}5$ foot tons. Then for the load of 3 tons acting alone, the end reactions would be 2·1 and 0·9 tons, and B.M. at the load $= 2{\cdot}1 \times 3 = 6{\cdot}3$ foot tons. Below the base line, at 3 feet from R_1 set down 6·3 foot tons and complete the diagram as shown. Then since the B.M. anywhere on the beam is the sum of the moments due to both the uniform and the concentrated load, the total B.M. at any point is given by a vertical ordinate extending from the curve above the base line, to the inclined boundary lines of the diagram below the base line. It is sufficient to roughly sketch the diagram. Vertical ordinates on this diagram represent *foot tons of moment,* and horizontal ordinates represent the length of the beam ; each to some convenient scale. Begiu the shearing force diagram at the left hand side of the base line, setting up 3·1 tons as shown. Now at 3 feet from R_1 the S.F. has been reduced by $\frac{3}{10} \times 2 = 0{\cdot}6$ ton due to the distributed load on 3 feet of the beam acting downwards. Draw $a\ b$ as shown.



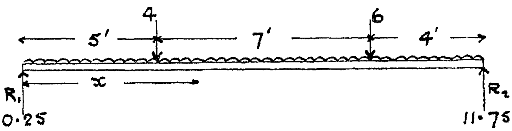
From b measure down $b\ c$ equal to 3 tons, cutting through the base line ; the ordinate below the base line is $3{\cdot}1 - 0{\cdot}6$

— 3 = — 0·5 ton. On the remaining 7 feet of the beam there is $\frac{7}{10} \times 2 = 1\cdot4$ tons acting down, and — 0·5 — 1·4 = — 1·9 tons at R_2. We already know the value of R_2 and this proves the S.F. diagram to be correct. Note that the sloping lines on the diagram will be parallel to each other, since the distributed load is uniform. Vertical ordinates on this diagram represent *tons of shearing force*, and horizontal ordinates represent the length of the beam ; each to some convenient scale.

Note that in problems of this kind, the end reactions must be determined *first*.

The question arises—where does the maximum B.M. occur ? The answer is :—The maximum B.M. occurs at the point on the beam where the S.F. *changes sign* from positive to negative. The proof of this statement is not given here. In the preceding problem, such a position occurs on the S.F. diagram at 3 feet from R_1 at the point of application of the 3 tons load, and the B.M. must be greatest at this point.

Example. A beam, 16 feet long, carries a uniform load of $\frac{3}{4}$ ton per foot run, a concentrated load of 4 tons at 5 feet from the left support, and a concentrated load of 6 tons at 4 feet from the right support. Determine the B.M. at the centre and at 5 feet from R_1 and find the position and amount of the maximum B.M. State the S.F. acting at each end and at the centre, and sketch the diagrams of B.M. and S.F.



The total distributed load is 12 tons.

Take moments round R_1 then :—

$R_2 \times 16 = 4 \times 5 + 6 \times 12 + 12 \times 8$

$R_2 = 11\cdot75$ tons ; $R_1 = 22 - 11\cdot75 = 10\cdot25$ tons.

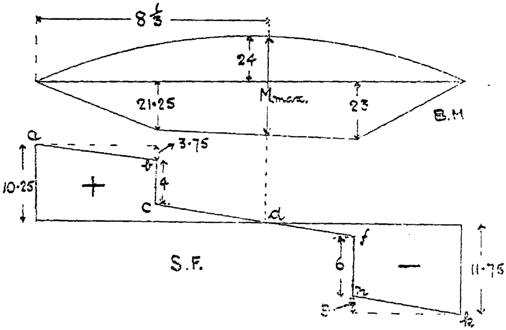
$M_{\text{centre}} = 10\cdot25 \times 8 - 4 \times 3 - 6 \times 4 = 46$ ft. tons of moment.

$M_{5\text{ feet}} = 10\cdot25 \times 5 - 5 \times \frac{3}{4} \times \frac{5}{2} = 41\cdot875$ ft. tons.

Proceeding to sketch the B.M. diagram as in the previous example, the maximum B.M. for the uniform load acting alone

is $\qquad \dfrac{12 \times 16}{} = 24$ ft. tons. The reactions for the other

loads if acting alone would be 4·25 and 5·75 tons, and the B.M. ordinates come as shown. Begin the S.F. diagram at the left end. Set up 10·25 tons. At 5 feet from R_1 the shear has fallen off by $\frac{3}{4} \times 5 = 3{\cdot}75$ tons due to the uniform load. Draw $a\ b$, set down $b\ c = 4$ tons, for the concentrated load. Then the point c is $10{\cdot}25 - 3{\cdot}75 - 4 = 2{\cdot}5$ tons on the force scale above the base line, and as the S.F. falls off at the rate of $\frac{3}{4}$ ton per foot, the S.F. will change sign at d at $2{\cdot}5 \div \frac{3}{4} = 3\frac{1}{3}$ feet from the 4 tons load. Now the diagram is easily completed. From c draw $c\ f$ parallel to $a\ b$, the point f being at the 6 tons load. Make $f\ n = 6$ tons, and draw $n\ k$ parallel to $c\ f$, and this must check up to $-$ 11·75 tons.



The maximum B.M. occurs where the S.F. changes sign ; in this problem it occurs where the S.F. = zero, for at d the S.F. is nothing.

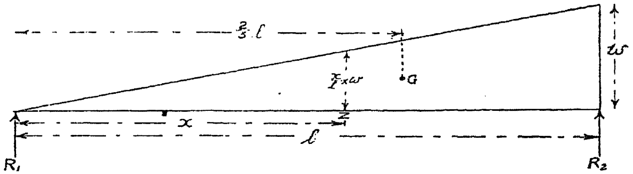
$$S.F._x = 10{\cdot}25 - 4 \qquad x = o$$

from which $x = 8\frac{1}{3}$ feet from R_1 and this is where the maximum B.M. occurs.

$$M_, \quad = 10{\cdot}25 \times 8\tfrac{1}{3} - 4(8\tfrac{1}{3} - 5) \qquad = 46{\cdot}04 \text{ ft. tons.} \quad \text{Ans.}$$

The position at which the maximum B.M. occurs in this problem is very close to the centre of the beam, and the B.M. at the two places is practically the same.

*__Beam supported at both ends,__ carrying a uniformly increasing load from zero at one end to w lb. per foot run at the other end.



The total load on the beam is represented by the area of the above diagram.

Total load $= \dfrac{w\ l}{2}$ lb., and the centre of gravity of this load is at $\frac{2}{3}\ l$ from R_1.

Taking moments about R_1 :—

$\dfrac{w\ l}{2} \times \frac{2}{3} l = R_2 \times l \ \therefore\ R_2 = \frac{2}{3} \times \dfrac{w\ l}{2}$ or $\dfrac{w\ l}{3}$ lb., therefore, the load carried by R_2 is two-thirds of the total load on the beam, and that carried by R_1 must be one-third of the total load on the beam, which is $\dfrac{w\ l}{6}$ lb.

The maximum bending moment occurs where the shearing force is zero, let this position be at Z, which is at x feet from R_1.

The intensity of the load at Z will be $\dfrac{x}{l} \times w$ lb. per foot run, and the load on the beam between this section and R_1 will be $\dfrac{x\ w}{l} \times \dfrac{x}{2} = \dfrac{w\ x^2}{2\ l}$ lb.

$$\therefore \text{ Shearing force at Z} = R_1 - \frac{w\ x^2}{2\ l} \qquad \frac{w\ l}{6} - \frac{w\ x^2}{2\ l}.$$

If this is to be zero, then $\dfrac{w\ l}{6} - \dfrac{w\ x^2}{2\ l} = 0.$

$$\frac{w\ l}{6} = \frac{w\ x^2}{2\ l} \qquad x^2 = \frac{2\ l^2}{6} \quad \therefore\ x = \frac{l}{\sqrt{3}} \text{ feet.}$$

The total load on the beam over the distance x feet is $\dfrac{w}{2\,l}$ lb.

and the centre of gravity of this piece of loading is at $\dfrac{x}{3}$ feet from Z, now taking moments about Z :—

$$M = R_1 \times x - \frac{w\,x^2}{2\,l} \times \frac{x}{3} = \frac{w\,l\,x}{6} - \frac{w\,x^3}{6}$$

$$= \frac{w}{6}\left(l\,x - \frac{x^3}{l}\right. \qquad \text{but } x = \frac{l}{\sqrt{3}}$$

$$\therefore\ M = \frac{w}{6}\left(\frac{l \times l}{\sqrt{3}} - \frac{l^3}{\sqrt{3} \times 3 \times}\right.$$

$$\frac{w}{6} \qquad \times 3 \qquad = \frac{w}{6} \times \qquad \sqrt{3} \times 3$$

$$\frac{w\ l^2}{9\,\sqrt{3}} \text{ foot lb.}$$

Hence, the maximum bending moment occurs at $\dfrac{l}{\sqrt{3}}$ feet from R_1 and its value is $\dfrac{w\,l^2}{9\,\sqrt{3}}$ foot lb., if l is in feet and w is the maximum intensity of the load in lb. per foot run.

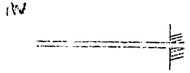
A practical example of this nature appears in Naval Architecture. It is the case of water pressure on a vertical bulkhead where, of course, the intensity of the water pressure is nothing at the free surface, increasing uniformly to a maximum at the bottom.

***Restrained or Fixed Beams.**

When a beam is restrained at the ends by being "built in" as shown, there is a moment at the ends of the beam. The previous methods cannot be applied to these cases, and the

value of the B.M. depends here upon the shape of the elastic line ; the proofs of the formulæ given for the next two cases are beyond the scope of the present Chapter.

Built in beam, with one load W at the centre.



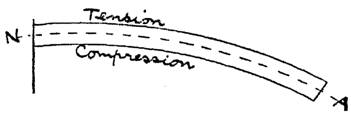
B.M. at centre and ends is $\frac{W\,l}{8}$

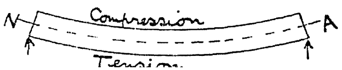
Built in beam, with a uniform load W. The maximum B.M. occurs at the ends.

$$\text{B.M.}_{\text{ends}} = \frac{W\,l}{12}$$

$$\text{B.M.}_{\text{centre}} = \frac{W\,l}{24}$$

Stresses in Beams.



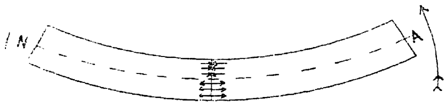


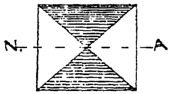
When a beam is distorted under load, it bends into a curve called the elastic line. In a loaded cantilever, the fibres of the top layer are in tension, and the fibres of the bottom layer are in compression. In the case of a beam supported at both ends, the top fibres are in compression and the bottom fibres in tension. As the strain is greatest at the outside fibres, it follows that the stresses are greatest there also. As the stress varies across the beam section from its maximum tensile value to its maximum compressive value, there must be a position in the section where the stress has zero value.

Neutral Axis.

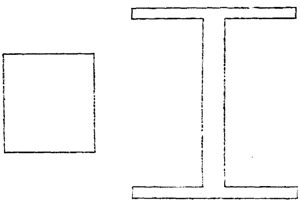
The stress, from its maximum values, passes through gradually decreasing values until a position of zero stress is reached. The plane or axis which passes through this position of zero stress is called the *Neutral Axis*. The Neutral Axis always passes through the centre of gravity of the beam section, no matter what shape that section may be. The strain varies directly as the distance from the neutral axis, and the stress varies as the strain. Note that the compressive and tensile forces acting are at right angles to a vertical section through the beam. In the case of a rectangular section, the C.G. is at the middle of the depth and the distribution of the stresses is shown in the

diagram. In this section, the layers near the neutral axis carry only a small stress and therefore the material is not economically disposed.





The material will be most economically disposed when the area of the cross section is arranged at the greatest possible distance from the neutral axis, that is, when a greater portion of the area carries the maximum stress. In the sketch, if the two figures have the same area, the I section will have a much greater resistance to bending, for reasons already given. The flanges carry most of the B.M., and the web takes the S.F.



The Moment of Resistance is the internal moment induced by the action of the externally applied bending moment; it is the moment which the beam is capable of enduring and transmitting. The moment of resistance is numerically equal to the bending moment.

Relation between Bending Moment and Stress.

The relation between bending moment, stress and strain in a beam is given by the equation :—

$$\frac{M}{I} = \frac{p}{y} = \frac{E}{R}$$

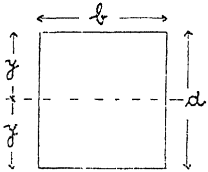
Where M = Bending moment in inch lb.
I = moment of inertia of the section, about the neutral axis in $(\text{inch})^4$ units.
p = stress in lb. per sq. inch.
y = distance from neutral axis where stress p occurs.
E = modulus of elasticity in lb. per sq. inch.
R = radius of curvature into which the beam is bent at the section considered.

For the purpose of this Chapter, we need only use the terms

$$\frac{M}{I} = \frac{p}{y}. \quad \text{Transposing we have } M = \frac{p\,I}{y}.$$

Now for any given section $\frac{I}{y}$ is constant, and it depends upon the fixed sizes of the section. To this quantity $\frac{I}{y}$, has been given the name of *Modulus of Section.* If the section is rectangular we have seen in the Chapter on Second Moments that I, or the moment of inertia about the C.G. axis is

$$\frac{b\,d^3}{12}; \text{ and } y \text{ is } \frac{d}{2}, \text{ then :—}$$

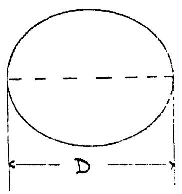


$$M = \frac{p\,I}{y} \quad \frac{p\,b\,d^3 \times 2}{12 \times d} \quad p\,b\,d^2$$

$$M = \frac{p\,b\,d^2}{6}, \text{ or } p = \frac{6\,M}{b\,d^2}$$

for a *rectangular* section.

For a circular section, we need the moment of inertia of a circular area about its diameter.



$$I = \frac{\pi\,D^4}{64} \text{ for a solid section, } y = \frac{D}{2}$$

$$M = \frac{p\,I}{y} \quad \frac{p\,\pi\,D^4\,2}{64\,D} \quad \frac{p\,D^3}{10{\cdot}2}$$

$$\text{or } p \quad \frac{10{\cdot}2\,M}{D^3}$$

The solid circular section is very uneconomical of material and the resistance it offers to bending is small, because a large portion of the area is concentrated near the neutral axis. For a hollow circular section, since $I = \frac{\quad}{64}\,[D^4$

$$M = \frac{p\ I}{y} = \qquad \frac{\cdot\ d^4]\ 2}{64\ D} \qquad \frac{p\ (D^4\ —}{10{\cdot}2\ D}$$

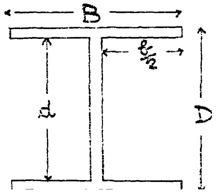
$$\text{or } p = \frac{10{\cdot}2\ M}{D^4 - d^4}$$

$$D$$

In the hollow circular section the material is disposed to better advantage.

The **Maximum** Stress is always needed, and as this occurs at the outside fibres, the maximum value of y must be used, this being the distance from N.A. to the outside fibres.

For a symmetrical section, such as a rolled joist, the moment of inertia must be determined.



$$I = \frac{B\ D^3 - b\ d^3}{12}$$

$$\text{Then } p = \frac{M\ y}{I} \text{ gives the stress,}$$

$$y \text{ being } \frac{D}{2}$$

Strength of Beams Compared.

For beams loaded in a similar manner, comparison of strength is often needed.

Since $M = p\ b$ –, then for the same stress, the Moment

which a beam will endure will vary as $b\ d^2$, the 6 in the formula being a constant. The moment which a beam will carry is a measure of its strength, therefore strength $\propto b\ d^2$, if the length is the same; but the longer a beam is made, the greater is the B.M. for the same load, and therefore the strength of a beam varies inversely as the length.

Therefore strength $\propto$ for rectangular sections.

For solid round sections, strength $\propto \dfrac{D^3}{L}$

For hollow round sections, strength $\propto \dfrac{D^4 - d^4}{D\,L}$

Flange Force.

An approximate method used to obtain the value of the moment of resistance is as follows. Assuming that in the case of an I section having similar flanges that the flanges take all the tension and compression, and that the stress is constant across the depth of each flange, then :—

Area of either flange $\times$ depth of section $\times$ stress $=$ B.M.

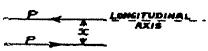
It is assumed that the web carries the shearing force. This method is only an approximation to the correct solution.

***Combined Bending and Direct Stress.**

Suppose a bar of material d inches diameter, rigidly held at one end, is subjected to a tensile load of P lb. applied through a point x inches from the longitudinal axis. We require to find the greatest and least stresses induced. Add and subtract P acting along the longitudinal axis and nothing is altered, but there is a re-distribution of the forces. We have the force

acting along the axis causing a direct stress of $\dfrac{}{A}$,

and also a couple of magnitude P $\times$ x which causes bending.



The bending moment, M $=$ P x, induces tensile stress in the lower fibres, and compressive stress in the upper fibres.

Now $\dfrac{M}{I} = \dfrac{p}{y}$, and I $= A\,k^2$ (Area $\times$ square of radius of gyration)

$$\therefore p = \frac{M\ y}{\ } = \frac{P\ x\ y}{A\ k^2}$$

$$\therefore \text{Maximum tensile stress} = \frac{\ }{A} + \frac{y}{A\ k^2}$$

$$= \frac{P}{A}\left\{1 + \frac{x\ y}{\ }\right.$$

and, Minimum tensile stress $= \dfrac{P}{A}\left\{1 - \dfrac{x\ y}{k^2}\right\}$. If this comes out negative then the least stress is compressive.

Example. A longitudinal stay in a double-ended boiler is 20 feet long and 3 inches diameter, and it carries a tensile load of 25 tons. If the stay sags $\frac{1}{4}$ inch below its end connections, find the greatest and least stresses in the stay.

Direct stress $= \dfrac{25 \times 2240}{3^2 \times \dfrac{\pi}{4}} = 7921$ lb. per sq. inch.

Bending moment $= 25 \times 2240 \times \frac{1}{4} = 25 \times 560$ inch lb.

I for a circular section $= \dfrac{\pi\ d^4}{64}\quad\dfrac{\pi \times 3^4}{64}$ inch^4 units.

$y = \frac{3}{2}$ inches.

$$\therefore \text{Bending stress } (p) = \frac{25 \times 560 \times 3 \times 64}{\pi \times 3^4 \times 2} = 5281 \text{ lb. per sq. inch.}$$

In the outermost fibres of the stay at the bottom, the stress is $7921 + 5281$
$= 13202$ lb. per sq. inch (tensile). Ans.

In the outermost fibres at the top, the stress is $7921 - 5281$
$= 2640$ lb. per sq. inch (tensile). Ans.

***Deflection of Beams.**

The formulæ which give the deflection of beams are as follows :—

Cantilever, with one load W at the end,

$$\text{deflection} = \frac{W\ l^3}{3\ E\ I} \text{ at the free end.}$$

Cantilever, with a uniform load W,

$$\text{deflection} = \frac{W\ l^3}{8\ E\ I} \text{ at the free end.}$$

Beam, supported at ends, one central load W,

$$\text{deflection} = \frac{W\ l^3}{48\ E\ I} \text{ at the middle of length.}$$

Beam, supported at ends, uniform load W,

$$\text{deflection} = \frac{5\ W\ l^3}{384\ E\ I} \text{ at the middle of length.}$$

The proofs of these formulæ are not necessary for this grade.

TEST EXAMPLES XVII.

1. A cantilever 10 feet long carries a load of 1 ton at its free end, and a uniform load of 200 lb. per foot. Find the value of the maximum bending moment. The beam is 3 inches broad and 10 inches deep, find the maximum stress. Sketch the B.M. and S.F. diagrams.

32,400 ft. lb.; 7,776 lb. per sq. inch. Ans.

2. A cantilever 8 feet long, carries a load of 1 ton at its free end and another load of 2 tons at 5 feet from the support. Find the depth of the beam if the maximum stress is not to exceed 6,000 lb. per sq. inch, the breadth being 3 inches.

12·7 inches. Ans.

3. A cantilever 8 feet long, carries a load of 4 cwt. per foot run. The maximum stress is not to exceed 3 tons per sq. inch. Find the size of rectangular section, the depth being 4 times the breadth.

8·482 inches deep; 2·121 inches broad. Ans.

4. A boat davit is 6 inches diameter of solid round section. The boat and crew weigh 3 tons and from the centre of the boat to the centre of the davit is 5 feet. Find the stress in the material of the davit. Each davit takes half the weight of the boat.

4·25 tons per sq. inch. Ans.

5. A beam 16 feet long is supported at each end, and carries a load of 4 tons at its centre. Find the value of the maximum bending moment. The load is now shifted 4 feet to one side of the centre, find the end reactions and the maximum bending moment. The beam is 3 inches wide and 8 inches deep, find the maximum stress in each case.

6 tons per sq. inch; 4·5 tons per sq. inch. Ans.

6. A rectangular beam 4 inches broad, 6 inches deep and 10 feet long, safely carries a concentrated load of 3 tons at its centre; find what load a beam of the same material will carry at its centre, its dimensions being 12 feet long, 3 inches wide and 8 inches deep.

$3\frac{1}{3}$ tons.

7. A rolled joist, fixed at one end with flanges $5\frac{1}{2}$ inches wide by $\frac{1}{2}$ inch thick, is 6 feet long. Find the necessary depth of this joist if it is to carry a load including the weight of the joist, of $\frac{1}{4}$ ton per foot, and the stress is limited to 6,000 lb. per sq. inch. Use the approximate method here.

7·33 inches deep. Ans.

8. A bar rests in equilibrium upon one support which is not in the centre of the bar. The bar is 10 feet long, a load of 500 lb. hangs from one end, and a load of 680 lb. hangs from the other end. Find the position of the support, the maximum bending moment, and design the section of the bar which is rectangular, the depth being 3 times the breadth, the allowed stress being 5,000 lb. per square inch. Sketch the B.M. and S.F. diagrams. 4·992 inches deep ; 1·664 inches broad.

9. A beam, 20 feet long, carries a concentrated load of 4 tons at 5 feet from its left end and another of 5 tons at 7 feet from its right hand end. Find the maximum bending moment, and the bending moment at the centre, and sketch the B.M. and S.F. diagrams. The beam is rectangular in section, 6 inches broad and 12 inches deep, find the maximum stress and the stress at the centre.

Max. stress = 2·48 tons sq. inch, and 2·3 tons sq. inch at centre. Ans.

10. A beam 16 feet long, supported at the ends, carries a uniform load of half ton per foot, a concentrated load of 4 tons at 3 feet from the left support, and a concentrated load of 6 tons at 5 feet from the other support. Find the position and amount of the maximum bending moment, find the bending moment and the shearing force at the centre, and sketch the diagrams of B.M. and S.F.

Max. B.M. 38·27 ft. tons ; B.M. at centre 37 ft. tons. Ans.

11. A round beam 10 feet long carries a uniform load of quarter ton per foot, and a concentrated load of 2 tons at 3 feet from one support, find the diameter of the beam, so that the maximum stress may not exceed $3\frac{1}{2}$ tons per sq. inch.

6·825 ft. tons ; d = 6·203 inches. Ans.

12. A lever safety valve is 20 inches long from the centre of the weight to the centre of the valve ; the lever is half inch broad and 2 inches deep, and the valve is fastened to the lever by a pin $\frac{5}{8}$ inch diameter. Find the stress on the lever at the pin when the weight at the end is 64 lb.

3961 lb. sq. inch. Ans.

13. A cantilever is 12 feet long, and it is a rolled joist 12 inches deep overall, with flanges 9 inches wide, the material being half inch thick throughout, and weighing 490 lb. per cubic foot. A load of $2\frac{1}{2}$ tons is hung at the free end. Find the stress in the beam when the weight of the beam is neglected, and when it is taken into account.

13,700 lb. per sq. inch ; 14,420 lb. per sq. inch. Ans.

14. A beam 16 feet long, supported at its ends, carries a concentrated load of 3 tons at 4 feet from one end and another of 4 tons at 5 feet from the other end. The beam is of I section half inch thick throughout, the flanges being each 8 inches wide. Find the depth of the girder, using the approximate method, so that the maximum stress may not exceed $3\frac{1}{2}$ tons per sq. inch, neglecting the weight of the beam.

15 inches. Ans.

15. An I section is 10 inches deep overall, having flanges 8 inches wide, the material being half inch thick throughout. Compare the strength of the section (*a*) when placed as an I section; (*b*) when placed as an |—| section.

As 1 : 0·2533. Ans.

16. A round beam is a solid section 2 inches diameter. Find the outer diameter of a hollow round section of the same area, the hole being 1 inch diameter. Compare the strengths of the two sections in bending.

34 per cent. stronger. Ans.

17. Find the size of rectangular section for the beam of Question 10, if the maximum stress is not to exceed 4 tons per square inch, and if the depth is three times the breadth.

12·735 inches deep; 4·245 inches broad. Ans.

*18. A beam 30 feet long is simply supported at each end, and carries a load which increases uniformly from zero at one end to 1,920 lb. per foot run at the other end. Find (*a*) the total load upon the beam, (*b*) the load carried by each support, (*c*) the position where the shearing force is zero, (*d*) the maximum bending moment.

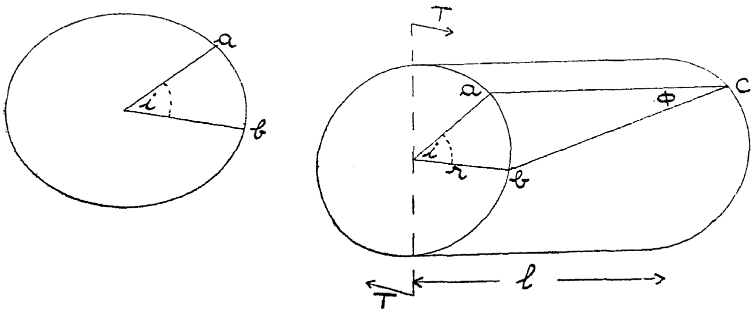
28,800 lb.; 9,600 lb.; 19,200 lb.; 17·32 feet from the lightly loaded end; 110,900 ft. lb. Ans.

CHAPTER XVIII.

TORSION.

A shaft transmitting Twisting Moment or Torque is said to be *in Torsion*, and the stress induced in the shaft is shear stress.

Consider the solid cylinder of length l. Let the line $a\ c$ be scribed on the surface, parallel to the central axis. If one end be fixed at c, and a Twisting Moment applied at the other end, then the point a moves to b, and the line $a\ c$ takes up a spiral form $b\ c$. If now we imagine that this cylinder, instead of being solid, is made up of a great number of thin circular discs strung upon an axis, then when the point a moves to b, each thin disc moves an angular distance across the face of its adjacent disc, and the torsion is, in effect, reduced to shear. The shear strain is measured by the angle of distortion ϕ, as previously explained.



$$\text{Shear Strain} = \frac{a\ b}{l}$$

The *angle of twist* is i at the centre of the shaft, and if we measure this in radians, then $i = \dfrac{a\ b}{r}$.

Just as in the case of direct tension and compression we have the relation :—

constant = **E**

Strain

so in the case of shear we have the relation :—

$$\frac{\text{Shear Stress}}{\text{Shear Strain}} = \text{constant} = C,$$

the symbol for this constant being C. It is called the Modulus of Rigidity, or the Transverse Modulus, and its value is about $\frac{2}{5}$ that of the Modulus of Elasticity (E) for wrought iron and mild steel; C is expressed generally as lb. per square inch. The symbol for shear stress is q.

The angle of twist in a shaft varies as the length of the shaft, for the same Twisting Moment.

The angle of twist in a shaft varies as the Twisting Moment, for the same length of shaft.

The angle of twist varies *inversely* as the (diameter)^4 for shafts of the same length and for the same Twisting Moment.

We have :—

$$\text{Shear Strain} = \frac{a\ b}{l}, \text{ and } i = \frac{a\ b}{r}, \text{ or } a\ b = i\ r$$

$$\text{also } \frac{\text{Shear Stress}}{\text{Shear Strain}} = C, \text{ or } \frac{q\ l}{a\ b} = C$$

$$\text{or } \frac{q\,l}{i\,r} = C\,; \text{ or } \frac{q}{r} = \frac{C\ i}{l} \quad . \quad . \quad . \quad . \quad . \quad (1)$$

It is seen from this equation, that q the stress, varies directly as the distance from the centre of the shaft.

The material of a *solid shaft* is not economically disposed, because only the outer fibres carry the maximum stress, the material near the centre carrying a low stress.

Relation between Twisting Moment, Stress and Angle of Twist.

$$\frac{T}{J} = \frac{q}{r} = \frac{C\ i}{l}$$

T = Torque in inch lb. of moment.
J = Polar Second Moment or Moment of Inertia of the shaft section about the axis.
q = Shear stress acting at r inches from shaft centre, in lb. per sq. inch.
C = Modulus of Rigidity, in lb. per square inch.
i = Angle of Twist in *radians*.
l = Length of the shaft in inches.

The Moment of Resistance is the twisting moment which the shaft is capable of transmitting, and it is numerically equal to the externally applied torque.

Taking the first two terms of the Torsion equation,

$$\frac{T}{\quad} = \frac{q}{\quad}, \text{ we have } T = \frac{q\ J}{\quad}. \quad \text{Now } J = \frac{\pi\ D^4}{32}, \text{ and}$$

$$\frac{D}{2} \qquad .\ T = \frac{\pi\ D^4 \times 2}{32 \times D} \times q \qquad \frac{\ D^3}{16}$$

$$\text{and since } \frac{\pi}{16} = \frac{1}{5{\cdot}1},\ T \qquad \frac{\quad}{5{\cdot}1} \times \text{stress, for a solid shaft.}$$

Example. A piston is 300 square inches in area, the crank is 21 inches long and the shaft is 10 inches diameter. Find the stress in the shaft when the steam pressure is 200 lb. per square inch.

As the length of the connecting rod is not given here, we may assume that the Maximum Torque is load on piston $\times$ length of crank.

$= 300 \times 200 \times 21 = 1260000$ inch lb. of moment.

$$T = \frac{D^3}{5{\cdot}1} \times q, \text{ or } q = \frac{T \times 5{\cdot}1}{D^3}$$

$$\frac{1260000 \times 5{\cdot}1}{10 \times 10 \times 10} = 6426 \text{ lb. per sq. inch.} \quad \text{Ans.}$$

Inspection of the twisting moment formula shows that the twisting moment, and therefore the strength of a solid shaft, varies as the diameter cubed, or as D^3.

Hollow Shaft.

Let D be the outside diameter and d the diameter of the hole, then :—

$$T = \frac{J\ q}{r}, \text{ and here } J = \frac{\pi}{32}\left[D^4 - d^4\right], \text{ and } r = \frac{D}{2}$$

$$T = \frac{\pi}{32}\ \frac{(D^4 - d^4)\ 2}{D} \times q, \text{ or } T = \frac{\pi}{16} \qquad \frac{\quad}{D} \qquad \times$$

In the case where the hole is half the outer diameter, $d = \frac{D}{2}$

$$\text{and } T = \frac{\pi}{16\ D}\left[D^4 - \left(\frac{D}{2}\right)^4\right] \times q$$

$$= \frac{\pi}{16\ D}\left[D^4 - \frac{D^4}{16}\right] \times q$$

$$\text{or } T = \frac{\pi}{16\,D} \times \tfrac{15}{16}\,D^4 \times q = \frac{\pi}{16} \times \tfrac{15}{16}\,D^3 \times q$$

Now if the shaft is solid, $T = \frac{\pi\,D^3}{16} \times q$, therefore it is

seen that by drilling a hole of diameter $\frac{D}{2}$ through the shaft,

only $\frac{1}{16}$th of the strength is taken away, while quarter of the material has been removed, since the areas are in the ratio of

$$\frac{D^2}{D^2 - \left(\frac{D}{2}\right)^2} \quad \text{or} \; - \qquad \text{or as } 1 : \tfrac{3}{4}.$$

Stiffness of a Shaft.

The stiffness of a shaft is a measure of the angle through which it twists under a given torque. Writing down the first and third terms of the torsion equation.

$\frac{T}{J} = \frac{C\ i}{l}$, we have $i = \frac{T\ l}{C\ J}$, and this tells us i

varies directly as $T\ l$ and inversely as $C\ J$.

$\frac{i\ C\ J}{T\ l}$ = constant, and for shafts of similar material

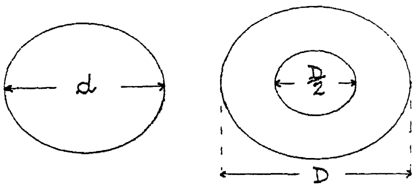
C is the same, $\therefore\ \frac{i\ J}{T\ l}$ = constant.

The stiffness may be written as $\frac{}{i}$, since the greater the angle of twist the less stiff is the shaft.

$$\text{Stiffness} = \frac{1}{i} = \frac{C\,J}{T\,l}$$

The stiffness varies as C, The Transverse Modulus.
 ,, ,, J the 2nd Moment.
 ,, varies inversely as the Twisting Moment.
 ,, varies inversely as the length.

Example. Two shafts have the same weight, and are the same length. One is solid, the other is hollow with its internal diameter half its external diameter. Compare the strengths of the two shafts, and compare the stiffness.



Let d = diameter of solid shaft.

D = diameter of hollow shaft.

Then since the weights and lengths are equal, the areas must be equal,

$$d^2 = D^2 - \left(\frac{D}{2}\right)^2 = \tfrac{3}{4}\,D^2$$

$$d = \sqrt{\tfrac{3}{4}}\,D$$

Solid Shaft.

$$T = \frac{}{16}\,d^3\,q$$

Strength $\propto d^3$

Hollow Shaft when Hole is $\frac{D}{2}$

$$T = \frac{\pi}{16} \times \tfrac{15}{16}\,D^3\,q$$

Strength $\propto \tfrac{15}{16}\,D^3$

Strengths are as $d^3 : \frac{15}{16} D^3$

as $[\sqrt{\frac{3}{4}} D]^3 : \frac{15}{16} D^3$

$$\frac{\sqrt{3} \times 3}{2 \times 2 \times 2} \qquad D^3$$

as $\sqrt{3}\, D^3 : \frac{5}{2} D^3$

or as $1 : \frac{2 \cdot 5}{\sqrt{3}}$, or as $1 : 1 \cdot 443$. Ans.

Weight for weight, for the same maximum stress, the hollow shaft is 44 per cent. stronger than the solid shaft.

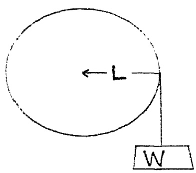
For the same T and l, the stiffnesses are

as J for solid : J for hollow

as

$\frac{9}{16} D^4 : \frac{15}{16} D^4$, or as $9 : 15$, or $1 : 1 \cdot 67$. Ans.

Relation between Horse Power and Twisting Moment.



Let W act at a leverage of L inches from the shaft centre, and let the shaft make n revolutions per minute.

Work per revolution $= 2\,\pi\,L\,W$ inch lb.

Work per minute $= 2\,\pi\,L\,W\,n$ inch lb.

But work also $=$ H.P. $\times$ 33,000 $\times$ 12 inch lb. per minute.

$\therefore 2\,\pi\,L\,W\,n = \text{H.P.} \times 33{,}000 \times 12$

Now $W \times L = T$,

$$\therefore T = \frac{\text{H.P.} \times 33{,}000 \times 12}{2\,\pi\,n} \qquad 63{,}000 \times \text{H.P.}$$

$$\therefore T = \frac{63{,}000 \times \text{H.P.}}{\text{revolutions}} \text{ inch lb. of moment.}$$

This is the *mean* twisting moment, and assumes a constant torque. In the case of an engine having cranks, the torque varies considerably, depending upon the number and arrangement of the cranks, therefore the formula for the mean T must be multiplied by a factor greater than unity, the value of which

is $\dfrac{\text{Maximum Torque}}{\text{Mean Torque}}$

$$\text{Maximum T} = \frac{63{,}000 \times \text{H.P.}}{\text{Revs.}} \times \text{ratio of Maximum T to Mean T.}$$

Comparision of Horse Power, Diameter, Revolutions and Stress.

$$\text{T} = \frac{63{,}000 \times \text{H.P.}}{\text{Revs.}}, \qquad \text{T} = \frac{D^3}{5\cdot 1} \times \text{stress}$$

$$\frac{}{5\cdot 1} \times \text{stress} = \frac{63{,}000 \times \text{H.P.}}{\text{Revs.}}$$

$$\therefore\ D^3 \times \text{stress varies as } \frac{\text{H.P.}}{\text{Revs.}}$$

$$\text{or, } \frac{D^3 \times \text{Stress} \times \text{Revs.}}{\text{H.P.}} = \text{constant.}$$

$$\text{or, } \frac{D_1^3\ \text{Stress}_1\ \text{R}}{\text{H.P.}_1} \qquad \frac{D_2^3\ \text{Stress}_2\ \text{Revs}_2}{\text{H.P.}_2}$$

Example. A shaft transmits 1,000 horse power at 80 revolutions per minute. Find the mean twisting moment. The maximum stress is not to exceed 7,700 lb. per square inch, and the maximum twisting moment is 1·5 times the mean, find the diameter of the shaft, if solid.

$$\text{Mean T} = \frac{63{,}000 \times \text{H.P.}}{\text{Revs.}} \qquad \frac{63{,}000 \times 1{,}000}{80}$$

$$= 787{,}500 \text{ inch lb. Ans.}$$

$$\text{Maximum T} = 787{,}500 \times 1\cdot5 \text{ inch lb.}$$

$$\text{and T} = \frac{D^3}{5\cdot1} \times \text{stress}$$

$$\frac{D^3}{5\cdot1} \times 7{,}700 = 787{,}500 \times 1\cdot5$$

$$D^3 = \frac{787{,}500 \times 1\cdot5 \times 5\cdot1}{7{,}700} = 782\cdot4$$

$$D = 9\cdot212 \text{ inches.} \quad \text{Ans.}$$

Example. A shaft 10 inches diameter transmits 1,000 horse power at 75 revolutions per minute. Find what horse power a shaft of 5 inches diameter can transmit at 550 revolutions per minute.

$$\frac{D_1^3 \text{ Stress}_1 \text{ Revs}_1}{\text{H.P.}_1} = \frac{D_2^3 \text{ Stress}_2 \text{ Revs}_2}{\text{H.P.}_2}$$

Since the stress is not given, we may assume it to be constant.

$$\frac{D_1^3 \text{ Revs}_1}{\text{H.P.}_1} \quad \frac{D_2^3 \text{ Revs}_2}{\text{H.P.}_2}$$

$$\text{H.P.}_2 = \frac{\times \text{ Revs}_2}{D_1^3 \times \text{Revs}_1}$$

$$\text{H.P.}_2 = \frac{1{,}000 \times 5 \times 5 \times 5 \times 550}{10 \times 10 \times 10 \times 75} = 916\cdot7. \quad \text{Ans.}$$

Example. Design a hollow shaft, having its internal diameter half its external diameter, to transmit 3,000 horse power at 75 revolutions per minute. The maximum shear stress is not to exceed 7,000 lb. per square inch, and the ratio of maximum to mean torque is 1·3.

$$\text{Maximum T} = \frac{63{,}000 \times 3{,}000}{75} \times 1\cdot3 \text{ inch lb.}$$

$$T = \frac{(D^4 - d^4)}{D} \times \frac{\text{Stress}}{5\cdot1}, \text{ and when } d = \frac{D}{2}$$

$$T = \tfrac{15}{16}\, D^3\, \frac{\text{Stress}}{5 \cdot 1}$$

$$\therefore \tfrac{15}{16}\, D^3 \times \frac{7{,}000}{5 \cdot 1} = \frac{63{,}000}{75} \times 3{,}000 \times 1 \cdot 3$$

$$D^3 = \frac{63{,}000 \times 3{,}000 \times 1 \cdot 3 \times 16 \times 5 \cdot 1}{15 \times 7{,}000 \times 75} = 2{,}546$$

$D = 13 \cdot 66$ inches outside diameter.

$d = 6 \cdot 83$ inches inside diameter. Ans.

Example. The shaft in the previous example is to be replaced by a solid one of the same material, and its strength must be the same. Find its diameter.

$$(\text{Diam. of Solid})^3 \times \frac{\text{Stress}}{5 \cdot 1} \qquad \frac{}{D} \times \frac{\text{Stress}}{5 \cdot 1}$$

As the stresses are the same, and $d = \dfrac{D}{2}$

$$(\text{Diam. of solid})^3 = \frac{D^4 - \left(\frac{D}{2}\right.}{D}$$

$(\text{Diam. of solid})^3 = \tfrac{15}{16} \times (13 \cdot 66)^3$.

Diam. of solid $= 13 \cdot 37$ inches. Ans.

Coupling Bolts.

It is usual to assume the same maximum stress in coupling bolts and shaft.

Moment of Resistance of bolts = Moment of Resistance of shaft.

Total area of bolts × stress × radius of bolt circle $= \dfrac{}{16} \times$ stress.

As the stress is the same then :—

$$\frac{}{4} \times N \times R = \frac{}{16}$$

$$\therefore d = \sqrt{\frac{}{4\ N\ R}} \qquad \frac{D}{2} \qquad \overline{D} \qquad \text{for solid shaft.}$$

Where D = diam. of shaft, d = diameter of bolts.

N = number of bolts. R = distance from centre of bolts to centre of shaft.

For hollow shaft, $d = \frac{1}{2}\sqrt{\frac{D^4 - d_2^4}{D\ N\ R}}$

where d_2 = diam. of hole in shaft.

Example. A shaft 12 inches diameter is fitted with 6 coupling bolts, at a radius of $9\frac{1}{2}$ inches from the centre of the shaft. Find the diameter of bolts, the shear stress in the bolts and the shaft being the same.

Moment of bolts = moment of shaft.

$$^2\ \frac{\pi}{4} \times 6 \times \frac{19}{2} = \frac{\pi D^3}{16}$$

$$\frac{12^3}{4} \times \qquad \times \frac{1}{6} =$$

$$d = \frac{12}{\sqrt{19}} = 2{\cdot}753 \text{ inches. Ans.}$$

Effect of Angularity on the Twisting Moment. This problem has already been solved in the Chapter on Forces ; another method is now given.

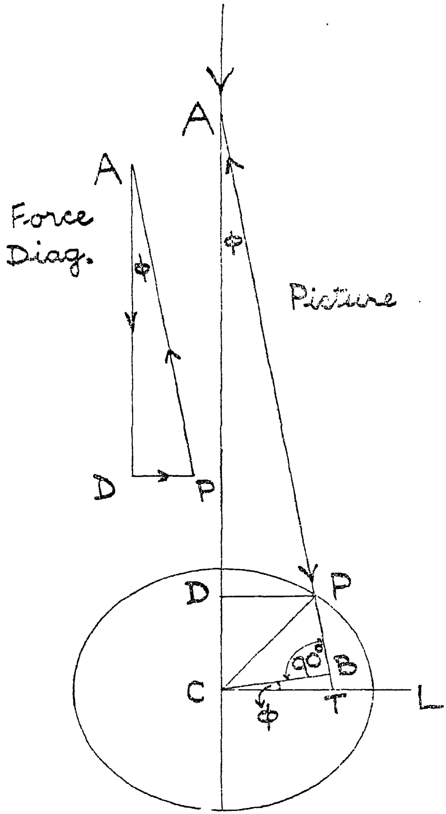
The line of action of the force in the connecting rod is in the direction A P. Produce A P to T, the line C T being at right angles to A C. Then the moment of the force in the connecting rod about C, is equal to the force in the connecting rod multiplied by the perpendicular distance B C.

Now angle A C B = 90° — ϕ

and angle A C T = 90°

$\therefore$ Angle B C T = 90° — (90° — ϕ) = ϕ

$$\text{Cos. } \phi = \frac{C\ B}{C\ T}, \text{ or } C\ B = C\ T \text{ Cos. } \phi \quad . \quad . \quad . \quad . \quad (1).$$



From the force diagram, where A D = force on piston and A P = force in connecting rod :—

$$\text{Cos. } \phi = \frac{A\ D}{A\ P}, \text{ or } A\ P = \frac{A\ D}{\text{Cos. } \phi}$$

$$\text{or Force in connecting rod} = \frac{\text{Force on piston}}{\text{Cos. } \phi} \quad . \quad . \quad . \quad (2)$$

Moment of Force in connecting rod about C = Force × C B.

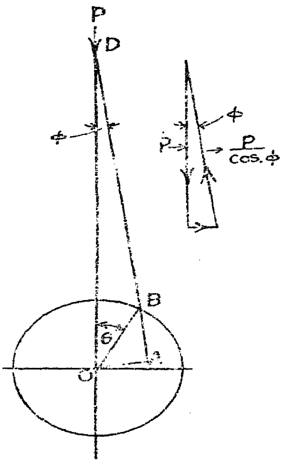
$$\text{Twisting Moment} = \frac{\text{Force on piston}}{\text{Cos. } \phi} \times \text{C T} \times \text{Cos.}\,\phi, \text{from (1) and (2)}$$

Twisting Moment = Force on piston × C T.

For any crank angle then, produce the connecting rod until it cuts the line C T (which is at right angles to A C), in the point T, then the *force on the piston* multiplied by the distance C T is the twisting moment, and this is true for any position of the crank. If the crank is more than 90° from the top centre, then the point T is where the connecting rod itself cuts the line C L. As previously stated, the Maximum Torque occurs when the connecting rod is at right angles to the crank.

It is sometimes necessary to draw a diagram representing the variation of the twisting moment throughout the revolution. To do this a diagram is drawn to scale for a number of crank angles, and the different lengths of C T are measured. The effective pressure on the piston for each corresponding crank angle is measured from indicator diagrams. The twisting moment for each position is calculated and the results plotted on a diagram, the base line representing angles turned through by the crank. This is fully dealt with at the end of the Chapter on Expansion of Steam, and under the heading Crank Effort Diagrams.

If it is required to calculate the twisting moment it may be done in this manner.



Let the angle of the crank from the top centre be θ; length of connecting rod l, length of crank c. and load on piston P.

In triangle O B D :—

$$\frac{l}{\text{Sine}} = \frac{c}{\text{Sine } \phi}$$

$$\text{Sine } \phi = \frac{c \text{ Sine } \theta}{l}$$

This gives angle ϕ.

From force diagram,

$$\frac{P}{\text{Force in connecting rod}} = \text{Cos.}$$

∴ Force in connecting rod

$$= \frac{P}{\text{Cos. } \phi}$$

In any triangle, if one side is produced the external angle formed is equal to the sum of the two opposite angles of the triangle.

$\therefore$ angle O B A $= \theta + \phi$.

$\therefore$ O A $= c$ Sine $(\theta + \phi)$.

Twisting moment $=$ Force in connecting rod $\times$ O A

$$= \frac{P}{\text{Cos. } \phi} \times c \text{ Sine } (\theta + \phi).$$

Example. The piston of an engine is 400 square inches area and the effective steam pressure is 150 lb. per square inch. The crank is 20 inches long and the connecting rod is 80 inches long. Find the twisting moment when the crank has turned through 50° from the top centre, and when it has turned through 120° from the top centre.

(*a*) $$\frac{80}{\text{Sine } 50^\circ} \qquad \frac{20}{\text{Sine } \phi} \qquad \frac{20 \times 0{\cdot}766}{80}$$

$$= 0{\cdot}1915.$$

$\therefore \phi = 11^\circ\ 2'$

$\theta + \phi = 50^\circ + 11^\circ\ 2' = 61^\circ\ 2'$

$$\text{Twisting moment} = \frac{400 \times 150 \times 20 \text{ Sine } 61^\circ\ 2'}{\text{Cos. } 11^\circ\ 2'}$$

$= 1{,}069{,}000$ inch lb. Ans.

(*b*) $$\frac{80}{\text{Sine } 120^\circ} \qquad \frac{20}{\text{Sine } \phi} \qquad \phi \qquad \frac{20 \times 0{\cdot}866}{80}$$

$$= 0{\cdot}2165$$

$\therefore \phi = 12^\circ\ 30'$.

$\theta + \phi = 120^\circ + 12^\circ\ 30' = 132^\circ\ 30'$

$$\text{Twisting moment} = \frac{400 \times 150 \times \text{Sine } 132^\circ\ 30' \times 20}{\text{Cos. } 12^\circ\ 30'}$$

$= 906{,}000$ inch lb. Ans.

Formulæ for Shafting.

The Board of Trade formula for the diameter of a solid crank shaft is :—

$$C\ P\ D^2$$

$$2 + \frac{D^2}{d^2} \Big)$$

Where S = diameter of shaft in inches.
P = absolute boiler pressure in lb. per sq. inch.
D = diameter of L.P. cylinder in inches.
d = diameter of H.P. cylinder in inches.
C = length of crank in inches.
f = a constant, depending upon the number and arrangement of the cranks.

From this formula it is seen that S^3 varies as $C \times P \times D^2$,

$$\frac{S^3}{C\,P\,D^2} = \text{constant, and as for any one engine C and D}$$

are constant, then to find the pressure for a shaft reduced in diameter by corrosion, we write :—

$$\frac{S^3}{\quad} = \text{constant, or } \frac{S_1^3}{\quad} = \frac{S_2^3}{\quad}$$

Angle of Twist.

A formula sometimes given for the twisting moment in terms of the angle of twist is :—

$$T = \frac{140 \times a \times d^4}{L}$$

where T = torque in ft. lb.
a = angle of twist in degrees.
d = diameter of shaft in inches.
L = length of shaft in feet.

This formula is derived from the Torsion Equation, $\dfrac{T}{J} = \dfrac{C\,i}{l}$

$$\text{or } T = \frac{C\,i\,J}{l}.$$

In this equation, T = torque in inch lb., and T (foot lb.) $\times$ 12 = T (inch lb.)

J = Polar second moment of the shaft, and this is $\dfrac{\pi\,d^4}{32}$ for a solid shaft.

C = Modulus of Rigidity (lb. per sq. inch.)

i = Angle of twist in radians $= \dfrac{a^\circ}{57{\cdot}3}$

l = Length of shaft in inches, and L (feet) $\times$ 12 = l (inches).

Substituting these values we have,

$$\text{T (ft. lb.)} \times 12 = \frac{\text{C} \times \overline{57{\cdot}3} \times 32}{\text{L (feet)} \times 12}$$

$$\text{T (ft. lb.)} = \frac{\text{C} \times \pi}{57{\cdot}3 \times 32 \times 12 \times 12} \quad \frac{a^\circ \times}{\text{L (feet)}}$$

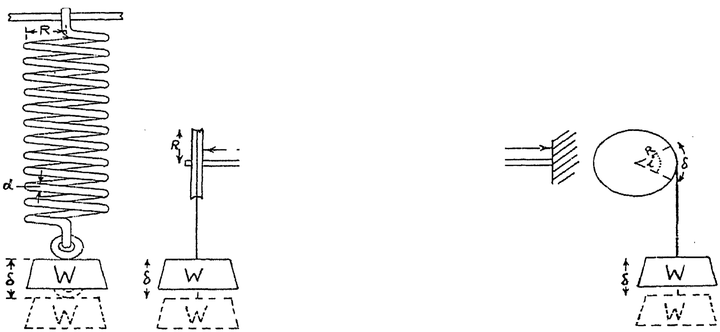
$$= \text{a constant} \times \frac{a^\circ \times d^4}{\text{L (feet)}}$$

If C is taken as 11·77 million lb. per sq. inch, and this is a probable value for mild steel, then the value of the constant is 140.

***Closely Coiled Helical Spring.**

When an axial load is applied to a spring of this type the wire is, to all intents and purposes, subjected to pure torsion. Stresses due to bending and to shearing force are small and may be disregarded.

Let the number of coils in the spring be N ; the diameter of the wire = d inches ; the mean radius of the spring = R inches ; the deflection, or stretch = δ inches when an axial load of W is applied.



Since the spring is closely coiled then the length of the wire is practically 2 π R N inches. If we imagine the wire to be straightened out and fitted with a pulley of radius R, then we see the twisting moment applied is W $\times$ R.

Also if i is the angle of twist in radians, then $R \times i = \delta$

$$\text{or } i = \frac{\delta}{R}$$

$$\text{Now } \frac{T}{J} = \frac{C\ i}{l} \qquad \text{and } J = \frac{\pi\ d^4}{32}$$

$$W \times R \qquad C \times \frac{\delta}{R}$$

$$32$$

$$\text{and } \delta = \frac{W \times R \times 2\,\pi\,R\,N \times R \times 32}{C\ d^4}$$

$$64\ W\ R^3$$

TEST EXAMPLES XVIII.

1. It is found experimentally that a load of 800 lb. at a leverage of 1 foot is required to break a wrought iron bar 1 inch diameter. What force at the end of a spanner 20 inches long will break an iron stud $\frac{3}{4}$ inch diameter at the bottom of the thread, and what is the breaking stress ?

202·5 lb. ; 48,960 lb. per sq. inch. Ans.

2. Two shafts are $14\frac{1}{2}$ inches and $13\frac{1}{2}$ inches diameter respectively. Find how much per cent. one is stronger than the other, and how much per cent. heavier.

24 per cent. stronger ; 15·4 per cent. heavier. Ans.

3. A shaft 14 inches diameter is subjected to torsion by a crank 28 inches long. The piston is 46 inches diameter and the effective pressure is 75 lb. per square inch. Find the stress in the shaft and state whether or not it is too high.

6,489 lb. per sq. inch. Ans.

4. A hollow shaft is 14 inches diameter with a 6 inch hole in it. How much per cent. is a 14 inch diameter solid shaft stronger than the hollow one ?

Solid shaft 3·5 per cent. stronger. Ans.

5. A shaft 14 inches diameter has 8 bolts at 10 inches radius. Find the diameter of the bolts, allowing the same stress in shaft and bolts.

2·928 inches. Ans.

6. A shaft 6 inches diameter safely transmits 95 horse power at 70 revolutions per minute. Find the horse power which a shaft 9 inches diameter can safely transmit at 90 revolutions per minute.

412·2 horse power. Ans.

7. A steel shaft 1·75 inches diameter, running at 2,500 revolutions per minute, safely transmits 270 horse power. A wrought iron shaft 9 inches diameter runs at 125 revolutions per minute. If the safe working stress for wrought iron is 70 per cent. of that for steel, what horse power can the iron shaft safely transmit ?

1,286 horse power. Ans.

8. An engine is to develop 3,000 horse power at 80 revs. per minute. The maximum twisting moment exceeds the mean by 30 per cent. If the stress is not to exceed 7,000 lb. per square inch, find the diameter of the crank shaft.

13·08 inches. Ans.

9. A hollow steel shaft $14\frac{1}{2}$ inches outside diameter weighs 410 lb. per foot of length. It is coupled up to a solid shaft 14 inches diameter. If the stress in the hollow shaft is 7,000 lb. per square inch when transmitting a certain power, find the stress in the solid shaft.

7,211 lb. per sq. inch. Ans.

10. What horse power can a 10 inch diameter shaft safely transmit at 100 revolutions per minute, if the stress is not to exceed 8,000 lb. per square inch, and what is the angle of twist in a length of 15 feet ? $C = 12{,}000{,}000$ lb. per sq. inch.

2,490 horse power ; 1·375°. Ans.

11. The stroke of an engine is 4 feet and the connecting rod is $8\frac{1}{2}$ feet long. The H.P. piston is 28 inches diameter, and the steam pressure 180 lb. per square inch gauge. Find the maximum twisting moment on the shaft, and if the total twisting moment due to all the engines is twice that due to the H.P. engine, find the diameter of crank shaft, using a stress of 7,500 lb. per square inch.

2,733,000 inch lb. ; 15·5 inches. Ans.

12.
$$S^3 = \frac{C\ P\ D^2}{\quad + \quad)}$$

S = Diameter of crank shaft in inches.
C = Length of crank in inches.
P = Absolute boiler pressure.
D = Diameter of L.P. in inches.
d = Diameter of H.P. in inches.
f = a constant, 1,110 for 3 cranks at 120°

Using this formula, find the least diameter of crank shaft for an engine having cylinders 27, 44 and 73 inches diameter, stroke 52 inches, and boiler pressure 180 lb. per square inch gauge, having three cranks arranged at 120°.

13·78 inches. Ans.

13. In a certain triple expansion engine, the shaft is 14 inches diameter, the low pressure engine being 75 inches diameter, and the steam pressure 195 lb. absolute. The crank is 25 inches long. If the cut-off takes place at 0·6 of the stroke in the H.P. engine, find the number of times the steam is expanded in the cylinders. Use the formula in question 12.

11·68. Ans.

14. A 2·75 inch diameter shaft transmits 100 horse power at 300 revolutions per minute. What diameter shaft will transmit 1,000 horse power at 100 revolutions per minute.

8·54 inches. Ans.

*15. A closely coiled helical spring has 10 coils of $\frac{5}{8}$ inch diameter steel, the mean diameter of the coils is $3\frac{1}{2}$ inches. Find what load it will carry if the maximum stress is not to exceed 25,000 lb. per sq. inch, and find also the deflection of the spring under this load. Take $C = 12 \times 10^6$ lb. per sq. inch.

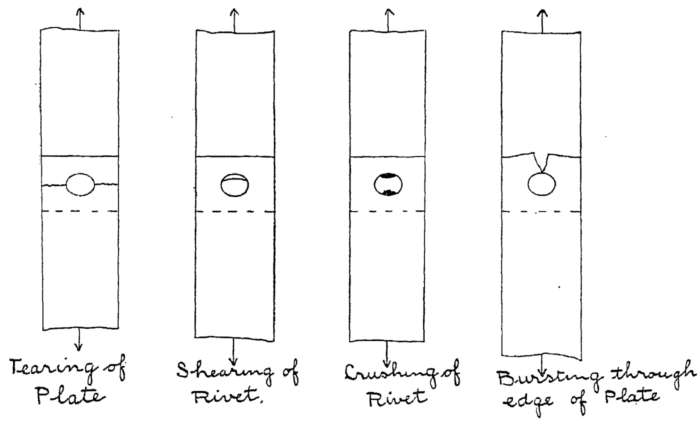
684·8 lb. ; 1·28 inches. Ans.

CHAPTER XIX.

RIVETED JOINTS, STRENGTH OF BOILERS.

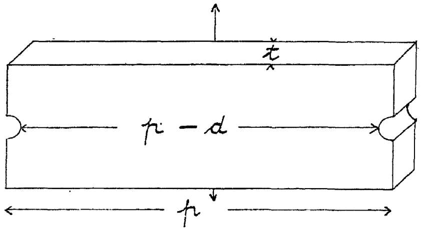
Riveted Joints may fail in any of the following ways :—

1. By tearing of the plate between the rivet holes.
2. By shearing of the rivets.
3. By crushing of the rivets through insufficient bearing surface.
4. By tearing of the plate from the rivet hole to the edge of the plate.



Crushing of the rivets may occur when the rivet diameter is less than the thickness of the plate. This form of failure does not occur in boiler joints or in ship riveting, because the rivet diameter is at least equal to the plate thickness and often greater. Bursting of the rivet through the edge of the plate may be avoided by allowing sufficient material between the edge of the rivet hole and the edge of the plate ; the distance from the centre of the rivet hole to the edge of the plate is generally $1\frac{1}{2}$ times the diameter of the rivet. It remains to consider the strength of the plate between the rivet holes, and the strength of the

rivets in shear ; the joint will be correctly designed when the strengths of these are equal. Let p = the pitch of the rivets, d = diameter of rivet, t = thickness of plate, f_t = tensile strength of plates, f_s = shearing strength of rivets.



Original strength of plate $= p \times t \times f_t$

(Note that this is area of section $\times$ stress).

Strength after drilling $= (p - d)\ t \times f_t$

$$\text{Fractional strength} = \frac{\text{Strength after drilling}}{\text{Original strength of solid plate}}$$

$$= \frac{(p - \quad t \times f_t}{p \times t \times f_t} = \frac{p - d}{p}$$

Expressing this as a percentage we write :—

$$\text{Strength of plate} = \frac{p - d}{p} \times 100.$$

Strength of Rivets.

In the lap joint and in the single butt strap joint the rivets are in single shear. Let n = number of rivets in one pitch.

Strength of one rivet in shear = area of rivet $\times f_s$

Strength of all rivets in one pitch = area of rivet $\times f_s \times n$. Comparing this with the strength of the solid plate we have :—

$$\text{Fractional strength} = \frac{\text{Area of rivet} \times f_s \times n}{p \times t \times f_t}$$

Expressing this as a percentage :—

$$\text{Rivet strength} = \frac{\qquad}{p \times t \times \qquad} \times 100.$$

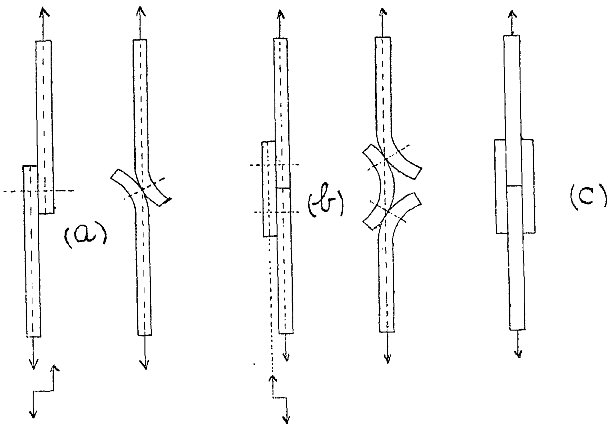
When the rivets are in double shear, as in a double butt strap joint, this equation is multiplied by $1\frac{7}{8}$, this being the allowance for double shear.

$$\text{Rivet strength} = \frac{\qquad}{p \qquad} \quad \frac{\times\ n}{\qquad} \times \qquad \times 100, \text{ for}$$

double shear.

Types of Joints.

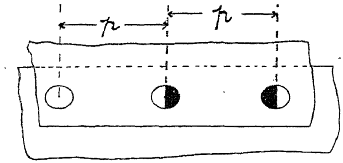
There are three types of joints (*a*) the lap joint ; (*b*) the single butt strap joint ; (*c*) the double butt strap joint.



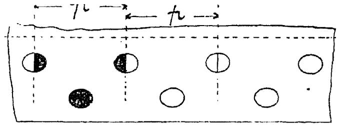
Distortion due to bending action occurs in the lap joint because the pull in the plates is not in the same straight line. The tendency is for the joint to distort as shown, until the pulls or tensile forces in the plates act along the same straight line. In the single butt strap joint, the same bending action occurs, since the pull in the plates must be carried across the butt by the strap.

In the double butt strap joint, no bending action occurs, each strap carries half the pull, which is balanced upon each side and no distortion occurs. This is the strongest and best type of joint.

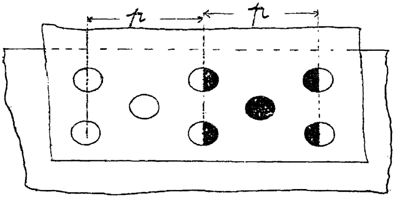
Arrangement of Rivets in Lap Joints.



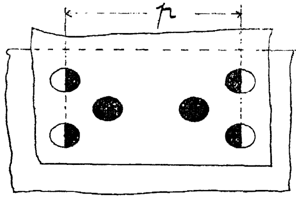
Single Riveted Lap Joint.
One Rivet per. Pitch.



Double Riveted Lap Joint.
Two Rivets per. Pitch.
Zig Zag Riveting.

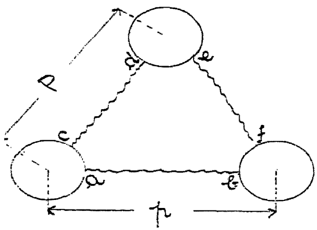


Treble Riveted Lap Joint.
Three Rivets per. Pitch.
Zig Zag Riveting.



Treble Riveted Lap Joint
of Special Design
Four Rivets per Pitch.

Lap joints may have one, two or three rows of rivets in boiler joints. The plates of the internal heating surfaces are single riveted, the rivets being either $\frac{7}{8}$ inch diameter pitched 2 to $2\frac{1}{8}$ inches apart or $\frac{15}{16}$ inch diameter pitched $2\frac{1}{8}$ to $2\frac{1}{4}$ inches apart. The end circumferential seams are double riveted, the rivets being generally the same diameter as the thickness of the shell. The mid-circumferential seam, if there is one, is treble riveted. A special type of treble riveted joint is shown, having the pitch of the middle row of rivets half the pitch of the other two rows. By this arrangement, it is possible to get four rivets in one pitch.



Distance between Rows when no rivets are omitted.

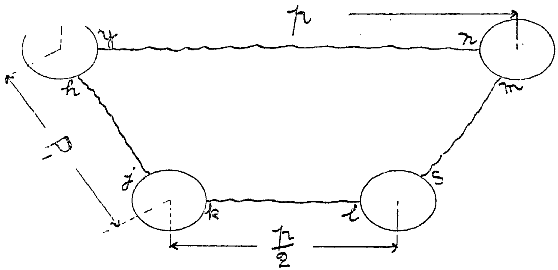
Let P = diagonal pitch.

Then the strength through the line *a b*, should equal the strength through the line *c* d, plus the strength through the line *e f* or, $P - d = 0{\cdot}5\ (p - d)$. It is found that the factor 0·5 is too small, and 0·6 is adopted instead.

$$P - d = 0{\cdot}6\ (p - d)$$
$$P = 0{\cdot}6\ p - 0{\cdot}6\ d + d$$
$$P = 0{\cdot}6\ p + 0{\cdot}4\ d, \text{ or } \frac{6\ p + 4\ d}{10}$$

and from this, the distance between the rows may be calculated. This formula gives the *diagonal pitch* between rivets for all kinds of joints, *provided that no rivets are omitted from any rows whatever.*

Distance between Rows for special riveting.



Treble riveted lap joint with every alternate rivet omitted in both the outer rows. Let P_1 = diagonal pitch. The strength through $y\ n$, should equal the strength through $k\ l$, plus strength through $h\ j$, plus strength through $s\ m$, or

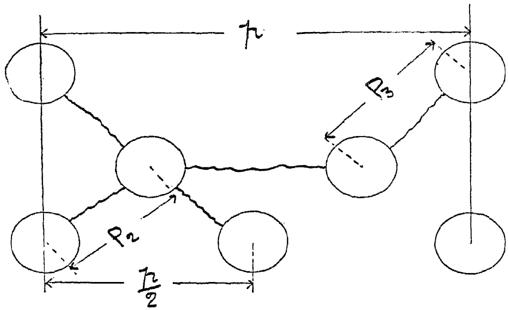
$$= \left(\frac{p}{2} - d\right) + 2\ (P_1 - d),$$

but experience suggests that only $\frac{5}{6}$ of the distance $(P_1 - d)$ is effective, therefore

$$p - d = \left(\frac{p}{2} - d\right) + 2 \times (P_1 - d) \times \tfrac{5}{6}$$

$$p - d = \frac{p}{2} - d + \tfrac{5}{3}\ P_1 \qquad \tfrac{5}{3}\ d, \text{ from which}$$

Treble riveted joints with every alternate rivet omitted in the outer row. This type of riveting is used in the double butt strap joint.



$$P_2 - d = 0{\cdot}6\left(\frac{p}{2} - d\right)$$

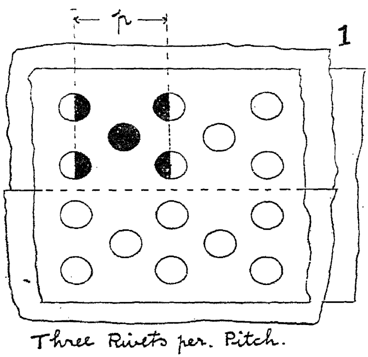
$$P_2 = 0{\cdot}3\ p - 0{\cdot}6\ d + d \qquad P_2 = \frac{3\ p + 4\ d}{10}$$

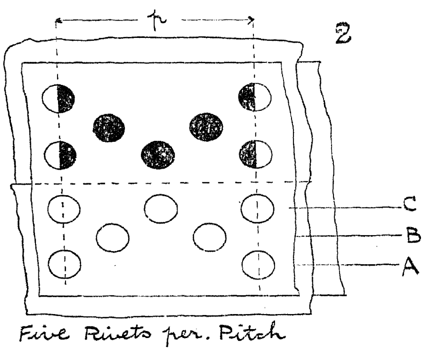
P_3 from the previous case $\qquad p + d.$

From these diagonal pitches the distances between the rows can be calculated. Formulæ, which give slightly different values for the distances between rows for both chain and zig-zag riveting, may be found in the rules of the Classification Societies.

Double Butt Strap Joints.

These are sometimes arranged with ordinary treble riveting on both sides of the butt; more often, and always when a joint of greatest strength is required, treble riveting with every alternate rivet in the outer row omitted, is adopted.





Consider figure 2.

Strength through outer row A $= \dfrac{p - d}{p}$

Consider the strength through the row B.

For the plate through B, strength $= \dfrac{p - 2\,d}{p}$, but before the plate can tear through this line as shown, one rivet must be sheared.

$\therefore$ Combined strength through B $= \dfrac{p - 2\,d}{p}$ strength of one rivet in shear.

The strength through row C is :—

$\dfrac{p - 2\,d}{p}$ + strength of 3 rivets in shear. As this strength is always greater than that of the solid plate, it need not be discussed further. In a joint of this type correctly designed, the strength through the outer row, and the strength through the middle row, should be equal. The Board of Trade in the rules state that, $\dfrac{p - 2\,d}{p}$ strength of one rivet in shear must not be less than $\dfrac{p - d}{p}$

Butt Straps.

The thickness of *single butt straps* is generally $1\frac{1}{4}$ times the thickness of the plates. Double butt straps, when fitted to joints in which no rivets are omitted from any rows, must be made each $\frac{5}{8}$ths of the thickness of the plates.

Butt straps for joints where rivets are omitted must be thicker than given by this rule. Consider the sketch in Figure 2, the weakest line of strength through the plate is $\dfrac{p - d}{p}$, but the line of least strength through the straps is $\dfrac{p - 2\,d}{p}$ through the

middle or inner rows, and the thickness of the straps is given by, $\frac{5}{8}\, t \times \left(\frac{p - d}{p - 2\,d}\right)$, where t is the thickness of the plates.

The thickness of the inner strap is given by :—

$$p - 2$$

The extra $\frac{1}{8}$ of an inch is an allowance for possible corrosion of the inner strap.

Example. Find the strength of a single riveted lap joint. The plates are $\frac{5}{8}$ inch thick, the rivets are $\frac{7}{8}$ inch diameter, and the pitch is $2\frac{1}{8}$ inches. Take f_t as 28 and f_s as 23 tons per sq. inch.

$$\text{Plate strength} = \frac{p \qquad d \qquad 2{\cdot}125 - 0{\cdot}875}{p \qquad\qquad 2{\cdot}125} = 0{\cdot}5882$$

$$\text{Rivet strength} = \frac{\text{Area} \times n \times f_s}{p \times t \times f_t}$$

$$= \frac{7}{8} \times \frac{7}{8} \times \frac{11}{14} \times \frac{23}{28} \times \frac{8 \times 8}{5 \times 17} = 0{\cdot}372$$

Now the strength of the joint will be the strength of its weakest part, and here the rivet strength is weaker than the plate, therefore :—

Strength of joint $= 0{\cdot}372$ of the solid plate, or $37{\cdot}2$ per cent Ans.

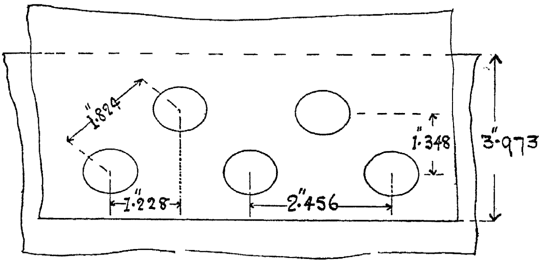
* *Example.* Design a double riveted lap joint for plates $\frac{5}{8}$ inch thick, rivets $\frac{7}{8}$ inch diameter. The joint is to be of maximum strength. The plate is 28 tons in tension and the rivets 23 tons in shear.

To design for maximum strength means that the plate and rivet strengths must be equal. The pitch is not given, we must therefore find it. Equating the formulæ for plate and rivet strength :—

$$\frac{p - d}{p} = d^2 \times \frac{11}{14} \times n \times \frac{23}{28} \times \frac{1}{p \times t}$$

$$p - \frac{7}{8} = \frac{7}{8} \times \frac{7}{8} \times \frac{11}{14} \times 2 \times \frac{23}{28} \times \frac{8}{5}$$

$$p = 1{\cdot}581 + 0{\cdot}875 = 2{\cdot}456 \text{ inches.}$$



The pitch must be 2·456 inches for maximum strength. Ans.

The Strength of Joint is $\dfrac{p - d}{p}$, since we have made plate and rivet strengths equal.

$$\text{Strength} = \frac{2{\cdot}456 - 0{\cdot}875}{2{\cdot}456} = 0{\cdot}6436$$

Strength of Joint is 64·36 per cent. Ans.

$$\text{Diagonal pitch} = \frac{6\,p + 4\,d}{10} = \frac{6 \times 2{\cdot}456 + 4 \times \frac{7}{8}}{10} = 1{\cdot}824 \text{ ins.}$$

$$\text{Distance between rows} = \sqrt{1{\cdot}824^2 - 1{\cdot}228^2} = 1{\cdot}348 \text{ inches.}$$

Ans.

$$\text{Width of lap} = 2 \times 1\tfrac{1}{2}\,d + 1{\cdot}348 = 2 \times 1\tfrac{1}{2} \times \tfrac{7}{8} + 1{\cdot}348$$
$$= 3{\cdot}973 \text{ inches. Ans.}$$

Example. Plates $1\frac{1}{4}$ inches thick, are connected by a treble riveted double butt strap joint. The rivets are $1\frac{1}{4}$ inches diameter pitched 6 inches apart. The plates have a strength of 28 tons in tension, and the rivets 23 tons in shear, the allowance for double shear being $1\frac{7}{8}$. Find the strength of the joint, give the thickness of the straps, and calculate the distance between the rows of rivets.

$$\text{Plate strength} = \frac{p \qquad d}{p} \qquad \frac{6 - 1\frac{1}{4}}{6} = 0{\cdot}791$$

$$= 0{\cdot}791 \text{ of solid plate, or } 79{\cdot}1 \text{ per cent.}$$

$$\text{Rivet strength} = d^2 \times \tfrac{11}{14} \times n \times \tfrac{23}{28} \times 1\tfrac{7}{8} \times$$
$$p \times$$

$$= \tfrac{5}{4} \times \tfrac{5}{4} \times \quad \times \quad \times \quad \times \tfrac{15}{8} \times \frac{}{6 \times 5}$$

$= 0{\cdot}756$ or $75{\cdot}6$ per cent.

The strength of the joint is $75{\cdot}6$ per cent. Ans.

The thickness of the straps is $\frac{5}{8} \times$ thickness of plate when there are no rivets omitted from any of the rows.

$\therefore$ Thickness of straps $= \frac{5}{8} \times \frac{5}{4} = \frac{25}{32}$ inch. Ans.

$$\text{Diagonal pitch} = \frac{6\,p + 4\,d}{10} \qquad \frac{6 \times 6 + 4 \times 1\tfrac{1}{4}}{10}$$

$= 4{\cdot}1$ inches.

Distance between rows $= \sqrt{4{\cdot}1^2 - 3^2} = 2{\cdot}795$ inches. Ans.

Example. Calculate the strength of a treble riveted double butt strap joint, having every alternate rivet in the outer row omitted. The plates are $1\frac{1}{2}$ inches thick, the rivets $1\frac{1}{2}$ inches diameter, the pitch is 10 inches and the allowance for double shear is $1\frac{7}{8}$. The ultimate strength of the plates is 28 tons in tension and of the rivets, 23 tons in shear. Find the distance between the rows, calculate the combined strength of plate and rivets at the middle row, and find the thickness of the butt straps.

$$\text{Plate strength} = \frac{p - d}{p} \times 100 = \frac{10 -}{10} \times 100$$

$= 85$ per cent.

$$\text{Rivet strength} = d^2 \times \quad \times n \times \tfrac{23}{28} \times \tfrac{15}{8} \times \frac{1}{p \times t} \times 100$$

$$= \tfrac{3}{2} \times \tfrac{3}{2} \times \quad \times 5 \times \quad \times \quad \times \frac{2}{10 \times 3} \times 100$$

$= 90{\cdot}75$ per cent.

Combined strength through middle row,

$$\frac{p - 2\,d}{p} \times 100 + \frac{90{\cdot}75}{5}$$

$\frac{75}{5}$ being the percentage strength of one rivet in shear.

$$\frac{10 -}{10} \times 100 + 18{\cdot}15 = 88{\cdot}15 \text{ per cent.}$$

The joint is weakest at the outer row where the strength of the plate is 85 per cent. Ans.

$$+\ 4 \qquad 30 + 6$$

2

$$10 \qquad 10$$

Distance between inner and middle rows

$$= \sqrt{3{\cdot}6^2 - 2{\cdot}5^2} = 2{\cdot}6 \text{ inches (say). Ans.}$$

$$P_3 = \tfrac{3}{10}\, p + d = 3 + 1\tfrac{1}{2} = 4\tfrac{1}{2} \text{ inches.}$$

Distance between middle and outer rows

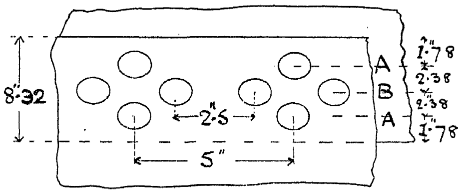
$$= \sqrt{4{\cdot}5^2 - 2{\cdot}5^2} = 3{\cdot}742 \text{ inches. Ans. (Say) } 3\tfrac{3}{4} \text{ inches.}$$

$$\text{Butt straps, thickness} = \tfrac{5}{8}\, t \times \left(\frac{p - d}{p - 2\, d}\right)$$

$$= \tfrac{5}{8} \times \tfrac{3}{2} \times \qquad 1{\cdot}14 \text{ inches (say). Ans.}$$

From centre of outer row to edge of strap $= 1\tfrac{1}{2}\, d = 2\tfrac{1}{4}$ inches.

Example. A treble riveted lap joint has the pitch of the middle row of rivets $2\tfrac{1}{2}$ inches. The pitch of the outer rows is 5 inches, and the diameter of the rivet is $1\tfrac{3}{16}$ inches. The plates are one inch thick, 28 tons in tension, and the rivets are 23 tons in shear. Find the strength of the joint, and give the distance between the rows.



Strength of plate at A

$$\begin{matrix} p - \\ \hline p \end{matrix} \times 100$$

$$\begin{matrix} 5 - 1\tfrac{3}{16} \\ \end{matrix} \times 100$$

$$= 76{\cdot}3 \text{ per cent.}$$

$$\text{Strength of rivets} = d^2 \times \tfrac{11}{14} \times n \times \tfrac{23}{28} \times \frac{1}{p \times t} \times 100$$

$$\tfrac{19}{16} \qquad \times \tfrac{11}{14} \times 4 \times \tfrac{23}{28} \times \frac{1}{5 \times 1} \times 100 = 72{\cdot}8 \text{ per cent.}$$

Per cent. strength of one rivet in shear $= \dfrac{72{\cdot}8}{4} = 18{\cdot}2.$

Combined strength of plate and rivet through row B

$$\frac{p - 2\,d}{p} \times 100 + 18{\cdot}2 \text{ per cent.}$$

$$\frac{5 - 2\frac{3}{8}}{5} \times 100 + 18{\cdot}2 = 70{\cdot}7 \text{ per cent.}$$

The joint is therefore weakest through the middle row, and the strength is 70·7 per cent. Ans.

For special riveting of this kind, we use the same rule for diagonal pitch as in the case of the outer row of rivets in the double butt strap joint.

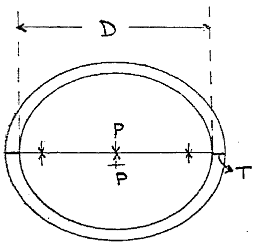
Diagonal pitch $= \frac{3}{10}\,p + d$

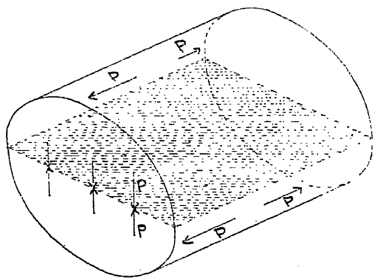
$= \frac{3}{10} \times 5 + 1\frac{3}{16} = 2{\cdot}6875$

Distance between rows $= \sqrt{2{\cdot}6875^2 - 1{\cdot}25^2} = 2{\cdot}38$ ins. Ans.

Width of lap $= 2 \times 1\frac{1}{2} \times 1\frac{3}{16} + 2 \times 2{\cdot}38 = 8{\cdot}32$ inches.

Strength of Boiler Shells.





It has already been shown, in the Chapter on Forces, that the total force due to the internal steam pressure in a boiler shell is equal to the pressure in lb. per square inch multiplied by the diameter in inches.

Let D = diameter of boiler (inside), in inches.
P = bursting pressure in lb. per square inch.
t = thickness of shell in inches.
s = ultimate strength of plates in lb. per sq. inch.
F = factor of safety.
W.P. = working pressure.

Force tending to cause rupture longitudinally $= \mathrm{P} \times \mathrm{D}$.

Force resisting rupture $= 2\,t\,s$.

$$\frac{2\,t\,s}{\mathrm{D}}$$

$$\text{Working Pressure} = \frac{2\,t\,s}{\mathrm{D} \times \mathrm{F}} \text{ lb. per square inch.}$$

$$\text{or Working Pressure} = \frac{2\,t \times \text{working stress allowed}}{\mathrm{D}}$$

Force tending to rupture circumferentially

$$= \frac{\pi}{4}\,\mathrm{D}^2 \times \mathrm{P}$$

This is resisted by a force $= \pi\,\mathrm{D}\,t\,s$

$$\therefore \mathrm{P} \times \frac{\pi}{4}\,\mathrm{D}^2 = \pi\,\mathrm{D}\,t\,s$$

$$\mathrm{P} = \frac{4\,t\,s}{\mathrm{D}}$$

Working pressure $= \dfrac{4\,t\,s}{\mathrm{D} \times \mathrm{F}}$, this means that the boiler is twice as strong circumferentially as it is longitudinally, so that we must use the formula which gives the least value :—

$$\mathrm{W.P.} = \frac{2\,t\,s}{\mathrm{D} \times \mathrm{F}}$$

This formula refers to an unbroken cylindrical shell, such as a solid drawn tube. When the plates of the shell are fastened together by a riveted joint, as they always are in boilers, then this formula must be multiplied by the efficiency of the riveted joint.

For Spherical Shells, such as the hemispherical top of a donkey boiler, the working pressure is given by :—

$$\text{W.P.} = \frac{4\ t\ s}{\text{D} \times \text{F}} \times \frac{\%\ \text{strength of joint}}{100}$$

The factor of safety usually given at the examination is 4·5 for boiler shells.

$$\text{W.P.} = \frac{2\ t\ s}{\text{D} \times} \times \frac{\%\ \text{strength of joint}}{100} \text{ for cylindrical shells.}$$

Since —— is the working stress allowed, we may write :-

$$\text{W.P.} = \frac{2 \times t \times \text{working stress}}{\text{D}} \times \frac{\%\ \text{strength of joint}}{100}$$

Example. The longitudinal treble riveted double butt strap joint in a boiler has every alternate rivet in the outer row omitted. The pitch of rivets is 10 inches, the plates are $1\frac{1}{2}$ inches thick and the rivets are $1\frac{1}{2}$ inches diameter, the plates having an ultimate strength of 28 tons in tension and the rivets 23 tons in shear. The allowance for double shear is $1\frac{7}{8}$. The boiler is 15 feet diameter. Calculate the working pressure, if the factor of safety is 4·5.

$$\text{Strength of plate} = \frac{p - d}{p} \times 100 = \frac{10\qquad}{10} \times 100$$

$$= 85\%$$

$$\text{Strength of rivets} = d^2 \times \tfrac{11}{14} \times n \times 1\tfrac{7}{8} \times \tfrac{23}{28} \times \frac{\qquad}{p \times t} \times 100$$

$$= \tfrac{3}{2} \times \tfrac{3}{2} \times \tfrac{11}{14} \times 5 \times \tfrac{15}{8} \times \tfrac{23}{28} \times \tfrac{2}{3} \times \tfrac{1}{10} \times 100 = 90{\cdot}7\%$$

Combined strength of plate and rivet through middle row is

so that the joint is weakest at the outer row, and the strength must be taken as 85%.

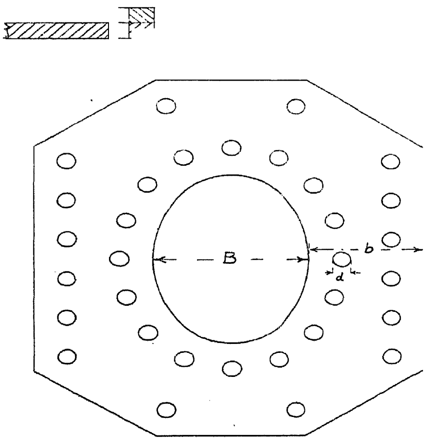
$$\text{W.P.} = \frac{2\ t\ s}{\text{D} \times \text{F}} \times \frac{\%\ \text{strength}}{100}$$

$$\text{W.P.} = \frac{2 \times 1\tfrac{1}{2} \times 28 \times 2240}{180 \times 4{\cdot}5} \times \tfrac{85}{100} = 197{\cdot}4\ \text{lb.}$$

per sq. inch. Ans.

Compensating Ring.

When an opening, such as a manhole, is cut in the shell of a boiler the sectional area of the metal in the direction of the longitudinal axis is reduced, and if compensation was not provided the stress in the metal remaining would be increased. Compensation is made by fitting a ring around the opening such that its strength is equal to the strength of the material removed by cutting the manhole.



A shell manhole is always cut with its minor axis parallel to the longitudinal axis of the boiler, and the usual size is 12 ins. × 16 ins.

Let B ins. = the breadth of the manhole ; t_s ins. = the thickness of the shell plates, and f_{ts} the tensile strength of the shell material.

Let b ins. = the breadth of the ring ; t_r ins. = the thickness of the ring, and f_{tr} the tensile strength of the ring material.

Let d inches = the diameter of the rivets, and let two rivets be placed in the direction of the minor axis.

Then strength lost by cutting the manhole =
$(\text{B} + 2\ d) \times t_s \times f_{ts}$ tons.

Strength restored by the ring =
$(b - d) \times 2 \times t_r \times f_{tr}$ tons.

These should be equal, and the breadth of the ring required may be determined. This gives the minimum breadth necessary.

The ring may not be less in thickness than the boiler shell. If a flat ring is fitted it may be made of the same grade of mild steel as the shell plates, and $f_{tr} = f_{ts}$ in its case. Also, this type of ring is usually riveted on the outside of the boiler shell. If a flanged ring is fitted it must be made of flanging steel, a milder quantity of mild steel than the boiler shell steel, and whose tensile strength is $\frac{26}{30}$ of that of the shell steel. A flanged ring is secured to the inside of the boiler shell, and the opening cut in the shell will be about 15 inches $\times$ 19 inches for a 12 ins. $\times$ 16 ins. manhole.

To determine the number of rivets required.

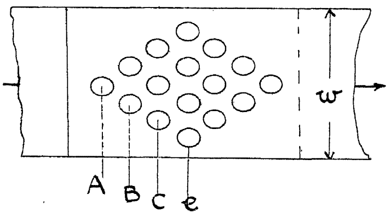
Let n = number of rivets in shear on either side of the minor axis, and f_s be the shearing strength of the rivet material.

Then $d^2 \times \frac{\quad}{4} \times n \times f_s$ tons is the shearing strength of the rivets, and this should be equal to the tensile strength of the ring. n is thus determined. The total number of rivets required will be $(2\,n + 2)$, and this is usually about 32. Approximately one half of the total number will be arranged around the periphery of the opening, at a pitch of about $3\frac{1}{2}$ inches, and the remainder in suitable positions so that there is no resulting weak line through the ring.

A manhole in the end plates of a boiler is usually 11 ins. $\times$ 15 ins. Compensation is provided by flanging the plate inwards, and the depth of the flange is given by $\sqrt{\text{plate thickness} \times \text{minor axis}}$, all dimensions being inches. This gives a depth of flange of about $3\frac{1}{4}$ inches.

*Lozenge Joint.

A special type of joint, called a lozenge joint, on account of the arrangement of the rivets, is used to connect plates when a joint of highest possible efficiency is required. This joint is often used to connect flat tie plates in structures, such as bridge girders, etc.



The rivets are arranged as shown in the sketch. Let it be required to design a lap joint for plates w inches wide and t inches thick for maximum strength.

Through A the strength is $(w - d)\, t \times f_t$ tons, and for maximum efficiency of joint, the total strength of the rivets in shear must equal the strength of the plate in tension, therefore if n = number of rivets :—

$$(w - d)\ t \times f_t = d^2 \times \tfrac{11}{14} \times f_s \times n$$

It is usual to be given w, d, f_t and f_s, and from this equation n may be calculated.

The efficiency of the joint at A is $\dfrac{w - d}{w} \times 100$, and this is also the rivet efficiency.

The efficiency of the joint at B, is :-

$$\frac{\qquad}{w} \times 100 + \%\ \text{strength of one rivet in shear,}$$

because before the plate can tear through the line B, one rivet must be sheared.

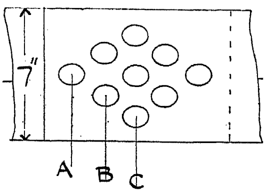
The efficiency at C is :—

$$\frac{w - 3\ d}{w} \times 100 + \%\ \text{strength of 3 rivets in shear.}$$

The efficiency at e is :—

$$\frac{w - 4\ d}{w} \times 100 + \%\ \text{strength of 6 rivets in shear.}$$

Example. Design a lap joint of maximum efficiency to connect tie plates 7 inches wide and $\frac{1}{2}$ in. thick. The plates have a strength of 28 tons in tension and the rivets a strength of 23 tons in shear. The rivets are $\frac{3}{4}$ inch diameter. Give all particulars.



$$(7 - \tfrac{3}{4}) \times \tfrac{1}{2} \times 28 =$$
$$\times \tfrac{3}{4} \times \tfrac{11}{14} \times n \times 23$$

$$\times 14 = \tfrac{99}{16} \times \frac{n}{14} \times 23$$

$n = 8{\cdot}6$, say 9 rivets arranged as shown.

Strength of solid plate $= 7 \times \frac{1}{2} \times 28 = 98$ tons.
Strength through A $= (7 - \frac{3}{4}) \times \frac{1}{2} \times 28 = 87{\cdot}5$ tons.

Strength of 1 rivet in shear $= \frac{3}{4} \times \frac{3}{4} \times \frac{11}{14} \times 23 = 10{\cdot}16$ tons.

Strength through B $= (7 - 2 \times \frac{3}{4}) \times \frac{1}{2} \times 28 +$ one rivet in shear.

$= 77 + 10{\cdot}16 = 87{\cdot}16$ tons.

Strength through C $= (7 - 3 \times \quad \times \frac{1}{2} \times 28 +$ 3 rivets in shear,

$= 66{\cdot}5 + 3 \times 10{\cdot}16 = 96{\cdot}98$ tons.

The weakest line is through B, where the % strength of plate and rivet combined is $\frac{87{\cdot}16}{98} \times 100 = 88{\cdot}9\%$

Note that through A the strength is $\frac{87{\cdot}5}{98} \times 100 = 89{\cdot}3\%$

and this is also the total strength of the rivets if we arrange 8·6 in the joint, which is impossible, the actual rivet strength

being $89{\cdot}3 \times \quad \times 100 = 93{\cdot}4\%$,

$\therefore$ Strength of Joint $= 88{\cdot}9\%$.

Taking the centres of the outer rivets through C as being $1\frac{1}{2}\,d$ from the edge of plate we have :—

Pitch of rivets $= \dfrac{7 - 2\,(1\frac{1}{2} \times}{} = 2{\cdot}375$ inches.

Diagonal pitch $= \cdot \quad 10 \quad \dfrac{6 \times 2{\cdot}375 + 4 \times}{10}$

$= 1{\cdot}725$ inches.

Distance between rows $= \sqrt{1{\cdot}725^2 - 1{\cdot}1875^2} = 1{\cdot}251$ inches.

Width of lap $= 2 \times 1\frac{1}{2} \times \frac{3}{4} + 4 \times 1{\cdot}251 = 7{\cdot}254$ inches.

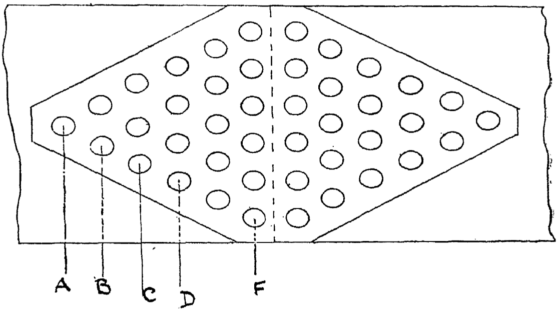
Example. Design a double butt strap joint for plates 20 inches wide and $\frac{3}{4}$ inch thick. The joint is to be of maximum efficiency. The plates have a strength of 28 tons in tension and the rivets a strength of 24 tons in shear. The rivets are to be $\frac{3}{4}$ inch diameter, the allowance for double shear $1\frac{7}{8}$.

$$(20 - \quad \times \tfrac{3}{4} \times 28 = \tfrac{3}{4} \times \tfrac{3}{4} \times \quad \times n \times 24 \times$$

$$\frac{77}{4} \times 28 = \frac{33 \times 45}{56} \times n$$

$$n = \frac{539 \times 56}{33 \times 45} = 20{\cdot}3, \text{ say } 21 \text{ rivets.}$$

In the double butt strap joint we must have this number of rivets on *each side* of the butt, therefore arrange as shown.



$$\text{Strength at A} = \frac{20 \qquad}{20} \times 100 = 96{\cdot}25\%$$

$$\text{Strength of 1 rivet} = \frac{96{\cdot}25}{21} = 4{\cdot}583\%$$

$$\text{Strength at B} = \frac{20 - 2 \times \qquad}{20} \times 100 + 4{\cdot}583 = 97{\cdot}08\%$$

If we examine the strength at C, D, etc., it will be found to increase and reach its greatest value at F.

Strength of joint $= 96{\cdot}25\%$

The pitch of rivets and distances between rows may be calculated as in the previous example.

Butt Straps.

The line of greatest weakness for the plate is at A, as shown above. The line of greatest weakness for the straps is at F.

$$\text{Thickness of straps} = \tfrac{5}{8} \times \tfrac{3}{4} \times \left[\frac{20 - \frac{3}{4}}{20 - 6 \times \frac{3}{4}}\right] = 0{\cdot}582 \text{ inch.}$$

Note that as we have equated the plate strength at A to the total rivet section in double shear, it is necessary only to find the plate strength at A to obtain the rivet strength. The rivet strength is really greater than this in the ratio of $\frac{21}{20\cdot3}$ which does not matter. Then the strength of one rivet in shear is $\frac{\text{Strength at A}}{20\cdot3}$, but we have taken it as only $\frac{\text{Strength at A}}{21}$, the difference being very small.

Escape of Steam from an Orifice.

The weight of steam in pounds which will escape through an orifice of area A square inches in one second is :—

$$\text{Weight} = \frac{\text{A} \times \text{Gross Pressure}}{70}, \text{ lb. per second.}$$

This is Napier's experimental value. Another rule, more accurate for high pressure, is :—

$$\text{Weight} = 0{\cdot}3 \text{ A} \sqrt{\frac{p}{v}} \text{ lb. per sec.}$$

where p = absolute pressure in lb. per square inch, and v is the volume of 1 lb. of steam at pressure p, A being the area in square inches.

Now the weight of steam which is generated in a boiler per second depends upon the area of the fire grate and the amount of coal burnt per square foot of grate, so that the weight may be calculated; then from either of the formulæ given, A the area of escape may be found. A small orifice, if the flow of steam through it is unrestricted, will relieve the pressure from a large boiler. Thus, a hole of about $1\frac{1}{8}$ inches diameter is large enough to relieve the pressure of steam from a single ended boiler of average size.

Spring Loaded Safety Valves.

The area of escape found from the formulæ given is not sufficient for a safety valve, because the valve would have to lift $\dfrac{D}{4}$ inches. Now the initial compression on the spring, necessary to keep the valve closed against the steam pressure is about $\dfrac{D}{4}$ inches, and to lift the valve a further $\dfrac{D}{4}$ inches would need a load equal to the load causing the initial compression, and the steam pressure would be greatly increased. The Board of Trade rule states that with bright fires, the accumulation of pressure, when testing the valves, shall not be more than 10 per cent. of the loaded pressure. As the initial compression of the spring is about $\dfrac{D}{4}$, and as the compression of a spring varies as the load, then an increase in the load of 10 per cent. will cause an increase in the compression of 10 per cent. and the lift of the valve will be $\dfrac{D}{4} \times \frac{1}{10} = \dfrac{D}{40}$

The area of a safety valve is about 10 times greater than that given by Napier's rule. This is because such a small lift is allowed.

The Board of Trade rule allows $\frac{1}{2}$ square inch of safety valve area per square foot of grate for a gross pressure of 75 pounds per square inch. Since the volume of steam varies inversely as the pressure, less area of safety valve is needed as the pressure increases. The formula is :—

Total area of valves =

$$\left(\frac{75}{\text{gross pressure}} \times \tfrac{1}{2}\right) \times \text{total grate area in square feet.}$$

$$= \frac{37 \cdot 5}{\text{gross pressure}} \times \text{total area of grate in square feet.}$$

When two valves are fitted, as generally, then each will have half the area found by the above rule.

Further, since the Board of Trade rule is arranged for a consumption not exceeding 20 lb. per square foot of grate per hour with natural draught, then if the consumption under conditions of forced draught exceeds this, the area of valves must be increased in the ratio of $\dfrac{\text{estimated consumption}}{20}$

Deadweight Safety Valves.

This type of safety valve is not suitable for marine installations. The total load of all the weights plus the weight of the valve and spindle, must equal the total upward steam load on the valve. From this the downward weight needed for a given boiler pressure is readily calculated.

TEST EXAMPLES XIX.

1. Find the percentage strength of a single riveted joint for $\frac{5}{8}$ inch plates, the rivets being $\frac{7}{8}$ inch diameter and $2\frac{1}{8}$ inches pitch. Plates 28 tons, rivets 23 tons. 37·2 per cent. Ans.

2. A double riveted lap joint is to be made for $\frac{3}{4}$ inch plates, the rivets being $\frac{7}{8}$ inch diameter. Calculate the pitch of the rivets, so that the plate strength equals the rivet strength, and give the efficiency of the joint. Plates 28 tons, rivets 23 tons. 2·1926 inches ; 60·1 per cent. Ans.

3. Find the strength of a double butt strap joint, the joint is treble riveted, the pitch being 5 inches and the rivet diameter $\frac{15}{16}$ inch, the plates being $\frac{3}{4}$ inch thick. The strength of the plates is 28 tons in tension and of the rivets 23 tons in shear, the allowance for double shear being $1\frac{7}{8}$. How would you proceed to find the working pressure of a boiler in which this was the longitudinal seam ? 81·25 per cent. Ans.

4. The longitudinal double butt strap joint in a boiler shell is treble riveted, with every alternate rivet in the outer row omitted. The plates are $1\frac{1}{4}$ inches thick, the rivets are $1\frac{1}{4}$ inches diameter and the pitch $8\frac{1}{2}$ inches. Find the working pressure of the boiler which is 13 feet diameter, the plates having an ultimate strength of 28 tons, and the rivets 23 tons, the allowance for double shear being $1\frac{7}{8}$, and the factor of safety 4·5. 190·5 lb. per sq. inch. Ans.

5. A solid drawn pipe is 8 inches diameter inside and $\frac{5}{16}$ inch thick, and works at a pressure of 210 lb. per square inch. If the tensile strength of the material is 22 tons per square inch find the factor of safety. 18·3. Ans.

6. A steel boiler is 15 feet diameter, the shell plates being $1\frac{1}{2}$ inches thick. The strength of the joint is 85 per cent., and the strength of the plates is 28 tons per square inch. The working pressure is 195 lb. per square inch, find the factor of safety.

4·556. Ans.

7. A treble riveted lap joint has the pitch of the middle row of rivets one-half of the outer pitch. The diameter of the rivets is $\frac{13}{16}$ inch, the plates being $\frac{11}{16}$ inch thick. The plate strength is 28 tons, and the rivet strength 23 tons. Find the pitch of the rivets and calculate the efficiency of the joint.

3·484 inches ; 71·12 per cent. Ans.

8. The shell of a vertical donkey boiler is made of three strakes of plating, the top and bottom strakes being $\frac{1}{2}$ inch thick, the middle strake being $\frac{5}{8}$ inch thick. If the joints are double riveted lap joints, with $\frac{7}{8}$ inch diameter rivets pitched $2\frac{3}{4}$ inches apart, find the strength of the seam and the working pressure if the diameter of the boiler is 7 feet. Strength of plates 28 tons, strength of rivets 23 tons, factor of safety 5.

101·8 lb. per sq. inch. Ans.

*9. Design a lap joint of the lozenge type to connect plates 18 inches wide and 1 inch thick. The joint must be of maximum strength, the diameter of rivet being $\frac{7}{8}$ inch. The plate strength is 30 tons, and the rivet strength 24 tons. Calculate the strength of the joint.

92·9 per cent. Ans.

*10. Design a double butt strap lozenge joint for plates 24 inches wide and $\frac{5}{8}$ inch thick, the rivets being $\frac{3}{4}$ inch diameter. The plates have a strength of 28 tons and the rivets 24 tons, the allowance for double shear being $1\frac{7}{8}$. Calculate the least strength of the joint and give the thickness of the straps.

96·87 per cent. ; straps 0·466 inch. Ans.

11. A safety valve is $3\frac{1}{2}$ inches diameter, and the spring is compressed 1 inch by a load of 1,800 lb. When in position on the boiler the compression of the spring is $\frac{7}{8}$ inch. Find the boiler pressure.

163·6 lb. per sq. inch. Ans.

12. A safety valve is $3\frac{1}{2}$ inches diameter. The spring is $\frac{3}{4}$ inch square section steel, its outer diameter being 4 inches. Find the pressure at which the valve lifts.

Note.—The Board of Trade rule for square steel for safety valves is $\dfrac{11000\ S^3}{D} = W$.

S = size of steel in inches.
D = diameter of coil, centre to centre.
W = total weight in lb. allowed on the spring.

148·4 lb. per sq. inch. Ans.

13. Find the diameter of spring made of $\frac{3}{4}$ inch square steel for a safety valve $3\frac{1}{4}$ inches diameter, for a boiler pressure of 180 lb. per sq. inch. Use the rule in Question 12.

3·857 inches. Ans.

14. A boiler has 6 furnaces, 6 feet long by 3 feet 3 inches diameter. Find the least diameter of the safety valves if two are fitted, also if three are fitted, and natural draught is used, the boiler pressure being 200 lb. per square inch.

3·602 inches ; 2·942 inches. Ans.

15. In Question 14 the consumption of coal is increased to 27 lb. per square foot of grate per hour, using forced draught. Find now the diameter of valves needed, if two are fitted, and if three are fitted.

4·183 inches ; 3·42 inches. Ans.

16. A safety valve is $3\frac{1}{2}$ inches diameter. The spindle, valve and spring, weigh 35 lb., the boiler pressure being 170 lb. per square inch. The spring has 14 coils of $\frac{3}{4}$ inch square steel, the mean diameter of the coil being 3 inches. Find the compression of the spring. If the accumulation of pressure when blowing off is 10 per cent., find the lift of the valve when blowing off.

Note.—Compression in inches $= \dfrac{D^3 \times L \times N}{d^4 \times C}$

D = mean diameter of coil.
C = a constant, 30 for square steel.
d = size of steel in $\frac{1}{16}$ths of an inch.
N = number of coils.
L = load on valve due to compression of spring.

0·972 inch ; 0·099 inch. Ans.

17. A furnace is 36 inches diameter and 6 feet 3 inches long, its thickness being $\frac{7}{16}$ inch, the longitudinal seam being welded. Find the working pressure. (Work both formulæ.) If the thickness of this furnace is corroded to $\frac{3}{8}$ inch, how much should the pressure be reduced ?

Note.—Board of Trade rule for circular furnaces with a butt strap, or longitudinally welded, is :—

Working Pressure =

$$\frac{99{,}000 \times \text{square of thickness of plate in inches}}{(\text{length in feet} + 1) \times \text{diameter in inches}}$$

The pressure must not exceed, $\dfrac{9{,}900 \times \text{thickness}}{\text{diameter in inches}}$

W.P. = 72·61 lb. sq. inch. Reduction = 19·28 lb. per sq. inch.
Ans.

18. A corrugated furnace is $\frac{9}{16}$ inch thick and 48 inches diameter. Find the working pressure using the formula :—

$$\text{Pressure} = \frac{15{,}000\ T}{D},$$ where T = thickness of plate in inches.
D = diameter in inches.

The wrapper plate for the combustion chamber bottom is 3 feet long. Find its thickness for the working pressure found from the following formula :—

$$\text{Working Pressure} = \frac{9{,}900\ T}{3\ D}\left(5 - \frac{L + 12}{60\ T}\right)$$

Where T = thickness in inches.
L = length in inches.
D = diameter in inches.

175·7 lb. per sq. inch ; 0·671 inch. Ans.

19. The Board of Trade rule for a copper steam pipe is

$$P = \frac{6{,}000\ (T - \frac{1}{16})}{D},$$ where P = boiler pressure.
T = thickness in inches.
D = diameter in inches.

Find the least thickness for a pipe 8 inches diameter for a pressure of 200 lb. per square inch.

0·3291 inch. Ans.

CHAPTER XX.

EXPANSION OF SOLIDS AND GASES. HEAT ENGINE CYCLES.

Temperature.

The temperature of a body is a measure of the intensity of heat contained in the body. For temperatures up to about 600° Fah. mercury thermometers are generally used. There are three scales by which temperatures are measured, the Fahrenheit scale, the Centigrade scale and the Réaumer scale. The freezing point of pure water and its boiling point at the normal pressure of the atmosphere, 14·7 lb. per square inch, are known to be constant temperatures, and these fixed points are used in the graduation of thermometers. The freezing points on these scales are :—Fah. 32°, Cent. 0°, Réau. 0°. The boiling points are :—Fah. 212°, Cent. 100°, Réau. 80°. Between freezing and boiling points there are 180° Fah., 100° Cent., and 80° Réau. When converting temperatures from one scale to another, the following rules are used :—

$$C = (F - 32) \times \tfrac{5}{9}\,;\ F = (C \times \tfrac{9}{5}) + 32\,;\ (R \times \tfrac{9}{4}) + 32 = F.$$

Example. Express 50° Fah., and — 5° Fah, as Cent. readings, and 60° C. and — 6° C. as Fah. readings.

$(F - 32)\ \frac{5}{9} = C.$ $(50 - 32)\ \frac{5}{9} = 10°\ C.$ Ans.	$(F - 32)\ \frac{5}{9} = C.$ $(-5 - 32)\ \frac{5}{9} = -20{\cdot}55°\ C.$ Ans.
$(C \times \frac{9}{5}) + 32 = F.$ $(60 \times \frac{9}{5}) + 32 = 140°\ F.$ Ans.	$(C \times \frac{9}{5}) + 32 = F.$ $(-6 \times \frac{9}{5}) + 32 = 21{\cdot}2°\ F.$ Ans.

Units of Heat.

The unit of heat in the British system is the British Thermal Unit, and it is defined as the quantity of heat necessary to raise the temperature of one lb. of water one degree Fah. The Centigrade heat unit is the quantity of heat necessary to raise the temperature of one lb. of water one degree Cent. The Gram-Calorie is the quantity of heat necessary to raise the temperature of one gram of water one degree Cent.

1,000 grams = 1 kilogram, and 2·20462 lb. = 1 kilogram,

$$\therefore \text{ One B.T.U.} = \frac{1000}{2{\cdot}20462} \times \frac{5}{9} = 252 \text{ gram-calories.}$$

Transfer of Heat.

Heat can be transferred in three ways, by Conduction, Convection, or Radiation.

Conduction of heat is the flow of heat through a body from a region of high temperature to a region of lower temperature. The rate at which heat flows through a body, i.e., the quantity per unit time, is directly proportional to the area exposed to the source of heat and to the temperature difference of its ends, and is inversely proportional to the distance through which the heat travels. Therefore we can write $Q = \dfrac{c\,a\,t\,\theta}{d}$ where Q = quantity of heat, a = area exposed to the source of heat, t = time of exposure, θ = temperature difference, d = distance of heat flow, and c = co-efficient of thermal conductivity. The value of this co-efficient will depend upon the material and is expressed in units which govern the units of Q, a, t, θ and d.

Example. A refrigerating chamber is 20 feet long, 18 feet wide, and 8 feet high. Every wall is covered with insulating material to a depth of 10 inches which has a co-efficient of thermal conductivity of 0·0003 gram-calorie per second per centimetre cube. Find the quantity of heat which must be taken away per minute, (*a*) in gram-calories, (*b*) in B.T.U., to maintain the temperature inside at — 3° Cent. when the external temperature is 22° Cent.

$$\text{Total area} = \{(20 \times 18) + (20 \times 8) + (18 \times 8)\} \times 2 = 1328 \text{ sq. feet.}$$

$$= 1328 \times 144 \times 2{\cdot}54^2 \text{ sq. centimetres.}$$

$$\text{Depth of insulation} = 10 \times 2{\cdot}54 \text{ centimetres.}$$

$$\text{Temperature difference} = 22 - (-3) = 25^\circ \text{ Cent.}$$

$$\text{Time} = 60 \text{ seconds.}$$

$$Q = \frac{c\,a\,t\,\theta}{d} = \frac{0{\cdot}0003 \times 1328 \times 144 \times 2{\cdot}54^2 \times 60 \times 25}{10 \times 2{\cdot}54}$$

$$= 21{,}860 \text{ gram-calories.} \quad \text{Ans.}$$

$$= \frac{21{,}860}{252} = 86{\cdot}74 \text{ B.T.U.} \quad \text{Ans.}$$

Convection of heat is the mode of transferring heat from one part of a substance to another part, by the movement of heated particles of the substance. Liquids and gases are heated in this way.

Radiation of heat takes place when the heat from a hot body is sent out in the form of rays, through space. These rays travel outwards in straight lines at approximately the same speed as that of light.

The darker the surface of the body the better will it radiate heat, and also the better will it be able to absorb radiant heat. A perfectly black surface is a perfect radiator, and a perfect absorber of radiant heat; a bright or polished surface is a poor radiator, and a poor absorber of radiant heat. In practice, the nearest approach to a perfect radiator is probably a rough surface coated with lamp black.

*Stefan's law of radiation states that the rate at which heat is radiated from a perfect radiator is proportional to the fourth power of its absolute temperature.

Then, quantity of heat radiated $= T^4 \times$ a constant.

This constant may be represented by K. Its value is found experimentally. Expressing the quantity of heat radiated in B.T.U. per hour per square foot of heating surface, and the absolute temperature in Fahrenheit degrees, then the value of the constant may be taken as 16×10^{-10}. If the temperature was given in Centigrade degrees absolute, then, since C° abs. $\times \frac{9}{5}$ = F° abs., the constant would be

$$16 \times (\tfrac{9}{5})^4 \times 10^{-10} = 168 \times 10^{-10}.$$

If the quantity of heat radiated was to be expressed in C.H.U. per square foot per hour, and the temperature given in Cent. degrees absolute, then the constant would be $168 \times 10^{-10} \times \frac{5}{9} = 93{\cdot}3 \times 10^{-10}$. This is usually given as 93×10^{-10}, but it must be remembered that the constants are experimentally determined.

Consider two perfectly black bodies radiating heat to each other, one has an absolute temperature of T_1 and the other an absolute temperature of T_2. The maximum heat radiated from the first body, if the temperature of its surrounds was 0° F. abs., would be $K\,T_1^4$. The maximum heat radiated from the second body, if the temperature of its surrounds was 0°F. abs., would be $K\,T_2^4$. Therefore, the maximum heat lost by one body and gained by the other is $K\,T_1^4 - K\,T_2^4 = K\,(T_1^4 - T_2^4)$.

Example. The flame in a boiler furnace has a temperature of 2240°F., and the surrounding surface of the furnace is at a temperature of 440°F. Find the maximum quantity of heat that can be given to the furnace per square foot of surface per hour.

$$\begin{aligned}
\text{Quantity} &= K\,(T \quad - T_2^4) \\
&= 16 \times 10^{-10} \times (2700^4 - 900^4) \\
&= 16 \times 10^{-10} \times (2700^2 + 900^2)\,(2700^2 - 900^2) \\
&= 16 \times 10^{-10} \times 8100000 \times 6480000 \\
&= 16 \times 10^{-10} \times 10^{10} \times 81 \times 64{\cdot}8 \\
&= 16 \times 81 \times 64{\cdot}8 \\
&= 83{,}980 \text{ B.T.U. per sq. ft. of surface per hour.}
\end{aligned}$$

Ans.

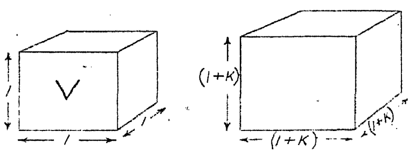
Expansion of Metals.

Most solids and liquids expand when heated. The effect of the application of heat on a solid with regard to its length is expressed by its "Co-efficient of Linear Expansion." This co-efficient, which can be represented by K, is the change in length per unit length for one degree change in temperature, hence the total change in length for a solid of length L, due to T degrees change in temperature, is K L T.

The "Co-efficient of Superficial Expansion" is the change in area per unit area for one degree change in temperature. Consider

an area of unit dimensions and let its temperature be raised by one degree, its dimensions will then become $(1 + K)$. Whereas its original area was 1, its expanded area is $(1 + K)^2 = 1 + 2K + K^2$. Now K has a small value (about 0·0000067 per degree Fahr. for steel) so the value of K^2 is very small indeed and is negligible, therefore the increase in area is $1 + 2K - 1 = 2K$. As 2 K is the increase in area per unit area for one degree rise, it is the co-efficient of superficial expansion, and is equal to twice the co-efficient of linear expansion. If A = original area, then, change in area $= 2\,K\,A\,T$.

The "Co-efficient of Cubical Expansion" is the change in volume per unit volume for one degree change in temperature.



Consider a volume of unit dimensions and let its temperature be raised by one degree, its dimensions will then become $(1 + K)$. Whereas its original volume was 1, its expanded volume is

$(1 + K)^3 = 1 + 3K + 3K^2 + K^3$. The values of $3K^2$ and K^3 are so small that they are negligible, the increase in volume is, therefore, $1 + 3K - 1 = 3K$. Now $3K$ is the increase in volume per unit volume for one degree rise in temperature, therefore it is the co-efficient of cubical expansion and is equal to three times the co-efficient of linear expansion. If V = original volume, then, change in volume $= 3KVT$.

Example. A block of steel measures 18 inches × 16 inches × 5 inches. Taking the co-efficient of linear expansion of steel to be 0·0000067 per degree Fahrenheit, find the increase in volume when heated through 50C°.

Rise in temperature $= 50 \times \frac{9}{5} = 90$F°.

Co-efficient of cubical expansion

$$= 3 \times 0{\cdot}0000067 \text{ per deg. F.}$$

∴ Increase in volume

$$= 3 \times 0{\cdot}0000067 \times 18 \times 16 \times 5 \times 90$$

$$= 2{\cdot}605 \text{ cubic inches. Ans.}$$

Specific Heat.

Different substances have different capacities for storing heat. While it takes one B.T.U. to raise the temperature of one lb. of water one degree F., it takes only 0·095 B.T.U. to raise the temperature of one lb. of copper one degree F.; so that the quantity of heat contained in one lb. of each of these substances although they may be at the same temperature, is not the same.

The Specific Heat of a substance is the quantity of heat in B.T.U. necessary to raise one lb. of a substance one degree Fah., compared with the heat needed to raise one lb. of water one degree Fah. A table of Specific Heats is given at the examination.

The **Water Equivalent** of a body is the weight of fresh water that could be raised one degree in temperature, by the same number of heat units that it takes to raise the temperature of that body one degree; it is, therefore, the weight of the body multiplied by its specific heat.

Joules' Equivalent.

Heat is a form of vibratory energy, and the first law of thermodynamics states :—"Heat and mechanical work are mutually convertible or interchangeable." One B.T.U. is equivalent to 778 foot lb. of work. This value was determined by Dr. Joule.

Properties of Gases.

A liquid of less volume than the vessel which contains it, will only partly fill the vessel. Any volume of a perfect gas, if put into a vessel of larger volume than itself, will at once fill every part of the vessel. The difference between a liquid and a gas lies essentially in their different elastic or expansive properties, and it is this elastic property of gases which is so useful in the cylinders of heat engines.

Now whereas all solids and liquids, when heated, expand at different rates (there are only a few exceptions to this rule), all gases expand at the *same* rate when heated, and contract at the *same* rate when cooled. One cubic foot of a perfect gas contracts $\frac{1}{492}$ part of its volume when cooled one degree F., at 32° F. This suggests that if a gas is cooled 492°F. below 32°F., that a state of heat called the Absolute Zero of temperature will be reached. This Absolute Zero is 460°F. below 0° on the Fah. scale, and 273°C. below 0° on the Cent. scale. In dealing with changes in pressures and volumes of perfect gases, we always work in absolute temperatures and absolute pressures. The absolute temperature is found by adding 460 to the reading on the Fah. scale, and by adding 273 to the reading on the Cent. scale.

Boyle's Law.

The pressure of a perfect gas varies inversely as its volume, if the temperature remains constant. Expressing this mathematically :—

$p\,v$ = constant, or $p_1\,v_1 = p_2\,v_2$.

Charles' Law.

The volume of a perfect gas varies directly as its absolute temperature, if the pressure remains constant ; also the pressure of a perfect gas varies as its absolute temperature if the volume remains constant, or we may write :—

$$\frac{\ }{\mathrm{T}} = \text{constant, and } \frac{p}{\mathrm{T}} = \text{constant.}$$

Combining Boyle's Law and Charles' Law we have :—

$\frac{p\,v}{\mathrm{T}}$ = constant. This is the law connecting the pressure, volume and temperature for a perfect gas.

Example. Ten cubic feet of air at a gauge pressure of 200 lb. per square inch expand to a volume of 25 cubic feet. Find the final gauge pressure.

$p_1\ v_1 = p_2\ v_2$, and working in absolute pressures,
$(200 + 15) \times 10 = p_2 \times 25$.

$$\frac{2150}{25} = 86 \text{ lb. per square inch absolute, or}$$

$p_2 = 86 - 15 = 71$ lb. per square inch gauge. Ans.

Law Connecting Pressure, Volume and Temperature of 1 lb. of air.

It is known that one pound of air at 32°F. and at 14·7 lb. per square inch absolute occupies a volume of 12·38 cubic feet.

$\dfrac{p}{T}$ = constant, and putting in the values above we have :—

$$\frac{14{\cdot}7 \times 144 \times 12{\cdot}38}{(32 + 460)} = 53{\cdot}2, \text{ and this constant is denoted by R.}$$

For any conditions of p, v and T, we have :

$\dfrac{p\ v}{T}$ 53·2 for one lb. of air, and $p\ v = R\ T$.

Note that p is in lb. per square foot.

Example. One lb. of air is at a pressure of 500 lb. per square inch absolute, its temperature being 400°F. Find its volume.

$$\frac{p\ v}{T} = 53{\cdot}2,\ v = \frac{53{\cdot}2 \times T}{p}$$

$$v = \frac{53{\cdot}2 \times (400 + 460)}{500 \times 144} = 0{\cdot}6354 \text{ cu. foot. Ans.}$$

Example. An air storage bottle has an internal diameter of 8 inches and its length is 8 feet. Find the weight of air it contains at 60°F. and at 1000 lb. per square inch absolute.

Vol. of bottle $= \frac{2}{3} \times \quad \times \quad \times 8 = 2{\cdot}8$ cu. feet nearly.

$$\text{Vol. of 1 lb. air in bottle} = \frac{53{\cdot}2 \times (460 + 60)}{1000 \times 144} = 0{\cdot}192 \text{ cu. foot.}$$

$$\text{Weight in bottle} = \frac{2{\cdot}8}{0{\cdot}192} = 14{\cdot}54 \text{ lb. Ans.}$$

The Internal Energy or, as it is sometimes called, the Intrinsic Energy of a perfect gas, is a measure of the heat energy contained in the gas. The internal energy depends upon the temperature of the gas only.

Isothermal Expansion.

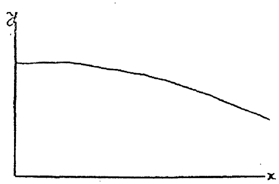
A gas, doing work by expanding, is said to expand isothermally when its temperature remains constant during the operation. During the expansion, a quantity of heat equal in amount to the external work done, must be supplied to the gas from an external source. During isothermal compression, heat must be taken from the gas to keep its temperature constant.

Now Boyle's Law states that the volume of a gas is inversely proportional to the pressure when the temperature is constant, or that $p\ v$ = constant. This therefore is the expression connecting pressure and volume for isothermal expansion or compression.

The curve representing isothermal expansion is called a hyperbola, and isothermal expansion is sometimes called hyperbolic expansion.

Adiabatic Expansion.

A gas, doing work by expanding, is said to expand adiabatically when the work is done at the expense of the internal energy of the gas, the temperature of the gas falling during the operation. During the expansion no heat is given to the gas, and no heat is lost by the gas, excepting that which is converted into work. During adiabatic compression, the work done on the gas is stored as internal energy, and the temperature of the gas rises.



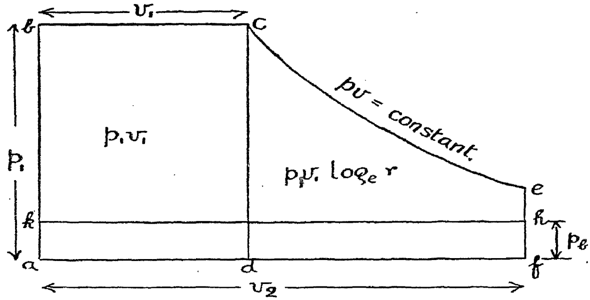
Consider a diagram which shows the relation between two quantities, say y and x, then the area between the curve and the axes represents that quantity which is in terms of the units of y multiplied by the units of x. Thus, suppose that the vertical ordinate represents velocity in feet per second, and the horizontal length represents time in seconds, then the area represents :—velocity × time = distance, and the units are :—

$$\text{Feet per second} \times \text{seconds} = \frac{\text{feet}}{\text{secs.}} \times \text{secs.} = \text{feet.}$$

Similarly, if a curve shows the relation between pressure in lb. per square foot and volume in cubic feet, then the area represents :—

lb. per sq. foot $\times$ cubic feet.

$$\frac{\text{lb.}}{(\text{feet})^2} \times (\text{feet})^3 = \text{lb.} \times \text{feet} = \text{work in ft. lb.}$$



Suppose a gas to be admitted to a cylinder at an absolute pressure p_1 and let the volume up to the "cut-off" be v_1. Let the volume at the end of the stroke be v_2. When cut-off is reached no more gas is admitted, so that as the volume increases from v_1 to v_2 the pressure falls. Suppose the expansion to be isothermal. Now up to the cut-off the pressure is constant so that the area $a\ b\ c\ d$ which represents the work done up to this point $= p_1\ v_1$. The area $c\ e\ f\ d$ represents the work done during the expansion and this area $= p_1\ v_1\ log_e\ r$, where r is the number

of times the gas is expanded $= \dfrac{v_2}{v_1}$

and $log_e\ r = 2{\cdot}3 \times$ common log r.

The proof that the area under the curve is $p_1\ v_1 \log_e r$, is beyond the scope of this book.

The total area is :—

$p_1\ v_1 + p_1\ v_1\ l$

$r)$

= work done on pressure side of piston throughout the stroke.

In practice there is always a "back" pressure on the other side of the piston, and the work done against this pressure is represented by the area $a\ k\ f\ h = p_b\ v_2$.

The nett work done per stroke is therefore :—

$$p_1\ v_1\ (1 + log_e\ r) - p_b\ v_2.$$

If the work done is required in foot lb. then p_1 and p_b must be in lb. per square foot absolute, and v_1 and v_2 must be in cubic feet.

The average height of the diagram will represent the average pressure.

$$\text{Average height} = \frac{\text{area}}{\text{length}}$$

$\therefore$ Average pressure $= \dfrac{p_1\ v_1}{\quad}$ $\quad$ $g_e\ r) - .$

$\dfrac{p_1}{r}$ $\quad$ $r) - p_b$

This gives the average effective pressure throughout the stroke when the expansion is isothermal. The average effective pressure is called the Mean Effective Pressure.

Example. Ten cubic feet of air at 100 lb. per sq. inch absolute, expand isothermally to 30 cubic feet. Find the work done during admission and during expansion. If the back pressure is 30 lb. per sq. inch absolute, find the nett work done and the mean effective pressure throughout the stroke.

Work during admission $= p_1\ v_1 = 144 \times 100 \times 10 = 144{,}000$ ft. lb. Ans.

Work during expansion $= p_1\ v_1\ \log_e\ r$

$$r = \frac{v_2}{v_1} = \tfrac{30}{10} = 3, \text{ and } \log_e\ 3 = 2{\cdot}3 \times 0{\cdot}4771.$$

$\therefore$ Work $= 144 \times 100 \times 10 \times 2{\cdot}3 \times 0{\cdot}4771 = 158{,}000$ ft. lb. Ans.

Work done against back pressure

$$= p_b\ v_2 = 144 \times 30 \times 30 = 129{,}600 \text{ ft. lb.}$$

$\therefore$ Nett work done $= 144{,}000 + 158{,}000 - 129{,}600 = 172{,}400$ ft. lb. Ans.

Mean effective pressure

$$\frac{\text{nett work}}{} \quad \frac{172{,}400}{30} = 5{,}747 \text{ lb. sq. foot.}$$

$$= \tfrac{5747}{144} = 39{\cdot}9 \text{ lb. sq. inch.} \quad \text{Ans.}$$

or :—

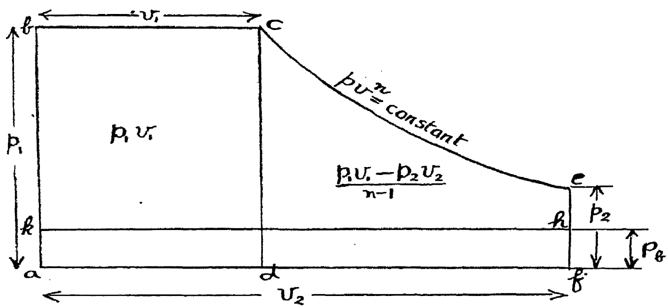
Mean effective pressure

$$= \frac{p_1}{r}(1 + \log_e r) - p_b$$

$$= \tfrac{100}{3}(1 + 2{\cdot}3 \times 0{\cdot}4771) - 30$$

$$= 39{\cdot}9 \text{ lb. per inch.} \quad \text{Ans.}$$

Work done during adiabatic expansion. The law of the expansion curve is $p\ v^n$ = constant, where n for air = 1·4, or $v_1^{1{\cdot}4} = p_2\ v_2^{1{\cdot}4}$



Work during admission = area $a\ b\ c\ d$ = $p_1\ v_1$

Work during expansion = area $c\ e\ f\ d$ = $\dfrac{p_1\ v_1}{n - 1}$

The proof of this is beyond the scope of this book.

Work against back pressure = $p_b\ v_2$

$$\therefore \text{ Nett work} = \qquad + \frac{p_1\ v_1 - p_2\ v_2}{\quad - 1}$$

If the expansion is continued down to the back pressure, $_b = p_2$ and the expression becomes :—

Nett work $= p_1 v_1 +$

$n - 1$

$- p_1 \qquad - p_2 v_2 - n p_2 v_2 +$

$n - 1$

$[p_1 v_1 \qquad p_2 v_2]$

1

In **expanding** a gas in a **cylinder** it is not usual to carry the expansion down to the back pressure, so that p_b is less than p_2.

In **compressing** a gas the compression must begin from the lowest pressure so p_b is then $= p_2$. In using the last expression for work done in a compressor, if p_2 is the suction pressure and v_2 the volume of gas taken into the cylinder, the calculated work done will be a negative amount. The negative sign indicates that the work has been done *on the gas.*

In a turbine the steam expands adiabatically. The value for n is 1·135 for saturated steam and 1·3 for superheated steam.

Example. Five cubic feet of air at 200°F. expand to 10 cubic feet, the expansion being adiabatic. If the initial pressure is 100 lb. sq. inch absolute, find the final pressure and temperature. If the back pressure is equal to the final pressure of expansion find the nett work done during admission and expansion.

$p_1 v_1^{1 \cdot 4} = p_2 v_2^{1 \cdot 4}$, $100 \times 5^{1 \cdot 4} = p_2 \times 10^{1 \cdot 4}$

$p_2 = 100 \times (\frac{5}{10})^{1 \cdot 4}$

$= 37 \cdot 89$ lb. sq. inch absolute. Ans.

also for a perfect gas $\dfrac{p\ v}{T} =$ constant

$$\frac{100 \times 5}{(460 + 200)} = \frac{37 \cdot 89 \times 10}{T_2}$$

Logs.
0·6990
1·4
―――
27960
6990
―――
0·9786
1·4
―――
$\bar{1} \cdot 5786$
2·0000
―――
1·5786 log. of 37·89

$$T_2 = \frac{37\cdot89 \times 10 \times 660}{500} = 500\cdot1^\circ\text{F absolute.}$$

Final Temperature $= 500\cdot1 - 460 = 40\cdot1^\circ$F. Ans.

$$\text{Nett work done} = \frac{1\cdot4}{1\cdot4 - 1} \times 144\ [100 \times 5 - 37\cdot89 \times 10] \text{ ft. lb.}$$

$$= 504 \times 121\cdot1 = 61{,}034 \text{ ft. lb.} \quad \text{Ans.}$$

Note.—In using the equations $p\ v^n = $ cons, and $\dfrac{p}{T} = $ cons., we may leave out the 144 to bring the pressure to lb. per square foot, as it cancels from both sides, but when finding the work done we must have the pressure as lb. per square foot.

Relation between temperature, pressure and volume during adiabatic operations.

For adiabatic operations $p\ v^n = $ constant.

$$\therefore\ p_1\ v_1^n = p_2\ v_2^n, \text{ and } \left(\frac{v_1}{v_2}\right)^n = \frac{p_2}{p_1}$$

Raising each side to the $\left(\dfrac{n-1}{n}\right)$ power we have

$$\left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

Now $\dfrac{p\ v}{T} = $ constant for any gas,

$$\therefore\ \frac{p_1\ v_1}{T_1} = \frac{p_2\ v_2}{T_2}, \text{ and } \frac{T_2}{T_1} = \frac{p_2\ v_2}{p_1\ v_1} = \frac{v_1^n}{v_2^n} \times \frac{v_2}{v_1}$$

$$= \frac{v_1^n \times v_1^{-1}}{v_2^n \times v_2^{-1}} = \left(\frac{v_1}{v_2}\right)^{n-1}$$

$$\therefore\ \frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{n-1} = \left(\frac{p_2}{p_1}\right)^{\frac{n-1}{n}}$$

The value of n for air is 1·4, and it is denoted by γ (gamma).

Example. A Diesel engine takes in air at 14 lb. per square inch absolute and 70°F., and compresses it to 510 lb. per square inch absolute. Find the temperature at the end of compression, assuming adiabatic compression, $n = 1\cdot4$.

From the equation already given, $\frac{T_2}{T_1} = \left(\frac{p_2}{p_1}\right)^{\frac{1\cdot4-1}{1\cdot4}}$

$$\frac{1\cdot4 - 1}{1\cdot4} \qquad \frac{0\cdot4}{1\cdot4}$$

$$\frac{T_2}{(460 + 70)} = \left(\frac{510}{14}\right)^{\frac{2}{7}} \text{ or } \frac{T_2}{530} = \left(\frac{510}{14}\right)^{\frac{2}{7}}$$

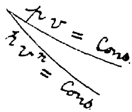
Log. T_2 = log. 530 + $\frac{2}{7}$ (log. 510 log. 14).

Log. T_2 = 3·1704

T_2 = 1480°F absolute.

Final Temp. = 1020°F. Ans.

Logs.
2·7076
1·1461
―――
1·5615
2
―――
7)3·1230
―――
0·4461
2·7243
―――
3·1704 log. of 1480



The work done during adiabatic expansion is less than the work done during isothermal expansion. For isothermal expansion the law of the curve is $p\ v$ = cons. For adiabatic expansion the law is $p\ v^n$ = cons., where n is greater than unity (1·4 for air). When the value of n is more than unity, the curve falls below the curve $p\ v$ = cons., and the area under the curve, which is a measure of the work done, is less.

***Specific Heat of Gases.**

A gas may be heated under constant volume conditions when its pressure will rise, or under constant pressure conditions when its volume will increase.

The specific heat at constant pressure is greater than the specific heat at constant volume because of the external work done.

The specific heat of air is :—

0·1691 B.T.U., or 131·6 ft. lb. per 1 lb. at constant volume. This is represented by K_v.

0·2375 B.T.U., or 184·8 ft. lb. per 1 lb. at constant pressure. This is represented by K_p.

If 1 lb. of air is heated at constant volume from T_1 to T_2, then since no heat is employed in doing external work because the volume does not change, the internal energy of the air must be increased by the amount of heat given.

$\therefore$ Change of internal energy $= K_v\ (T_2 - T_1)$; or the change of internal energy is always equal to the specific heat at constant volume multiplied by the change of temperature.

Again, if 1 lb. of air is heated at constant pressure p, from temperature T_1 to temperature T_2, and the volume increases from v_1 to v_2, then

Heat given = Change of internal energy + External work done.

$$(T_2 - T_1) = \quad (T_2 - T_1) + p\ (v_2 - v_1).$$

But $p\ v_2 = R\ T_2$, and $p\ v_1 = R\ T_1$, because $p\ v = R\ T$,
$K_p\ (T_2 - T_1) = K_v\ (T_2 - T_1) + R\ (T_2 - T_1)$
and from this,

$$= K_v + R, \text{ and } R = \quad - K_v$$

Putting in the values for K_p and K_v,
$R = 184{\cdot}8 - 131{\cdot}6 = 53{\cdot}2$ ft. lb. per 1 lb. of air.

It has been shown that $K_p = K_v + R$. Divide all by K_v

$$\frac{K_p}{K_v} \quad \frac{K_v}{K_v} + \frac{R}{K_v} \text{ and } \therefore \frac{K_p}{K_v} = 1 + \frac{R}{K_v}$$

- is the ratio of the specific heats. This ratio is important

and is given the symbol γ (gamma).

$$\therefore \gamma = 1 + \frac{R}{K_v} \text{ and } K_v = \frac{R}{\gamma - 1}$$

The value of γ for air is $\dfrac{0{\cdot}2375 \text{ B.T.U.}}{0{\cdot}1691 \text{ B.T.U.}}$ or $\dfrac{184{\cdot}8 \text{ ft. lb.}}{131{\cdot}6 \text{ ft. lb.}} = 1{\cdot}4$

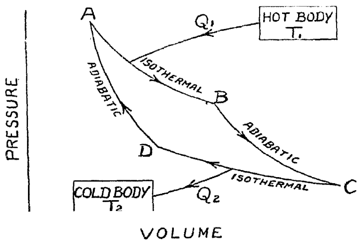
***Heat Engine Cycles.**

If a quantity of a substance is at certain conditions of temperature, pressure and volume, and if it goes through thermal operations involving changes of temperature, pressure and volume and eventually returns to its initial conditions of temperature, pressure and volume it is said to have passed through a cycle of operations.

***Carnot Cycle.**

This ideal heat engine cycle was suggested by a French scientist, Sadi Carnot, about the beginning of the 19th century, as a standard for the comparision of the performance of actual heat engines.

The operations may be regarded as taking place in a cylinder. Consider there is a hot body whose temperature is T_1 absolute, and which remains at that temperature no matter how much heat is taken from it. Also, a cold body at T_2 absolute, which remains at that temperature independent of the quantity of heat given to it.



At A let there be 1 lb. of gas at pressure p_A, temperature T_1 and volume v_A and let this expand isothermally to B, the conditions then being, pressure $= p_B$; temperature $= T_1$ and volume $= v_B$.

In order that the temperature shall remain constant during this isothermal expansion, an amount of heat, equivalent to the work done, must be supplied to the gas by the hot body. Let this be Q_1.

Now the work done under an isothermal curve has already been stated to be $p_1\, v_1 \log_e r$. In this case it is $p_A\, v_A \log_e \dfrac{v_B}{v_A}$

$$\text{But } p_1\, v_1 = R\, T_1 \text{ and therefore } p_A\, v_A = R\, T_1$$

$$\text{Thus, the work done and the heat given} = Q_1 = R\, T_1 \log_e \frac{v_B}{v_A}$$

At point B adiabatic expansion commences and is continued to C, the temperature of the gas falling to T_2, because the work done during adiabatic expansion is done at the expense of the internal energy of the gas. At C isothermal compression commences and is continued to point D, a quantity of heat Q_2, being rejected to the cold body in order to keep the temperature T_2 constant.

The work done under this curve is $p_D\, v_D \log_e \dfrac{v_C}{}$, which

equals $R\, T_2 \log_e$ $\qquad$ $Q_2 = R\, T_2 \log_e$

At D adiabatic compression starts and is continued to A, when the gas has returned to its initial conditions of pressure, temperature and volume, and a cycle of operations has been completed.

The cycle must be a closed one, therefore the isothermal compression must finish at D, a point so chosen that the adiabatic through D will pass through A.

Considering the adiabatic B to C, $\frac{T_1}{T_2} = \left(\frac{v_C}{v_B}\right)^{n-1}$

Considering the adiabatic D to A, $\frac{T_1}{T_2} = \left(\frac{v_D}{v_A}\right)^{n-1}$

$$\therefore \left(\frac{v_C}{v_B}\right)^{n-1} = \left(\frac{v_D}{v_A}\right)^{n-1}, \quad \frac{v_C}{v_B} = \frac{v_D}{v_A} \text{ or } \frac{v_C}{v_D} = \frac{v_B}{v_A}$$

Now, $\frac{v_C}{v_D}$ is the ratio of isothermal compression, and $\frac{v_B}{v_A}$ is the ratio of isothermal expansion. It has been shown that these are equal, therefore in order that the cycle shall close $\frac{v_C}{v_D}$ must equal $\frac{v_B}{v_A}$. Let this ratio be r.

$$Q_1 = R\,T_1 \log_e \frac{v_B}{} = R\,T_1 \log_e r$$

$$Q_2 = R\,T_2 \log_e \frac{v_C}{v_D} = R\,T_2 \log_e r$$

The difference between the heat given and the heat rejected is the work done during the cycle.

$$\text{Heat, or thermal efficiency} = \frac{\text{Heat given} - \text{heat rejected}}{\text{Heat given}}$$

$$\frac{\text{Work done}}{\text{Heat given}} \quad \frac{Q_1 - Q_2}{Q_1}$$

$$\text{Heat, or thermal efficiency} = \frac{r - R\,T_2 \log_e r}{T_1 \log_e r}$$

$$T_1 - T_2$$

The efficiency is dependent upon the temperatures only, and is independent of the nature of the gas. All the heat is given at the higher temperature T_1, and all the heat rejected is rejected at the lower temperature T_2. Therefore it is not possible for any heat engine, working between the temperature limits of T_1 and T_2, to have a higher efficiency than is given by $\dfrac{T_1 - T_2}{T_1}$

The efficiency of all actual heat engines must be less than that of this ideal cycle.

The Carnot cycle is reversible. Starting at D isothermal expansion could occur and continue to C, a quantity of heat Q_2 being taken in from the cold body. From C to B there would be adiabatic compression, raising the temperature of the gas from T_2 to T_1. From B to A isothermal compression, accompanied by the rejection of heat equal to Q_1 to the hot body; and finally adiabatic expansion from A to D, the temperature falling from T_1 to T_2. An engine working on this reversed cycle would be acting as a heat pump taking in heat at the lower temperature T_2 and delivering it at the higher temperature T_1, mechanical work being done to bring this about. This is, in brief, the function of a refrigerating machine. The measure of the efficiency, referred to as the co-efficient of performance is

$$\frac{\text{Heat extracted}}{\text{Work done}} \qquad \frac{Q_2}{Q_1 - Q_2} \qquad \frac{T_2}{T_1 - T_2}$$

Example. A refrigerating machine works between the temperature limits of 67°F. and 12°F. and is driven by a motor of 2 horse power. How much ice at 28°F. could this machine make per hour from water at 60°F. ?

Sp. heat of ice = 0·5. Latent heat of water = 143 B.T.U. per lb.

Coefficient of performance

$$= \frac{(460 + 12)}{67 - 12} = \frac{472}{55} = 8{\cdot}58 = \frac{\text{Heat extracted}}{\text{Work done}}$$

$\therefore$ Heat extracted = 8·58 × Work done.

Now one H.P. hour has a heat equivalent of 2545 B.T.U. and 2 H.P. = 5090 B.T.U.

$\therefore$ the machine is capable of extracting 8·58 × 5090 B.T.U. per hour.

Heat to be taken from 1 lb. of water at 60°F. to convert to ice at 28°F.

$$= (60 - 32) + 143 + 0{\cdot}5\ (32 - 28)$$

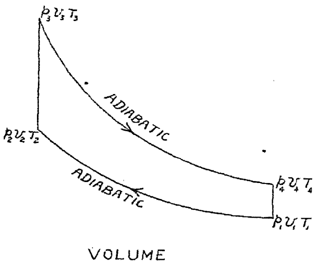
$$= 28 + 143 + 2 = 173 \text{ B.T.U.}$$

$$\therefore \text{ ice made per hour} = \frac{8{\cdot}58 \times 5090}{173} = 252{\cdot}5 \text{ lb.} \quad \text{Ans.}$$

*Note :—The co-efficient of performance of the ideal reversed cycle engine is 8·58, but that of an actual refrigerating machine might not be more than 0·6 of this value.

In that case the ice per hour would be 0·6 of 252·5 = 151·5 lb.

*Otto Cycle, or Constant Volume Cycle.



The cycle of operations starts with a volume of air v_1 at pressure p_1 and absolute temperature T_1. The air is compressed adiabatically to volume v_2, the pressure rising to p_2 and the temperature to T_2. Heat is now given at constant volume, causing the pressure to rise to p_3 and the temperature to T_3. Adiabatic expansion then occurs to v_4, the pressure falling to p_4 and the temperature to T_4.

Note that $v_4 = v_1$; also that $v_3 = v_2$.

The cycle is completed by rejection of heat at constant volume, the initial thermal conditions of pressure, volume and temperature being regained.

Let the ratio of compression, i.e. —— be r.

$$\text{Thermal efficiency} = \frac{\text{Heat given} - \text{Heat rejected}}{\text{Heat given}}$$

$$\frac{\ldots - T_2) - K_v\ (T_4 - \ldots}{K_v\ (T_3 - T_2)} = 1 - \frac{\ldots}{(T_3 - \ldots}$$

$$\text{Now } \frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{\gamma - 1}; \text{ also } \frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma-1} = \left(\frac{v_2}{v_1}\right)^{\gamma - 1}$$

because $v_3 = v_2$ and $v_4 = v_1$.

$$\therefore \frac{T_1}{T_2} = \frac{T_4}{T_3}$$

Let $T_4 = c\,T_1$ and $T_3 = c\,T_2$

$$\text{then } \frac{(T_4 - T_1)}{(T_3 - T_2)} = \frac{c\,T_1 - T_1}{c\,T_2 - T_2} = \frac{T_1\,(c - 1)}{T_2\,(c - 1)} = \frac{T_1}{T_2} \text{ or } \frac{T_4}{T_3}$$

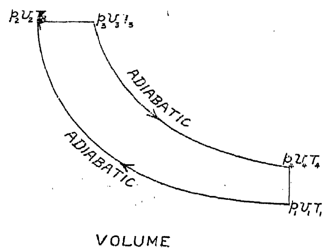
$$\text{Thermal efficiency} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)} = 1 - \frac{T_1}{T_2} \text{ or } 1 - \frac{T_4}{T_3}$$

$$\text{But } \frac{T_1}{T_2} = \frac{T_4}{T_3} = \left(\frac{v_2}{v_1}\right)^{\gamma - 1} = \left(\frac{1}{r}\right)^{\gamma - 1}$$

$$\therefore \text{Thermal efficiency} = 1 - \frac{T_1}{T_2} = 1 - \left(\frac{1}{r}\right)^{\gamma - 1}$$

This is the air standard cycle efficiency.

***Diesel Cycle, or Constant Pressure Cycle.**



The cycle of operations starts with a volume of air v_1 at pressure p_1 and temperature T_1. The air is compressed adiabatically to v_2, the pressure rising to p_2 and the temperature to T_2. Heat is now given at constant pressure, the volume increasing to v_3 and the temperature to T_3, but the pressure remains constant, therefore

The air now expands adiabatically to v_4, v_4 being equal to v_1, the pressure falls to p_4 and the temperature to T_4. The cycle is completed by rejection of heat at constant volume, and the initial thermal conditions have been regained.

Let r = ratio of compression = $\frac{1}{\;}$ or $\frac{v_4}{\;}$

Let r_1 = ratio of expansion $= \dfrac{v_4}{v_3}$ or $\dfrac{v_1}{v_3}$

$$\text{Thermal efficiency} = \frac{\text{Heat given} - \text{Heat rejected}}{\text{Heat given}}$$

$$= \frac{K_p\ (T_3 - T_2) - K_v\ (T_4 - T_1)}{K_p\ (T_3 - T_2)}$$

$$= 1 - \frac{K_v}{K_p} \left\{\frac{T_4 - T_1}{T_3 - T_2}\right\}$$

$$= 1 - \frac{1}{\gamma} \left\{\frac{T_4 - T_1}{T_3 - T_2}\right\}$$

Express all temperatures in terms of T_1

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{\gamma - 1} = r^{\gamma - 1}, \quad \therefore\ T_2 = T_1 \times r^{\gamma - 1}$$

$$\frac{T_3}{T_2} = \frac{v_3}{v_2} = \frac{v_3}{v_4} \times \frac{v_4}{v_2} = \frac{r}{r_1},$$

$$\therefore\ T_3 = T_2 \times \frac{r}{r_1} = T_1 \times r^{\gamma-1} \times \frac{r}{r_1} = T_1 \times \frac{r^\gamma}{r_1}.$$

$$\frac{T_4}{T_3} = \left(\frac{v_3}{v_4}\right)^{\gamma - 1} = \frac{1}{r_1{}^{\gamma-1}}$$

$$\therefore\ T_4 = T_3 \times \frac{1}{r_1{}^{\gamma-1}} = T_1 \times \frac{r^\gamma}{r_1} \times \quad 1 \qquad\qquad \times \quad r^\gamma$$

$$\text{Thermal efficiency} = 1 - \frac{1}{\gamma} \left\{\frac{T_4 - T_1}{T_3 - T_2}\right\}$$

$$= 1 - \frac{1}{\gamma} \left\{\frac{T_1 \times \dfrac{r^\gamma}{r_1{}^\gamma} - T_1}{T_1 \times \dfrac{r^\gamma}{r_1} - T_1 \times r^{\gamma-1}}\right\}$$

Thermal efficiency $= 1 - \cdot \qquad \dfrac{\dfrac{r^{\gamma}}{r_1} - r^{\gamma-1}}{}$

$$= 1 \quad \frac{1}{\gamma} \times \frac{1}{r^{\gamma-}}$$

Now it has already been shown that $\dfrac{v_3}{\quad} = \dfrac{r}{\quad}$. We may call $\dfrac{3}{\quad}$ the cut off ratio, or the ratio of burning. Let $\dfrac{r}{\quad} = r_c$

Thermal efficiency $= 1 - \dfrac{1}{\gamma} \times \dfrac{1}{r^{\gamma-1}} \qquad r\,\Big)^{\gamma} - 1$

$$= 1 - \frac{1}{r^{\;\gamma-1}} \times \frac{1}{\quad} \qquad r_c -$$

which may be written as $1 - \left\{\dfrac{1}{r}\right\}^{\gamma-1} \times \dfrac{1}{\gamma} \left\{\dfrac{r_c^{\gamma} - 1}{r_c - 1}\right\}$,

and is thus comparable to the efficiency of the Otto Cycle.

Suppose $r = 14$ and $r_1 = 7$, then $r_c = \dfrac{r}{\quad} = \dfrac{14}{\quad} = 2$.

Then $\dfrac{1}{\gamma} \left\{\dfrac{r_c^{\gamma} - 1}{r_c - 1}\right. \qquad \dfrac{1}{1{\cdot}4} \left(\dfrac{2^{1\cdot4} - 1}{2 - 1}\right.$

$$\frac{1}{1{\cdot}4} \left(\frac{2{\cdot}638 - 1}{2 - 1}\right. = 1{\cdot}17$$

Now r_c must always be greater than 1, and therefore

$\dfrac{\quad}{\gamma} \left(\dfrac{\gamma - 1}{r_c -}\right\}$ must always be greater than 1.

$$\therefore 1 - \left(\frac{1}{r}\right)^{\gamma-1} \times \frac{1}{\gamma}\left\{\frac{r_c^{\gamma} - 1}{r_c - 1}\right\}$$ will always have a value less than that given by $1 - \left(\frac{1}{r}\right)^{\gamma - 1}$

Stated in another way, for the same ratio of compression (r) the constant volume cycle has, theoretically, a greater thermal efficiency than the constant pressure cycle. In practice, however, the reverse is the case because in the constant pressure, or Diesel engine, the ratio of compression may be high since air only is compressed; whilst engines using the constant volume cycle, such as petrol and gas engines, compress the charge and the ratio of compression is limited due to the danger of pre-ignition.

Example. Determine the efficiency of an ideal engine working on the constant volume cycle. The initial volume is 4, and the volume after compression is 1. The initial pressure is 15 lb. per sq. inch absolute, and the temperature 80°F. The value of γ is 1·4, and both the compression and expansion curves follow the law $p\,v^{\gamma}$ = constant. The highest pressure during the cycle is 250 lb. per sq. inch absolute.

$$r = \tfrac{4}{1} = 4.$$

$$\text{Efficiency} = 1 - (\tfrac{1}{4})^{1\cdot4-1} = 1 - (\tfrac{1}{4})^{0\cdot4} = 1 - (\tfrac{1}{4})^{\frac{2}{5}}$$
$$= 1 - 0\cdot5744 = 0\cdot4256, \text{ or } 42\cdot56\%. \quad \text{Ans.}$$

Suppose, however, the law of compression was stated to be $p\,v^{1\cdot35}$ = constant, and the law of expansion to be $p\,v^{1\cdot3}$ = constant, it would be necessary to use the expression,

$$\text{Efficiency} = 1 - \frac{(T_4 - T_1)}{(T_3 - T_2)}$$

Determine the temperatures T_2, T_3 and T_4.

$$\frac{T_2}{T_1} = \left(\frac{v_1}{v_2}\right)^{1\cdot35}, \quad \therefore T_2 = (80 + 460) \times (4)^{0\cdot35}$$
$$= 877°\text{F. abs.}$$

$$\frac{p_1 v_1}{T_1} = \frac{p_2 v_2}{T_2}, \therefore p_2 = \frac{p_1\ v_1\ T_2}{v_2\,T_1} = \frac{15 \times 4 \times 877}{1 \times 540}$$
$$= 97\cdot5 \text{ lb. sq. inch.}$$

$$\frac{p_2}{T_2} = \frac{p_3}{T_3}, \therefore T_3 = \frac{p_3 T_2}{p_2} = \frac{250 \times 877}{97 \cdot 5} = 2250^\circ \text{F. abs.}$$

$$\frac{\quad}{\quad} = \left(\frac{\quad}{\quad}\right) \quad , \therefore T_4 = 2250 \times (\tfrac{1}{4})^{0 \cdot 3} = 1484^\circ \text{F. abs.}$$

$$\text{Efficiency} = 1 - \frac{(1484 - 540)}{(2250 - 877)} = 1 - 0 \cdot 6874 = 0 \cdot 3126$$
or $31 \cdot 26\%$ Ans.

Example. In an engine, working on the Diesel cycle, air is taken in at 14 lb. per sq. inch absolute and 100° F. The ratio of compression is 14 and the ratio of expansion is 6·35. The compression curve and the expansion curve both follow the law $p\,v^\gamma$ = constant, and $\gamma = 1 \cdot 4$. Determine the ideal efficiency. If an actual engine working on this cycle uses 0·41 lb. of oil per B.H.P. hour, of calorific value 18,900 B.T.U. per lb. determine the relative thermal efficiency of the engine.

$$\text{Cut-off ratio} = r_c = \frac{14}{6 \cdot 35} = 2 \cdot 205.$$ This means that the volume, after reception of heat at constant pressure, is 2·205 times the volume before.

$$\text{Efficiency} = 1 - \left(\frac{\quad}{r}\right) \quad \times \frac{\quad}{\gamma} \left\{\frac{\quad}{r_c - 1}\right\}$$

$$,, = 1 - \left\{\frac{1}{14}\right\}^{1 \cdot 4 - 1} \times \frac{1}{1 \cdot 4} \left\{\frac{2 \cdot 205^{1 \cdot 4} - 1}{2 \cdot 205 - 1}\right\}$$

$$,, = 1 - 0 \cdot 348 \times \frac{1}{1 \cdot 4} \left\{\frac{3 \cdot 024 - 1}{2 \cdot 205 - 1}\right\}$$

$$,, = 1 - 0 \cdot 348 \times \frac{1}{1 \cdot 4} \times \frac{2 \cdot 024}{1 \cdot 205}$$

$$= 1 - 0 \cdot 4177 = 0 \cdot 5823, \text{ or } 58 \cdot 23\%. \quad \text{Ans.}$$

$$\text{Thermal efficiency of actual engine} = \frac{2545}{0 \cdot 41 \times 18900}$$

$$= 0 \cdot 3284, \text{ or } 32 \cdot 84\%.$$

$$\text{Relative thermal efficiency} = \frac{\text{Effy. of actual engine}}{\text{Effy. of ideal engine}}$$

$$\frac{0{\cdot}3284}{0{\cdot}5823} = 0{\cdot}564, \text{ or } 56{\cdot}4\%. \quad \text{Ans.}$$

TEST EXAMPLES XX.

1. Convert — 20°F., and 229°F. to Cent. readings. Convert — 20°C. and 112°C. to Fah. readings.

— 28·8°C. ; 109·4°C. ; —4°F. ; 233·6°F. Ans.

2. Find the volume of 6 lb. of air at 120°F. and 250 lb. per square inch absolute.

5·142 cu. feet. Ans.

3. Air at 100°F. and at a pressure of 35 lb. per square inch absolute has its temperature increased to 380°F., its volume remaining constant. Find the final pressure.

52·5 lb. per sq. inch. Ans.

4. A certain weight of air occupying 2·5 cubic feet is at a temperature of 130°F. It is heated to 500°F., the pressure remaining the same. Find its final volume.

4·067 cu. feet. Ans.

5. If the temperature of the products of combustion is 1200° F. at the back ends of the tubes and 675° F. at the front ends, and if the velocity of the gases is 1,300 feet per minute at the back end, find the velocity at the front end. What practical result due to this is observed at the end of a voyage ?

888·8 ft. per min. Ans.

6. Find the weight of 1 litre of air at a pressure of 75 kilos. per sq. cm. gauge, and a temperature of 45°C., if 12·4 cubic feet of air at 32°F. and 14·7 lb. per square inch absolute weigh 1 lb.

0·1796 lb. Ans.

7. Air is taken into an oil engine cylinder at a pressure of 15 lb. per square inch absolute, and compressed to 75 lb. per square inch gauge. Find the clearance as a fraction of the stroke, assuming the compression to be isothermal.

$\frac{1}{5}$ of the stroke. Ans.

8. An air storage bottle is 22 cm. internal diameter and 2 metres long. Find the weight of air it contains at a pressure of 100 kilos. per square cm. gauge and a temperature of 30° C. Give the answer in lb.

19·09 lb. Ans.

9. Three cubic feet of air at 120 lb. per square inch gauge and 100°F., expand to 12 cubic feet isothermally. Find the nett work done and the mean effective pressure during the stroke, and state how many B.T.U. must have been given to the gas. Back pressure 10 lb. gauge.

95,880 ft. lb. ; 55·49 lb. sq. inch. ; 123·2 B.T.U. Ans.

10. Ten lb. of air at 40°F. and at 15 lb. square inch absolute are heated to 300°F. at constant volume. Find the heat given in B.T.U. and in work units, and find the final pressure. The specific heat of air at constant volume is 0·169 B.T.U. per lb.

439·3 B.T.U. ; 341,800 ft. lb. ; 22·8 lb. per square inch. Ans.

*11. A Diesel engine takes in air at atmospheric pressure and at 85° F. The clearance space is 7·5 per cent. of the working stroke. Find the final pressure and temperature, and the mean effective pressure during compression. The compression is adiabatic, and n is to be taken as 1·4. Take the atmospheric pressure as 15 lb. per square inch.

623·6 lb. sq. inch absolute ; 1121°F. ; 61·6 lb. sq. inch. Ans.

CHAPTER XXI.

PROPERTIES OF STEAM.

Sensible Heat.

When heat is given to a substance, if the temperature of the substance rises, the heat given is said to be *sensible heat*; if sensible heat is taken from a substance, the temperature of the substance falls.

Change of State.

The same substance may exist in several forms. Water may be converted from its liquid form into the solid form of snow or ice; or it may be converted into the gaseous form of steam.

In converting water at 32°F. into ice at 32°F., the temperature of the substance does not alter; but to change its physical state from the liquid to the solid form, heat must be taken from it. The heat taken from water at 32°F. to convert it into ice at 32°F. is called the *latent heat of water.*

Fresh water, heated at atmospheric pressure, boils at a temperature of 212°F. After boiling point is reached, although heat may still be applied to it, the temperature of the water does not rise above 212°F. The heat put into the water at its boiling point of 212°F, goes to convert it from the liquid form to the gaseous form, and is called the *latent heat of steam* at atmospheric pressure

Latent Heat is the heat given to a substance to change its physical state, the temperature remaining the same during the change of state

The latent heat of water is 143 B.T.U. per pound. This statement means that to convert one pound of ice at 32°F. into water at 32°F., 143 B.T.U. must be supplied to it. The specific heat of ice is 0·5, so that for every degree one pound of ice is cooled below 32°F., 0·5 B.T.U. must be taken from it. The latent heat of steam at atmospheric pressure is 966 B.T.U. per pound; this means that 966 B.T.U. must be given to one pound of water boiling at atmospheric pressure, to convert it into steam at 212°F. It follows that 966 B.T.U. must be taken from one pound of steam at 212°F. to convert it into water at 212°F.

Properties of Saturated Steam.

Saturated steam is steam in the presence of the water from which it is generated; it contains no particles of water in suspension, it is invisible and is often called *dry steam.*

When water is heated in a closed vessel, its boiling point rises with the pressure, the steam generated having the same temperature as the water. Steam in the presence of the water from which it is evaporated can have only *one temperature* for a *given pressure.* There is no simple law connecting the temperature and pressure of saturated steam ; temperatures and the corresponding pressures are tabulated in the steam tables.

For temperatures above 212°F., the latent heat of steam is less than 966 B.T.U. per pound. For temperatures below 212°F., it is greater than 966 B.T.U. per pound. The latent heat of steam for any temperature is given in the steam tables but its value may be found from the approximate formula :—

Latent heat $= 966 - 0{\cdot}7\ (t - 212)$ B.T.U. per lb. where t is the temperature of the steam in Fah. degrees. This is the formula given at the examination.

Example. Find the latent heat of steam at 300°F., and also at 150°F.

$$966 - 0{\cdot}7\ (300 - 212) = 966 - 0{\cdot}7 \times 88$$
$$= 904{\cdot}4 \text{ B.T.U. per lb. Ans.}$$
$$966 - 0{\cdot}7\ (150 - 212) = 966 - 0{\cdot}7\ (-\ 62)$$

Note that $-\ 0{\cdot}7 \times (-\ 62) = +\ 43{\cdot}4$,
and $966 + 43{\cdot}4 = 1009{\cdot}4$ B.T.U. per lb. Ans.

√*Example.* Find the total heat supplied to one pound of feed water at 120°F., to convert it into steam at 360°F.

The total heat supplied is given in two stages. Sensible heat must be given to the water to raise it from 120°F. to 360°F., then latent heat must be given to the water at 360°F. to convert it into steam at 360°F.

Sensible heat supplied $= (360 - 120) = 240$ B.T.U. per lb.
Latent heat supplied $= 966 - 0{\cdot}7\ (360 - 212) = 862{\cdot}4$ B.T.U. per lb.

Total heat per lb. $= 240 + 862{\cdot}4 = 1102{\cdot}4$ B.T.U. Ans.

Wet Steam.

When steam contains moisture, or particles of water in suspension, it is said to be *wet steam.* This means that the evaporation of the water is incomplete. In one pound of wet steam, if there is x of a pound of water, then there is $(1 - x)$ of a pound of steam, and we say that the *dryness fraction* of the steam is $\left(\dfrac{1 - x}{1}\right)$. Steam of dryness fraction 0·9, has 0·1 of its weight in the form of water.

To generate wet steam, therefore, takes less heat than to generate dry steam at the same temperature. To generate steam of dryness fraction 0·9, takes only 0·9 of the *latent heat* necessary for the formation of dry steam at the same temperature.

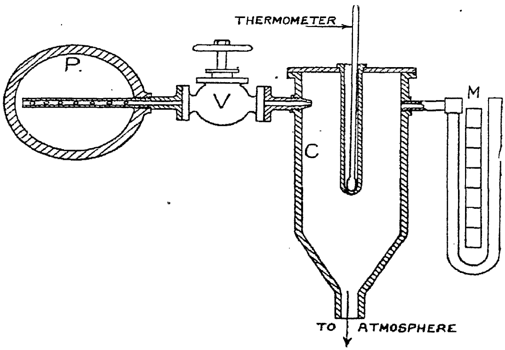
Example. Find the total heat necessary to convert one pound of water at 150°F. into steam of dryness 0·88 at 350°F.

Sensible heat $= 350 - 150 = 200$ B.T.U. per lb.

Latent heat $= 0{\cdot}88\ [966 - 0{\cdot}7\ (350 - 212)\ ] = 765$ B.T.U. per lb.

Total heat $= 200 + 765 = 965$ B.T.U. per lb. Ans.

Throttling Calorimeter.



One method of determining the dryness fraction of steam is by the use of a throttling calorimeter. The instrument consists of a vessel, or calorimeter C with an exit to the atmosphere, a thermometer pocket and thermometer, and a mercury U tube pressure gauge, M. The inlet to the calorimeter is by way of a small orifice, $\frac{1}{16}$ inch or less in diameter, controlled by a valve V.

In order to obtain a fair sample of the steam passing along the main steam pipe P, a perforated pipe, about $\frac{1}{2}$ inch diameter is arranged in the main pipe the perforations being directed towards the steam flow. It is a matter of doubt whether a more correct sample is obtained by arranging the pipe horizontally, as shown in the sketch, or vertically entering from below.

The principle of operation depends upon the fact that if steam is reduced in pressure from a high pressure p_1 to a lower pressure p_2 by a wire-drawing or throttling process, no external

work is done, therefore no heat is converted into work, and the steam at p_2 has exactly the same quantity of heat as at p_1. Now dry saturated steam at p_1 contains more heat than dry saturated steam at p_2. If then dry steam at p_1 was throttled to p_2, the difference in the total heats would be available for superheating the steam at p_2. For example, steam at 200 lb. per sq. inch absolute contains 1201·8 B.T.U. per lb. measured from 32°F., whilst steam at 20 lb. per sq. inch absolute contains 1156·6 B.T.U. per lb., the difference being 45·2 B.T.U. If 1 lb. of dry steam at 200 lb. per sq. inch was reduced in pressure by throttling to 20 lb. per sq. inch this 45·2 B.T.U. would appear as superheat in the lower pressure steam. At, and around, atmospheric pressure the specific heat of superheated steam is 0·48, therefore the steam would be superheated $\dfrac{45 \cdot 2}{0 \cdot 48} = 94 \cdot 2$ F degrees. If the higher pressure steam was not quite dry then the final degree of superheat would be less, because some of the surplus heat would be used in drying the steam. If the higher pressure steam was very wet then it would be partially dried, in other words the dryness fraction would be made higher. The action of a reducing valve is usually to partially dry the steam.

To use the instrument, first note the pressure p_1 in the main pipe and from steam tables obtain its sensible heat h_1 and its latent heat L_1. Open valve V and allow steam to flow. Note the pressure p_2 shown by the mercury gauge and from tables obtain its temperature t_2, its sensible heat h_2 and its latent heat L_2. Observe the thermometer reading t_3. It is essential that t_3 shall be higher than t_2, because then we definitely know the steam is finally superheated and in consequence must be dry. If the condition that t_3 is higher than t_2 cannot be obtained, then no calculation can be made of the initial dryness. Assuming the necessary conditions have been obtained, then the calculation is made thus, C_p being the specific heat of superheat:—

Heat before throttling = Heat after throttling

$$h_1 \qquad L_1 = C_p\ (t_3 - \qquad h_2$$

$$+ L_2$$

$$x =$$

The throttling calorimeter is only suitable over a limited range of steam wetness. If the initial dryness is less than 0·95 or 0·94 it may not be possible to obtain final superheat.

At 200 lb. per sq inch abs. $h_1 = 355{\cdot}2$ B.T.U.
and $L_1 = 846{\cdot}6$ B T.U.

At 20 lb. per sq. inch abs. $h_2 = 196{\cdot}1$ B.T.U.
and $L_2 = 960{\cdot}5$ B.T.U.

Suppose steam at 200 lb. per sq. inch and x dryness was throttled to 20 lb. per sq. inch. and that the steam was just dry and saturated at the latter pressure, then

$$355{\cdot}2 + 846{\cdot}6\,x = 196{\cdot}1 + 960{\cdot}5$$

$$846{\cdot}6\,x = 1156{\cdot}6 - 355{\cdot}2 = 801{\cdot}4$$

$$\frac{801{\cdot}4}{846{\cdot}6} = 0{\cdot}946.$$

0·946 is the limiting dryness, under the conditions suggested, for which this method of determining the dryness would be possible.

The values of sensible heats and of latent heats in the foregoing example have been taken from steam tables. Steam tables are not supplied to candidates at the B. of T. examinations and the values would have to be worked out by the approximate formulæ.

Example. Steam at 140 lb. per sq. inch absolute, temperature 353°F., was passed through a throttling calorimeter and reduced in pressure to 18 lb. per sq. inch absolute, saturation temperature 222·4°F. The final temperature of the steam was observed to be 249·8°F. If the specific heat of superheat is 0·48, what was the initial quality of the steam ?

$h_1 = 353 - 32 = 321$ B.T.U. per 1 lb. above 32°F.
$L_1 = 966 - 0{\cdot}7\,(353 - 212) = 867{\cdot}3$ B.T.U. per lb., if dry.
$h_2 = 222{\cdot}4 - 32 = 190{\cdot}4$ B.T.U.
$L_2 = 966 - 0{\cdot}7\,(222{\cdot}4 - 212) = 958{\cdot}72$ B.T.U.
Superheat $= 249{\cdot}8 - 222{\cdot}4 = 27{\cdot}4$ F°.

Then $321 + x \times 867{\cdot}3 = 0{\cdot}48 \times 27{\cdot}4 + 190{\cdot}4 + 958{\cdot}72$

$$x \times 867{\cdot}3 = 841{\cdot}27$$

$$x = \frac{841{\cdot}27}{867{\cdot}3} = 0{\cdot}97.$$

Specific Volume of Steam.

When water is evaporated into steam it becomes endowed with a new property. The liquid has practically no elasticity, the gaseous vapour called steam into which it is converted has great elasticity, and may be expanded or compressed through a great change of volume.

In changing from the liquid to the gaseous form the substance has its volume greatly increased. The specific volume is the number of cubic feet of steam at a given pressure, generated by the complete evaporation of one lb. of water. The volume of one lb. of steam at atmospheric pressure is about 26·6 cubic feet; the volume of one lb. at 400 lb. per square inch is only 1·167 cubic feet. The volume of one lb. of water is 0·016 cubic foot approximately, so that even in the case of high pressure steam, the increase in volume in passing to the gaseous form is very great. The following formulæ give the relation between the pressure and volume of dry saturated steam.

$$P\,v^{1 \cdot 0646} = 479.$$

$$v = \frac{410 + \frac{P}{4}}{P + 1}, \qquad \text{and } w = \frac{P + 1}{410 + \frac{P}{4}}$$

P = absolute pressure in lb. per square inch.

v = volume in cubic feet of one pound weight of steam.

w = weight in lb. of one cubic foot of steam.

The first of these formulæ is the better, if the volume is required over a large range of pressure; but the second one gives fairly accurate results for ordinary boiler pressures, and has the advantage of being easy to solve. For the pressures at which condensers work, the first formula gives the best results.

The Relative Volume of steam is the volume of steam compared with the volume of the same weight of water, and is given by :—

$$\text{Relative volume} = \frac{\text{Vol. of steam}}{\text{Vol. of water}}$$

$$= \frac{\text{Specific Vol.}}{\frac{1}{62 \cdot 5}}$$

$$= \text{Specific Volume} \times 62 \cdot 5.$$

The volume of one lb. of wet steam is found by multiplying the Specific Volume by the dryness fraction of the steam, the volume of water present, being very small, is neglected.

Superheated Steam.

When dry saturated steam is taken from a boiler and has additional heat applied to it, the steam is said to become *superheated*. Superheated steam may be regarded as a perfect gas, and as long as its temperature is above the temperature of saturated steam at the same pressure, it will follow the same laws as a perfect gas. Steam is always superheated at *constant pressure,* and its volume increases. The specific heat of superheated steam is generally taken as 0·48, but it varies both with the temperature and the pressure.

Example. Steam at 380°F. and 195 lb. per sq. inch absolute has a dryness fraction of 0·9, find the heat needed to generate this steam from feed at 200°F. Find also the additional heat to convert this wet steam into superheated steam of temperature 600°F. Calculate the volume of both the wet and the superheated steam.

Latent heat at 380°F. = 966 — 0·7 (380 — 212) = 848·4 B.T.U. per lb.

Total heat per lb. = 380 — 200 + (0·9 × 848·4) = 943·6 B.T.U. (wet steam).

$$\text{Specific Vol.} = \frac{+\ \dfrac{195}{4}}{195 + 1} = 2{\cdot}34 \text{ cu. feet per lb.}$$

Vol. of steam of dryness 0·9 = 2·34 × 0·9

= 2·106 cu. feet per lb.

As 0·1 lb. of water is present per lb. of steam, to convert this into steam will take 0·1 × 848·4 = 84·84 B.T.U. This makes the steam just dry.

Heat to Superheat Steam = 0·48 (600 — 380) = 105·6 B.T.U. per lb.

Additional heat = 84·84 + 105·6 = 190·44 B.T.U. per lb.

$$\text{Vol.} = 2{\cdot}34 \times \frac{600 + 460}{380 + 460} = 2{\cdot}95 \text{ cu. feet per lb.} \quad \text{Ans.}$$

Example. Find the total heat necessary to convert 12 lb. of ice at 22°F. into steam at 370°F.

To raise the ice to 32°F. takes 0·5 (32 — 22) = 5 B.T.U. per lb.

To convert the ice to water at 32°F. takes 143 B.T.U. per lb.

To raise the water to 370°F. takes (370 — 32) = 338 B.T.U. per lb.

To convert water at 370°F. into steam at 370°F., we must supply latent heat.

Latent heat = 966 — 0·7 (370 — 212) = 855·4 B.T.U. per lb.

Total heat = 5 + 143 + 338 + 855·4 = 1341·4 B.T.U. per lb.

Total heat for 12 lb. = 1341·4 × 12 = 16096·8 B.T.U. Ans.

Temperature of Mixtures.

When quantities of water at different temperatures are mixed together, the final temperature may be found by taking the sum of the sensible heats in each quantity and then dividing by the total weight. The sensible heat is here reckoned from 0°F. In mixtures of ice and water, the latent heat of water must be subtracted from the total sensible heat, because the ice when melting must receive latent heat. The total sensible heat is therefore reduced by the amount of latent heat given to the ice.

Example. 40 lb. of water at 80°F., and 100 lb. of water at 100°F. are mixed together. Find the final temperature.

$$\text{Final Temp.} = \frac{\text{Total heat in B.T.U. from } 0°\text{F.}}{\text{Total weight in lb.}}$$

$$= \frac{(40 \times 80) + (100 \times 100)}{100 + 40} = 94{\cdot}28°\text{F.} \quad \text{Ans.}$$

Example. 3 lb. of ice at 32°F. are put into 34 lb. of water at 58°F. Find the final temperature of the mixture.

$$\text{Final temperature} = \frac{\text{Total heat in B.T.U.}}{\text{Total weight}}$$

$$\frac{(3 \times 32) + (34 \times 58) - (3 \times 143)}{3 + 34} = 44{\cdot}3°\text{F. nearly.}$$ A

Example. 2 lb. of steam at 360°F. are blown into 12 gallons of water at 50°F. Find the final temperature.

Latent heat $= 966 - 0{\cdot}7\ (360 - 212) = 862{\cdot}4$ B.T.U. per lb.

Total heat in steam $= 360 + 862{\cdot}4 = 1222{\cdot}4$ B.T.U. per lb. from 0°F.

$$\text{Final Temp.} = \frac{\text{Total heat in B.T.U.}}{\text{Total weight}}$$

$$\frac{(12 \times 10) \times 50 + (2 \times 1222{\cdot}4)}{(12 \times 10) + 2} = 69{\cdot}2°\text{F.} \quad \text{Ans.}$$

Example. Steam from an evaporator at 230°F. is blown into the feed water, raising its temperature from 120°F. to 200°F. How many pounds of feed water are heated by one pound of steam ?

Let x lb. of feed water be heated per lb. of steam.

$$\text{Final Temp.} = \frac{\text{Total heat in B.T.U.}}{\text{Total weight}}$$

Latent heat $= 966 - 0{\cdot}7\ (230 - 212) = 953{\cdot}4$ B.T.U. per lb.

$$\text{Final Temp.} = 200° = \frac{1 \times (230 + 953{\cdot}4) + (x \times 120)}{1 + x}$$

$$200 + 200\,x = 1183{\cdot}4 + 120\,x$$

$$80\,x = 983{\cdot}4,\ x = 12{\cdot}29 \text{ lb.} \quad \text{Ans.}$$

The method already given may be used for mixtures of water and steam, or the following method may be used :—

Heat lost by steam $=$ heat gained by water.

This formula may be used to determine the dryness fraction of wet steam. A pipe carrying the wet steam is led into a vessel containing a known quantity of water at a known temperature. Steam is blown into the water, the temperature of which rises, and at a certain temperature the steam is shut off and the weight of water in the vessel carefully noted.

Let t_1 = original temp. of the water.

t_2 = temp. of the steam.

t_3 = final temp. of the water.

L = latent heat of the steam.

x = dryness fraction of the steam.

W = original weight of water.

w = weight of steam blown in.

Heat lost by steam = heat gained by water.

$$w\,[t_2 - t_3 + x\,\mathrm{L}] = \mathrm{W}\,[t_3 - t_1]$$

$$(t_2 - t_3) + x\,\mathrm{L} = \frac{\mathrm{W}}{w}\,(t_3 - t_1)$$

$$x\,\mathrm{L} = \frac{\mathrm{W}}{w}\,(t_3 - t_1) - (t_2 - t_3)$$

$$x = \frac{\frac{\mathrm{W}}{w}\,(t_3 - t_1) - (t_2 - t_3)}{\quad}$$

Example. Into 40 lb. of water at 50°F. are blown 2 lb. of wet steam at 312°F. The final temperature of the mixture is 101·7°F. Find the dryness fraction of the steam.

Latent heat at 312°F. = 966 — 0·7 (312 — 212) = 896 B.T.U. per lb.

$$x = \frac{\frac{40}{2}\,(101{\cdot}7 - 50) - (312 - 101{\cdot}7)}{896} = 0{\cdot}92 \text{ nearly.}$$

Ans.

In one lb. of this wet steam there is 0·92 lb. of steam and 0·08 lb. of water.

When a quantity of any metal of known weight and temperature is put into water of known weight and temperature, the final temperature is given by :—

Heat lost by metal = heat gained by water.

Example. 2 lb. of cast iron at 500°F., specific heat 0·13, are put into 10 lb. of water at 60°F. Find the final temperature.

Heat lost by metal = heat gained by water.

Then, if T = final temperature,

$$0{\cdot}13 \times 2\,(500 - T) = 10 \times 1\,(T - 60)$$

$$130 - 0{\cdot}26\,T = 10\,T - 600$$

$$10{\cdot}26\,T = 730,\quad T = 71{\cdot}15^\circ F. \quad \text{Ans.}$$

When several pieces of metal, each having different specific heats, are added to a quantity of water, it may be convenient to reduce them to their *Water Equivalents*. Taking the specific heat of copper as 0·09, then 1 lb. of copper has the same capacity for heat as 0·09 lb. of water; the water equivalent is equal to the weight of the substance multiplied by its specific heat. The water equivalent of 3 lb. of copper is $3 \times 0{\cdot}09 = 0{\cdot}27$ lb. of water. After reducing the various substances to their water equivalents, the problem is the same as that of a mixture of several quantities of water at different temperatures.

Example. 4 lb. of copper (S.H. 0·09) at 500°F., and 2 lb. of platinum (S.H. 0·03) at 900°F., are added to 10 lb. of water at 35°F. Find the final temperature of the mixture.

Water equivalent of copper $= 4 \times 0{\cdot}09 = 0{\cdot}36$ lb.

Water equivalent of platinum $= 2 \times 0{\cdot}03 = 0{\cdot}06$ lb.

$$\text{Final Temp.} = \frac{\text{Total B.T.U.}}{\text{Total weight}}$$

$$(0{\cdot}36 \times 500) + (0{\cdot}06 \times 900) \quad (10 \times 35)$$

$$0{\cdot}36 + 0{\cdot}06 + 10$$

$$\frac{584}{10{\cdot}42} = 56{\cdot}04^\circ F. \quad \text{Ans.}$$

Efficiency of a Boiler.

The efficiency of a boiler is the ratio of the heat it gives to the steam, compared with the heat supplied to it.

$$\text{Boiler Efficiency} = \frac{\text{Total heat given to steam per lb. of coal}}{\text{Total heat per lb. of coal}}$$

Boilers may work under different conditions of feed temperature; the dryness fraction of the steam may vary, and the pressure may vary. In comparing the evaporative efficiencies of different boilers, all these conditions must be taken into account.

Equivalent Evaporation.

In dealing with boiler performances it is usual to reduce them all to a common standard. Under atmospheric conditions, to convert 1 lb. of feed at 212°F. into steam at 212°F. takes 966 B.T.U. and the *equivalent evaporation from and at* 212°F. means the number of pounds of water which would be evaporated by the same number of heat units from feed at 212°F. to steam at 212°F.

Equivalent evaporation from and at 212°F.

$$= \frac{\text{Total heat given to steam per lb. of coal}}{966}$$

Example. A boiler generates 9·5 lb. of dry steam at 370°F. from feed at 150°F., per lb. of coal. The calorific value of the coal is 14,000 B.T.U. per lb. Another boiler generates 9 lb. of steam at 360°F., dryness 0·95, from feed at 160°F. Find the efficiency of each boiler, and the lb. evaporated from and at 212°F.

Latent heat at 370°F. $= 966 - 0{\cdot}7\ (370 - 212) = 855{\cdot}4$ B.T.U. per lb.

Latent heat at 360°F. $= 966 - 0{\cdot}7\ (360 - 212) \doteq 862{\cdot}4$ B.T.U. per lb.

Boiler efficiency, first boiler $\doteq$

$$\frac{(370 - 150 + 855{\cdot}4)\ 9{\cdot}5}{14000} \quad \frac{1075{\cdot}4 \times 9{\cdot}5}{14000} = 0{\cdot}729.$$ Ans.

$$\text{Pounds from and at } 212° = \frac{1075{\cdot}4 \times 9{\cdot}5}{966} = 10{\cdot}57 \text{ lb.}$$ Ans.

Boiler efficiency, second boiler =

$$\frac{(360 - 160 + 862{\cdot}4 \times 0{\cdot}95)\ 9}{14000} \quad \frac{1019 \times 9}{14000} = 0{\cdot}655$$

$$\text{Pounds from and at } 212° = \frac{1019 \times 9}{966} = 9{\cdot}493 \text{ lb.}$$ Ans.

TEST EXAMPLES XXI.

1. If 100 lb. of water at 70°F., 1 cwt. of water at 80°F., and 15 lb. of ice at 20°F. are mixed together, find the final temperature. 62·57°F. Ans.

2. Find the total heat necessary to convert 20 lb. of ice at 30°F. into steam at 320°F. 26,448 B.T.U. Ans.

3. Five lb. of cast iron (Sp.H. 0·13) at 500°F., 3 lb. of copper (Sp.H. 0·095) at 800°F., are put into 20 lb. of water at 41°F. Find the resultant temperature. 65·58°F. Ans.

4. A surface heater is supplied with steam at 365°F., and the drain water from the tubes is 200°F. The hotwell is 125°F., and one lb. of feed requires 0·08 lb. of heating steam. Find the temperature of the feed water.
206·9°F. Ans.

5. If 20 lb. of steam at a temperature of 390°F. and a pressure of 205 lb. per square inch are blown into a tank containing 2,000 lb. of water at 100°F., find the final temperature.
111·1°F. Ans.

6. Find the total heat needed to convert 10 kilograms of ice at − 5°C. into dry saturated steam at 195°C.
29585·6 B.T.U. Ans.

7. Find the total heat given per lb. to generate steam of dryness fraction 0·9 and temperature 380°F. from feed at 160°F. What additional heat is needed to superheat the steam to 580°F.?
983·56 B.T.U. per lb. ; 180·84 B.T.U. per lb. Ans.

8. Half a pound of wet steam at 300°F. is blown into 10 lb. of water at 50°F., and the final temperature is 101°F. Find the dryness fraction of the steam. 0·907. Ans.

9. One gallon of water at 65°F. has steam at atmospheric pressure blown into it until the temperature is 185°F., and the quantity has increased to 9 pints of water. Find the dryness fraction of the steam. 0·965. Ans.

10. A boiler gives 8·9 lb. of dry steam at 325°F. from feed at 100°F. per lb. of coal of calorific value 13,000 B.T.U. per lb. Find the boiler efficiency and the equivalent evaporation from and at 212°F.
0·761 ; 10·24 lb. Ans.

11. The total heat of 1 lb. of steam at a certain temperature is 1197·5 B.T.U., reckoned above 32°F. The feed temperature is 130°F. If 15 lb. of this steam are used per horse power hour, find the theoretical efficiency of the engine.

15·49 per cent. Ans.

12. The boiler steam has a temperature of 373°F., and 4 per cent. of the total steam is taken direct to the feed heater and the remainder goes to the engines. The heating steam mixes directly with the hotwell water which is at 120°F., find the temperature of the feed.

164·25°F. Ans.

13. The discharge water is at 98°F. when the sea is at 60°F. The sea water now rises to 70°F. Find the temperature of the discharge if the quantity of the injection water is increased by 15 per cent.

103·04°F. Ans.

14. An engine of 4,000 I.H.P. uses 16 lb. of steam per horse power hour. The temperature of the exhaust steam is 120°F. The sea water is at 60°F., and the discharge is 101°F. Find the tons of circulating water used per minute, assuming that the temperature of the condensed water is the same as that of the discharge.

12·13 tons. Ans.

15. The total heat supplied to one lb. of feed water at 160°F. to convert it into steam 0·9 dry is 961 B.T.U. Find the temperature of the steam.

319°F. Ans.

16. The steam is at 380°F. and part of the steam is taken direct to the contact heater. The feed water is raised from 120°F. to 200°F., what percentage of the total steam goes to the engines ?

92·79 per cent. Ans.

17. The capacity of a refrigerating machine is calculated from the weight of ice it can make in 24 hours from water at 32°F. What weight of ice would be made in a two ton machine from water at 57°F. into ice at 24°F. in a watch of 4 hours ?

620·7 lb. Ans.

18. Steam at 379·2°F. is supplied to the coils of a feed heater, and the drain water from the coils is at 208°F. The feed water is raised from 126°F. to 178°F. How many lb. of feed water are heated per lb. of steam ?

19·6 lb. Ans.

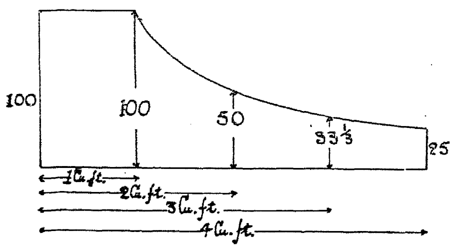
CHAPTER XXII.

THE EXPANSION OF STEAM.

The potential energy in steam may be converted into work by allowing the steam to expand behind a piston ; the pressure and the temperature of the steam diminish during expansion, the volume increases. The range of expansion in any one cylinder is limited mostly by the difference between the temperature of the steam entering the cylinder and the temperature of the exhaust steam. In reciprocating engines of ordinary design, steam is admitted and exhausted by the same port. During the exhaust stroke the metal of the cylinder, including the clearance space and the ports, becomes cooled to the exhaust temperature. During admission on the next stroke, the entering steam, being hotter than the metal of the clearance spaces is cooled somewhat, and part of it is condensed. Steam is not a perfect gas, and its condition after admission to a cylinder will depend upon the design of the cylinder, and the arrangements for lagging, jacketing and draining away of the water of condensation. Every cylinder will have its own conditions of working, and the law by which the steam expands will depend partly upon these conditions. It is usual to assume that steam expands according to the simple law $p\ v$ = constant, and to correct this by a factor suggested by experience. This factor is called the *diagram factor*.

The Hyperbolic Curve.

The equation $p\ v$ = constant, may be represented by a curve.



Let one cubic foot of steam at 100 lb. per square inch absolute xpand to 4 cubic feet. Calculate the pressures when the volumes re 2, 3 and 4 cubic feet respectively.

$p_1 \; v_1 = p_2 \; v_2$
$100 \times 1 = p_2 \times 2, \qquad p_2 = 50$ lb. per sq. inch absolute.

$p_1 \; v_1 = p_3 \; v_3.$
$100 \times 1 = p_3 \times 3, \qquad p_3 = 33\frac{1}{3}$ lb. per sq. inch absolute.

$p_1 \; v_1 = p_4 \; v_4.$
$100 \times 1 = p_4 \times 4, \qquad p_4 = 25$ lb. per sq. inch absolute.

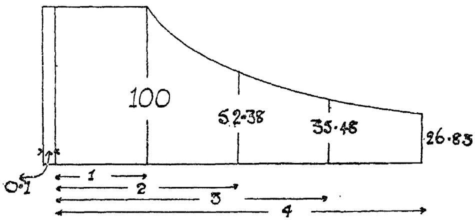
The pressures may be set up to a convenient scale at 1, 2, 3 and 4 cubic feet from the line of zero pressure and a curve drawn through the points.

$$\text{The Ratio of Expansion} = \frac{\text{Final volume}}{\text{Initial volume}}$$

$$= \frac{4}{1} = 4.$$

Taking Clearance into Account.

Let an engine have a stroke of 4 feet, the area of the cylinder being one square foot. Let the clearance volume be $\frac{1}{40}$ of the volume swept out by the piston. As before, let the initial pressure be 100 lb. per square inch absolute. Calculate the pressures at half stroke, three quarters stroke, and at the end of the stroke, when the cut off takes place at quarter stroke.



$p \; v = $ constant.

Now at cut off, the volume of steam present is 1 cubic foot plus 0·1 cubic foot, because the clearance space has to be filled.

Therefore $100 \times 1{\cdot}1 = p_2 \times 2{\cdot}1, \; p_2 = 52{\cdot}38$ lb. per sq. inch absolute.

and $100 \times 1{\cdot}1 = p_3 \times 3{\cdot}1, \; p_3 = 35{\cdot}48$ lb. per sq. inch absolute.

and $100 \times 1{\cdot}1 = p_4 \times 4{\cdot}1, \; p_4 = 26{\cdot}83$ lb. per sq. inch absolute.

The pressures are higher for the same positions of the piston than in the last example, this is due to the clearance.

$$\text{Ratio of expansion} = \frac{\text{Final volume}}{\text{Initial volume}} \quad \frac{4\cdot1}{1\cdot1} = 3\cdot727$$

We may state the relation between the pressure and volume as follows :—

$$p_1 \times (v_1 + c) = p_2 \times (v_2 + c)$$

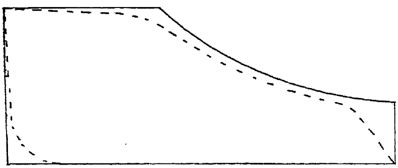
$p_2 = p_1 \times \left(\frac{v_1 + c}{v_2 + c}\right)$, where v_1 and v_2 are volumes swept out by the piston.

The pressures given here are absolute.

The effective pressure, however, would not be lb. per sq. inch *absolute*. It is the pressure per sq. inch on the piston which causes the piston to move in the cylinder ; and the difference between the *gauge* pressure at any of the given points and the *gauge* back pressure would give the same effective pressure.

The Diagram Factor.

In practice, the actual curve drawn by the indicator does not quite correspond to the curve $p\ v$ = constant. By reason of the cylinder conditions and the imperfections of the valve motion, the perfect or theoretical diagram is considerably modified. The area, which represents work done, is less than the area under the theoretical curve. As the mean effective pressure is



$\frac{\text{Area of card}}{\text{Stroke of piston}}$, the mean effective pressure will be less than that for the theoretical case. The mean effective pressure is found for the case of perfect hyperbolic expansion and the result is multiplied by a factor less than unity. This diagram factor varies from about 0·57 in small engines to about 0·72 in large and good engines. Values for marine engines are generally given between 0·65 and 0·7.

The mean effective pressure is then given by :—

$$p_m = \left[\frac{p_1}{r}(1 + \log_e r) - \text{back pressure}\right] \times \text{a diagram factor,}$$

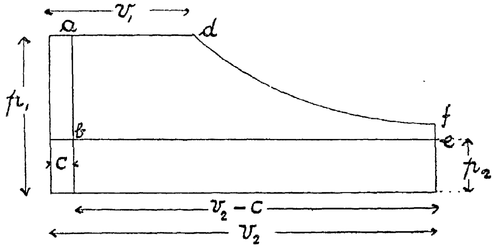
where p_m is the mean effective pressure, r the ratio of expansion and p_1 is the absolute initial pressure.

The back pressure must also be absolute. This formula assumes that there is no clearance.

If a diagram factor is not given in the question, it is assumed that the theoretical mean pressure is required.

When clearance is given the following method should be used.

Area of whole diagram $= p_1\, v_1\, (1 + \log_e\, r)$.



The effective work is the area $a\ b\ e\ f\ d$, this being the area of the theoretically perfect card.

Area $a\ b\ e\ f\ d = p_1\, v_1\, (1 + \log_e\, r) - p_1\, c - p_2\, (v_2 - c)$

$$p_m = \frac{\text{Area } a\,b\,e\,f\,d}{(v_2 - c)} = \frac{p_1\, v_1\, (1 + \log_e\, r) - p_1\, c}{v_2 - c} - \frac{p_2\ (v_2 - c)}{v_2 - c}$$

$$p_m = \frac{p_1\, v_1 (1 + \log_e\, r) - p_1\, c}{v_2 - c} - p_2$$

where p_2 is the back pressure.

If a diagram factor is given, then p_m found by this formula must be multiplied by the factor given.

Example. Steam at 185 lb. per square inch gauge is admitted to a cylinder and cut off at 0·35 of the stroke. The back pressure is 35 lb. per square inch gauge. Find the mean effective pressure, neglecting clearance.

$$r = \frac{\text{Final volume}}{\text{Volume at cut off}} = \frac{1}{0{\cdot}35} = 2{\cdot}857$$

$p_1 = 185 + 15 = 200$ lb. per sq. inch absolute.
Back pressure $= 35 + 15 = 50$ lb. per sq. inch absolute.

$$p_m = \frac{p_1}{r}[1 + \log_e r] - \text{back pressure}$$

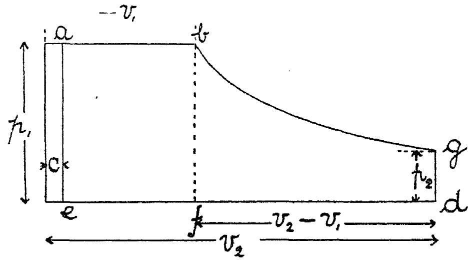
$$p_m = \frac{200}{2{\cdot}857}[1 + \log 2{\cdot}857 \times 2{\cdot}3] - 50$$

$$= 70\,[1 + 0{\cdot}4559 \times 2{\cdot}3] - 50$$

$$= 70\,[1 + 1{\cdot}048] - 50 = 70 \times 2{\cdot}048 - 50$$

$= 93{\cdot}36$ lb. per sq. inch. Ans.

Example. Steam is admitted to a cylinder at 205 lb. per square inch gauge. The cut off is at 0·3 of the stroke, and the clearance volume is 5 per cent. of the volume swept out by the piston. Find (*a*) the mean gross pressure during expansion; (*b*) the mean gross pressure during the stroke; (*c*) the mean effective pressure if the back pressure is 40 lb. per square inch gauge.



Work done during expansion, neglecting back pressure $= p_1 v_1 \log_e r$, represented by the area $b\ f\ d\ g$.

$$\text{Mean gross pressure during expansion} = \frac{p_1 v_1 \log_e r}{v_2 - v_1}$$

$p_1 = 205 + 15 = 220$ lb. per square inch absolute.
$v_1 = 0{\cdot}3 + 0{\cdot}05 = 0{\cdot}35$; $v_2 = 1 + 0{\cdot}05 = 1{\cdot}05$

$$r = \frac{1{\cdot}05}{0{\cdot}35}$$

$$P_m \text{ gross, during expansion} = \frac{220 \times 0{\cdot}35 \times \log_e \dfrac{1{\cdot}05}{0{\cdot}35}}{1{\cdot}05 - 0{\cdot}35}$$

$$\frac{220 \times 0{\cdot}35 \times 1{\cdot}097}{0{\cdot}7} = 120{\cdot}7 \text{ lb. per sq. inch.}$$

The mean gross pressure during the whole stroke is given by the area $a\ b\ g\ d\ e$ divided by $(v_2 - c)$

Area $a\ b\ f\ e = p_1\ (v_1 - c)$

Area $b\ f\ d\ g = 120{\cdot}7 \times (v_2 - v_1)$

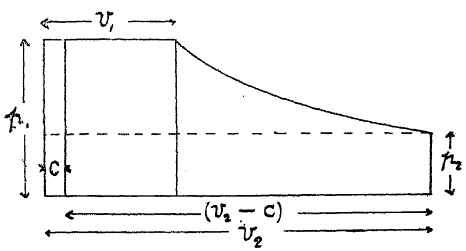
Mean gross pressure during stroke

$$\frac{p_1\ (v_1 - c) + 120{\cdot}7\ (v_2 - v_1)}{}$$

$$= \frac{220 \times 0{\cdot}3 + 120{\cdot}7 \times 0{\cdot}7}{1} = 150{\cdot}5 \text{ lb. per sq. inch.}$$ Ans.

Mean effective pressure $= 150{\cdot}5 - (40 + 15)$
$= 95{\cdot}5$ lb. per sq. inch. Ans.

Example. The M.P. cylinder of an engine is 44 inches diameter, the stroke being 50 inches. The clearance volume at each end of the cylinder is 10 per cent. of that swept out by the piston. The steam pressure up to the point of cut off is 58 lb. per square inch gauge, and the back pressure is 10 lb. per square inch gauge. The cut off is at $25\frac{1}{4}$ inches of the stroke, and the engine turns 70 revolutions per minute. Find the mean effective pressure, and the indicated horse power taking a diagram factor of 0·7.



$p_1 = 58 + 15 = 73$ lb. per sq. inch absolute.
$p_2 = 10 + 15 = 25$ lb. per sq. inch absolute.
$c = \frac{10}{100} \times 50 = 5$ inches.
$v_1 = 25{\cdot}25 + 5 = 30{\cdot}25$ inches.
$v_2 = 50 + 5 = 55$ inches.

$$r = \frac{55}{30{\cdot}25}$$

$$p_m = \frac{73 \times 30{\cdot}25 \left(1 + \log_e \dfrac{55}{30{\cdot}25}\right) - (73 \times 5)}{55 - 5} - 25$$

$$p_m = \frac{3526 - 365}{50} - 25$$

$p_m = 63{\cdot}22 - 25 = 38{\cdot}22$ lb. per sq. inch, theoretically.
Actual $p_m = 38{\cdot}22 \times 0{\cdot}7 = 26{\cdot}754$ lb. per sq. inch. Ans.

$$\text{I.H.P.} = \frac{\text{Area of piston} \times \text{revs.} \times 2 \times \text{stroke} \times p_m}{33000}$$

$$= \frac{44 \times 44 \times 11 \times 50 \times 70 \times 2 \times 26{\cdot}75}{14 \times 33000 \times 12}$$

$= 719{\cdot}4.$ Ans.

Note as the stroke is 50 inches, we divide by 12 to bring it to feet, as one horse power is 33000 *ft. lb.* of work per minute.

Indicated Horse Power.

The work done per stroke is the mean force on the piston multiplied by the length of stroke in feet. The work per minute is the work per stroke multiplied by the number of strokes per minute.

Work per stroke = mean effective pressure × area of piston × stroke
$= p_m \times A \times L$

Where p_m = mean effective pressure. Sq inch
A = area of piston in sq. inches.
L = length of stroke in feet.
N = number of strokes per minute.

Work per min. $= p_m \times A \times L \times N$

$$\text{I.H.P.} = \frac{p_m \text{ A.L.N.}}{33000}$$

Example. The diameter of a steam cylinder is 25 inches, and the stroke is $3\frac{1}{2}$ feet. The mean effective pressure is 80 lb. per square inch, and the revolutions are 60 per minute. Find the indicated horse power.

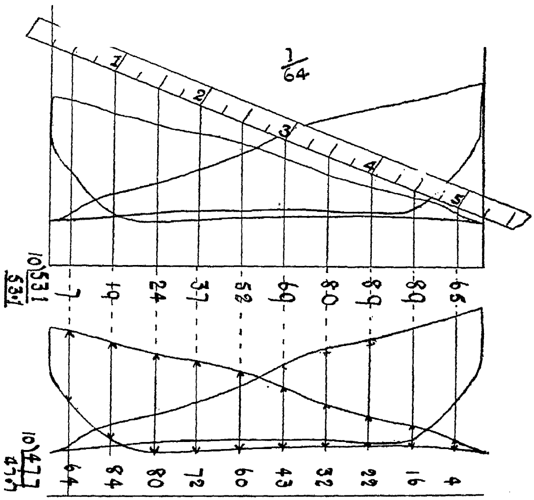
$$\text{I.H.P.} = \frac{p_m \text{ A L N}}{33000}$$

$$\text{I.H.P.} = \frac{80 \times 25 \times 25 \times 11 \times 3{\cdot}5 \times 2 \times 60}{33000 \times 14} = 500. \quad \text{Ans.}$$

The student should notice that in the equation

$\text{I.H.P.} = \dfrac{p_m \text{ A L N}}{33000}$, any one term may be the unknown. Thus p_m, A, L, and I.H.P may be given to find N ; or N, A, L, and I H.P. may be given to find p_m

Mean Effective Pressures from Actual Cards.



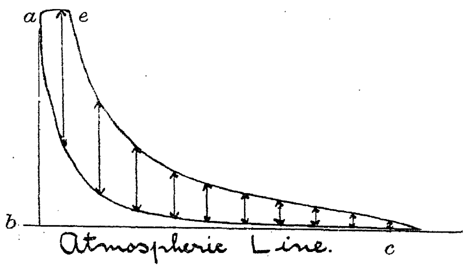
$$p_m = \frac{53{\cdot}1 + 47{\cdot}7}{2} = 50{\cdot}4 \text{ lbs sq. in.}$$

Erect lines at the ends of the diagrams perpendicular to the atmospheric line. Divide the length between these perpendiculars into 10 equal parts, then draw vertical lines midway between the 10 points. A convenient method of doing this is illustrated. Place an ordinary ruler or straight-edge as shown, mark points on the paper at every $\frac{1}{2}$ inch. Note that the distance on the inclined ruler between the perpendiculars is 5 inches, beginning at $\frac{1}{4}$ inch, and ending at $5\frac{1}{4}$ inches. Dropping vertical lines from the $\frac{1}{2}$ inch, 1 inch, $1\frac{1}{2}$ inch, etc., marks gives the mid ordinates of 10 equal spaces.

Now measure to the correct scale of pressures the lengths of the ordinates, from the steam line of the card to the exhaust line, and put down the values as shown. Add up the values and divide by 10. This is done for each card separately, and the mean of both cards taken finally. If a pressure scale rule is not available, the ordinates may be measured in inches and then multiplied by the scale of the spring. In the example given, this is 64.

Cards from Oil Engines.

Mark off and measure the card exactly as shown for the steam card, take the mean of the 10 ordinates as mean effective pressure. To find the indicated horse power use the same formula I.H.P. $= \dfrac{p_m \text{ A L N}}{33{,}000}$, the symbols p_m,

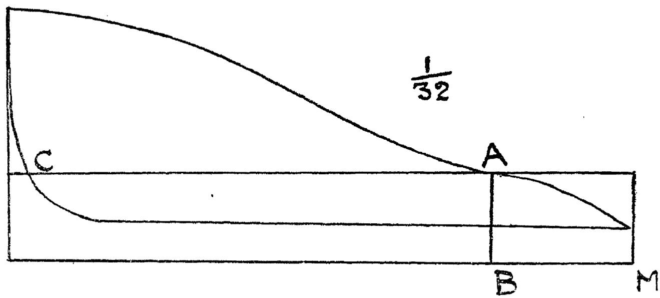


A, and L, have the same meaning as given before; N is now the number of cycles per minute, which is the same thing as the number of working strokes per minute. For two stroke cycle engines N is the number of revolutions, as there is one complete cycle per revolution. For four stroke cycle engines N is the number of revolutions per minute divided by 2, as there is one complete cycle every 2 revolutions. Note that

the work done during combustion and expansion is given by the area *a e c b*, and the work lost during compression is given by the area *a b c*, the difference between these areas is the actual work done in one cycle.

Weight of Steam from the Indicator Diagram.

On the card shown let L M be the atmospheric line. Choose a point A on the expansion curve, as near as possible to the point of release, but before release occurs on the card. Draw



through A the horizontal line D E, and draw the vertical line A B. Mark the point C on the compression curve. Measure A C and E D carefully. Then the volume of steam used per stroke is $\frac{A\ C}{E\ D}$ × stroke volume at a gauge pressure corresponding to A B on the scale of pressure. Measure A B carefully and find the gauge pressure at this point of the stroke, add 15 to this to get the gross pressure. Apply the formula given by the Board of Trade for the weight of one cubic foot of steam :—

$$\text{Wt. of one cubic foot of steam} = \frac{p + 1}{410 + \frac{p}{4}} \text{ lb.}$$

where p = gross pressure.

Suppose that in this example the cylinder is 30 inches diameter, and that A C measures 3·5 inches and that E D measures 5 inches. Let A B measure 1 inch, which is equivalent to 32 lb. per square inch. Let the stroke be 4 feet. Then, vol. of steam

$$\text{per stroke} = \frac{4 \times 30 \times 30}{144} \times \frac{11}{14} \times \frac{3 \cdot 5}{5} = 13 \cdot 75 \text{ cu. ft.}$$

Weight of one cubic foot at $(32 + 15)$ lb. per square inch gross $= \dfrac{47}{410} + 1$ lb.

Weight of one cubic foot $= 48 \qquad = 0{\cdot}1138$ lb.

Weight per stroke $= 0{\cdot}1138 \times 13{\cdot}75 = 1{\cdot}565$ lb.

Now take the card from the other side of the piston, and find the weight of steam used for that stroke. Adding the two results gives the weight of steam used per revolution. The steam used per hour may now be found by multiplying the steam per revolution by the revolutions per minute, and then by 60. The steam used per hour divided by the horse power gives the steam used per I.H.P. hour. The weight of steam found per I.H.P. hour from any one of the cylinders of a triple or quadruple expansion engine is the weight per I.H.P. hour for the whole engine, as the weight of steam in each cylinder is the same, neglecting losses by leaks, condensation, etc.

The most correct result for the weight of steam used per I.H.P. per hour by a multi-cylinder engine would be obtained from the diagram taken from the engine where the steam is dryest. This will be the diagram from the H.P. engine.

It is important to notice that this gives only the theoretical weight of steam, on the assumption that the steam in the cylinder is dry at the pressure and volume measured; actually, the weight of steam shown by the card should be divided by the dryness fraction to give the total weight of wet steam present in the cylinder.

Another method of determining the indicated weight of steam is given on the following pages.

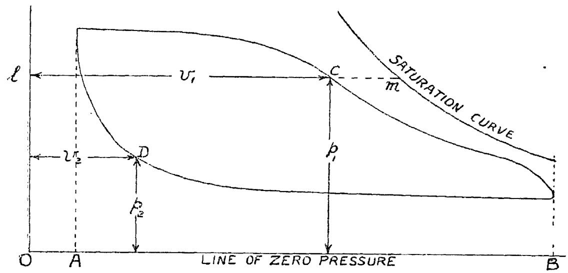
Saturation Curve, or curve of constant steam weight.

By the aid of saturated steam tables, a curve may be plotted whose co-ordinates are absolute pressure and specific volume, or volume per 1 lb. of steam. This curve is a pressure-volume curve, or a curve of constant steam weight, and its equation is $p\ v^{1\cdot0646} =$ constant.

***Missing Quantity.**

Let the figure on page 365 represent the indicator diagram from an engine whose stroke volume is known. The length A B represents the stroke volume to scale, and let O A represent the clearance volume to the same scale.

Let observations be taken, whilst the engine is running, of the weight of water discharged by the air pump over a period of say one hour. Dividing this weight by the number of strokes made by the engine gives the *cylinder feed* per stroke, assuming no leakage takes place past the piston or slide valve. Let the cylinder feed be W lb. per stroke.



Choose some point C on the expansion curve soon after cut off has taken place, measure the pressure p_1 to the scale of the diagram, and determine the volume represented by v_1. This is

$$\frac{l\,\mathrm{C}}{\mathrm{A\,B}} \times \text{stroke volume.}$$

From steam tables obtain the specific volume at p_1. Let this be V. Then $\dfrac{v_1}{\mathrm{V}} = w_1$ lb. is the weight of steam present at C, or the *indicated steam.*

Choose a point D after the valve has closed to exhaust. Measure p_2 and determine the volume represented by v_2. Let the specific volume at p_2 be S. Then $\dfrac{v_2}{\mathrm{S}} = w_2$ lb. is the weight of steam present at D. This is the *cushion steam.*

$w_1 - w_2$ is the weight of steam used per stroke, as measured from the indicator diagram.

At point C there should be present in the cylinder the cushion steam w_2 + the cylinder feed W, whilst the indicated weight is w_1.

Then $(w_2 + W) - w_1$ is the *missing quantity.* This is principally due to condensation.

Draw a saturation curve for $(w_2 + W)$ lb. of steam. Then C m represents the volume of this missing quantity at the pressure p_1.

Also $\dfrac{l\,C}{l\,m}$ is the dryness fraction at C.

The dryness fraction is also

W)

or $$\frac{\text{indicated weight}}{\text{cushion steam} + \text{cylinder feed}}$$

It may be observed that due to re-evaporation during the latter part of the expansion period, the missing quantity near the point of release is less than that immediately after cut off.

Referred Mean Pressure.

In compound, triple and quadruple expansion engines, the expansion of the steam is divided up into stages. This reduces the temperature range in each cylinder, and diminishes initial condensation. The low pressure cylinder is capable of developing alone the whole of the power, and must be large enough to accommodate the volume of the steam after expansion is complete. The referred mean pressure is that pressure which acting on the low pressure piston is capable of developing the total power. If we have a compound engine expanding the steam eight times, then the same power could be obtained by admitting high pressure steam to the low pressure cylinder alone, and cutting off at one-eighth of the stroke. In the case of triple expansion engines, as each engine indicates approximately one-third of the total power, then the actual mean effective pressure in the low pressure cylinder will be one-third of the referred mean pressure. In the other two cylinders, the mean effective pressure will be greater than that in the low pressure, in inverse ratio to their volumes.

Example. A triple expansion engine has cylinders 20 inches, 36 inches and 56 inches diameter. The mean effective pressures are 65 lb. per square inch in the H.P., 25 lb. per square inch

in the M.P., and 10 lb. per square inch in the L.P. cylinder. Find the mean pressure all referred to the low pressure cylinder.

$$\frac{\text{L.P. Volume}}{\text{H.P. Volume}} = \frac{56^2}{20^2} = 7{\cdot}84$$

$$\frac{\text{L.P. Volume}}{\text{M.P. Volume}} = \frac{56^2}{36^2} = 2{\cdot}42$$

Mean Pressure in H.P. referred to L.P. $= \dfrac{65}{7{\cdot}84} = 8{\cdot}29$ lb. sq. inch.

Mean Pressure in M.P. referred to L.P. $= \dfrac{25}{2{\cdot}42} = 10{\cdot}3$ lb. sq. inch.

Mean Pressure in L.P. $= 10$ lb. per sq. inch.

Mean Pressure referred all to the L.P. $= 8{\cdot}29 + 10{\cdot}3 + 10$ $= 28{\cdot}59$ lb. per sq. inch. Ans.

Example. The cylinder areas in a quadruple expansion engine are 154, 320, 665 and 1,386 square inches. The mean pressure referred to the L.P. cylinder is 36 lb. per square inch. Find the mean effective pressure in each cylinder to develop equal powers.

Mean effective pressure in L.P. $= \frac{36}{4} = 9$ lb. per sq. inch. Ans.

Mean effective pressure 2nd M.P. $= 9 \times \frac{1386}{665} = 18{\cdot}76$ lb. sq. inch. Ans.

Mean effective pressure 1st M.P. $= 9 \times \frac{1386}{320} = 38{\cdot}98$ lb. sq. inch. Ans.

Mean effective pressure H.P. $= 9 \times \frac{1386}{154} = 81$ lb. sq. inch. Ans.

Example. A triple expansion engine has cylinders 27, 44 and 73 inches diameter, the stroke being 4 feet. The initial pressure is 180 lb. per square inch gauge, the back pressure being 4 lb. per square inch absolute, and the diagram factor is 0·7. Find the horse power developed at 80 revolutions per minute. The cut off in the H.P. cylinder is at 0·6 of the stroke.

$$\frac{\text{Final Volume}}{\text{Initial Volume}} = \frac{\text{Volume of L.P.}}{\text{Volume of H.P. up to cut off}}$$

$$r = \frac{(\text{Diam. of L.P.})^2}{(\text{Diam. of H.P.})^2 \times \text{cut off in H.P.}}$$

$$r = \frac{73^2}{27^2 \times 0{\cdot}6} = 12{\cdot}2 \text{ very nearly.}$$

$$p_m = \left[\frac{p_1}{r}(1 + \log_e r) - p_2\right] \times \text{diagram factor}$$

$$p_m = \left[\frac{195}{12{\cdot}2}(1 + \log.\ 12{\cdot}2 \times 2{\cdot}3) - 4\right] \times 0{\cdot}7$$

$$p_m = [15{\cdot}98\ (1 + 2{\cdot}498) - 4] \times 0{\cdot}7 = 36{\cdot}33 \text{ lb. per sq. inch.}$$

Now this pressure if acting on the L.P. piston will give the total power.

$$\text{I.H.P.} = 73 \times 73 \times \tfrac{11}{14} \times \frac{80 \times 2 \times 4 \times 36{\cdot}33}{33{,}000}$$

$$= 2955. \quad \text{Ans.}$$

Note that the diameter of the M.P. is not used to determine the total power.

***Determination of Cylinder Diameters.**

For a proposed engine, the I.H.P., the piston speed, the initial and terminal pressures, the number of expansions and the diagram factor are generally known. The referred mean pressure is first determined. Then the low pressure cylinder diameter may be found, and from the equation $r = \dfrac{\text{Final Volume}}{\text{Initial Volume}}$ the diameter of the H.P. cylinder may be determined. In the case of triple expansion engines the relation between the cylinder diameters may be taken as $\dfrac{d}{x} = \dfrac{x}{\text{D}}$, where x is the diameter of the intermediate cylinder or $x = \sqrt{\text{D} \times d}$.

For the cylinders of quadruple expansion engines the relation between the cylinder diameters may be taken as

$$\frac{d}{x} = \frac{x}{y} = \frac{y}{\text{D}},$$

where x and y are the diameters of the first and second intermediate cylinders respectively. When d and D, the H.P. and L.P. diameters are determined, x and y may be found. We have :—

$$\frac{d}{x} = \frac{x}{y} = \frac{y}{\mathrm{D}}$$

from the first and last terms, $y = \dfrac{\mathrm{D}\ d}{x}$

from the first and second terms, $x^2 = d\ y$

$$\therefore\ x^2 = d \times \frac{\mathrm{D}\ d}{x}, \text{ or } x^3 = \mathrm{D}\ d^2$$

This determines x, and finally y may be found. There is, however, considerable difference of opinion as to the best diameters of cylinders to place between the H.P. and L.P cylinders, and most engine builders have their own rules for these diameters.

Example. Determine the cylinder diameters for a triple expansion engine to develop 2,500 I.H.P. at 65 revolutions per minute. The initial pressure is 180 lb. per square inch gauge, the back pressure 4 lb. per square inch absolute. The stroke is to be 4 feet and the cut off in the H.P. cylinder is at 0·6 of the stroke. The steam is expanded 12 times, and the diagram factor may be taken as 0·68.

$$p_m = [\tfrac{195}{12}(1 + \log_e 12) - 4] \times 0{\cdot}68 = 35{\cdot}76 \text{ lb. per sq. inch.}$$

$$\text{I.H.P.} = \frac{p_m \mathrm{L\,A\,N}}{33{,}000}$$

$$2{,}500 \times 33{,}000 = 35{\cdot}76 \times 4 \times \mathrm{D}^2 \times \tfrac{11}{14} \times 65 \times 2$$

$$\mathrm{D}^2 = \frac{33{,}000 \times 2{,}500 \times 14}{35{\cdot}76 \times 88 \times 65} = 5{,}645.$$

$$\mathrm{D} = \sqrt{5645} = 75{\cdot}13, \text{ say 75 inches diameter.}$$

$$\text{Also } r = \frac{\mathrm{D}^2}{d^2 \times \text{cut off in H.P.}} \qquad \frac{\text{Final Volume}}{\text{Initial Volume}}$$

$$12 = \frac{5645}{d^2 \times 0{\cdot}6} \qquad d = \sqrt{\frac{5645}{12 \times 0{\cdot}6}}$$

$d = 27{\cdot}98$, say 28 inches diameter.

M.P. diameter $= \sqrt{27{\cdot}98 \times 75{\cdot}13} = 45{\cdot}83$, say 46 inches diameter.

Cylinders are 28, 46, 75 inches diameter. Ans.

Example. A quadruple expansion engine is to develop 5,000 I.H P at 80 revolutions per minute, the stroke being 5 feet. The initial pressure is 215 lb. per square inch gauge, the back pressure being 4 lb. per square inch absolute. The steam is expanded 14 times, the cut off in the H.P. cylinder is at 0·72 of the stroke, and the diagram factor is 0·7. Determine the diameters of the cylinders.

$p_m = [\frac{230}{14}(1 + \log_e 14) - 4] \times 0{\cdot}7 = 38{\cdot}98$ lb. per square inch.

$$5{,}000 \times 33{,}000 = D^2 \times \tfrac{11}{14} \times 38{\cdot}98 \times 5 \times 2 \times 80$$

$$D^2 = \frac{5{,}000 \times 33{,}000 \times 14}{110 \times 38{\cdot}98 \times 80} = 6{,}735$$

$D = \sqrt{6735} = 82{\cdot}06$, say 82 inches diameter.

$$D^2 \qquad = \sqrt{\frac{6735}{14 \times 0{\cdot}72}}$$

$d = 25{\cdot}84$, say 26 inches diameter.

1st M.P. diameter $= \sqrt[3]{D \times d^2} =$
$= 38{\cdot}09$, say 38 inches.

$$\text{2nd M.P. diameter} = \frac{D \times d}{\text{1st M.P.}}$$

$$\frac{82{\cdot}06 \times 25{\cdot}84}{38{\cdot}09} = 55{\cdot}63, \text{ say } 56 \text{ inches.}$$

Diameters 26, 38, 56 and 82 inches. Ans.

The power transmitted to the shaft is less than the indicated horse power, some of the power being lost in friction at the bearings, etc.

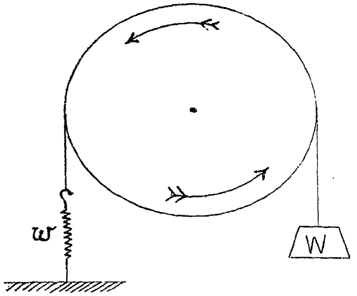
The Brake Horse Power is the actual horse power given out by the shaft. The relation between I.H.P. and B.H.P. depends upon the mechanical efficiency of the engine.

$$\text{Mechanical efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}}$$

The mechanical efficiency of steam engines is from 88 to 92 per cent., and from about 80 to 85 per cent. for internal combustion engines.

The brake horse power may be stated as :—

Brake Horse Power = I.H.P. — horse power lost in friction.



To obtain the B.H.P. of a small internal combustion engine, a rope brake may be used. One end of the rope is fastened to a spring balance fixed on the floor, and a weight W hangs from the other end. The direction of rotation is shown on the sketch. Let the load indicated on the spring balance be w lb. Then the pull on the rope is $(W - w)$, and the work done per revolution is equal to :—

Pull in rope $\times$ circumference of pulley.

$$\text{B H P} = \frac{- w) \times \text{circum.} \qquad \text{revs. per min.}}{33{,}000}$$

$$\text{B.H.P.} = \frac{\cdot\, w) \times \text{speed of rim in feet per min.}}{33{,}000}$$

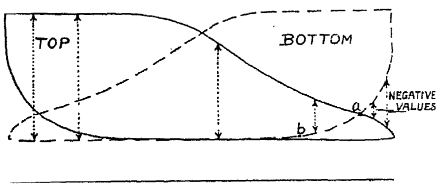
Power Transmitted by Belts.

If T is the tension in lb. in the tight side of the belt and S the tension in lb. in the slack side, then :—

$$\text{Horse power} = \frac{(T - S) \times \text{speed of belt in feet per min.}}{33{,}000}$$

Effective Pressure from Indicator Diagrams.

Referring to the sketch of a pair of indicator diagrams, the full line diagram from the top side of the piston shows the variation of pressure on that side of the piston throughout a revolution whilst the broken line diagram shows the variation of pressure on the under side of the piston. The effective pressure at any point in the stroke is the difference between the pressures on the two sides of the piston, and in order to obtain this difference reference must be made to the two diagrams.

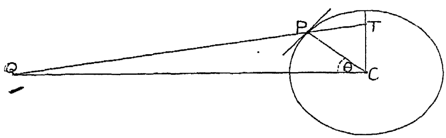


At any point in the stroke, the effective pressure is measured from the steam line of one diagram to the exhaust line of the other. Thus, the verticals on the sketch indicate the effective pressures on the down stroke at various points. At a there

is equality of pressure on each side, and after this point the effective pressures are negative. At *a* cushioning effect really begins on the under side of the piston, but compression may have begun at point *b*.

***Crank Effort Diagram.**

Questions have been given at the examination asking candidates to describe the construction of a crank effort diagram, and the manner of doing this is here dealt with in detail.



The action of the steam pressure on the piston causes a force to act in the connecting rod Q P. This force may be resolved at the crank pin P into components, one tangential to the crank pin circle and the other acting along the centre line of the crank. The latter puts the crank webs alternatively in tension and in compression, but it exercises no turning moment about the shaft axis.

The tangential component is called the *Crank Effort*, and is here denoted by S.

S × C P, or S × crank length, is the turning moment imposed on the shaft. It has already been proved (refer to page 282) that the turning moment for any crank angle θ is equal to the effective piston load × C T,

$$\therefore \text{S} \times \text{crank length} = \text{effective piston load} \times \text{C T}$$

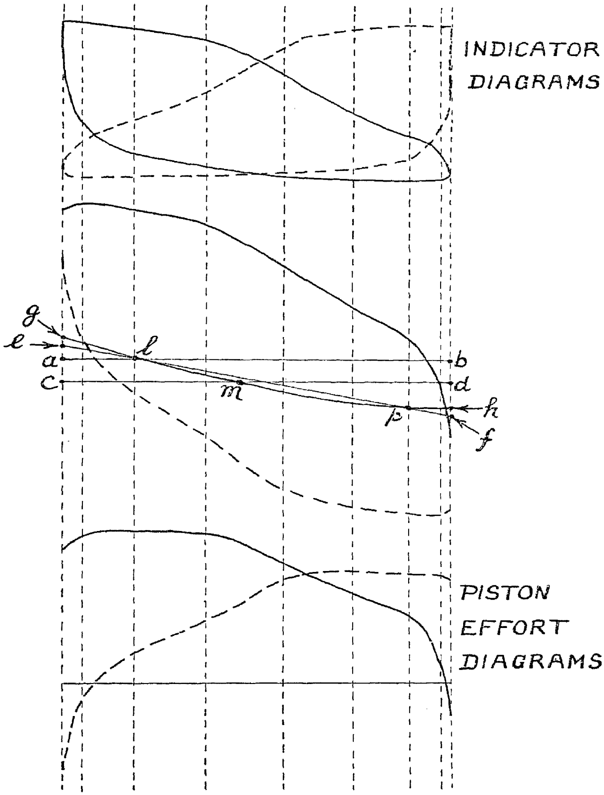
$$\text{S (crank effort)} = \frac{\text{effective piston load} \times \text{C T}}{\text{crank length}}$$

Now the effective piston load is equal to the effective steam pressure × the area of the piston. The effective steam pressure is not that measured directly from the indicator diagram for these reasons.

At the beginning of the stroke the reciprocating parts must be accelerated, and steam pressure is required to do this. This part pressure is not available at the crank pin. After about half stroke the reciprocating parts are retarded, and this increases the pressure at the crank pin.

Also, in an inverted cylinder engine the weight of the reciprocating parts assists the engine on the down stroke, and has the opposite effect on the up stroke.

Before a crank effort diagram can be constructed it is necessary to construct a "piston effort" diagram, or a diagram from which the true effective pressure, for various crank angles, may be measured.



To construct the piston effort diagram, let the indicator diagrams on page 373 be taken from an inverted cylinder engine, the diameter of the cylinder being 18 inches and the stroke 2 feet, the full line diagram being from the top side of the piston and the broken line diagram from the bottom side. Revolutions per min. = 80. Weight of reciprocating parts per sq. inch of piston area = 4 lb.

Let the connecting rod length be 4 feet, then $\dfrac{\text{Con. rod length}}{\text{Crank length}} = \frac{4}{1} = 4$, and let this be denoted by n.

The vertical ordinates are piston positions for successive crank angles of $22\frac{1}{2}°$, obtained as shown by the diagram on page 375.

On a base line $a\ b$ set up above $a\ b$ the effective pressures, measured from the indicator diagrams, for the top side of the piston. Below $a\ b$ set down the effective pressures from the under side of the piston. Draw a new base line $c\ d$ below $a\ b$, the vertical distance between them being equal to the weight of the reciprocating parts per sq. inch of piston area, and make it to the scale of the indicator diagrams.

Now if the piston moved with simple harmonic motion, the acceleration at the ends of the stroke would be $\omega^2 r$ feet per sec.^{2} and the force to accelerate the reciprocating parts would be, per sq. inch of piston area, $\frac{4}{g} \times \omega^2\ r$ lb.

$$\omega = \frac{80 \times 2\ \pi}{60} = \frac{8\ \pi}{3} \text{ rads. per sec.}$$

$\therefore$ Accel. force per sq. inch of piston area

$$\frac{}{32 \cdot 2} \times \qquad \times 1 = 5 \cdot 5 \text{ lb.}$$

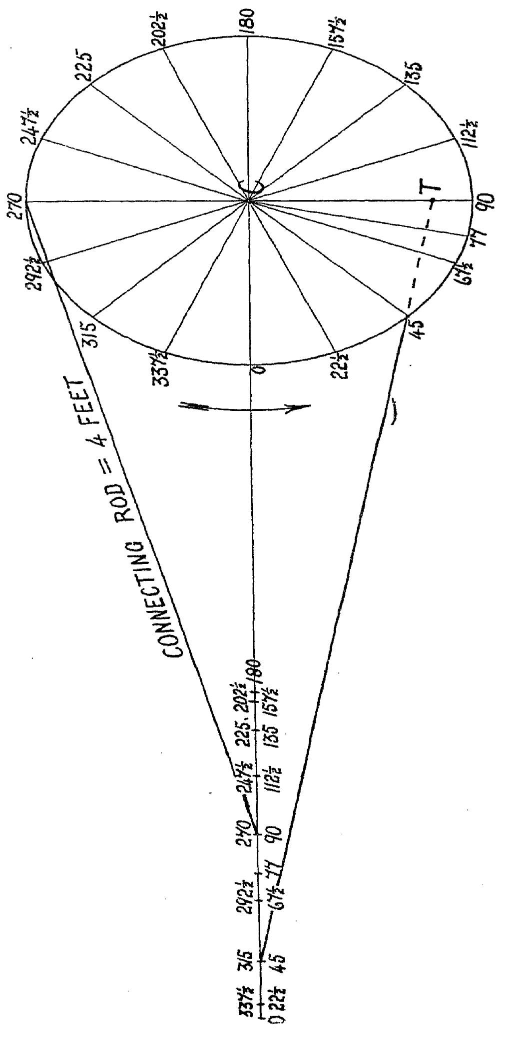
Set up $c\ e = 5 \cdot 5$ lb. and set down $d\ f = 5 \cdot 5$ lb., using the scale of the indicator diagrams. Join $e\ f$ by a straight line. This straight line would represent the conditions regarding the accelerating pressure if the piston had S.H.M., because then the acceleration and therefore the accelerating forces would be directly proportional to the distance from mid-stroke.

But the motion of the piston is a modification of S.H.M. due to the connecting rod having a finite length. The acceleration at the top, or "in" end, is $\omega^2\ r \left\{1 \qquad \frac{}{n}\right\}$, and at the bottom, or "out" end, it is $\omega^2\ r \left\{1 - \frac{}{n}\right.$

The actual accelerating force per sq. inch of piston area when at the top end is

$$\frac{4}{32 \cdot 2} \times \omega^2\ r\ (1 + \tfrac{1}{4}) = 5 \cdot 5 + \frac{5 \cdot 5}{4} = 5 \cdot 5 + 1 \cdot 4 \text{ lb.}$$

and at the bottom end it is $5 \cdot 5 - 1 \cdot 4$ lb.


CONNECTING ROD = 4 FEET
T
0
45
90
135
180
225
270
315

Therefore set up $e\,g = 1{\cdot}4$ lb., and set up $f\,h = 1{\cdot}4$ lb.

Now at crank angles of 45° from the top or bottom centres, the piston acceleration would be the same for S.H.M., or for the modification of S.H.M. These points are indicated by l and p on the straight line $e\,f$. When n is 4 the piston has zero acceleration, and the reciprocating parts zero accelerating force at 77° from the top centre, or approximately when the crank and connecting rod are at 90°. This position is indicated by m.

Draw a curve through $g\ l\ m\ p\ h$. The vertical distances from this curve to the full line diagram, and to the broken line diagram give the true effective pressures for down stroke and up stroke respectively. These should be measured off, and plotted to give the "piston effort" diagram.

The values of C T for the various crank angles are required, and these are obtained by measurement from the diagram on page 375. Tabulate the values as shown.

Crank angle from top centre ...	0°	22½°	45°	67½°	90°	112½°	135°
Effective pressures (lb. per sq. inch)	25·5	28	28·5	28·25	22·5	17·25	13·25
Value of C T (feet)	0	0·46	0·83	1·01	1	0·83	0·58
Product of effective pressure and C T	0	12·9	23·6	28·6	22·5	14·3	7·7

Crank angle from top centre ...	157½°	180°	202½°	225°	247½°	270°	292½°	315°	337½°	360°
Effective pressures (lb. per sq. inch)	3·75	—6·75 +20	21	21	21·25	19·5	12	6	—3	—16·5
Value of C T (feet)	0·3	0	0·3	0·58	0·83	1	1·01	0·83	0·46	0
Product of effective pressure and C T	1·12	0	6·3	12·2	17·6	19·5	12·15	5	—1·38	0

It has been shown that crank effort (S)

$$= \frac{\text{Effective piston load} \times \text{C T}}{\text{Crank length}}$$

$$= \frac{\text{Effective pressure} \times \text{C T} \times \text{Area of piston}}{\text{Crank length}}$$

$$= (\text{Effective pressure} \times \text{C T}) \times \text{a constant}.$$

If therefore we plot (effective pressure $\times$ C T), on a base line of angles turned through by the crank, we have a *rectilinear crank effort diagram.*

The ordinates of the crank effort diagram are,

$$\text{Effective pressure} \times \text{C T} \times \frac{\text{Area of piston}}{\text{Crank length}}$$

but since $\dfrac{\text{Area of piston}}{\text{Crank length}}$ is constant, then the form of the diagam cannot be affected by simply plotting (effective pressure $\times$ C T).

Now the dimensions of the crank effort are

$$\frac{\text{lb.}}{\text{sq. inch}} \times \text{length} \times \frac{\text{sq. inches}}{\text{length}} = \text{lb.}$$

Also $2\pi \times$ crank is the distance moved through in one revolution ; therefore the area of the rectilinear crank effort diagram represents the work done in one revolution.

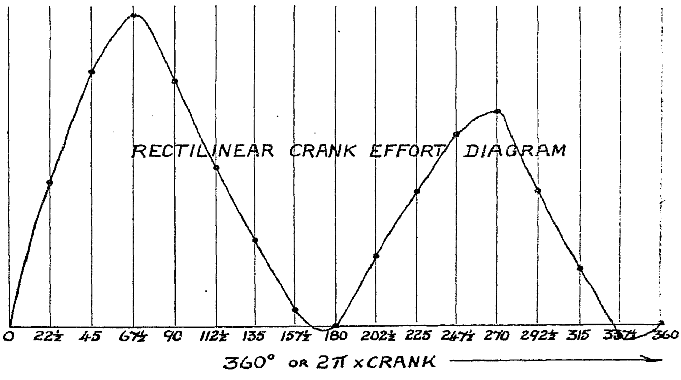
Work per revolution =

$$\frac{\text{Mean val. of (effect. press.} \times \text{C T)} \times \text{area of piston}}{\text{Crank length}} \times 2\pi \times \text{crank}$$

$$= \text{Mean value of (effect. press.} \times \text{C T)} \times \text{area of piston} \times 2\pi$$

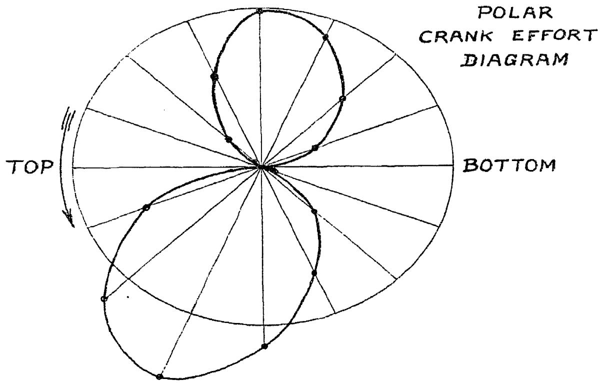
$$= \text{Mean value of (effective piston load} \times \text{C T)} \times 2\pi$$

$$= \text{Mean value of twisting moment} \times 2\pi.$$



Instead of plotting the values of crank effort on a base line of angles turned through by the crank, they may be plotted from the centre of the crank pin path along radial lines at the respective crank angles. The diagram thus drawn is a *polar crank effort diagram.*

It should be noted that the scale of the diagrams on page 373 has not been indicated, because it would not be possible to maintain that scale in reproducing the diagrams.



Steam Turbines.

In the reciprocating engine the steam acts in much the same manner as a load or weight placed upon the piston, that is the action of the steam is "static." In the steam turbine the force exerted on the blades by the steam is due to the velocity of the steam, and to the fact that the curved blades, by changing the direction of the steam, receive a force or impulse.

The action of the steam in this case is said to be "dynamic." In all types of turbines the steam must first acquire velocity, and this is done by allowing the steam to expand from a high pressure to a lower one. Some of the potential heat energy of the steam at the high pressure is changed into kinetic energy at the lower pressure; the heat energy given up by the steam during expansion makes its re-appearance in urging the steam along at a high velocity.

Example. Steam is allowed to expand in passing through a properly designed nozzle, and the heat energy given up, called the "heat drop," is 49 B.T.U. per lb. If 88% of the heat drop is available for creating velocity, calculate the velocity of exit from the nozzle.

Kinetic energy acquired by the steam = Heat energy given up.

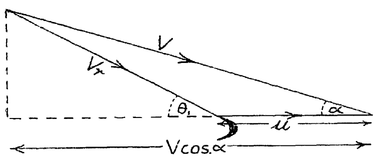
Considering 1 lb. of steam.

$$\frac{1 \times v^2}{2\,g} = 1 \times 49 \times 0{\cdot}88 \times 778$$

$$\therefore\ v^2 = 2\,g \times 49 \times 0{\cdot}88 \times 778,$$

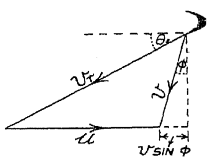
and $v = 1470$ feet per sec. Ans.

The blading of the turbine absorbs this kinetic energy, and takes up part of the velocity of the steam. Turbine blades are curved in such a manner that the steam jet, directed upon them, enters without shock, but there is always some loss of energy by friction of the steam upon the surface of the blades.



Consider a jet of steam, having a velocity of V feet per sec. to impinge upon a row of blades moving at u feet per sec. and let the steam jet make an angle α with the direction of movement of the blades.

Drawing a velocity vector diagram, then the velocity of the steam jet relative to the moving blades is V_r, and its direction is at θ_1 to the direction of movement of the blades. In order that the steam shall enter the blades without shock, the entrance edge of the blades must be in the direction of the relative velocity V_r, and θ_1 is the *entrance angle* of the blades. Also, V cos. α is the component of the velocity of the steam jet in the direction of motion of the blades. V cos. α is the "velocity of whirl at entrance."



Let the steam leave the blades at an angle of θ_2, that is θ_2 is the exit angle of the blades, and let the velocity of the steam relative to the moving blades be v_r. Drawing the velocity vector diagram, v is the absolute velocity of the steam at exit, and ϕ is the angle at which the steam leaves the casing.

Also, v sin. ϕ is the component of the final absolute velocity of the steam, and v sin. ϕ is the "velocity of whirl at exit."

As shown here, v sin. ϕ is in the opposite direction to V cos. α, therefore the total velocity of whirl, and the change of velocity of the steam is V cos. α + v sin. ϕ.

If v sin. ϕ had been in the same direction as V cos. $\propto$, then the change of velocity would have been V cos. $\propto$ — v sin. ϕ.

Now the direction of motion of the steam has been changed as it passes through the blade channel. The blades must have exerted a force upon the steam to cause this change, and since action and re-action are equal and opposite, a force of equal magnitude, but opposite in direction, must have been exerted by the steam upon the blades.

Let W lb. of steam be used per sec. then,

Change of momentum per sec.

$$= \text{Mass} \times \text{change of velocity per sec.}$$

$$= W\ (V \text{ cos. } \propto + v \text{ sin. } \phi).$$

But, force = change of momentum per sec.

$\therefore$ Force exerted on the blades

$$= W\ (V \text{ cos. } \propto + v \text{ sin. } \phi) \text{ poundals.}$$

$$= \frac{W}{g}\ (V \text{ cos. } \propto + v \text{ sin. } \phi) \text{ pounds.}$$

Work done on the blades per sec.

$$= \text{Force} \times \text{distance moved by blades per sec.}$$

$$= \frac{W}{g}\ (V \text{ cos. } \propto + v \text{ sin. } \phi) \times u \text{ ft. lb.}$$

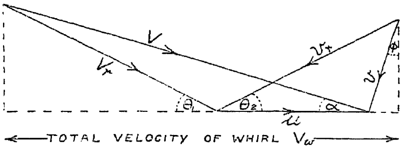
$$\text{and Horse Power} = \frac{W\ (V \text{ cos. } \propto + v \text{ sin. } \phi) \times u}{g \times 550}$$

Note, one horse power = 33,000 ft. lb. per min. = $\frac{33000}{60}$ = 550 ft. lb. per sec.

With impulse blading, the entrance angle and the exit angle are often the same, that is $\theta_1 = \theta_2$.

If friction of the steam upon the blade surface is neglected, then $v_r = V_r$. But with ordinary blading v_r is usually about 0·8 V_r.

Instead of drawing the velocity vector diagrams separately they may be combined, and this is very often a convenient form of diagram to draw.



From this diagram we see that,

$V_w = V \cos. \alpha + v_r \cos. \theta_2 - u$

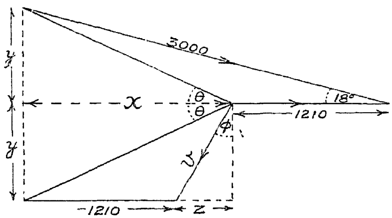
and if $\theta_2 = \theta_1$ say θ

also if $v_r = k\ V_r$, then V_w

or $V_w = V_r \cos. \theta + k\ V_r \cos. \theta = V_r \cos. \theta\ (1 + k)$

If friction is neglected, then $k = 1$.

Example. In a simple impulse turbine the entrance and exit angles of the blades have the same value. The steam leaves the nozzle at 3,000 feet per sec. and at 18° to the plane of the wheel. The mean diameter of the blade ring is 15 inches, and the wheel runs at 18,480 revs. per minute. Find (*a*) the blade angle at entrance, (*b*) the absolute velocity of the steam and its direction as it leaves the turbine (neglect frictional losses), (*c*) the horse power if 1·8 lb. of steam are used per minute, (*d*) the efficiency of the blade arrangement.



$$\text{Blade speed} = u = \frac{18480 \times 15\ \pi}{12 \times 60}$$

$$= 1210 \text{ feet per sec.}$$

$y = 3000 \sin. 18° = 927$ feet per sec.

$x = 3000 \cos. 18° - 1210 = 1643$ feet per sec.

$$\text{Tan. } \theta = \frac{927}{1643} = 0·5643$$

$\theta = 29° \ 26'$, the entrance and exit angle. Ans. (*a*)

$z = 1643 - 1210 = 433$ feet per sec.

$$\text{Tan. } \phi = \frac{433}{927} = 0·4671$$

$$= 25^\circ\ 2'$$

$$\frac{927}{\text{Cos.}\ \theta} \quad \frac{927}{0{\cdot}9061} = 1023 \text{ feet per sec.}$$

Absolute velocity of the steam at exit = 1023 ft. per sec. and its direction is 25° 2′ to the axis of the turbine. } Ans. (*b*)

Change of velocity of the steam = 1643 + 1210 + 433 = 3286 feet per sec.

$$\text{Horse power} = \frac{1{\cdot}8 \times 3286 \times 1210}{60 \times 32{\cdot}2 \times 550} = 6{\cdot}721. \quad \text{Ans. } (c)$$

$$\text{Energy in steam supplied to blades} = \frac{1{\cdot}8 \times 3000^2}{60 \times 2 \times 32{\cdot}2} \text{ ft. lb. per sec.}$$

Energy given to blades = 6·721 × 550 ft. lb. per sec.

$$\therefore \text{Efficiency of blading} = \frac{\text{Energy given to blades}}{\text{Energy in steam}}$$

$$= \frac{6{\cdot}721 \times 550 \times 60 \times 64{\cdot}4}{1{\cdot}8 \times 3000^2} = 0{\cdot}8837$$

$$= 88{\cdot}37\%. \quad \text{Ans. } (d).$$

The efficiency may be obtained thus :—

$$\text{Energy in steam at entrance} = \frac{W\ V^2}{2\ g}$$

$$\text{,, \quad ,, \quad at exit} = \frac{W\ v^2}{2\ g}$$

$$\text{,, \quad given to blades} = \frac{W}{2\ g}\left[V^2 - v^2\right]$$

$$\text{Efficiency of blading} = \frac{\frac{W}{2\ g}\left[V^2 - v^2\right]}{\frac{W\ V^2}{2\ g}} = \frac{V^2 - v^2}{V^2}$$

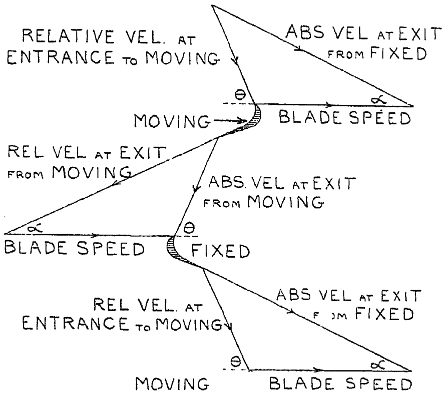
$$= 1 - \left(\frac{v}{V}\right)^2 \quad \text{or} \quad \frac{(V + v)\ (V - v)}{V^2}$$

$$= 1 - \left(\frac{1023}{3000}\right)^2 = 1 - 0{\cdot}1162 = 0{\cdot}8837,$$

as before.

Reaction Turbine.

In the impulse turbine the steam expands whilst passing through fixed nozzles, and there is no expansion whilst it passes through the moving blades. All generation of velocity occurs in the nozzles.



In a reaction turbine expansion of the steam takes place in both fixed and moving blades, and there is increase in velocity of the steam during its passage through both.

The absolute velocity of the steam leaving the fixed blades is greater than it was at entrance, and the velocity of the steam relative to the moving blades is greater at exit from those blades than it was at entrance.

If the fixed and moving blades are similar in form and have equal frictional losses, and if the heat drop in each is the same, then the absolute velocity at exit from the fixed blades is equal to the relative velocity of the steam at exit from the moving blades. Also, the absolute velocity at exit from the moving blades is the absolute velocity at entrance to the fixed blades, and this is equal to the relative velocity at entrance to the moving blades.

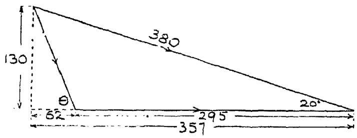
The velocity vector diagrams for fixed and moving blades will be identical.

θ is the entrance angle of the blades.

α is the exit angle from the blades.

Example. At a certain stage of a reaction turbine, the exit angle from the fixed blades is 20° and the steam leaves them at a velocity of 380 feet per second. The mean speed of the moving blades is 295 feet per second, and they are similar in all respects to the fixed blades.

Determine the entrance angle of the blades, and the horse power developed per pound of steam supplied per second to this stage.



$380 \sin 20° = 129{\cdot}96$ (say 130) ft. per sec.

$380 \cos 20° = 357{\cdot}08$ (say 357) ft. per sec.

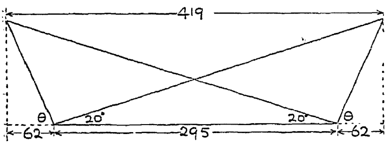
$357 - 295 = 62$

$$\text{Tan } \theta = \frac{130}{62} = 2{\cdot}0968,$$

$$\theta = 64° \; 30'.$$

Entrance angle of blades is $64° \; 30'$. Ans.

Combine the velocity vector diagrams for fixed and moving blades by using a common base of blade speed.



Velocity of whirl (v_w)
$= 62 + 295 + 62 = 419$ ft. per sec.

Force per lb. of steam per sec.

$$= \frac{1}{32{\cdot}2} \times$$

Work done per lb. of steam per

$$\frac{419\cdot}{\cdot 2}$$

$$\text{H.P.} = \frac{419 \times 295}{32{\cdot}2 \times 550} = 6{\cdot}978. \quad \text{Ans.}$$

Nominal Horse Power of Reciprocating Steam Engines.

This name was given to a quantity which depended only upon the sum of the squares of the cylinder diameters. The N.H.P. was determined by dividing the sum of the squares of the cylinder diameters by a constant which varied from about 22 for triples to about 30 for compounds. As this quantity did not take account of the stroke and the steam pressure, it gave no real estimate of the power of the engine. For engines of the same piston speed working at the same boiler pressure, the nominal horse power was a rough standard of comparison. A formula now given by the Board of Trade for nominal horse power takes account of both the stroke and the steam pressure.

$$\text{N.H.P.} = \frac{(3\text{ H} + \text{D}^2 \times \qquad \times}{700}$$

H $=$ Heating surface of boilers in square feet.
D^2 $=$ Square of L.P. cylinder diameter.
S $=$ Stroke in inches.
P $=$ Boiler pressure.

The heating surface is approximately 35 times the grate surface for natural draught, and 40 times the grate surface for forced draught.

TEST EXAMPLES XXII.

1. A cylinder is 20 inches diameter and the stroke is 3 feet. Steam is admitted for 12 inches of the stroke at a pressure of 160 lb. per square inch gauge. Find the terminal pressure and the mean effective pressure (*a*) neglecting clearance, (*b*) if the clearance is 10 per cent. of the working stroke. The back pressure is 26 lb. per square inch absolute.

(*a*) 43·33 lb. gauge ; 96·3 lb. sq. inch. (*b*) 53·93 lb. gauge ; 102·7 lb. sq. inch. Ans.

2. Steam is admitted to a cylinder at 200 lb. per square inch gauge, cut off taking place at 0·32 of the stroke. The clearance volume is 8 per cent. of the working stroke, find the terminal pressure and the mean effective pressure if the back pressure is 60 lb. per square inch gauge.

64·62 lb. sq. inch gauge ; 79·1 lb. sq. inch. Ans.

3. A piston is 1,600 square inches in area, and the mean effective pressure is 24 lb. per square inch. The piston speed is 480 feet per minute, and the revolutions 80 per minute. Find the work done per stroke and the indicated horse power.

115,200 ft. lb. ; 558·5 H.P. Ans.

4. An engine exhausts at a pressure of 20 lb. per square inch gauge, the stroke is 24 inches and the clearance is 10 per cent of the volume swept out by the piston. The initial pressure is 75 lb. per square inch gauge. At what point must the exhaust close so that the steam may be compressed to the initial pressure ?

15·71 per cent. of stroke from end. Ans.
3·77 inches from the end of the stroke.

5. Steam is admitted at 185 lb. per square inch gauge, and cut off at 0·4 of the stroke, the clearance being 8 per cent. of the stroke. Find the mean effective pressure during expansion alone, and the mean effective pressure for the whole stroke. The cylinder is 24 inches diameter and 42 inches stroke, find the I.H.P. at 80 revolutions per minute. The diagram factor is 0·7 and the back pressure is 45 lb. per square inch gauge.

69·6, 97·76 lb. sq. inch, I.H.P. 525·3. Ans.

6. Steam is admitted at 180 lb. per square inch gauge and cut off at one-third of the stroke. The gauge pressure at three-quarters stroke is 82·5 lb. per square inch. Find the clearance as a percentage of the stroke.

8·33 per cent. Ans.

7. Find the weight of steam, neglecting clearance, admitted to the cylinder during one stroke, in question 1. Use the formula :—

$$\text{Vol. of one pound} = \frac{410 + \frac{p}{4}}{p + \quad} \qquad p = \text{gross pressure.}$$

0·8461 lb. Ans.

8. A triple expansion engine has cylinders 26, 42, and 68 inches diameter. The mean pressure all referred to the L.P. is 30 lb. per square inch. Find the mean effective pressure in each cylinder, if each engine develops the same power.

68·4, 26·2, 10 lb. sq. inch. Ans.

9. A quadruple expansion engine has cylinders 25, 36, 54 and 80 inches diameter. The mean effective pressures are 74 lb. per square inch in the H.P., 38 lb. per square inch in the 1st M.P., 17 lb. per square inch in the 2nd M.P. and 8 lb. per square inch in the L.P. Find the mean pressure all referred to the L.P.

30·67 lb. sq. inch. Ans.

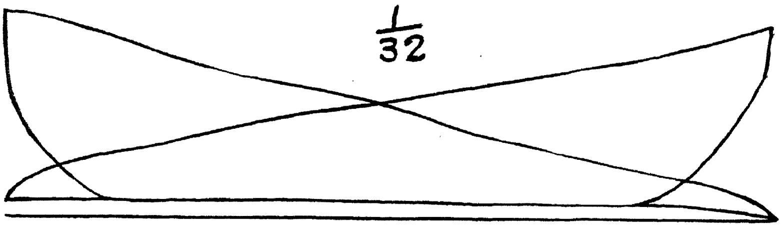
10. A set of indicator cards are 4 inches long, and the sum of the areas of the two cards is 9·5 square inches. The scale of the spring is $\frac{1}{80}$, or one inch of height on the card equals 80 lb. per square inch. Find the mean effective pressure and the I.H.P. if the cylinder is 25 inches diameter, 3·5 feet stroke, and the revolutions 75 per minute.

95 lb. sq. inch ; 742·2 H.P. Ans.

*11. Calculate the cylinder diameters for a triple expansion engine to develop 2,200 I.H.P. at 72 revolutions per minute, stroke 42 inches. The initial pressure is 180 lb. per square inch gauge, the back pressure 4 lb. per square inch absolute. The cut off is at 0·6 of the stroke in the H.P. cylinder, the total number of expansions is 12, and the diagram factor is 0·7.

26·5, 43 and 71 inches. Ans.

12. Calculate the horse power from the given cards. The scale is $\frac{1}{32}$, the cylinder is 42 inches diameter and the revolutions 70 per minute, and the stroke 4 feet.

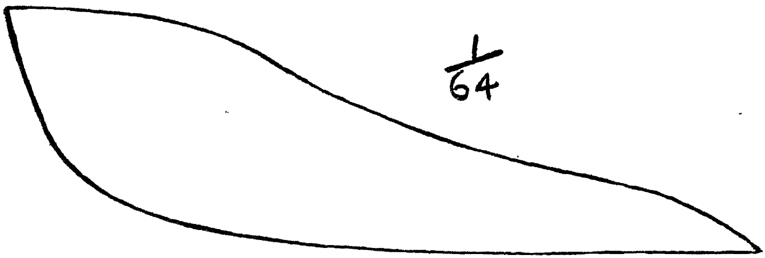


433 H.P. Ans.

13. Calculate the weight of steam used in tons per day from the diagram given. The diagram is from the high pressure cylinder which is 26 inches diameter and 48 inches stroke, the revolutions being 60 per minute. Both cards are the same as the one given.

Note.—Weight of 1 cubic foot of steam in lb. at P lb. per sq. inch absolute is given by :—

$$\text{Wt.} = \frac{P + 1}{410 + \frac{P}{4}}$$



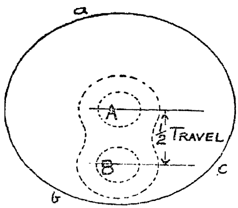
186·7 tons. Ans.

CHAPTER XXIII.

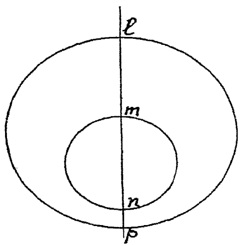
THE SLIDE VALVE : SIZE OF PORTS, Etc.

The Slide Valve.

An eccentric sheave is a form of crank, and its function is to convert rotary motion into reciprocating motion. In the figure let B be the shaft and A the crank pin.



Let the diameter of the crank pin be greatly enlarged, by drawing the circle a b c round the centre of A. Then for the eccentric sheave a b c, the distance between the centre of the sheave and the centre of the shaft is the half travel of the valve, or, as it is sometimes called, the *throw* of the sheave.



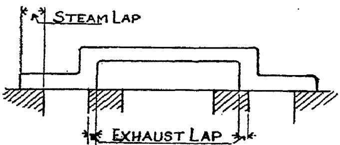
The travel of the valve may be found by taking the difference between l m the thickest part of the sheave, and n p the thinnest part of the sheave :—

Travel of valve $= l\,m - n\,p$

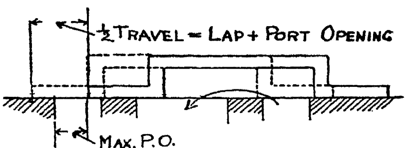
Half travel or throw $=$

$l\,m - n\,p$

A slide valve is shown in mid position, that is, at the middle of its stroke. The steam lap is the distance by which the valve overlaps its steam ports when in mid position.



VALVE IN MID POSITION.



VALVE AT BOTTOM OF STROKE.

In moving from mid position to the end of the stroke :—First the valve moves a distance equal to the lap, and this brings the edge of the valve to the edge of the steam port ; the valve moves a further distance until the port is open an amount called the *maximum port opening*, so that :—

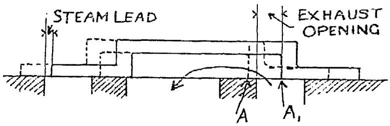
Half travel = lap + maximum port opening.

Travel = 2 (lap + maximum port opening).

The valve is usually designed so that the lap is greater at the top end of the valve than on the bottom, but (lap at top + maximum port opening at top) must equal (lap at bottom + maximum port opening at bottom), and each of these quantities must equal the half travel of the valve.

The lead is the distance the steam port is open when the crank is on the dead centre.

The exhaust lap is the amount by which the exhaust edge of the valve overlaps the exhaust bar, when the valve is in mid position.



Slide valves generally have no exhaust lap. The valve is next shown open to lead at one end, and exhausting at the other end. The valve has moved from mid position by an amount equal to (lap + lead), and the exhaust edge A has moved to A_1, a similar distance. The opening to exhaust is therefore (lap + lead — exhaust lap). When the engine is on the top centre, the valve has opened the top port an amount equal to the lead, and the opening to exhaust at the bottom port is (lap + lead — exhaust lap at bottom). Should the valve have minus exhaust lap or exhaust lead, then the opening to exhaust when the crank is on the dead centre is (lap + lead + exhaust lead). It should be noted that the valve does not open the steam port to its full depth, but generally to about three-quarters of the depth ; the port is, however, opened to its full depth when exhausting. This is because the steam after expanding has a much greater volume.

Example. The lap of a slide valve is $1\frac{1}{4}$ inches at top and $1\frac{1}{8}$ inches at bottom. The lead is $\frac{1}{8}$ of an inch at top and $\frac{1}{4}$ of an inch at bottom. There is $\frac{1}{16}$ of an inch exhaust lap at the top and $\frac{1}{8}$ of an inch exhaust lap at the bottom. What is the opening to exhaust when the engine passes (*a*) the top centre, (*b*) the bottom centre ?

Distance valve is from mid position when engine passes centre = lap + lead = $1\frac{1}{4} + \frac{1}{8}$ or $1\frac{1}{8} + \frac{1}{4} = 1\frac{3}{8}$.

Note that (top lap + top lead) = (bottom lap + bottom lead).

Exhaust opening = distance of valve from mid position — exhaust lap.

Exhaust opening when engine on top $= 1\frac{3}{8} - \frac{1}{8} = 1\frac{1}{4}$ inches. Ans.

Exhaust opening when engine on bottom $= 1\frac{3}{8} - \frac{1}{16} = 1\frac{5}{16}$ inches. Ans.

Example. The lap of a slide valve is $1\frac{3}{4}$ inches, and the depth of the steam port, which is 1·8 times the maximum opening to steam, is $2\frac{1}{4}$ inches. What is the travel of the valve, and why is such a deep port necessary?

Max. Port Opening $\times$ 1·8 = 2·25

$$\text{Max. Port Opening} = \frac{2\cdot25}{1\cdot8} = 1\cdot25 \text{ inches.}$$

$\frac{1}{2}$ travel = Lap + Max. Port Opening.

$\frac{1}{2}$ travel = 1·75 + 1·25 = 3 inches.

Travel = 3 $\times$ 2 = 6 inches. Ans.

The port opens the full $2\frac{1}{4}$ inches to exhaust, so that the larger volume of the exhaust steam may escape without wire drawing.

Example. The travel of a slide valve is 5 inches. The lap at the top is $1\frac{1}{2}$ inches, find the maximum port opening at the top.

The lead at the top is $\frac{1}{8}$ inch, and the lead at the bottom is $\frac{1}{4}$ inch. Find the lap and the maximum port opening at the bottom.

Half travel $= \frac{5}{2}$ = 2·5 inches.

Max. port opening = half travel — lap.

= 2·5 — 1·5 = 1 inch at top. Ans.

Lap + lead = 1·5 + 0·125 = 1·625 inches.

Lap at bottom + lead at bottom = 1·625 inches.

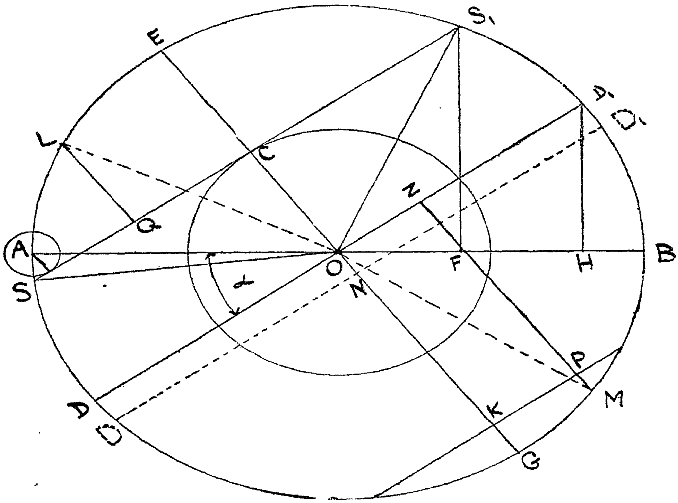
Lap at bottom = 1·625 — 0·25 = 1·375 inches. Ans.

Max. port opening at bottom = 2·5 — 1·375 = 1·125 inches. Ans.

The Reuleaux Valve Diagram.

The travel of the valve is represented by A B, the throw or eccentricity of the sheave is therefore O B or A O.

The lap is the radius O C of the circle described about O. The lead is the radius of the small circle which has its centre at A. The line S S_1, is drawn as a tangent to the lap and lead circles ; S S_1 is the steam line.



Now A B also represents the stroke of the piston to some scale ; S_1 F is perpendicular to A B, and cut off takes place

at $\dfrac{\text{A F}}{\text{A B}}$ as a fraction of the stroke of the piston. This neglects

the effect of the angularity of the connecting rod. For any position of the crank while the valve is open to steam, such as O L, the steam port is open a distance L Q. The maximum steam port opening is C E, where O E is perpendicular to S S_1. Since O C is the lap and C E the maximum port opening, then O C + C E = O E or A O, or as before :—

Lap + maximum port opening = half travel.

$D\ D_1$ is always parallel to $S\ S_1$ and when the exhaust lap is zero, $D\ D_1$ passes through the point O. If there is exhaust lap $D\ D_1$ is drawn as shown dotted, O N being the exhaust lap. For any crank angle, while the port is open to exhaust from D_1 to D, such as O M, the opening to exhaust is Z M, if the steam port is as deep as the half travel. As this is not generally the case, O K is made equal to the depth of the steam port, and the nett opening to exhaust is Z P.

The angle A O D or α, is the angle of advance, and the angle A O S is the angle of pre-admission. The piston moves along A B, and the valve moves along E G.

When the crank is at S, the steam opening is zero.

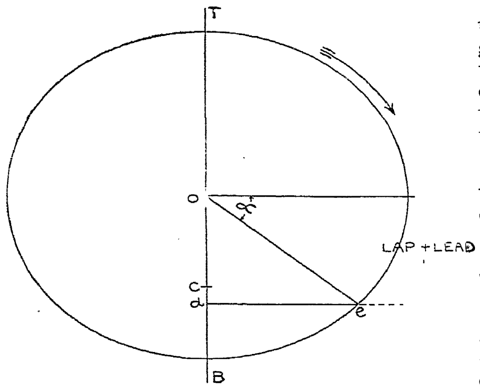
,,	,,	A	,,	,,	is equal to the lead
,,	,,	L	,,	,,	is L Q.
,,	,,	E	,,	,,	is C E (max).
,,	,,	S_1	,,	,,	is zero.

For the angle swept by the crank from S_1 to D_1 the port is closed and the steam expands.

When the crank is at D_1 the exhaust opening is zero.

,,	,,	M	,,	,,	P Z.
,,	,,	D	,,	,,	zero.

Diagrams for the Solution of Valve Problems.



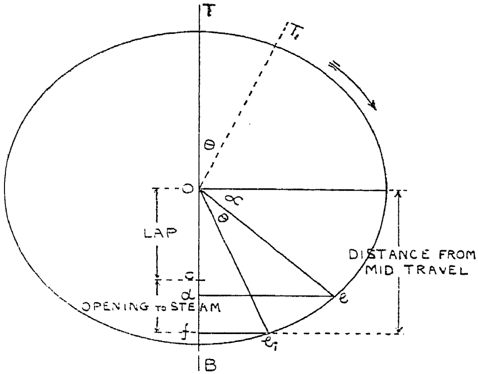
Let the radius of the circle in this diagram be equal to the throw, or half travel of the valve. T is the top centre, and B is the bottom centre.

Set down *o c* equal to the steam lap and *c d* equal to the lead.

Draw *d e* at right angles to T B. Join *o e*, then *e* is the centre of the eccentric, and α is the angle of advance of the eccentric.

If the valve takes steam on the outside the crank pin is at T; if on the inside then it is at B, and the direction of rotation of the engine is clockwise. The valve is *o d* from its mid-position.

$$\text{Sin. } \alpha = \frac{\text{lap} + \text{lead}}{\text{half travel}}$$



Consider now that the crank pin moves from T to T_1 through the angle θ. The eccentric moves from e to e_1 through an equal angle θ.

The valve is now of from its mid-travel. $\text{Sin.}(\propto + \theta) = \dfrac{of}{\text{half travel}}$

$\therefore \; of = \frac{1}{2}\text{ travel} \times \text{Sin}(\propto + \theta)$

But the distance from mid-travel = lap + port opening to steam.

$\therefore$ lap + opening to steam $= \frac{1}{2}$ travel $\times \sin(\propto + \theta)$,
and opening to steam $= \frac{1}{2}$ travel $\times \sin(\propto + \theta)$ — lap.

In dealing with $\text{Sin}(\propto + \theta)$, $\propto$ and θ, each in degrees, must be added together and the sine of the whole angle taken from the tables. Note that $\text{Sin}(\propto + \theta)$ is *not* equal to $\text{Sin}\propto + \text{Sin}\,\theta$.

Example. The travel of a valve is $5\frac{1}{2}$ inches, the maximum port opening is $1\frac{1}{4}$ inches, and the lead is $\frac{1}{8}$ of an inch. Find the angle of advance.

Lap $= \frac{1}{2}$ travel — max. port opening.

$$\text{Lap} = \frac{5{\cdot}5}{2} - 1{\cdot}25 = 1{\cdot}5 \text{ inches.}$$

Lap + lead $= 1{\cdot}5 + 0{\cdot}125 = 1{\cdot}625$ inches.

$$\text{Sine of angle of advance} = \frac{\text{lap} + \text{lead}}{\frac{1}{2}\text{ travel}} = \frac{1{\cdot}625}{2{\cdot}75}$$

Sin. of angle of advance $= 0{\cdot}5909$.

Angle of advance $= 36^\circ\ 13'$.

Example. The travel of a valve is 6 inches. The lap is $1\frac{1}{2}$ inches and the lead $\frac{1}{4}$ inch. The steam port is $2\frac{1}{2}$ inches deep, and the exhaust lap is $\frac{1}{4}$ inch. Find the angle of advance, the maximum port opening and the opening when the crank is at 90° from the dead centre.

Maximum port opening $=$ half travel $-$ lap.

Maximum port opening $= 3 - 1\frac{1}{2} = 1\frac{1}{2}$ inches. Ans.

$$\text{Sine of angle of advance} = \frac{\text{lap} + \text{lead}}{\frac{1}{2}\text{ travel}} = \frac{1{\cdot}5 + 0{\cdot}25}{3}$$

$= 0{\cdot}5833.$

Angle of advance $= 35° \; 41'$. Ans.

Displacement of valve from mid position
$= \frac{1}{2}$ travel $\times$ Sin. $(\propto + \theta)$.

Or displacement of valve
$=\frac{1}{2}$ travel $\times$ Sin. (angle of advance $+$ crank angle from dead centre)

$= 3 \times \text{Sin.}\ (35° \; 41' + 90°) = 3\ \text{Sin.}\ 125° \; 41'.$

Now the Sine of an angle is the Sine of its supplement, and the supplement $= 180° - 125° \; 41' = 54° \; 19'$.

Valve is 3 Sin. $54° \; 19'$, or $3 \times 0{\cdot}8123 = 2{\cdot}4369$ inches from mid position.

Opening to steam $=$ displacement from mid position $-$ lap $= 2{\cdot}4369 - 1{\cdot}5 = 0{\cdot}9369$ inch. Ans.

Example. The travel of a slide valve is 7·75 inches and the angle of advance of the eccentric is 30°. The port opening to steam when the crank is 100° past the centre is 1·25 inches. Find the lead.

Displacement of valve from mid position
$= \frac{1}{2}$ travel $\times$ Sin. $(100° + 30°)$

$$\frac{7{\cdot}75}{2} \times \text{Sin. } 130°.$$

Sin. 130° is the same as Sine $(180° - 130°)$, or Sin. 50°

$$= \frac{7{\cdot}75}{2} \times 0{\cdot}766 = 2{\cdot}968 \text{ inches.}$$

Lap $=$ displ. from mid position $-$ port opening.

Lap $= 2{\cdot}968 - 1{\cdot}25 = 1{\cdot}718$ inches.

$$\text{Sin. of angle of advance} = \frac{\text{Lap} + \text{lead}}{\frac{1}{2}\text{ travel}}$$

$$\text{Sin. } 30° = \frac{1{\cdot}718 + \text{lead}}{3{\cdot}875}, \qquad \text{Sin. } 30° = 0{\cdot}5.$$

$$\text{Lead} = (3{\cdot}875 \times 0{\cdot}5) - 1{\cdot}718$$

$$= 0{\cdot}2195 \text{ inch. Ans.}$$

Example. A slide valve has a travel of $8\frac{1}{2}$ inches. Steam lap $1\frac{7}{8}$ inches, lead $\frac{1}{8}$ inch. How far is the valve from the bottom of its travel when the crank has passed through 45° from the top centre ? The valve takes steam from the outside.

$$\text{Lap} + \text{lead} = 1\tfrac{7}{8} + \tfrac{1}{8} = 2 \text{ inches.}$$

Sine of angle of advance =

$$\frac{\text{Lap} + \text{lead}}{\frac{1}{2}\text{ travel}} \quad \frac{2}{4{\cdot}25} = 0{\cdot}4706$$

Angle of advance = 28° 4′.

Displacement from mid position = $4{\cdot}25 \times \text{Sin. } (28° \; 4' + 45°)$

$= 4{\cdot}25 \times \text{Sin. } 73° \; 4' = 4{\cdot}25 \times 0{\cdot}9566$

Displacement from mid position = $4{\cdot}0655$

Distance from bottom of stroke = $4{\cdot}25 - 4{\cdot}0655$

$= 0{\cdot}1845$ inch. Ans.

Size of Steam Ports.

It is usual to allow steam speeds of from 5,000 to 8,000 feet per minute through main steam pipes and steam ports. If the pressure is to remain constant, then area multiplied by velocity must be constant.

∴ Area of cylinder × piston speed = area of port × speed through port.

The *maximum* speed of the piston should be used in the equation, and the maximum piston speed is approximately equal to the speed of the crank pin. Assuming the width of the port to be about 0·8 of the cylinder diameter, then $\dfrac{\text{Area}}{0{\cdot}8\,D}$ gives the maximum port opening. The maximum port opening is about three-quarters of the total depth of the steam port.

Example. A cylinder is 25 inches diameter, the stroke being 4 feet. The cut off is at 0·6 of the stroke, and the engine turns 70 revolutions per minute. Allowing 8,000 feet per minute through the steam pipe, and 6,000 feet per minute through the ports, find the area of the steam pipe and of the steam ports. If the ports are 21 inches wide, find the maximum port opening.

Maximum speed of the piston $= 2\pi \times 2 \times 70 = 880$ feet per min.

Area of cyl. × max. piston speed = area of port × speed through port.

$25 \times 25 \times \frac{11}{14} \times 880 =$ area of port $\times$ 6,000.

$$\text{Area of port} = \frac{491 \times 880}{6{,}000} = 72{\cdot}02 \text{ sq. inches.} \quad \text{Ans.}$$

$$\text{Maximum port opening} = \frac{72{\cdot}02}{21} = 3{\cdot}43 \text{ inches.} \quad \text{Ans.}$$

This would be $\overset{3\cdot43}{}$ $= 1{\cdot}715$ inches for each port of a double ported valve.

Total depth of port $= 3{\cdot}43 \times \frac{4}{3} = 4{\cdot}573$ inches.

$$\text{Area of pipe} = \frac{491 \times 880}{8{,}000} = 54{\cdot}015 \text{ sq. inches.} \quad \text{Ans.}$$

TEST EXAMPLES XXIII.

1. The travel of a slide valve is 7 inches. The leads are 0·2 of an inch at top and 0·37 of an inch at bottom. The exhaust laps are $-$ 0·2 of an inch at top and $+$ 0·2 of an inch at bottom. The opening to exhaust when the engine is on the bottom centre is 2·38 inches. Find the angle of advance of the eccentric.
$38^\circ\ 33'$. Ans.

2. An outside steam slide valve has a travel of 7 inches. The steam lap is 2 inches and the lead $\frac{1}{4}$ inch. How far is the valve from its mid position when the crank has passed through 30° from the top centre? 3·289 inches. Ans.

3. A slide valve has a lap of $1\frac{1}{4}$ inches, lead $\frac{1}{4}$ inch, and the port is $2\frac{1}{4}$ inches open to steam when the crank is 90° from the dead centre. Find the travel of the valve. 7·616 inches. Ans.

4. The travel of a slide valve is 3·2 inches and the lap is 0·7 inch. When the crank has passed through 90° from the dead centre, the port opening is 0·7 inch. Find the lead.

0·0746 inch. Ans.

5. A cylinder is 30 inches diameter, 36 inches stroke and cut off is at 0·65 of the stroke. The engine makes 75 revolutions per minute. The speed of the steam is to be 6,000 feet per minute through the ports and the same through the steam pipe. Find the areas of port and steam pipe. The width of the ports is 25 inches, find the maximum port opening, and if this is three-quarters of the total depth of the port, find the total depth.

83·34 sq. inches. 3·334 inches. 4·445 inches. Ans.

COMBUSTION OF FUEL.

Combustion is a chemical combination taking place between the combustibles of a fuel and oxygen, heat energy being evolved during the process. All the oxygen required for combustion is obtained from the atmosphere, which is a simple mixture of two chief gases, nitrogen and oxygen. Other gases, and water vapour, are also present but only in very small quantities, and for practical purposes air may be regarded as consisting of nitrogen and oxygen alone.

The proportions by weight are, Nitrogen 77%; Oxygen 23%

or by volume, Nitrogen 79%; Oxygen 21%

Since air consists of approximately 23% by weight of oxygen,

then 1 lb. of oxygen is contained in $\frac{100}{23} = 4{\cdot}35$ lb. of air.

The oxygen is the active element in the process of combustion, nitrogen takes no active part; it dilutes the products of combustion, reduces the temperature due to combustion and, being raised in temperature, carries away heat. The fuels used for marine purposes are coal and oil, and in all fuels whether they are solid, liquid or gaseous the principal combustibles are Carbon and Hydrogen. Sulphur, also, is combustible; and most fuels contain a small amount of this element; but the calorific value of sulphur is low, it contributes little to the heating value of the fuel and may generally be disregarded.

The Calorific value, or heating value of a substance is the amount of heat evolved during the complete combustion of unit weight of the substance.

Carbon, if supplied with sufficient oxygen, will burn completely to carbon dioxide (CO_2) gas, and in doing so will evolve 14,500 B.T.U. per 1 lb. of carbon. CO_2 may be referred to as the gas of complete combustion. Should the supply of oxygen be insufficient then carbon monoxide (CO) gas will be formed instead, and only about 4,400 B.T.U. will be evolved per 1 lb. of carbon. This involves the loss of about 10,100 B.T.U. for each 1 lb. of Carbon; sufficient air must always be supplied to avoid the formation of CO, the gas of incomplete combustion.

Hydrogen combines chemically with oxygen to form water (H_2O), and for each 1 lb. of hydrogen so combined 62,000 B.T.U. are evolved.

Sulphur in chemical combination with oxygen forms sulphur dioxide (SO_2) gas. The calorific value of sulphur is about 4,200 B.T.U. per 1 lb. The atomic weights of the elements involved in the combustion of fuels are :—

Hydrogen, 1 ; Oxygen, 16 ; Nitrogen, 14 ; Carbon, 12 ; Sulphur, 32.

An equation that represents the complete combustion of Carbon is :—

$C + O_2 = CO_2$, and putting in the atomic weights
$12 + (2 \times 16) = 44$, and dividing throughout by 12
$1 + 2\frac{2}{3} = 3\frac{2}{3}$.

This means that 1 lb. of Carbon requires $2\frac{2}{3}$ lb. of Oxygen for its complete combustion, and the weight of CO_2 formed is $3\frac{2}{3}$ lb.

The combustion of Hydrogen may be represented by :—

$H_2 + O = H_2O$, and putting in the atomic weights
$(2 \times 1) + 16 = 18$, and dividing by 2
$1 + 8 = 9$.

This means that 1 lb. of hydrogen requires 8 lb. of Oxygen for its combustion, and the weight of water formed is 9 lb.

Water formed by the combustion of hydrogen in the fuel used in the furnace of a boiler, or in the cylinder of an internal combustion engine, cannot exist as water, due to the high temperature, but must be in the form of superheated steam. The steam passes away in the waste gases, and each 1 lb. of steam will carry away about 1,000 B.T.U. Now 9 lb. of water (or steam) are formed from the combustion of 1 lb. of hydrogen, therefore about 9,000 B.T.U. are carried away in the waste gases, and the heating value of hydrogen for useful purposes is about 62,000 — 9,000 = 53,000 B.T.U. per lb.

62,000 B.T.U. per lb. is termed the *higher calorific value of hydrogen.*
53,000 ,, ,, ,, *lower calorific value of hydrogen.*

The analysis of a fuel generally shows the presence of some oxygen, and this should be assumed as combined with some of the hydrogen in the form of water in the fuel. For instance, the analysis of a sample of coal as stated by the analyst might be Carbon 82% ; Hydrogen 5% ; Oxygen 5%, the remainder being ash, etc. But the sample probably contained some moisture,

which is not shown by the analysis. The 5% of oxygen should be assumed as combined with $\frac{1}{8}$ of its weight of hydrogen in the form of moisture in the fuel.

Therefore the hydrogen actually available for combustion is

$$- \frac{O}{\quad} \Big)$$, where H and O represent the amounts of hydrogen

and oxygen respectively. In the sample suggested the available hydrogen would be $(5 - \frac{5}{8}) = 4{\cdot}375\%$, the remaining $0{\cdot}625\%$ being taken as already combined with the 5% of oxygen as moisture in the fuel. An expression for the calorific value of a fuel is then,

$$\text{Calorific value} = 14{,}500\ C + 62{,}000 \left(H - \frac{O}{8}\right) \text{ B.T.U. per lb.}$$

$$= 14{,}500 \left\{ C + 4{\cdot}28 \left(H - \frac{\quad}{\quad} \right) \right\} \text{B.T.U. per lb.}$$

Where C, H and O are the weights of carbon, hydrogen and oxygen respectively in 1 lb. of the fuel.

Example. A sample of coal has the following composition, Carbon 82% ; Hydrogen $4\frac{1}{2}\%$; Oxygen 6%, the remainder being ash, etc. Calculate the calorific value of this coal.

In 1 lb. there is 0·82 lb. of C. ; 0·045 lb. of H, and 0·06 lb. of O.

$$\text{Available H} = 0{\cdot}045 - \frac{0{\cdot}06}{8} = 0{\cdot}0375 \text{ lb.}$$

$$\therefore \text{ Heat given out by Hydrogen} = 0{\cdot}0375 \times 62{,}000 = 2325 \text{ B.T.U.}$$

$$\text{Heat given out by Carbon} = 0{\cdot}82 \times 14{,}500 = 11890 \text{ ,,}$$

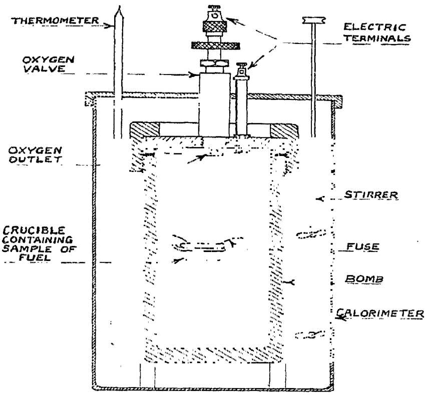
$$\text{Total} = 14{,}215 \text{ ,,}$$

The calorific value is 14,215 B.T.U. per lb. Ans.

The calorific value of a fuel is best determined experimentally, and a **Bomb Calorimeter** may be used for the purpose.

The bomb calorimeter consists of a strong inner vessel or bomb of monel metal, which is fitted with a rubber-jointed monel metal cover to which is secured (1) a support for the crucible or capsule, which holds a measured quantity of fuel, (2) an oxygen inlet valve which can be shut when the required pressure of

oxygen (usually about 25 atmospheres) has been supplied, (3) an insulated electrode to which is attached a fine fuse which dips into the fuel and thence to the crucible support. The bomb is placed inside a calorimeter and a known weight of water completely surrounds and covers bomb. The calorimeter with bomb is placed inside an insulated outer calorimeter. The fuse is heated by a battery (about 12 volts) which ignites the fuel; the heat given out during combustion causes the bomb fittings and surrounding water to rise in temperature. Uniform heating of the parts is ensured by stirring the water continually during the experiment. Careful note is taken of the temperature rise by a finely graduated thermometer, and the calorific value calculated thus :—



Heat given out by fuel = Heat absorbed by water and fittings.

Wt. of fuel × its calorific value = (Wt. of water + water equiv. of fittings) × rise in temperature.

Example. 0·78 gram of coal was burnt in a bomb calorimeter, whose water equivalent is 460 grams; the weight of the water in which the bomb was immersed was 2,400 grams, and the temperature was observed to rise through 2·21 Centigrade degrees. Determine the calorific value of this coal in (a) gram calories per gram, (b) B.T.U. per lb., and state whether the result is the higher or lower calorific value.

$$\text{Heat given out by coal} = \text{Heat absorbed by water and fittings}$$

$$0{\cdot}78 \times \text{C.V.} = (2400 + 460) \times 2{\cdot}21$$

$$\frac{2860 \times 2{\cdot}21}{0{\cdot}78}$$

$$= 8103 \text{ gram calories per gram (a).}$$

8103 gram calories per gram

$$= 8103 \times \tfrac{9}{5} \text{ B.T.U. per lb.}$$

$$= 14590 \text{ B.T.U. per lb. (b)}$$

The result is the higher calorific value because the steam formed during combustion condenses inside the bomb and gives up its latent heat to the fittings and surrounding water.

Air required for Combustion.

The weight of air required for combustion may be calculated if the analysis of the fuel is known.

It has been shown that 1lb. of Carbon requires $2\frac{2}{3}$ lb. of Oxygen for its complete combustion, and that 1 lb. of Hydrogen requires 8 lb. of Oxygen. Also that the weight of Hydrogen that is available for combustion is given by $\left(H - \frac{O}{8}\right)$

$\therefore$ Weight of Oxygen required per 1 lb. of fuel

$$= 2\tfrac{2}{3}\,C + 8$$

and, weight of Air required $= \frac{100}{23} \times \left\{2\tfrac{2}{3}\,C + 8\left(H - -\right.\right.$

$$= 11{\cdot}6\,C + 34{\cdot}8\left(H - \frac{O}{8}\right)\text{lb. per 1 lb. of fuel.}$$

This gives the theoretical weight of air required. In practice about twice this weight would be required for natural draught, and about $1\frac{1}{2}$ or $1\frac{3}{4}$ times the theoretical weight would be necessary for forced draught.

Example. A certain fuel oil contains 84% Carbon, 14% Hydrogen and 1·5% Oxygen. Find the calorific value, and the theoretical weight of air required for complete combustion.

Calorific value $= 14{,}500\,C + 62{,}$

$$= 14{,}500 \times 0{\cdot}84 + 62{,}000\left(0{\cdot}14 - \frac{0{\cdot}015}{8}\right)$$
$$= 14{,}500 \times 0{\cdot}84 + 62{,}000 \times 0{\cdot}1382$$
$$= 20{,}748 \text{ B.T.U. per lb.} \quad \text{Ans.}$$

Theoretical weight of air required $= 11{\cdot}6\,C + 34{\cdot}8\left(H - -\right.$

$$= 11{\cdot}6 \times 0{\cdot}84 + 34{\cdot}8 \times 0{\cdot}1382$$
$$= 14{\cdot}55 \text{ lb. per 1 lb. of oil.} \quad \text{Ans.}$$

The composition of the funnel gases may also be estimated, and the method may be best shown by an example.

Let 23 lb. of air be supplied to each pound of oil in the foregoing example.

The weight of gases passing up the funnel for each 1 lb. of oil burnt will be $= 23 + 0{\cdot}84 + 0{\cdot}14 + 0{\cdot}015 = 23{\cdot}995$ lb., say 24 lb.

Weight of Nitrogen in 23 lb. of air $= \dfrac{77}{100} \times 23 = 17{\cdot}71$ lb.

Oxygen $= \dfrac{23}{100} \times 23 = 5{\cdot}29$,,

Weight of Oxygen to burn 0·84 lb. of C $= 2\frac{2}{3} \times 0{\cdot}84 = 2{\cdot}24$ lb.

,, ,, CO_2 formed $= 0{\cdot}84 + 2{\cdot}24 = 3{\cdot}08$ lb.

,, ,, Oxygen to burn 0·1382 lb. of H $= 8 \times 0{\cdot}1382 = 1{\cdot}1056$ lb.

,, ,, H_2O present in the gases $= 9 \times 0{\cdot}14 = 1{\cdot}26$ lb.

Note that all the hydrogen present in the fuel will appear, combined with oxygen, as water in the gases.

Weight of oxygen necessary for combustion $= 2{\cdot}24 + 1{\cdot}1056 = 3{\cdot}3456$ lb.

$\therefore$ surplus Oxygen $= 5{\cdot}29 - 3{\cdot}3456 = 1{\cdot}944$ lb.

The gases resulting from the combustion of 1 lb. of oil consist of :—

17·71 lb. of Nitrogen ; 3·08 lb. of Carbon dioxide ; 1·26 lb. of Water vapour ; 1·944 lb. of free Oxygen, a total of 23·994 lb.

The composition of the gases is therefore approximately,

Nitrogen $= \dfrac{17{\cdot}71}{24} \times 100 = 74\%$

Carbon dioxide $= \dfrac{3{\cdot}08}{24} \times 100 = 12{\cdot}8$

Water $= \dfrac{1{\cdot}26}{24} \times 100 = 5{\cdot}25\%$

Free oxygen $= \dfrac{1{\cdot}944}{24} \times 100 = 8{\cdot}1\%$

TEST QUESTIONS XXIV.

1. The analysis of a certain coal is 83% Carbon; 4% Hydrogen; 5·6% Oxygen, the remainder being ash, etc. Calculate the calorific value of this coal; also the overall thermal efficiency of a steam installation which uses 1·45 lb. of this coal per horse power per hour.

14,081 B.T.U. per lb. 12·47%. Ans.

2. A sample of fuel oil contains 85% Carbon; 12% Hydrogen; 1·5% Oxygen and 1·5% impurities. Estimate the theoretical weight of air required to burn 1 lb. of this fuel.

13·82 lb. Ans.

3. A boiler burns 1,400 lb. of oil per hour. The oil is composed of 84% Carbon; 13% Hydrogen; 2% Oxygen and 1% other matters. If the weight of air supplied is 70% in excess of the theoretical amount, find the weight of gases which pass up the funnel every hour.

35,112 lb. Ans.

*4. A fuel oil is composed of 86% Carbon; 11% Hydrogen; 1·6% Oxygen and 1·4% impurities. If 24 lb. of air are supplied per 1 lb. of oil burnt in the furnace, estimate the composition of the funnel gases by weight.

CO_2 = 12·62%; N = 73·97%; H_2O = 3·96%; Oxygen = 9·45%. Ans.

5. In a test to determine the calorific value of a fuel oil, a Bomb Calorimeter, whose water equivalent was 450 cubic centimetres, was used. The sample of fuel weighed 0·8 gram, and the volume of the water in which the calorimeter was immersed was 1800 c.c. The temperature was observed to rise from 27°C. to 30·8° C. Determine the calorific value of the fuel.

19,460 B.T.U. per lb. Ans.

6. A six cylinder 2 stroke single acting Diesel engine runs at 92 revs. per minute, and develops 4,000 I.H.P. The consumption of fuel is 0·36 lb. per I.H.P. per hour, and its analysis is 85% Carbon; 13% Hydrogen; 1% Oxygen; 1% other matters. 20 cubic feet of scavenge air at 90°F., and 3 lb. per square inch gauge pressure, enter each cylinder every cycle. Find (*a*) the theoretical weight of air required per cylinder per cycle, (*b*) the actual weight of air supplied per cycle, (*c*) the per cent. excess air supplied.

(*a*) 0·623 lb. (*b*) 1·742 lb. (*c*) 179·5%. Ans.

CHAPTER XXV.

PROPULSION OF SHIPS.

The resistance to propulsion is made up of :—

1. Frictional resistance.
2. Eddy making resistance.
3. Wave making resistance.

The frictional resistance of the skin of a vessel is the greatest part of the total resistance, and may represent as much as 80% of the total resistance. The eddy making and wave making resistances depend upon the under water form of the vessel and upon the speed. In this Chapter only the frictional resistance is dealt with.

Fluid Friction.

Fluid friction depends upon the condition of the surface.
,, ,, is proportional to the total immersed area.
,, ,, ,, ,, $(\text{velocity})^n$
,, ,, ,, ,, the density of the liquid.
,, ,, is independent of the fluid pressure.

The friction force varies directly as the area exposed to the fluid, and varies directly as $(\text{velocity})^n$; these two rules are important. The value of n is often taken as 2, a more correct value is 1·83, and this value refers to clean painted surfaces.

Froude found experimentally that a force of 0·25 lb. is required per square foot of surface at a speed of 600 feet per minute, a speed which is approximately 6 knots. This is for fresh water.

There are many rules for determining the approximate wetted surface of a vessel. Denny's rule is easy to apply :—

$$\text{Wetted surface in square feet} = 1{\cdot}7\ \text{L}\ \text{D} + \frac{\text{V}}{\text{D}}$$

L is the length in feet ; D is the draught in feet, and V is the under water volume in cubic feet.

Example. A vessel is 400 feet long, its draught is 25 feet and its displacement 11,100 tons. The vessel is to be driven at 12 knots. Find the total wetted surface and the total frictional resistance, allowing ¼ lb. per square foot at 6 knots.

Volume of displacement $= 11{,}100 \times 35$ cu. feet.

$$\text{Wetted surface} = 1{\cdot}7 \times 400 \times 25 + \frac{11{,}100 \times 35}{25}$$

$$= 17{,}000 + 15{,}540 = 32{,}540 \text{ sq. feet.} \quad \text{Ans.}$$

Taking the friction force varying as V^2.

Frictional resistance per sq. foot $= \frac{1}{4} \times (\frac{12}{6})^2 = 1$ lb.

Total frictional resistance $= 32{,}540 \times 1 = 32{,}540$ lb. Ans.

If the frictional resistance was $\frac{1}{4}$ lb. per sq. foot at 6 knots, in fresh water, and the frictional resistance varies directly as the density, what would be the total resistance of this vessel when in sea water of density 1,026 ounces per cu. foot ?

Fresh water weighs 1,000 ozs. per cu. foot.

$$\text{Total frictional resistance} = 32540 \times \frac{1{,}026}{1{,}000} = 33{,}386 \text{ lb.} \quad \text{Ans.}$$

Thrust Horse Power.

Approximately from 50 to 60 per cent. only of the indicated horse power is available at the thrust. The horse power at the thrust is often called the tow rope horse power, and this is the power to apply externally by means of a tow rope to give the required speed.

Example. Find the indicated horse power necessary to propel the vessel in the previous example if only 55 per cent. of the I.H.P. is available at the thrust. Assume that the frictional resistance is 78 per cent. of the total resistance.

$$\text{Horse power at thrust} = \frac{32{,}540 \times 100}{78} \times \frac{12 \times 6{,}080}{60 \times 33{,}000}$$

$= 1{,}537.$

I.H.P. $= 1{,}537 \times \quad = 2{,}795.$ Ans.

Another method of estimating the horse power is by the formula $\dfrac{D^{\frac{2}{3}} \times V^3}{\text{I.H.P.}} =$ a constant, where D is the displacement in tons, V is the speed in knots, the constant being a number determined from the previous performance of similar ships.

This constant is often called the "Admiralty Co-efficient," and for cargo vessels at ordinary speeds the value of the constant ranges from 250 to 300.

The formula $\dfrac{D^{\frac{2}{3}} \times V^3}{\text{I.H.P.}}$ = a constant, may be derived in this manner. The surfaces of similar solids are proportional to the squares of their corresponding dimensions, and their volumes are proportional to the cubes of their corresponding dimensions.

$$\text{Volume of displacement} \propto (\text{Length})^3$$

$$\therefore\ (\text{Displ.})^{\frac{1}{3}} \propto \text{Length, and } (\text{Displ.})^{\frac{2}{3}} \propto (\text{Length})^2$$

$$\text{Wetted surface} \propto (\text{Length})^2$$

$$\text{,,} \quad \text{,,} \quad \propto (\text{Displ.})^{\frac{2}{3}}$$

$$\text{I.H.P.} \propto \text{Total frictional resistance} \times \text{Distance per min.}$$

$$\text{Total frictional resistance} \propto \text{Wetted surface} \times (\text{Velocity})^2$$

also Distance per minute $\propto$ Velocity.

$$\therefore\ \text{I.H.P.} \propto \text{Wetted surface} \times (\text{Velocity})^2 \times (\text{Velocity})$$

$$\text{I.H.P.} \propto (\text{Displ.})^{\frac{2}{3}} \times (\text{Velocity})^3,$$

$$\text{or } \frac{D^{\frac{2}{3}} \times V^3}{\text{I.H.P.}} = \text{a constant.}$$

Now I.H.P. $\propto$ Consumption,

$$\frac{D^{\frac{2}{3}} \times V^3}{\text{Consumption}} = \text{a constant.}$$

The expression $\dfrac{D^{\frac{2}{3}} \times V^3}{\text{Consumption}}$ might be used to compare the performances of various vessels, and the value derived could be regarded as a *figure of merit.* This has been suggested by Mr. H. R. Cullen, late Principal of the Marine School.

Example. A ship displaces 7,000 tons and has a speed of $8\frac{1}{2}$ knots on a consumption of 25 tons of coal per day. Another ship displaces 9,000 tons and steams 9 knots on 36 tons of coal per day. Compare these performances.

$$\text{1st vessel, } \frac{(7000)^{\frac{2}{3}} \times (8{\cdot}5)^3}{25} = 8{,}990.$$

$$\text{2nd vessel, } \frac{(9000)^{\frac{2}{3}} \times 9^3}{36} = 8{,}760.$$

The *figure of merit* of the 1st vessel is slightly better than that of the 2nd, and her performance could be regarded as the better.

Example. Find the horse power necessary to drive a vessel of 11,100 tons at 12 knots, using the formula,

$$\frac{D^{\frac{2}{3}} \times V^3}{\text{constant}} = \text{I.H.P., taking the constant as 280.}$$

$$\frac{\times V^3}{\text{constant}} = \text{I.H.P.}$$

$$\frac{(11{,}100)^{\frac{2}{3}} \times 12^3}{280} = 3{,}071. \quad \text{Ans.}$$

Force on the Thrust.

The pressure on the thrust will vary according to the external conditions such as weather, state of the hull, etc. If the vessel steams into a head sea, the speed is reduced ; but if the horse power remains the same, then the pressure on the thrust must increase because :—

Horse power $\propto$ force $\times$ feet per minute,

and if the horse power is the same but the speed is reduced, then the force or pressure on the thrust must increase. The pressure on the thrust varies directly as the horse power and inversely as the speed of the vessel.

Or if H = horse power, V = speed in knots, and P = pressure,

$$\text{then} \quad P_1 V_1 \qquad P_2 V_2$$

Example. In a certain ship the engines indicate 2,000 I.H.P. at 70 revolutions per minute, and the pressure on the thrust is 40 lb. per square inch. The vessel now steams into a head sea, and the engines indicate 1,800 I.H.P. at 60 revolutions per minute, find the pressure on the thrust.

$$\frac{40 \times 70}{2{,}000} \qquad \frac{P_2 \times 60}{1{,}800}$$

$$\frac{40 \times 70 \times 1{,}800}{60 \times 2{,}000} = 42 \text{ lb. per sq. inch.} \quad \text{Ans.}$$

Relation between Speed and Horse Power.

Since force varies as Resistance at the thrust, and Resistance varies as V^2, and as Horse Power varies as force $\times$ distance in unit time,

$\therefore$ Horse Power varies as $V^2 \times V$.

Or H.P. varies as V^3, and as the consumption varies as the horse power, we may write that

Consumption varies as V^3.

Note that to increase the speed from 10 to 12 knots will increase the consumption in the ratio of $(\frac{12}{10})^3$ or 1·728, that is the consumption is increased 72·8 per cent.

If the speed is decreased from 10 to 8 knots, the consumption is decreased in the ratio $(\frac{8}{10})^3$ or 0·512, or 48·8 per cent. This is the explanation of the saving in coal at reduced speed.

The consumption varies as the $(\text{speed})^3$. This statement refers to time, or :—

Consumption per hour or per day $\propto V^3$.

The consumption, however, for a given distance or voyage varies as V^2.

Consumption per voyage $\propto$ coal per hour $\times$ hours on voyage

but coal per hour $\propto V^3$,

and hours on voyage $= \frac{\text{Distance}}{\text{--}}$

$$\text{Consumption per voyage} \propto V^3 \times \frac{\text{Distance}}{V}$$

Consumption per voyage $\propto V^2 \times D$.

So that for a given distance or voyage, consumption varies as V^2.

Example. A vessel steams at 10 knots indicating 2,200 I.H.P. What I.H.P. is required to drive the vessel at 11 knots ?

I.H.P. at 11 knots $= 2{,}200 \times (\frac{11}{10})^3 = 2{,}928$. Ans.

Example. A ship steaming at 10 knots is 1,600 miles from port and has coal for 900 miles only at 10 knots. At what speed must she be driven to reach port ?

$$\text{Consumption} \propto V^2 \times \text{distance}.$$

$$\therefore \frac{\text{Consumption}_1}{V_1^2 \times d_1} = \frac{\text{Consumption}_2}{V_2^2 \times d_2}$$

But here the coal used will be the same.

$$10^2 \times 900 = V_2^2 \times 1{,}600$$

$$V_2^2 = \tfrac{900}{1600} \times 10^2$$

$$V_2 = \sqrt{\tfrac{900}{1600}} \times 10 = 7{\cdot}5 \text{ knots.} \quad \text{Ans.}$$

Example. A vessel has coal for 6 days at 10 knots, but is 8 days from port if going 10 knots. Find the speed at which to run to reach port.

Here, consumption $\times$ time is constant, and as the *distance* gone is the same whatever the speed

$$\therefore \text{Consumption} \propto V^2$$

If x tons is the daily consumption at 10 knots
Coal needed at 10 knots $= 8\,x$ tons
Coal aboard is only $6\,x$ tons
Consumption $\propto V^2$

$$\frac{8\,x}{10^2} \qquad 6\,x \qquad V_2 = \qquad \times \sqrt{\frac{6\,x}{8\,x}}, \; x \text{ cancels}$$

$$V_2 = 8{\cdot}66 \text{ knots.} \quad \text{Ans.}$$

Example. A ship is running at 10 knots and requires 500 tons of coal to reach port at this speed. Find the speed at which to run if only 400 tons of coal are available.

As the distance gone at any speed to reach port is the same,

$$\text{Consumption per distance} \propto V^2$$

Consumption$_1$ Consumption$_2$

$$\frac{500}{10^2} \quad \frac{400}{V_2^2} \qquad \frac{400 \times 10^2}{500}$$

$$V_2 = \sqrt{\tfrac{4}{5}} \times 10 = 8{\cdot}944 \text{ knots. Ans.}$$

Relation between Mean Pressure and Revolutions.

Let p_m = mean pressure, R = revs., P = pitch, V = speed, D = L.P. diam.

I.H.P. $\propto p_m \times R \times D^2$

But I.H.P. also $\propto V^3$, and $V = P \times R$ (neglecting slip),

$\therefore$ I.H.P. $\propto P^3 \times R^3$

$\therefore P^3 R^3 \propto p_m R D^2$

or $\dfrac{P^3 R^3}{\qquad} = \text{constant,}$

or $\dfrac{P_1^3 \cdot R_1^2}{p_n}$

which gives the relation between pitch of propeller, revolutions, mean pressure and cylinder diameter. As the pitch of the propeller and the L.P. diameter generally remain constant we may write :—

$$\frac{\quad}{p_{m1}} \quad \frac{R_2^2}{p_{m2}} \quad \text{or,}$$

the mean pressure varies directly as the (revolutions)^2.

Example. The revolutions were 60 per minute when the mean pressure was 38 lb. per square inch. Find the mean pressure when the revolutions are reduced to 48 per minute.

$$\frac{\quad}{p_{m1}} \quad \frac{R_2^2}{p_{m2}} \quad \text{or} \quad \frac{60^2}{38} \quad \frac{\quad}{p_{m2}}$$

$$p_{m2} = 38 \times \left(\tfrac{48}{60}\right)^2 = 24{\cdot}32 \text{ lb. sq. inch. Ans.}$$

This relation holds good when external conditions such as weather and state of the hull are unchanged only. Suppose that the horse power in the first case in the above example was 2,000, then to find the horse power at reduced speed :—

$$\text{H.P.} \propto p_m \times \text{R}, \qquad \therefore \frac{\text{H.P.}}{p_m \text{R}} = \text{cons.}$$

$$\frac{\text{H.P.}_1}{p_{m1}\,\text{R}_1} = \frac{\text{H.P.}_2}{p_{m2}\,\text{R}_2}, \quad \text{or} \quad \frac{2000}{38 \times 60} = \frac{\text{H.P.}_2}{24{\cdot}32 \times 48}$$

From which $\text{H.P.}_2 = 1{,}024$. Ans.

If the external conditions are not the same, and if the horse power remains constant with the revolutions reduced, then p_m R is constant or,

$$p_{m1}\,\text{R}_1 = p_{m2}\,\text{R}_2$$

Thus in the previous example, if the reduction in speed was due to head wind or sea, and if the horse power remained the same, then :—

$$38 \times 60 = p_{m2} \times 48$$

$$p_{m2} = 38 \times \tfrac{60}{48} = 47{\cdot}5 \text{ lb. sq. inch.}$$

Pitch of Propeller.

The pitch of a propeller is the distance the propeller would advance in one revolution, if working in an unyielding medium. The distance advanced in a revolution is less than the pitch, by an amount called the *slip* The slip is always expressed as a percentage of the theoretical speed, that is as a percentage of the speed if the propeller worked in an unyielding fluid.

$$\text{Theoretical speed} = \frac{\text{pitch} \times \text{revs.}}{6{,}080} \quad 60$$

$$\text{Percentage slip} = \frac{\text{Theoretical speed} - \text{speed of ship}}{\text{Theoretical speed}} \times 100$$

Example. A vessel at 80 revolutions per minute runs at 14 knots, the pitch of the propeller being 20 feet. Find the percentage slip.

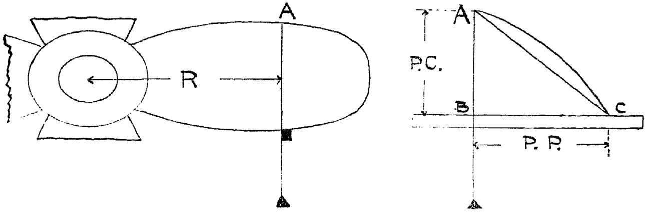
$$\text{Theoretical speed} = \frac{20 \times 80}{6{,}080} \times 60 = 15{\cdot}79 \text{ knots.}$$

$$\text{Percentage slip} = \frac{15{\cdot}79 - 14}{15{\cdot}79} \times 100$$

$$= 11{\cdot}33 \text{ per cent. Ans.}$$

To Measure the Pitch.

Turn the shaft and place one blade of the propeller in a horizontal position as shown.



Suspend a plumb line from a point A on the top edge of the blade, the point A being at a distance of R feet from the shaft centre ; for propellers of average dimensions, from about 4 to 6 feet is a convenient radius to choose for R. Place a straight-edge in a horizontal position touching the bottom edge of the blade and the plumb line ; the straight edge lying fore and aft. Measure the distances A B and B C in feet and inches. Now in one revolution, a point at R from the centre of the shaft describes a distance of $2\pi R$ feet, and this is called the *whole circumference* at radius R.

When the propeller turns through a distance A B, the theoretical advance of the ship is B C ; when the propeller turns one revolution the theoretical advance is equal to the pitch.

$$\therefore AB : BC :: 2\pi R : \text{Pitch}.$$

$$\text{or Pitch} = \frac{BC}{AB} \times 2\pi R.$$

A B is called the *part circumference*, and B C the *part pitch*, and we may state :—

Part Circum. : Whole Circum. : : Part Pitch : Whole Pitch.

Let R be 5 feet, let A B measure 3 feet 6 inches, and let B C measure 2 feet 1 inch.

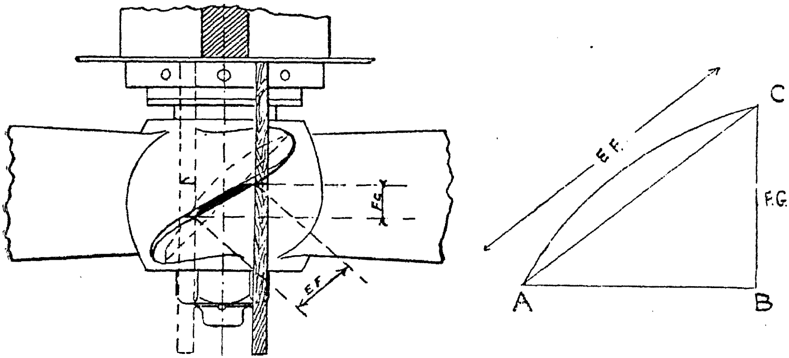
$$\text{Pitch} = \frac{BC}{AB} \times 2\pi R.$$

$$\text{Pitch} = \frac{25}{12} \times \frac{2}{7} \times 2 \times \frac{22}{7} \times 5$$

$$\text{Pitch} = 18{\cdot}71 \text{ feet}.$$

Another method of taking the pitch is as follows :—

Turn the shaft so that one blade is in a vertical position above the shaft. At a suitable radius R from the centre of the shaft, measure by means of a horizontal lath, the distance of the leading edge of the blade from the stern frame. Now place the lath on the other side of the blade as shown, and measure the distance from the following edge of the blade to the stern frame.



Take the difference between the two distances found as described, and this gives F G, the part pitch. Now measure the width of the blade across the ahead side, this gives the distance E F. Next determine A B the part circumference.

$$\text{A B} = \sqrt{(\text{E F})^2 - (\text{F G})^2}$$

Then, as before :—

$$\text{Pitch} = \frac{\text{B C}}{\text{A B}} \times 2\,\pi\,\text{R}.$$

Example. A propeller blade is $2\frac{1}{2}$ feet wide at 5 feet from the centre of the shaft. The leading edge is 1 foot 6 inches, and the following edge 2 feet 10 inches from the stern post respectively. Find the pitch of the propeller.

Part pitch $= 2$ ft. 10 ins. $-$ 1 ft. 6 ins. $=$ 1 ft. 4 ins.

Part circum. $= \sqrt{2\frac{1}{2}^2 - 1\frac{1}{3}^2} = 2{\cdot}115$ feet.

$$\text{Then Pitch} = \frac{1\frac{1}{3}}{2{\cdot}115} \times 2 \times \tfrac{22}{7} \times 5 = 19{\cdot}82 \text{ ft.} \quad \text{Ans.}$$

Force at the Thrust.

When a nozzle, through which a jet of water is driven at high velocity, is directed by the hand against a wall, the jet exerts a force on the wall and an equal force in the opposite direction on the hand holding the nozzle. In much the same way a column of water is fed through the propeller axially, and this column is driven astern by the propeller with a velocity equal to the speed of the slip. It is the forward reaction to the backward thrust of this column of water, acting on the after side of the propeller blades, which is the forward thrust.

Let A = area of stream through propeller in sq. feet.
$= \frac{11}{14}$ [(Diam. of propeller)^2 — (Diam. of Boss)^2].
K = speed of propeller in knots.
k = speed of slip in knots.
W = weight of water dealt with by propeller per sec.

One cubic foot of sea water weighs 64 lb.

$$W = A \times \frac{K \times 6080}{3{,}600} \times 64, \text{ or volume of stream } \times 64$$

$$W = A\,K \times 108{\cdot}1 \text{ (nearly).}$$

$$\text{Velocity of slip in feet per sec.} = \frac{k \times 6080}{3{,}600} = k \times 1{\cdot}688$$

Force = change of momentum per sec.

$$\text{Force} = \frac{W \times v}{g}$$

And here v = velocity of slip.

$$\text{Force or Thrust} = \frac{A\,K \times 108{\cdot}1 \times k \times 1{\cdot}688}{32{\cdot}2}$$

$$\text{Thrust} = 5{\cdot}67\ A \times K \times k = \text{force in pounds.}$$

This is the formula given by the Board of Trade for the thrust of a screw propeller.

As we have seen already :—

$$\text{Thrust horse power} = \frac{\text{Thrust in lb.} \times \text{Speed of ship in ft. per min.}}{33{,}000}$$

Losses in Marine Engines.

The losses which occur through loss of heat to the and by radiation in boilers represents in the case of natural draught boilers about 28 per cent. of the heat supplied by the coal on the grate, so that about 72 per cent. is supplied to the engine. Of this 72 per cent. about 60 is lost by rejecting heat to the condenser, by radiation and friction; this means that 12 per cent. of the total energy in the coal is given out by the shaft. This is sometimes called the overall efficiency of the boilers and engines.

Of the 12 per cent. delivered to the shaft, about 7 is usefully employed at the propeller, or taking the propulsion efficiency as about 58 per cent. $12 \times \frac{58}{100} = 6{\cdot}96$ or roughly 7 per cent. of the total heat given is available at the propeller, or the overall efficiency of the whole plant is only about 7 per cent., the chief sources of loss being the rejection of heat to the condenser and funnel. In the Diesel engine, the chief sources of loss are the loss of heat to the water jacket and to the exhaust, these amounting to about 60 per cent. of the heat supplied, so that the efficiency of the engine may be 40 per cent. approximately, and the overall efficiency of engines and propeller may be as high as 24 per cent.

TEST EXAMPLES XXV.

1. A piece of thin plate is drawn edgeways through the water and the friction per square foot at 600 feet per minute is 0·25 lb. If a vessel travels at 10 knots and has an under-water surface of 15,000 square feet, what horse power is expended in overcoming friction? 328·1 H.P. Ans.

2. The frictional resistance of a plate whose surface is similar to that of a ship's hull, is 0·3 lb. per square foot at 600 feet per minute. A certain vessel is towed through the water, and the horse power transmitted along the tow rope is 2,000. The wetted surface of the vessel is 18,000 square feet. What is the speed? 16·18 knots. Ans.

3. The frictional resistance of a plate similar to that of a ship's hull is 0·3 lb. per sq. foot when towed through fresh water at 600 feet per minute. The frictional resistance is proportional to $(\text{speed})^{2 \cdot 2}$, and also to the density of the water. A steamer has a wetted surface of 25,000 sq. feet, and a speed of 10·5 knots

in water of density 1,026 ozs. per cu. foot. If 70% of the total I.H.P. is expended in overcoming frictional resistance, find (*a*) the frictional resistance per sq. foot, (*b*) the total I.H.P.

1·085 lb. per sq. foot ; 1,250 I.H.P. Ans.

4. Find the tow rope horse power for a vessel whose wetted surface is 35,970 sq. feet, if the speed is to be $12\frac{1}{2}$ knots, the friction force being 0·25 of a lb. per square foot at 6 knots, and the frictional resistance being 80 per cent. of the total resistance. The propulsive efficiency is 55 per cent. Find the I.H.P.

1,873 ; 3,404 H.P. Ans.

5. The I.H.P. of a certain ship is 5,000 and the propulsive efficiency is 45 per cent. Find the total force on the thrust block at 12 knots.

61,060 lb. Ans.

6. The mean radius of the thrust collars is $6\frac{1}{2}$ inches, and 15 horse power is lost at the thrust by friction at 100 revolutions per minute. Find the co-efficient of friction if the total force on the thrust is 14,000 lb.

0·1039. Ans.

7. Find the I.H.P. to drive a vessel of 2,880 tons displacement.

Use the formula $\dfrac{D^{\frac{2}{3}}\ V^3}{C} = \text{I.H.P.}$ taking C as 250. The speed is to be 9 knots.

590·3. Ans.

8. A vessel has 3 boilers of equal heating surface. With 3 boilers the speed is 12 knots. Find the speed when 2 boilers only are in use.

10·48 knots. Ans.

9. A twin-screw vessel travels at 22 knots. One engine breaks down. Find the speed with one propeller working, assuming that the revolutions are the same as before.

17·46 knots. Ans.

10. On one occasion the engine indicated 2,000 I.H.P., the speed being 12 knots. On another occasion, when steaming into a head sea, the speed was 10 knots and the I.H.P. 1,800. The pressure was 40 lb. per square inch on the thrust in the first case. Find the pressure in the second case.

43·2 lb. sq. inch. Ans.

11. The mean effective pressure was 31 lb. per square inch when the revolutions were 62 per minute. The engine speed is changed to 56 revolutions per minute, find the mean effective pressure. The horse power was 2,500 in the first case, what is it in the second case ?

25·29 lb. sq. inch ; H.P. 1,842. Ans.

12. A steamer indicates 1,200 H.P. at 60 revolutions per minute, the mean pressure being 30 lb. per square inch. The weather changes and the vessel steams into a head wind, the revolutions falling to 56 per minute. Find the mean pressure, assuming that the horse power remains constant.

32·14 lb. sq. inch. Ans.

13. At a radius of 6 feet from the centre of the shaft the breadth of a propeller blade is 4 feet. The fore and aft measurement between leading and following edge is ·2 feet. Find the pitch of the propeller.

21·77 feet. Ans.

14. The diameter of a propeller is 16 feet, the pitch is 18 feet and the boss is $2\frac{1}{2}$ feet diameter. The slip is 10 per cent., and the revolutions are 70 per minute. Find the speed of the ship. Using the formula 5·67 A K k, find the total thrust in pounds.

11·19 knots ; 17,150 lb. Ans.

15. When the draught of a steamer is increased by $1\frac{1}{2}$ feet, the increase in the wetted surface is 1,150 square feet. Calculate the length of the steamer from the formula :—

$$\text{Wetted surface} = 1{\cdot}7 \text{ L D} + \frac{\text{V}}{\text{D}}$$

451 feet. Ans.

16. The thermal efficiency of the boiler is 65 per cent., and the thermal efficiency of the engine 17 per cent. ; the mechanical efficiency of the engine is 88 per cent., and the efficiency of the propeller is 70 per cent. What percentage of the heat in the coal is actually used in the propulsion of the ship ?

6·806 per cent. Ans.

17. The calorific value of the coal is 14,000 B.T.U. per lb. The combined efficiency of boilers and engines is 11·4 per cent. Find the coal used per horse power per hour.

1·595 lb. Ans.

CHAPTER. XXVI.

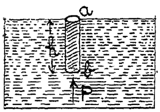
HYDROSTATICS.

Fluid Pressure.

Liquids such as water and oil are almost incompressible. Their change of volume under pressure is so slight that it may be neglected, and for ordinary purposes water may be regarded as incompressible.

The Pressure depends only on the Head.

The vertical height of water above any given point is called the *head of water*; this is the same as saying that the vertical distance from the free surface of the water to a point below the surface is the *head.*



Let $a\ b$ be a thin cylinder of water of area A square feet and length h feet, and let w be the weight of one cubic foot of water. Let the pressure at the depth h be P lb. per square foot. The small cylinder of water is in equilibrium under two forces, its own downward weight and the pressure P acting upwards on the base b.

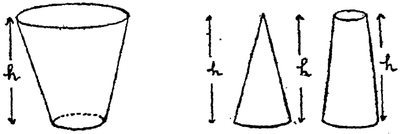
Weight of cylinder of water $= h \times A \times w$.

Upward Force $= P \times A$.

For equilibrium $P \times A = h \times A \times w$, from which

$P = h \times w$, since A cancels.

The pressure then, at any depth h is independent of the extent of the free surface of the water, and depends only upon the head. Let the four vessels, filled with water as shown, have the same



vertical height h and the same area of base. It follows that the total force upon the base of each vessel is the same, since the force on the base of each is pressure $\times$ area, and the pressure depends only on the head h, which is the same for all, and is independent of the area of the free surface of the water.

At a depth of one foot below the free surface, the pressure per square foot $P = w\,h$, and w the weight of a cubic foot of fresh water is 62·5 lb.

$$P = 62{\cdot}5 \times 1 = 62{\cdot}5 \text{ lb. per square foot.}$$

$$\text{or } \frac{62{\cdot}5}{144} \text{ lb. per square inch.}$$

$$\text{or pressure per square inch} = \frac{w\,h}{144}$$

The head required to give a pressure of one lb. per square inch is found as follows :—

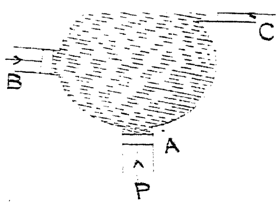
$$1 = \frac{62{\cdot}5\,h}{144}, \text{ or } h = \frac{144}{62{\cdot}5} = 2{\cdot}3 \text{ feet (say).}$$

One cubic foot of sea water weighs 64 lb., and the head to give one lb. per square inch is :—

$$1 = \frac{64\ h}{144}, \quad h = \frac{144}{64} = 2\tfrac{1}{4} \text{ feet.}$$

Transmission of Pressure.

When a pressure is applied to the surface of a liquid, the pressure is transmitted to all parts of the liquid, equally and in all directions. Let a vessel of any shape be filled with liquid, and let small cylinders of equal area into which pistons fit, be arranged round the walls of the vessel as shown. When a force P is applied at the piston A, it is found experimentally that exactly equal forces are required at the pistons B and C to prevent them moving outwards. This proves that the pressure has been transmitted to all parts of the liquid. Further, let the piston and cylinder at C be at *any* angle to the wall of the vessel, it is found that the force needed at C is always the same, and this proves that the pressure transmitted by a liquid always acts at right angles to the surface with which the liquid is in contact.



Specific Gravity.

The specific gravity of a substance is *the ratio* of the weight of the substance to the weight of an equal volume of fresh water ; this quantity is therefore a pure number.

The weight of a cubic foot of cast iron is found to be 7·21 times the weight of a cubic foot of fresh water, and therefore the specific gravity of cast iron is the number 7·21.

Substance	Specific Gravity	Weight per Cubic Inch in Lb.	Weight per Cubic Foot in Lb.	Cubic Feet per Ton
Fresh Water	1	0·03617	62·5	35·84
Salt Water	1·024	0·037	64	35
Cast Iron	7·21	0·26	450	4·977
Wrought Iron ...	7·78	0·281	486	4·61
Steel	7·86	0·283	490	4·57
Brass	8·4	0·303	524	4·274
Lead	11·4	0·412	712	3·146
Mercury	13·6	0·491	849	2·638

It follows from the definition given that :—

$$\text{Sp. G.} = \frac{\text{Weight of the substance}}{\text{Weight of the same volume of fresh water}}$$

also, Weight $=$ Volume $\times$ Sp. G. $\times\ w$.

$$\text{Volume} = \frac{\text{Weight}}{\text{Sp. G.} \times w},$$ where w is the weight of a cubic foot of fresh water.

Example. Find the volume of a piece of cast iron weighing 1,000 lb., the specific gravity being 7·21.

$$\text{Weight of same volume of fresh water} = \frac{1{,}000}{7{\cdot}21}\text{ lb.}$$

$$\text{Volume} = \frac{1{,}000}{7{\cdot}21 \times 62{\cdot}5} = 2{\cdot}219 \text{ cubic feet. Ans.}$$

Example. An oil tank is 2 feet square and 3·8 feet deep and holds 800 lb. of a certain oil. Find the specific gravity of the oil.

Vol. of oil $= 2 \times 2 \times 3{\cdot}8 = 15{\cdot}2$ cubic feet.

Weight $=$ volume $\times$ Sp. G. $\times$ w.

$$800 = 15{\cdot}2 \times \text{Sp. G.} \times 62{\cdot}5.$$

$$\text{Sp. G.} = \frac{800}{15{\cdot}2 \times 62{\cdot}5} = 0{\cdot}8422.$$

Specific Gravities of Mixtures.

When volumes of different substances, whose specific gravities are known, are mixed together, to find the specific gravity of the mixture, we proceed as follows :—

Let V_1, V_2, and V_3 be the volumes, Sp. $G._1$, Sp. $G._2$. Sp. $G._3$ be the specific gravities, w the weight of one cubic foot of fresh water.

The weights of the substances are V_1 Sp. $G._1\, w$, V_2 Sp. $G._2\, w$, V_3 Sp. $G._3\, w$.

Original total weight $= V_1$ Sp. $G._1\, w + V_2$ Sp. $G._2\, w +$ V_3 Sp. $G._3\, w$.

And if the volumes do not alter during mixing :—

Final weight $=$ Sp. G. $\times$ $w\,(V_1 + V_2 + V_3)$ where Sp. G. is the specific gravity after mixing.

$$\therefore \text{Sp. G.} \times w\,(V_1 + \qquad V_3$$

$$= V_1\,\text{Sp. G.}_1\, w + V_2\,\text{Sp. G.}_2\, w + V_3\,\text{Sp. G.}_3\, w$$

$$\text{Sp. G.} = \frac{V_1\,\text{Sp. G.}_1 + V_2\,\text{Sp. G.}_2 + V_3\,\text{Sp. G.}_3}{V_1 + V_2 + V_3}$$

When weights of different substances are mixed together, the Specific Gravities of the substances being known, to find the Specific Gravity of the mixture :—

Let W_1, W_2, W_3 be the weights and Sp. $G._1$, Sp. $G._2$, Sp. $G._3$ the specific gravities, then the Volumes are :—

$$\text{Sp. G.}_1\, w \qquad \text{Sp. G.}_2\, w \qquad \text{Sp. G.}_3\, w$$

and if the volumes do not alter after mixing, then :—

$$\text{Final Vol.} = \frac{W_1}{\text{Sp. G.}_1\, w} \qquad \frac{W_2}{\text{Sp. G.}_2\, w} \qquad \frac{W_3}{\text{Sp. G.}_3\, w}$$

and weight = Vol. × Sp. G. × w, and as the final weight is the same as the original weight, then

$$\mathrm{W}_2 \qquad \mathrm{W}_3$$

$$\text{Sp. G.}_1\, w \qquad\qquad w \qquad \left. \frac{\mathrm{W}_3}{\text{.. G.}_3\, w} \right) \times \text{Sp. G.} \times w$$

$$\therefore \text{Sp. G.} = \qquad \frac{\mathrm{W}_2 \qquad \mathrm{W}_3}{\dfrac{\mathrm{W}_2}{\text{Sp. G.}_2} \qquad \dfrac{\mathrm{W}_3}{\text{Sp. G.}_3}}$$

$$\text{Sp.}$$

Mixtures of three substances are shown here, but the same method applies to a mixture of two substances or more.

Example. 10 cubic feet of a liquid of Sp. G. 1·31 are mixed with 8 cubic feet of a liquid of Sp. G. 0·98, find the Sp. G. of the mixture.

$$\text{Sp. G.} = \frac{(10 \times 1{\cdot}31) + (8 \times 0{\cdot}98)}{10 + 8} = 1{\cdot}163. \quad \text{Ans.}$$

Example. How much pure water must be added to 32 ounces of a liquid of Sp. G. 1·08, so that the final Sp. G. of the mixture may be 1·05 ?

Let x = weight of pure water required.

Then final weight $= 32 + x$

$$\text{and final Vol.} = \frac{32 + x}{1{\cdot}05}$$

$$\text{Original Vol.} = \frac{32}{1{\cdot}08} + \frac{x}{1}$$

Then assuming no change in volume after mixing

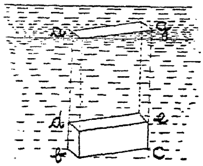
$$\frac{32 + x}{1{\cdot}05} = \frac{32}{1{\cdot}08} + \frac{x}{1}, \text{ multiply by } 1{\cdot}05$$

$$32 + x = \frac{32 \times 1{\cdot}05}{1{\cdot}08} + 1{\cdot}05\,x$$

$$32 + x = 1{\cdot}05\,x + 31{\cdot}111$$

$$0{\cdot}05\,x = 0{\cdot}889, \; x = 17{\cdot}78 \text{ ozs.} \quad \text{Ans.}$$

of Weight by Immersion in Water.



Let a body be immersed as shown. Project vertical lines up to the surface of the water. Before the body is immersed, the column of water *a b c g* is in equilibrium, and if this column of water be lifted out, then the *upward* thrust on the area *b c* is exactly equal to the weight of the column *a b c g*. The weight of the column of water *a d e g* acts *downwards*. The actual upward thrust on the body then is the difference between the weight of the column of water *a b c g* and the weight of the column of water *a d e g*, and this is equal to the weight of water displaced by the body *d b c e*. Therefore the *loss of weight in water is equal to the weight of the displaced water.*

Principle of Archimedes.

If a body is immersed in a fluid, then the total resultant upward pressure of the fluid on the body is equal to the weight of fluid displaced by the body.

To determine the specific gravity of a body which sinks in water, it is first weighed in air, then it is immersed in water by a string hung from a balance, and the weight noted. The difference between the weight in air and the weight in water is equal to the weight of the displaced water, and this displaced water has exactly the same volume as the body immersed, then :—

$$\text{Specific Gravity} = \frac{\text{Weight of body in air}}{\text{Weight of an equal vol. of fresh water}}$$

$$= \frac{\text{Weight of body in air}}{\text{Weight of body in air} - \text{Weight of body in fresh water}}$$

Example. A piece of metal is found to weigh 10 lb. in air and 8·6 lb. in sea water. Find its specific gravity.

Weight of displaced water $= 10 - 8{\cdot}6 = 1{\cdot}4$ lb.

Weight of an equal vol. of fresh water $= 1{\cdot}4 \times \frac{62{\cdot}5}{64} = 1{\cdot}367$ lb.

Sp. G. $= \frac{10}{1{\cdot}367} = 7{\cdot}316$. Ans.

Example. A rectangular block of material is 2 feet by 3 feet by 4 feet, and its specific gravity is 3·8. Find the pull on the chain suspending it in oil, one cubic foot of which weighs 52 lb.

Weight of block $= 2 \times 3 \times 4 \times 62{\cdot}5 \times 3{\cdot}8 = 5{,}700$ lb.

Weight of displaced oil $= 2 \times 3 \times 4 \times 52 = 1{,}248$ lb.

Pull on chain $= 5.700 - 1{,}248 = 4{,}452$ lb. Ans

Floating Bodies.

A body floating in water is in equilibrium under the action of two vertical forces; its own weight acting downwards, and the upward thrust of the displaced water. It follows that the weight of the displaced water must equal the weight of the floating body. If a body has a specific gravity of 0·75, it will float in fresh water with $\frac{3}{4}$ of its volume immersed, and in salt water with $\frac{3}{4} \times \dfrac{62{\cdot}5}{64}$ of its volume immersed, since the weight of the displaced water, in each case, must equal the weight of the body.

Example. A piece of wood one foot square and 6 inches deep, floats at a draught of $4\frac{1}{2}$ inches in salt water. Find the specific gravity of the wood.

$$\text{Weight of displaced water} = 1 \times 1 \times \frac{4{\cdot}5}{12} \times 64 = 24 \text{ lb.}$$

Wt. of displaced water = Wt. of floating body = 24 lb.

Wt. of vol. of fresh water equal to vol. of wood $= 1 \times 1 \times \frac{6}{12} \times 62{\cdot}5 = 31{\cdot}25$ lb.

$$\text{Sp. G.} = \frac{\text{Weight of body}}{\text{Weight of equal vol. of fresh water}} = \frac{24}{31{\cdot}25}$$

Sp. G. = 0·768. Ans.

Example. A piece of cast iron weighing 15 lb. is fastened to the bottom of a piece of wood one foot square and 8 inches deep. When floating in sea water, the draught is 7 inches, find the specific gravity of the wood. The specific gravity of cast iron is 7·21.

Wt. of fresh water of same vol. as 15 lb. of cast iron

$$= \frac{15}{7{\cdot}21} \text{ lb.}$$

$$\text{Wt. of salt water of same vol.} = \frac{15}{7{\cdot}21} \times \frac{64}{62{\cdot}5} = 2{\cdot}13 \text{ lb.}$$

Wt. of 15 lb. of iron when submerged $= 15 - 2{\cdot}13 = 12{\cdot}87$ lb.
Wt. of water displaced by wood $= 1 \times 1 \times \frac{7}{12} \times 64 = 37{\cdot}33$ lb.
Wt. of wood $= 37{\cdot}33 - 12{\cdot}87 = 24{\cdot}46$ lb.
Wt. of fresh water of same vol. as wood $= 1 \times 1 \times \frac{8}{12} \times 62{\cdot}5 = 41{\cdot}66$ lb.

$$\text{Sp. G. of wood} = \frac{24{\cdot}46}{41{\cdot}66} = 0{\cdot}5873. \text{ Ans.}$$

Problems on Pumps.

The normal pressure of the atmosphere is about 14·7 lb. per sq. inch, and this corresponds to the pressure exerted by a column of mercury 30 inches high. As the specific gravity of mercury is 13·6, then a column of fresh water $\dfrac{30 \times 13{\cdot}6}{12} = 34$ feet high, would exert the same pressure, and this is the height of the water barometer corresponding to a pressure of 14·7 lb. per square inch. This is the depth from which a pump would draw the water, if a perfect vacuum could be maintained in the suction pipe. Theoretically a pump should be able to draw fresh water from a depth of 34 feet; on account of the difficulty of creating a perfect vacuum, a pump can draw water from a depth of about 26 feet. One lb. per square inch would be exerted by a column of mercury $\dfrac{30}{14{\cdot}7}$ or 2·04 inches high. It is sufficiently accurate to take this as 2 inches; if the vacuum in a pump barrel is 20 inches, and the barometer stands at 30 inches, then the absolute pressure in the pump barrel is $\dfrac{30 - 20}{} = \cdot 5$ lb per square inch.

Work done in pumping against Pressure.

Since a head of 2·3 feet of fresh water gives a pressure of one lb. per square inch, then to pump against a pressure of 200 lb. per square inch is the same as pumping against a head of $200 \times 2{\cdot}3$ or 460 feet, and the work done will be given by multiplying the weight of water delivered, by the equivalent head Another method is as follows :—Imagine a pump having an area of bucket of one square foot; the total load on the bucket is (pressure per sq inch $\times$ 144) lb., and the work done to deliver one cubic foot of water against the pressure is the work done by the bucket when it moves a distance of one foot along its stroke.

Work done = Pressure per square foot $\times$ Volume delivered in cubic feet.

The student should note carefully that as we generally need work done in ft. lb., that the pressure must be in *lb. per sq. foot*, and the volume in *cubic feet*.

Example. Find the work done per pound of feed water in pumping it into the boiler at a pressure of 200 lb. per square inch.

Work done per cubic foot of water pumped in = Pressure per sq. foot × Volume in cubic feet.

$= 200 \times 144 \times 1 = 28{,}800$ ft. lb. per cubic foot, but 1 lb.

is only $\dfrac{1}{}$ of a cubic foot.

$\therefore$ Work per lb. of feed $= 28{,}800 \times \dfrac{}{62{\cdot}5}$

$= 460{\cdot}8$ ft. lb. Ans.

Example. A pump delivers 1,000 gallons of feed into a boiler at 180 lb. per square inch, in 17 minutes. The efficiency of the pump is 69 per cent., find the horse power to drive the pump.

$$\text{Cubic feet pumped in} = \frac{1{,}000}{6{\cdot}25} = 160 \text{ cu. feet.}$$

$$\text{Work done in 17 mins.} = 180 \times 144 \times 160 = 4{,}147{,}000 \text{ ft. lb.}$$

$$\text{Work per min.} = \frac{4{,}147{,}000}{17} \text{ ft. lb.}$$

$$\text{Horse power of pump} = \frac{4{,}147{,}000}{17 \times 33{,}000} \times \tfrac{100}{69} = 10{\cdot}72. \quad \text{Ans.}$$

Pumps working together.

When two or more pumps work together pumping out a tank, then if one pump working alone can empty the tank in n hours, and if the other pump can empty the same tank in m hours, then taking the capacity of the tank as 1,

$$\text{Quantity pumped per hour by 1st pump} = \frac{}{n} \text{ of the tank.}$$

$$\text{Quantity pumped per hour by 2nd pump} = \frac{1}{m} \text{ of the tank.}$$

Quantity per hour when working together

$$= \frac{1}{n} + \frac{1}{m} = \frac{n + m}{n \times }$$

Time to pump out when working together

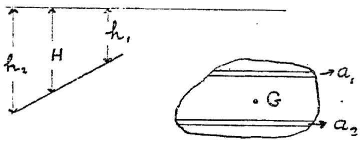
$$= \frac{n \times m}{n + m} \text{ hours.}$$

$n +$

$n \times m$

Pressure on an Immersed Area.

Let the area of the immersed figure be A square feet, let the centre of gravity be at G, at a *vertical distance* H feet below the free surface of the water. Let the surface, which may have *any inclination* to the vertical be divided up into many small strips, a_1, a_2, etc., the depths of these strips being h_1 and h_2 feet below the surface.



Then force on strips $= w\, h_1\, a_1 + w\, h_2\, a_2 +$ etc., the term "etc." including the force on all strips not mentioned.

But $w\, h_1\, a_1 + w\, h_2\, a_2 +$ etc. $= w\,(h_1\, a_1 + h_2\, a_2 + \ldots)$ and the term in the bracket is the sum of the first moments of all the strips of area about the water level. By the principle of the centre of gravity already explained,

$$w\,(h_1\, a_1 + h_2\, a_2 + \ldots) = w \text{ H A},$$

$\therefore$ Total pressure on whole area $=$ H A w, where H is always the vertical distance from the centre of gravity of the *wetted area* to the *free surface of the water* in feet, A the wetted area in square feet, and w the weight of one cubic foot of the water.

Example. A bulkhead 40 feet wide, 20 feet deep, has sea water on one side to a height of 16 feet. Find the total load on the bulkhead.

$$\text{Total load} = \text{H A } w = \frac{8 \times 16 \times 40 \times 64}{2240} = 146{\cdot}3 \text{ tons.}$$

Ans.

Example. A tank in a double bottom is 20 feet square and 4 feet deep. Find the load upon the top, bottom and one side of the tank, (*a*) when the tank is just full, (*b*) when the sounding pipe is filled to a height of 10 feet.

(*a*) Load on top $= 0$, because the head is 0. Ans.

$$\text{Load on bottom} = \text{H A } w = \frac{4 \times 20 \times 20 \times 64}{2240}$$

$= 45{\cdot}71$ tons. Ans.

$$\text{Load on one side} = \text{H A } w = \frac{2 \times 20 \times 4 \times 64}{2240}$$

$= 4{\cdot}571$ tons. Ans.

(*b*) Note the head must now be measured from the free surface of the water in the pipe.

$$\text{Load on top} = \text{H A } w = \frac{10 \times 20 \times 20 \times 64}{2240}$$

$= 114{\cdot}28$ tons. Ans.

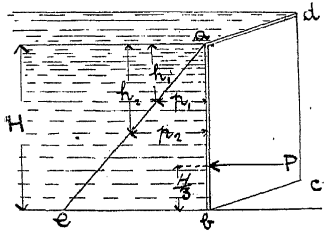
$$\text{Load on bottom} = \text{H A } w = \frac{14 \times 20 \times 20 \times 64}{2240}$$

$= 160$ tons. Ans.

$$\text{Load on one side} = \text{H A } w = \frac{12 \times 20 \times 4 \times 64}{2240}$$

$= 27{\cdot}43$ tons. Ans.

The Centre of Pressure of an immersed surface is that point through which the total pressure on the whole surface may be regarded as acting.



In the case of rectangular areas submerged, the Centre of Pressure is generally easy to find. Let a rectangular surface *a b c d* be vertical, and let the depth of water on one side be *a b* or H feet. Since the pressure is proportional to the head of water, the pressures for any heads may be represented by a straight line *a e*, and the triangle *a e b* is the *load diagram*. Now the centre of gravity of the load diagram or of the triangle *a e b* is at one-third the vertical height above the base, or at $\frac{H}{3}$. The student should note that this is not the

same as the centre of gravity of the wetted surface, which is $\frac{H}{2}$ for the rectangle $a\ b\ c\ d$. The moment of the total water load above the centre of pressure must be equal to the moment of the total water load below the centre of pressure. The centre of pressure is the position at which to apply a shore to help the surface, which may be a bulkhead in a vessel's hold, to sustain the water pressure on the other side.

Example. A bulkhead is 50 feet wide, 25 feet high, and has sea water on one side to a depth of 20 feet. Find the total load on the bulkhead, and state the position of the centre of pressure below the water level.

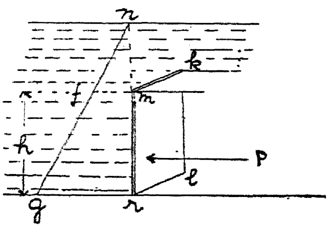
$$\text{Total load} = \text{H A } w = \frac{10 \times 20 \times 50 \times 64}{2240} = 285{\cdot}7 \text{ tons.}$$

Ans.

Centre of Pressure is at $\frac{1}{3}$ the wetted height *above* the bottom for a rectangle, or at $\frac{2}{3}$ the wetted height below the water level.

Centre of Pressure is at $\frac{2}{3} \times 20 = 13\frac{1}{3}$ feet below the surface.

The student must note that the previous paragraphs refer to rectangular areas, when the top edge of the wetted area lies at the water level. If the top edge of the immersed rectangular area is some distance below the water level, the centre of pressure is not at $\frac{H}{3}$ from the bottom of the area. Let the top edge



of the rectangular surface $m\ r\ l\ k$, be at a depth $n\ m$ below the water level. The load diagram for the area $m\ r\ l\ k$ is the trapezium $f\ g\ r\ m$, and the centre of pressure is at the centre of gravity of this trapezium. Now we might find the pressures at the depths $n\ m$ and $n\ r$, but as the pressure is proportional to the head, we may use the head to find the centre of pressure.

C.G. of the trapezium $= \frac{h}{2}$ $(f\ m) + (g\ r)$

this may be written as,

$$\frac{h}{3}\left[\frac{2 \times (n\ \overset{h}{m}) + (n\ \overset{h}{\ \ })}{(n\ m) + (n\ r)}\right],$$ where h is the height of the wetted area.

Example. A rectangular sluice door is 5 feet wide and 6 feet high. It lies in a vertical position with its top edge 10 feet below the level of the water, which weighs 64 lb. per cubic foot. Find the total load on the door, and the position of the centre of pressure.

Load on door $=$ H A w. Now H is the vertical height from the centre of gravity of the area to the water level, and is here $10 + \frac{6}{2} = 13$ feet.

$$\text{H A } w = \frac{13 \times 5 \times 6 \times 64}{2240} = 11\tfrac{1}{7} \text{ tons. Ans.}$$

10′

Centre of Pressure

$$= \frac{6}{3}\left[\frac{2 \times 10 + 16}{10 + 16}\right] \text{above base}$$

$$= 2 \times \tfrac{36}{26} = 2{\cdot}77 \text{ feet (nearly).}$$

This is $16 - 2{\cdot}77 = 13{\cdot}23$ feet below water level. Ans.

The general formula for immersed surfaces which lie in a vertical plane is :—

$$\text{Centre of Pressure below water level} = \frac{k^2 + \text{H}^2}{\text{H}},$$ where k is the radius of gyration of the surface about its centre of gravity, and H is the vertical distance from the centre of gravity to the water level. This formula is applicable, whether the top edge of the area lies on the water level, or at some distance below the water level.

$$k^2 \text{ for rectangular area about its C.G.} = \frac{(\text{Depth of rectangle})^2}{12}$$

$$\text{for circular area about its diam.} = \frac{(\text{Diam.})^2}{16}$$

$$k^2 \text{ for triangular area about its C.G.} = \frac{(\text{Vert. height of triangle})^2}{18}$$

An important expression, and one that is easy to apply is :—

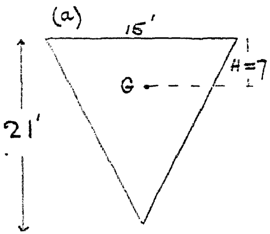
Centre of Pressure below water level =

$$\frac{\text{2nd moment about water level}}{\text{1st moment about water level}}$$

Example. A fore peak collision bulkhead is of triangular shape. It is 21 feet high, and its greatest breadth is 15 feet. Find the load on the bulkhead and the position of the centre of pressure (*a*) when the peak is just full ; (*b*) when there is 10 feet of water in the sounding pipe above the top of the peak.

$$(a)\ \text{Total load} = \text{H A } w = \frac{7 \times 15 \times 21}{2240 \times 2} \times 64 = 31{\cdot}5 \text{ tons.}$$

Ans.

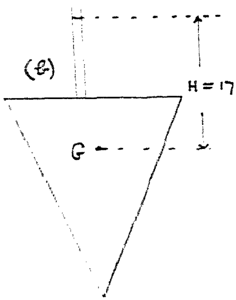


Centre of Press. = $\dfrac{H^2}{H}$

below water level.

$$\frac{21^2}{18} + 7^2 \qquad 24{\cdot}5 \qquad 49$$

$$= \frac{73{\cdot}5}{} = 10{\cdot}5 \text{ feet below water level.} \quad \text{Ans.}$$



(*b*) Total load = H A w

$$= \frac{17 \times 15 \times 21 \times 64}{2240 \times 2}$$

$= 76{\cdot}5$ tons. Ans.

Centre of Press. =

$$\frac{\frac{21^2}{18} + 17^2}{17} = \frac{24{\cdot}5 + 289}{17}$$

$$\frac{313{\cdot}5}{17} = 18{\cdot}44 \text{ feet below water level, or}$$

$18{\cdot}44 - 10 = 8{\cdot}44$ feet below top of peak. Ans.

The semi-width of a vertical bulkhead is 9 feet at the top, and 1·9 feet at the bottom. The semi-widths taken at intervals of 2 feet apart are :—8·8 feet ; 8·6 feet ; 8·3 feet ; 7·7 feet ; 6·7 feet ; 5·4 feet ; 3·9 feet. The bulkhead is 16 feet high, and is exposed on one side to sea water. Find the load carried and the position of the centre of pressure.

Semi-ordinates	Simpson's Multipliers	Functions of ordinates	Distance of ordinates from top	Functions for 1st moments	Distance of ordinates from top	Functions for 2nd moments
9·0	1	9·0	0	0	0	0
8·8	4	35·2	2×1	$2 \times 35{\cdot}2$	2×1	$2^2 \times 35{\cdot}2$
8·6	2	17·2	2×2	$2 \times 34{\cdot}4$	2×2	$2^2 \times 68{\cdot}8$
8·3	4	33·2	2×3	$2 \times 99{\cdot}6$	2×3	$2^2 \times 298{\cdot}8$
7·7	2	15·4	2×4	$2 \times 61{\cdot}6$	2×4	$2^2 \times 246{\cdot}4$
6·7	4	26·8	2×5	$2 \times 134{\cdot}0$	2×5	$2^2 \times 670{\cdot}0$
5·4	2	10·8	2×6	$2 \times 64{\cdot}8$	2×6	$2^2 \times 388{\cdot}8$
3·9	4	15·6	2×7	$2 \times 109{\cdot}2$	2×7	$2^2 \times 764{\cdot}4$
1·9	1	1·9	2×8	$2 \times 15{\cdot}2$	2×8	$2^2 \times 121{\cdot}6$
		Sum = 165·1		Sum = $2 \times 554{\cdot}0$		Sum = $2^2 \times 2594{\cdot}0$

Area of bulkhead $= \frac{2}{3} \times 165{\cdot}1 \times 2$ sq. feet.

1st moment about top of bulkhead $= \frac{2}{3} \times 2 \times 554 \times 2$

$$\text{C.G. from top} = \frac{\text{1st moment about top}}{\text{Area}}$$

$$= \frac{\frac{2}{3} \times 2 \times 554 \times 2}{\frac{2}{3} \times 165{\cdot}1 \times 2} = \frac{1108}{165{\cdot}1} = 6{\cdot}709 \text{ feet.}$$

$$\text{Load} = \frac{2 \times 165{\cdot}1 \times 2 \times 6{\cdot}708 \times 64}{3 \times 2240} = 42{\cdot}2 \text{ tons.} \quad \text{Ans.}$$

2nd moment about top $= \frac{2}{3} \times 2^2 \times 2594 \times 2$

$$\text{Centre of pressure from top} = \frac{\text{2nd moment about top}}{\text{1st moment about top}}$$

$$= \frac{\frac{2}{3} \times 2^2 \times 2594 \times 2}{\frac{2}{3} \times 2 \times 554 \times 2} = \frac{5188}{554}$$

$= 9{\cdot}365$ feet. Ans.

Example. A rectangular bulkhead is 18 feet high, and the vertical stiffeners are spaced at regular intervals of 27 inches. The bulkhead is exposed to sea water. Find the shearing force on the stiffeners at top and bottom of their length, and the position where the shearing force is zero.

Since the load on a surface immersed in a liquid varies directly as the depth of immersion, the load on the bulkhead, and therefore the load on each stiffener increases uniformly from zero at the top to a maximum at the bottom.

Each stiffener may be regarded as a beam, supported at its ends, and carrying a load which increases uniformly from 0 at the top to $18 \times \frac{27}{12} \times 1 \times 64 = 2{,}592$ lb. per ft. at the bottom.

$$\text{Total load} = \frac{2592 \times 18}{2} = 23{,}328 \text{ lb.}$$

Reaction at top, and shearing force at top

$$23{,}328$$

$$= 7{,}776 \text{ lb.} \quad \text{Ans.}$$

Reaction at bottom, and shearing force at bottom

$$= 7{,}776 \times 2 = 15{,}552 \text{ lb.} \quad \text{Ans.}$$

$$\text{Shearing force is zero at } \frac{18}{\sqrt{3}} = 10{\cdot}392 \text{ feet from top.}$$

The student should refer to page 259 for the investigation of the case of a beam loaded in this manner.

Boiler Density.

When sea water is evaporated in a boiler or in an evaporator, the solid matter is left behind, the steam generated being pure water vapour. One gallon of fresh water weighs 10 lb. or 160 ozs.; one gallon of sea water weighs about 165 ozs., so that roughly there are 5 ozs. of solid matter in 160 ozs. of fresh water or $\frac{5}{160}$ or $\frac{1}{32}$ as a fraction of the weight of fresh water present. The density of sea water is said to be $\frac{1}{32}$, this means 5 ozs. of solids per gallon; a density of $\frac{3}{32}$ means 15 ozs. of solids per gallon. When a boiler is using sea water or part sea water as

feed, the density of the water in the boiler rises as the solids are left behind by evaporation, and a limit to this density must be fixed because of the danger to the heating surface by precipitation of the solids. The usual limit is $\frac{4}{32}$ or 20 ozs. of solids per gallon. After this limit has been reached, the boiler must have a quantity of the dense water blown out, this water being replaced by sea water, or by feed which is partly sea water. To keep the density constant, then in any given time, the amount of solids blown out must equal the amount taken in by the feed water, and this may be written as :—

Amount of feed $\times$ feed density $=$ Amount blown out $\times$ boiler density.

This assumes that water is being constantly blown out of the boiler. Sometimes the water in a boiler has its density reduced by first blowing out a certain quantity, and then pumping up with fresh water, or perhaps with sea water. The density will be reduced by pumping in water of less density than the original boiler water density.

Example. A boiler is worked at $\frac{4}{32}$, and the feed water contains 3 ozs. of solids per gallon. What amount of water must be blown continuously out of the boiler, as a fraction of the feed pumped in ?

Amount feed $\times$ Feed density $=$ Amount blown out $\times$ boiler density.

Let the feed pumped in $= 1$, and expressing the density in both cases as ozs. per gallon :—

$1 \times 3 =$ Amount blown out $\times\ 20$.

Amount blown out $= \frac{3}{20}$ of the feed taken in. Ans.

Example. A boiler contains 40 tons of water at a density of $3\frac{1}{2}$ times the density of sea water ; 8 tons of water are blown out and replaced by sea water. What is now the density of the boiler ?

$x =$ final density, then

$(40 \times 3\frac{1}{2}) - (8 \times 3\frac{1}{2}) + (8 \times 1) = 40 \times x$

$140 - 28 + 8 = 40\,x$

$x = 3$ times the density of the sea. Ans.

Note that all the densities must be in the same units. Here the densities are all expressed as multiples of the density of the sea.

Example. An engine of 1,500 I.H.P. uses 15 lb. of steam per I.H.P. hour. The boilers are filled with 50 tons of sea water at the beginning of the voyage, and after 12 days steaming the density has risen to 17 ozs. per gallon. Find the average density of the hotwell in ozs. per gallon.

$$\text{Feed used in 12 days} = \frac{1500 \times 15 \times 24 \times 12}{2240} \text{ tons.}$$

Total feed $\times$ Average feed density = Water in boilers $\times$ rise in density.

$$\therefore \text{Feed density} = \frac{\text{Water in boilers} \times \text{Rise in density}}{\text{Total feed}}$$

$$\text{Feed density} = \frac{50 \times 2240 \times (17 - 5)}{1500 \times 15 \times 24 \times 12} \qquad \frac{28}{15 \times 9}$$

Feed density = 0·2074 ounce per gall. Ans.

Example. An evaporator is filled with 0·96 ton of sea water, and is worked without blowing for $2\frac{3}{4}$ hours, the density being then $\frac{3}{32}$. Find the rate at which fresh water is made in gallons per hour.

Rise in density $= \frac{3}{32} - \frac{1}{32} = \frac{2}{32}$.

No. of times water has been changed $= \dfrac{\frac{2}{32}}{\;} = 2$.

Total feed $= 0{\cdot}96 \times 2 = 1{\cdot}92$ tons.

$$\text{Weight of fresh water made} = 1{\cdot}92 \times \frac{62{\cdot}5}{64} \text{ tons.}$$

$$\text{Water made} = 1{\cdot}92 \times \frac{62{\cdot}5}{64} \times \frac{2240}{10} \times \frac{\;}{2{\cdot}75} \text{ galls per hour.}$$

Water made = 152·7 gallons per hour. Ans.

Note the evaporator was full when starting, and is supposed to have the same amount of water in at the finish; therefore the whole of the feed pumped in has been evaporated.

Example. Steam is supplied to an engine at 180 lb. per sq. inch gauge, temp. 379°F. The feed temp. is 160°F. and the coal consumption is 25 tons per day. The surface condenser starts to leak, and the average feed water density is 0·15 of the

sea water density. The boilers are kept at 3·25 times the sea water density by resorting to blowing down. Estimate the coal consumption in these circumstances.

Heat units necessary to form 1 lb. of steam before blowing down,

$$= (379 - 160) + 966 - 0{\cdot}7\ (379 - 212) = 1068{\cdot}1 \text{ B.T.U.}$$

Fraction of the total feed that must be blown out to maintain

$$\text{a constant boiler density} = \frac{\text{Feed density}}{\text{Boiler density}} = \frac{0{\cdot}15}{3{\cdot}25} = \tfrac{3}{65}$$

and the fraction of the feed that is left to form steam

$$= 1 - \tfrac{3}{65} = \tfrac{62}{65}.$$

This means that for every 62 lb. of steam formed, 65 lb. of feed water must be supplied and 3 lb. must be blown out.

Or, to form 1 lb. of steam, $\frac{65}{62}$ lb. of feed must be supplied, and $\frac{3}{62}$ lb. will be blown out.

Heat units necessary to form 1 lb. of steam when blowing down $= \frac{65}{62}\ (379 - 160) + 966 - 0{\cdot}7\ (379 - 212) = 1078{\cdot}7$ B.T.U.

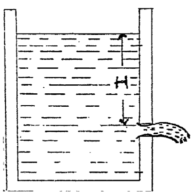
Note.—Sensible heat must be given to all the feed, but latent heat only to that fraction which is evaporated.

$$\text{Coal consumption} = \frac{1078{\cdot}7}{1068{\cdot}1} \times 25 = 25{\cdot}25 \text{ tons per day.}$$

Ans.

Discharge of Water from an Orifice.

When water is delivered or discharged through a sharp edged circular orifice, the area of the stream of water delivered is less than the area of the orifice. This is due to the bad form of the sharp edged entrance to the orifice, which causes eddying of the water. For a head of water H feet the velocity of water should be the same as that of a body falling freely through the vertical height H, or since $v^2 = 2\ g$ H, then $v = \sqrt{2\ g\ H}$. Due to frictional losses, the actual velocity is about 0·97 of that due to free fall. The area of the stream is reduced to about 0·64 of the area of the orifice. The co-efficient of discharge is therefore 0·97 × 0·64 = 0·62 about, and the quantity discharged per second will be :



Quantity discharged $= 0{\cdot}62 \times$ Area $\times$ Velocity.

Where A = Area of orifice in square feet.

v = Velocity in feet per second.

Quantity $= 0{\cdot}62 \times A \times \sqrt{2\ g\ H}$ cubic feet per second.

Example. Find the weight of fresh water delivered per minute through a sharp edged circular orifice of 2 square inches area, the head above the orifice being 10 feet of fresh water.

Cubic feet per min. $= 0{\cdot}62 \times \frac{2}{144} \times \sqrt{64{\cdot}4 \times 10} \times 60 = 13{\cdot}11$

Weight per minute $= 13{\cdot}11 \times 62{\cdot}5 = 819{\cdot}4$ lb. Ans.

For water flowing through pipes of constant bore, the quantity discharged will be Area $\times$ Velocity, when the pipe is full; when the pipe is only partly full the co-efficient of area should be given.

Example. In order to obtain values of the co-efficients of velocity, contraction of area and discharge, water is allowed to issue from a circular sharp edged orifice of area 1 sq. inch, fitted in the side of a tank. A constant head of 8 feet is maintained over the orifice, and the jet is observed to pass through a ring, whose centre is 6 feet horizontally from the orifice, and 1·2 feet below the orifice. 36·7 gallons are discharged per minute.

From this data determine the co-efficients.

The jet has no vertical velocity immediately it issues from the orifice. The jet moves over a horizontal distance of 6 feet in the same time that it falls a vertical distance of 1·2 feet.

$$\therefore\ 1{\cdot}2 = \tfrac{1}{2}\ g \times t^2, \qquad t = \sqrt{\frac{1{\cdot}2}{16{\cdot}1}} = 0{\cdot}273 \text{ sec.}$$

Horizontal velocity of jet $\times\ 0{\cdot}273 = 6$

$$\text{,,} \qquad \text{,,} \qquad \text{,,} \qquad = \frac{6}{0{\cdot}273} = 21{\cdot}98 \text{ feet per sec.}$$

Theoretical velocity $= \sqrt{2\ g \times 8} = 22{\cdot}7$ feet per sec.

$$\therefore \text{ Coefficient of velocity } = \frac{21{\cdot}98}{22{\cdot}7} = 0{\cdot}968. \quad \text{Ans.}$$

Quantity discharged = Area of orifice × Coeff. of discharge × Theoretical velocity × time.

$$\frac{36{\cdot}7}{6{\cdot}25} = \frac{1}{144} \times \text{Coeff.} \times 22{\cdot}7 \times 60$$

$$\therefore \text{ Coefficient of discharge } = \frac{36{\cdot}7 \times 144}{3{\cdot}25 \times 22{\cdot}7 \times 60}$$

$= 0{\cdot}6208$. Ans.

Coeff. of velocity × Coeff. of contraction = Coeff. of discharge.

$$\therefore \text{ Coefficient of contraction of area } = \frac{0{\cdot}6208}{0{\cdot}968}$$

$= 0{\cdot}641$. Ans.

*Energy of Water. Bernouilli's Theorem.

The total energy of a quantity of water consists of :—

1. Potential energy, or energy due to position.
2. Pressure energy.
3. Kinetic energy or energy due to velocity.

Consider 1 lb. of water, and let w lb. be the weight per cubic foot. Let p lb. be the pressure per sq. foot ; v feet per sec. be the velocity, and let the water be at h feet above datum level.

The potential energy of 1 lb. is h ft. lb.

The pressure energy is $\frac{p}{w}$ ft. lb. because the vol. of 1 lb. is $\frac{1}{w}$ cu. foot, the capability of doing work due to the pressure of p lb. per sq. foot is pressure × volume. Pressure energy of

$$1 \text{ lb.} = p \times \frac{1}{w} = \frac{p}{w} \text{ ft. lb.}$$

The kinetic energy is $\frac{\quad}{2\,g}$ ft. lb.

$$\therefore \text{ the total energy } H = h + \frac{p}{w} + \frac{\quad}{2\,g}.$$

Now if all particles of the water move with the same velocity at any instant ; if there is no frictional resistance to the motion ; if the water is regarded as incompressible, and if no work is done by the water or done upon the water, then

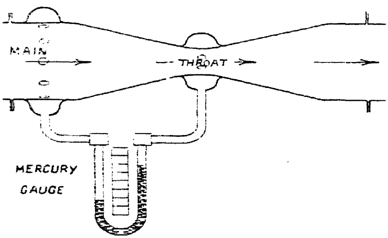
$$H = h + \frac{p}{w} + \frac{v^2}{2\,g}$$ is a constant quantity, although

h, p and v may vary.

This is Bernouilli's theorem.

*Venturi Water Meter.

This instrument is used to measure the flow of large quantities of water, and its principle of operation is based upon Bernouilli's theorem. Essentially, the meter consists of a short length of pipe, tapering to a "throat" at the middle of its length ; and means of measuring the difference between the pressure in the main and the pressure at the throat.



The throat is usually about $\frac{1}{3}$ of the diameter of the main. Annular belts are cast around main and throat, and these belts communicate with the inside of the meter by a number of small holes. The belts are connected to a U gauge containing mercury, and hence the difference in pressure is determined. The centre line of the meter must be arranged horizontally.

At the main let a_1 be the area in sq. feet ; p_1 be the pressure in lb. sq. foot and v_1 be the velocity in feet per sec.

At the throat let the conditions be a_2 and

The quantity of water passing along the main is the same as that passing through the throat.

$$\text{and } v_1 = v_2 \times \frac{a_2}{a_1}, \quad \text{or } v_2 = v_1 \times \frac{a_1}{a_2}$$

By Bernouilli's theorem,

$$h_1 + \frac{p_1}{w} + \frac{v_1^2}{2\,g} = h_2 + \frac{p_2}{w} + \frac{v_2^2}{2\,g},$$ but $h_1 = h_2$ since the meter is horizontal.

$$\therefore \frac{p_1}{w} + \frac{v_1^2}{2\,g} = \frac{p_2}{w} + \frac{v_2^2}{2\,g}$$

$$\frac{p_1 - p_2}{w} = \frac{v_2^2 - v_1^2}{2\,g}$$

If the value of v_1 is substituted,

$$\frac{p_1 - p_2}{w} = \frac{v_2^2 - v_2^2 \times \left(\frac{a_2}{a_1}\right)^2}{2\,g}$$

If the value of v_2 is substituted,

$$\frac{p_1 - p_2}{w} \quad \frac{v_1^2 \times \left(\frac{a_1}{a_2}\right)^2 - v_1^2}{2\,g} \quad \frac{v_1^2 \left\{ \left(\frac{a_1}{a_2}\right)^2 - 1 \right\}}{2\,g}$$

Hence the velocity of flow at either the throat, or at the main is found, and the quantity passing per sec. = (Area $\times$ velocity) cubic feet. $(p_1 - p_2)$ is the difference in pressure at main and throat, expressed in lb. per sq. foot. Note that the pressure in the main is greater than that at the throat. The kinetic energy at the throat is greater than that in the main, but the sum of pressure energy and kinetic energy is the same at both parts.

Suppose the question given stated that the pressure difference was equivalent to 8 feet head of water. Now 1 cu. foot of water weighs 62·5 lb., and therefore 1 foot head of water gives a pressure of 62·5 lb. per sq. foot.

8 feet head $= 8 \times 62{\cdot}5$ lb. per sq. foot,

$$\frac{p_1 -}{w} \qquad \times 62$$

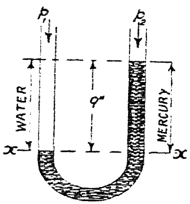
If the question stated that the pressure difference was equivalent to 9 inches of mercury of specific gravity 13·6.

1 cu. ft. of mercury weighs $13 \cdot 6 \times 62 \cdot 5$ lb., and 1 ft. head of mercury would give $13 \cdot 6 \times 62 \cdot 5$ lb. pressure per sq. foot.

$$\therefore 9 \text{ inches head of mercury} = \frac{13 \cdot 6 \times 62 \cdot 5 \times 9}{12} \text{ lb. sq. ft.}$$

$$\text{and } \frac{p_1 - p_2}{w} = \frac{13 \cdot 6 \times 62 \cdot 5 \times 9}{62 \cdot 5 \times 12} = 10 \cdot 2.$$

If, however, the question stated that the mercury gauge showed a difference of levels of 9 inches, and the gauge is full of water up to the mercury level.



Consider the state of equilibrium at the level $x\ x$.

p_1 + pressure due to 9″ of water = p_2 + press. due to 9″ of mercury.

$\therefore p_1 - p_2 =$

press. due to 9″ of mercury
— press. due to 9″ of water.
= press. due to $(9 \times 13 \cdot 6)''$ of water
— press. due to 9″ of water.
= press. due to $(9 \times 12 \cdot 6)$ ins. of water.

$$\frac{9 \times 12 \cdot 6 \times 62 \cdot 5}{12} \text{ lb. per sq. foot.}$$

$$\text{and } \frac{p_1 - p_2}{w} = \frac{9 \times 12 \cdot 6 \times 62 \cdot 5}{62 \cdot 5 \times 12} =$$

Example. The diameter of the main of a Venturi water meter is 12 inches and the throat is 5 inches diameter. If the pressure difference between main and throat is equivalent to 10 feet head of water, find the gallons of water passing through the meter per minute.

$$\frac{p_1 - p_2}{w} \qquad \frac{w_1}{}) - 1$$

$$- p_2 \qquad 10 \times 62{\cdot}5$$

$$w \qquad 62{\cdot}5$$

$$\left(\frac{12^2}{5^2}\right)^2 = \frac{20{,}736}{625}$$

$$\therefore \left(\frac{a_1}{a_2}\right)^2 - 1 = \frac{20{,}736}{625} - 1 = \frac{20{,}111}{625}$$

$$10 = \frac{v_1^2 \times \dfrac{20{,}111}{625}}{2\,g}, \quad v_1 = \sqrt{\frac{10 \times 2\,g \times 625}{20{,}111}} = 4{\cdot}474$$

feet per sec. (velocity in main).

$$\text{Gallons per minute} = \frac{12^2 \times 0{\cdot}7854}{144} \times 4{\cdot}474 \times 60 \times 6{\cdot}25 = 1{,}318 \text{ gallons. Ans.}$$

Work out the foregoing example if the pressure difference is equivalent to 8·5 inches of mercury, of Sp. Gr. 13·6.

$$p_1 - p_2 \qquad 8{\cdot}5 \times 13{\cdot}6 \times 62{\cdot}5$$

$$w \qquad 12 \times 62{\cdot}5$$

$$v_1 = \sqrt{\frac{\quad}{20{,}111}} = 4{\cdot}39 \text{ feet per sec. in main.}$$

Quantity per minute $\propto$ velocity

$$\text{Gallons per minute} = 1{,}318 \times \frac{4{\cdot}39}{4{\cdot}474} = 1{,}293 \text{ gallons. Ans.}$$

If the difference in level of the mercury in the gauge was 8·5 inches, and the gauge was full of water to the mercury level, what would be the quantity per minute?

$$\frac{p_1 - p_2}{w} \qquad \frac{8{\cdot}5 \times 12{\cdot}6 \times 62{\cdot}5}{12 \times 62{\cdot}5} = 8{\cdot}925$$

$$v_1 = \sqrt{\frac{8 \cdot 925 \times 2\,g \times 62 \cdot 5}{20,111}} = 4 \cdot 227 \text{ feet per sec. in main.}$$

$$\text{Gallons per minute} = 1,318 \times \frac{4 \cdot 227}{4 \cdot 474} = 1,246 \text{ gallons.} \quad \text{Ans.}$$

TEST EXAMPLES XXVI.

1. The specific gravity of mercury is 13·6, find the volume of one lb. 2·032 cu. ins. Ans.

2. A plank of wood 12 feet long, 12 inches wide and 6 inches deep, floats in fresh water at a mean draught of 4 inches, find its specific gravity and its weight, and find the draught in sea water. $\frac{4}{6}$; 250 lb.; 3·906 ins. Ans.

3. A drum made of sheet steel weighing 1·75 lb. per square foot is $2\frac{1}{2}$ feet diameter and 4 feet long. It is filled with oil of specific gravity 0·88. Find its draught when floating in fresh water with its axis vertical. 3·753 feet. Ans.

4. A plank weighs 25 lb., and floats in sea water. A weight of 8 lb. placed on the plank at its mid length just submerges it. Find the specific gravity of the plank. 0·7756. Ans.

5. A cylindrical buoy weighs 2 tons and floats in fresh water with its axis vertical at a draught of 4 feet. A piece of cast iron weighing half a ton in air, is suspended from beneath the buoy. Find the increase in draught.

0·8611 foot. Ans.

6. A piece of a certain metal weighs 3 lb. in air, and 2·75 lb. when suspended in fresh water. Find the weight of one cubic foot of the metal.

750 lb. Ans.

7. What must be the height of a column of oil weighing 54·8 lb. per cubic foot, to balance a height of 30 inches of mercury? 38·73 feet. Ans.

8. A piece of brass weighing 5 lb., of specific gravity 8·4, is submerged in oil of specific gravity 0·82. Find the pull on the string. 4·512 lb. Ans.

9. A chain 15 feet long weighs 12 lb. per foot run and its specific gravity is 7·6. The chain lies at the bottom of a dock containing fresh water, the depth of which is 10 feet. Find the work done to lift the chain by one end until it is just clear of the water.

2,913·1 foot lb. Ans.

10. Feed water is delivered into a boiler against a pressure of 220 lb. per square inch, find the work done in pumping in one cubic foot. If the engine indicates 1,000 I.H.P. and uses 18 lb. of steam per I.H.P. hour, find the horse power of the pump.

31,680 ft. lb. ; 4·608 H.P. Ans.

11. A feed pump can pump out a tank in 15 hours, and the ballast pump can pump the same tank out in 6 hours. When both pumps work together, in what time can the tank be pumped out ?

$4\frac{2}{7}$ hours. Ans.

12. A ballast tank is 60 feet long, 40 feet wide and 4 feet deep, and is filled with sea water. Find the pressure on the top, bottom and on one long side when (*a*) the tank is just full, (*b*) when the sounding pipe has 10 feet of water in it above the tank top.

(*a*) 0 ; $274\frac{2}{7}$; $13\frac{5}{7}$ tons. Ans.
(*b*) $685\frac{5}{7}$; 960 ; $82\frac{2}{7}$ tons. Ans.

13. A dock gate is 20 feet wide by 16 feet deep, and has fresh water on one side to a depth of 10 feet. Find the total load on the gate and the position of the centre of pressure.

27·9 tons ; $3\frac{1}{3}$ feet from bottom. Ans.

*14. A sluice door 5 feet wide, 10 feet high, has its top edge 20 feet below the surface of sea water ; find the total load on the door and the position of the centre of pressure.

35·71 tons ; $4\frac{2}{3}$ feet from bottom. Ans.

15. A cylindrical tank has its axis vertical. It is 15 feet diameter and 15 feet high. The tank is filled by a pump working at 50 lb. per square inch, and the bottom of the tank is 15 feet above the pump. The pipe is 4 inches diameter. Find the time taken to fill the tank.

6·84 mins. Ans.

16. The surface condenser of a marine engine is leaking and in consequence the feed water density is 0·25 of the sea density. Blowing down from the boilers is resorted to, to maintain the boilers at a certain density. The steam temperature

is 390°F., and the feed water temperature is 190°F., whilst the increase on the coal consumption is found to be 3 per cent. What boiler density is being maintained?

1·85 times the density of the sea. Ans.

17. In a certain vessel, the boilers need 12 tons of extra feed per day. The evaporator holds three-quarters of a ton of water and is run continuously, being filled up at the beginning and blown out at the end of the watch of 4 hours. Find the density in ozs. per gallon of the water in the evaporator at the end of the watch. 18·65 ozs. per gallon. Ans.

18. An evaporator holds 0·75 of a ton of sea water and it is worked without blowing down for 4 hours. The water is maintained at a constant level in the gauge glass, and the density is found to be 19 ozs. per gallon at the end of the 4 hours. How many tons of water have been evaporated?

2·1 tons. Ans.

19. A boiler contains 25 tons of fresh water, and it is run without blowing down until a density of 18 ozs. per gallon is reached. If 1,000 ozs. of sea water contains 23 ozs. of sodium chloride, 1·4 ozs. of sulphate of lime and 0·33 oz. of carbonate of lime, find the amount of scale forming matter taken into the boiler.

474 lb. Ans.

20. A boiler contains 50 tons of water at a density of 20 ozs. per gallon, what weight must be blown out and replaced by sea water to reduce the density to 17 ozs. per gallon? Also what weight must be blown out and made up with fresh water to reduce the density to 17 ozs. per gallon?

10 tons: 7·5 tons. Ans.

CHAPTER XXVII.

BUOYANCY AND STABILITY OF SHIPS.

The **Co-efficient of Water Plane Area** of a ship is the ratio of the area of the water plane to that of a rectangle, which has the same extreme dimensions.

$$\text{Co-efficient of water plane area} = \frac{\text{Area of water plane}}{\text{Length} \times \text{Beam}}$$

Example. The semi-ordinates measured across a vessel at her load water line are : 0·2, 9, 15·5, 19·5, 21, 20, 18, 12·2, and 2 feet respectively. The length of the vessel is 360 feet and the maximum breadth is 42 feet. Find the co-efficient of water plane area.

By Simpson's Rule :—

Semi-ordinates	Simpson's Multipliers	Functions of Semi-ordinates
0·2	1	0·2
9	4	36
15·5	2	31
19·5	4	78
21	2	42
20	4	80
18	2	36
12·2	4	48·8
2	1	2

Sum = 354·0

Common interval between ordinates $= \frac{360}{8} = 45$ feet.

Half area $= \frac{45}{3} \times 354$ **sq. ft.**

$\therefore$ **Area** $= \frac{45}{3} \times 354 \times 2 = 10{,}620$ **sq. ft.**

Co-efficient of water plane area =

$$\frac{\text{Area of water plane}}{\text{Length} \times \text{Beam}} = \frac{10{,}620}{360 \times 42} = 0{\cdot}702. \quad \text{Ans.}$$

The Co-efficient of Immersed Midship Section is the ratio of the cross-sectional area of the immersed part of the ship amidships, to that of a rectangle which has the same extreme dimensions.

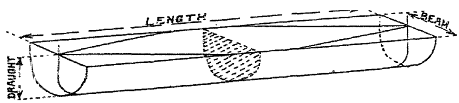
Co-efficient of immersed midship section =

$$\frac{\text{Area of immersed midship section}}{\text{Draught} \times \text{Beam}}$$

The **Block Co-efficient** or **Co-efficient of Fineness of Displacement** is the ratio of the immersed volume of the ship to that of a rectangular block, which has the same extreme dimensions.

$$\text{Block co-efficient} = \frac{\text{Under water volume of ship}}{\text{Length} \times \text{Beam} \times \text{Draught}}$$

The **Prismatic Co-efficient** is the ratio of the immersed volume of the ship to that of a rectangular prism, whose length is equal to the length of the ship and whose constant cross-section is equal to the immersed midship section of the ship.



Prismatic co-efficient =

$$\frac{\text{Under water volume of ship}}{\text{Immersed midship sectional area} \times \text{Length}}$$

Example. A vessel, which is 400 ft. long and 50 feet broad, has a draught of 25 feet in sea water, when her displacement is 10,800 tons. The immersed midship section is 1,148 sq. feet. Find (*a*) the co-efficient of immersed midship section, (*b*) block co-efficient, (*c*) prismatic co-efficient.

Unless otherwise stated, one cubic foot of sea water may be taken as weighing 64 lb., that is, 35 cubic feet = 1 ton.

Volume of water displaced $= 10{,}800 \times 35 = 378{,}000$ cu. ft.

$$\text{Co-effic. of immersed midship section} = \frac{1{,}148}{25 \times 50}$$

$$= 0{\cdot}918. \quad \text{Ans. } (a)$$

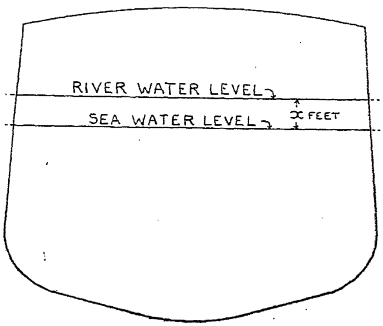
$$\text{Block co-efficient} = \frac{378{,}000}{400 \times 50 \times 25} = 0{\cdot}756. \quad \text{Ans. } (b).$$

$$\text{Prismatic co-efficient} = \frac{378{,}000}{1{,}148 \times 400} = 0{\cdot}823. \quad \text{Ans. } (c).$$

Change in Draught with Change in Density of Water.

The total downward weight of the ship and cargo is exactly equal to the upward force of the displaced water, and therefore the draught of a vessel will depend upon the density of the water in which it floats. The density of the water is measured by the hydrometer (a salinometer is a form of hydrometer). The hydrometer is graduated to show the density in ounces per cubic foot. The reading for fresh water is 1,000, as one cubic foot of fresh water weighs 1,000 ounces ; and for sea water the reading is about 1,026. As river water is often partly salt, readings will be between 1,000 and 1,026.

When a ship moves from river water to sea water, or vice versa, and the tons displacement does not alter, there will be a change in draught because there is a change of water density. The change will be a small amount, and it is therefore reasonable to assume that the water plane area at the river water level is the same as at the sea water level.



Let T = tons displacement.

W.P.A. = water plane area (sq. feet).

Then $T \times 2240 \times 16$ = displ. of ship in ozs.

and $\dfrac{T \times 2240 \times 16}{\text{Hydr. reading}}$ = cu. feet of water displaced.

If H_r = hydrometer reading in river water

H_s = ,, ,, ,, sea ,,

x = change of draught in feet.

$$\text{W.P.A.} \times x = \frac{T \times 2240 \times 16}{H_r} - \frac{T \times 2240 \times 16}{}$$

$$x = \frac{T \times 2240 \times 16}{\text{W.P.A.}} \quad \frac{1}{H_r}$$

Example. A vessel has a water plane area of 15,000 square feet and a displacement of 7,000 tons. In passing from sea water into a river, where the hydrometer reading was 1,005, the draught increased by 3·9 inches. Find the hydrometer reading of the sea water.

Change of draught in inches =

$$\frac{T \times 2{,}240 \times 16 \times 12}{\text{W.P.A.}} \left\{ \frac{1}{\text{Hydr. river}} \quad \text{Hydr. sea} \right\}$$

$$= \frac{7{,}000 \times 2{,}240 \times 16 \times 12}{15{,}000} \quad \frac{1}{1{,}005} \quad \frac{1}{\text{Hydr. sea}}\Big\}$$

$$\frac{3\cdot9 \times 15{,}000}{7{,}000 \times 2{,}240 \times 16 \times 12} \;:\; \frac{1}{1{,}005} \quad \frac{1}{\text{Hydr. sea}}$$

$$0\cdot00001944 = 0\cdot0009951 - \frac{1}{\text{Hydr. sea}}$$

$$\frac{1}{\text{Hydr. sea}} = 0\cdot0009951 - 0\cdot00001944 = 0\cdot0009757$$

$$\text{Hydr. sea} = \frac{1}{0\cdot0009757} = 1{,}025.$$

Hydrometer reading of sea water = 1,025 ozs. Ans.

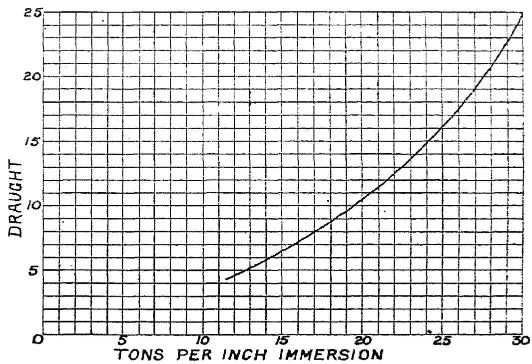
The **Tons per Inch Immersion** is the weight in tons, which, if placed on board, would sink the vessel one inch ; or, if lightened by the same amount, would allow the vessel to rise one inch. Again, letting A represent the water plane area in square feet then when the vessel sinks one inch the increased volume of

water displaced will be $A \times \frac{1}{12}$ cubic feet, which is $\frac{A}{12 \times 35}$ tons of sea water. Hence the tons per inch immersion $= \frac{A}{12 \times 35} \quad \frac{A}{420}$

As the water plane areas are different for all different draughts, then so is the tons per inch immersion different. A curve representing the tons per inch immersion with respect to the draught is a handy means of determining how many tons of cargo can be taken aboard for a given change of draught. The graph is plotted so that the horizontal measurements represent tons per inch immersion, and the vertical measurements represent draught. Areas within the curve therefore represent displacement.

Example. At the load water line the draught of a vessel is 25 feet and the area of the water plane is 12,600 sq. feet. The areas of parallel waterplanes below this, at regular intervals of 5 feet, are 11,550, 10,080, 8,190 and 5,250 sq. feet respectively. Draw a curve of tons per inch immersion and from this graph estimate the weight of cargo, that would increase the draught from 16 to 18 feet.

At 25 ft. draught, tons per inch immersion	$= \frac{12600}{420}$	30
,, 20 ft. ,, ,, ,,	$= \frac{11550}{420}$	27·5
,, 15 ft. ,, ,, ,,	$= \frac{10080}{420}$	24
,, 10ft. ,, ,, ,,	$= \frac{8190}{420}$	19·5
,, 5 ft. ,, ,, ,,	$= \frac{5250}{420}$	12·5



From the graph, obtain the mean tons per inch immersion between 16 and 18 feet draughts, this is 25·6. Therefore to put the ship down from 16 to 18 feet, i.e., 24 inches, the cargo taken aboard will be $25·6 \times 24 = 614·4$ tons. Ans.

Note that in this example, the curve between 16 and 18 feet draught is practically a straight line, and the mean tons per inch immersion is that for 17 feet draught. If, however, we are to find the mean tons per inch immersion over a greater range of draught, say, from 14 to 20 feet, it would be better to find the mean tons per inch immersion by, first, obtaining from the graph the tons per inch immersion for 14, 15, 16, 17, 18, 19 and 20 feet respectively, and then putting these values through Simpson's rule.

Reserve Buoyancy.

The buoyancy of a vessel is the weight of water which is displaced, and the watertight volume of the vessel above its water line is reserve buoyancy. If a compartment was holed and open to the sea a certain amount of buoyancy is lost, and for the ship to remain afloat the reserve of buoyancy will be decreased by the same amount. If the reserve of buoyancy is less than the buoyancy lost due to bilging the compartment, then the vessel will sink.

Example. A box barge is 180 feet long, 30 feet broad, 14 feet deep, and floats at a draught of 8 feet. Two transverse bulkheads, extending up to the deck, form a central compartment 25 feet long. If this compartment is opened to the sea find the new draught.

The original water plane area was 180×30 sq. feet.

The final water plane area is $(180 - 25) \times 30$ sq. feet.

The weight of the barge does not change and therefore it must still displace the same volume. Let x feet be the new draught, then,

$$(180 - 25) \times 30 \times x = 180 \times 30 \times 8$$

$$x = \frac{180 \times 8}{155} = 9·29 \text{ feet. Ans.}$$

Suppose this central compartment was fitted with a watertight deck at 4 feet above the bottom of the barge, and this lower space was opened to the sea, what would be the new draught ?

The water plane now remains constant at 180×30 sq. feet.

Loss of buoyancy $= 30 \times 25 \times 4$

Gain of buoyancy $= 180 \times 30 \times y$, y being the increase of draught.

$$180 \times 30 \times y = 30 \times 25 \times 4$$

$$\therefore y = \frac{100}{180} = 0{\cdot}555 \text{ foot.}$$

New draught $= 8{\cdot}555$ feet. Ans.

The **Permeability** of a compartment is a measure of the proportion of the total volume of the compartment to which water could gain access, if the compartment became flooded. As ordinarily stowed, the coal in coal bunkers occupies 44 cubic feet per ton. The specific gravity of coal is about 1·28, and one ton of solid coal occupies $\dfrac{2{,}240}{1{\cdot}28 \times 62{\cdot}5} = 28$ cubic feet.

The permeable space is therefore $44 - 28 = 16$ cubic feet, and the permeability is $\frac{16}{44} \times 100 = 36{\cdot}4\%$. This is the permeability of full bunkers, but their average condition between the start and the finish of a voyage is half full. If, therefore, we consider $\frac{1}{2}$ ton of solid coal in the space into which one ton of coal was stowed, then the permeable space is $44 - 14 = 30$ cubic feet, and the average permeability is $\frac{30}{44} \times 100 = 68{\cdot}2\%$.

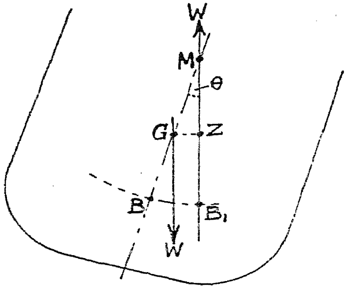
The permeability of bunker spaces is often taken to be 60%, but that of machinery spaces will be higher, it may be 80% to 85%.

Stability.

The **Centre of Gravity** (G) of a ship, is that position where the whole weight of the ship (including everything she has on board) may be considered as acting. Thus, the weight of a ship may be considered as a downward force acting through its centre of gravity.

The **Centre of Buoyancy** (B) is the centre of gravity of the displaced water. Now, the law of Archimedes states that a floating body displaces an amount of water equal to its own weight, the weight of water displaced is therefore equal to the weight of the ship. Buoyancy is therefore an upward force supporting the ship, and this upward force may be regarded as acting through the centre of buoyancy.

When a ship heels the volume of water displaced does not change, but the shape of the transverse section of this volume does change, and the centre of buoyancy takes up a new position.



If the positions of B, for various small angles of heel, are plotted, they are found to lie on a curve, and this is the **Curve of Buoyancy.** If the angle of heel is kept within the limits of say 10° or 12° on either side of the vertical, the curve of buoyancy is practically the arc of a circle. The centre of the circle which practically coincides with the curve of buoyancy is the **Transverse Metacentre.** It is denoted by M. For small angles of heel the transverse metacentre may be regarded as a fixed point, and the centre of buoyancy is vertically below it.

The position of G, the centre of gravity of the ship, does not alter if there is no movement of weights or cargo aboard the ship, and the distance separating G from M is the **Transverse Metacentric Height.** This is denoted by $\overline{GM}$.

Consider a ship heeled to an angle θ from the vertical. The weight of the ship (W) acts vertically down through G whilst the upward force due to the weight of water displaced (W) acts vertically up through B_1 and through M. These two equal and opposite forces constitute a couple of magnitude $W \times \overline{GZ}$ which tends to restore the ship to its vertical position. This couple is the **Righting Moment** or **Moment of Statical Stability.**

Now, $\overline{GZ} = \overline{GM} \sin. \theta$, but for a small angle, the sine, tangent, and the circular measure, i.e., the angle expressed in radians, have all the same value.

Thus, sine 5° = 0·0872 ; tangent 5° = 0·0875 ; and 5° = $\frac{5}{360} \times 2\pi$ = 0·08727 radian. Again, sine 10° = 0·1736 ; tangent 10° = 0·1763 ; and 10° = 0·17454 radian ; here there is a difference, but it is not large.

Therefore, for small angles of heel $\overline{GZ} = \overline{GM}\,\theta$, and the righting moment $= W\,\overline{GM}\,\theta$. If a ship is to be stable when floating upright, that is, if it is to be able to return to the vertical condition after being heeled, then G must be below M, or we may say $\overline{GM}$ must have a positive value. If G was above M

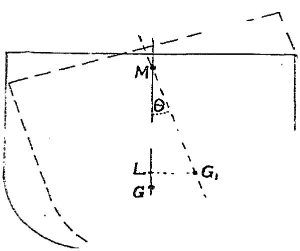
then the metacentric height would be negative, and the ship could not float in the vertical condition. She might, however, heel over to port or starboard, to say 15° or 20° and be quite stable in that position, having then a positive metacentric height. Examples of this are seen in timber laden vessels with large deck cargoes, which float with quite a considerable list, and are perfectly stable in that position. They then have a positive $\overline{GM}$, whereas when the ship is vertical the $\overline{GM}$ is negative, and the ship is quite unable to maintain that position.

It should be remembered that for small angles of heel, the curve of buoyancy is practically the arc of a circle, and M is its centre. This, however, does not apply to large angles of heel.

The position of G, the centre of gravity of the ship, with respect to the metacentre is very important. If heavy cargo is stowed low down in a ship, G will be nearer the keel than if a light cargo was on board. If G is low down in the ship, then $\overline{GM}$ and also the righting moment will have a large value; the ship will be "stiff," and when heeled by wave action will return quickly to the upright. If G is high in the ship, then $\overline{GM}$ and the righting moment will be small; the ship will be more easily heeled and may be said to be "tender," but the return to the vertical will be slow and the motion more comfortable.

Cargo vessels have a positive $\overline{GM}$ of about one foot. Sailing vessels which have to carry a large spread of canvas, and war vessels which require steady gun platforms, may have metacentric heights of about 4 feet.

The work done in heeling a vessel is called **Dynamical Stability.**



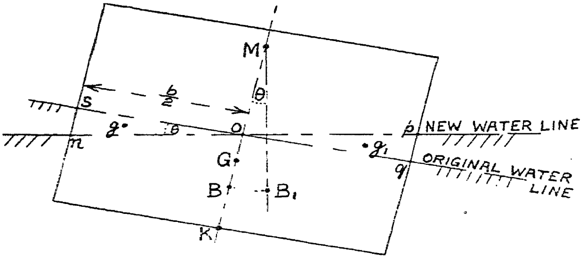
If when the vessel is heeled through a small angle θ the centre of gravity G shifts to G_1, the work done in heeling = Weight of vessel × vertical rise of centre of gravity.

Dynamical stability

$$= W \times \overline{GL}$$
$$= W(\overline{GM} - \overline{GM}\ \text{Cos.}\ \theta)$$
$$= W\,\overline{GM}\,(1 - \text{Cos.}\ \theta)$$

Consider a vessel of rectangular shape, length l feet and breadth b feet, and let it be heeled through a small angle θ. For a small angle of heel the new water line will intersect the old water line on the vertical centre of the vessel. The volume displaced

by the vessel remains the same, but the transverse section has changed. A wedge $n\ s\ o$ has emerged and a wedge $o\ p\ q$ has been immersed, and the centre of buoyancy has shifted from B to B_1.



The moment required to bring about this change is the weight equivalent of one wedge multiplied by $g\ g_1$ the distance between the centres of gravity of the two wedges.

Now let $s\ n$ or $p\ q = \frac{b}{2} \tan\theta = \frac{b}{2}\theta$ for a small value of θ.

Let w = the density of the water, i.e., its weight per unit volume.

$$\text{Weight equivalent of the wedge} = \tfrac{1}{2}\left(\frac{b}{2} \times \frac{b}{2}\theta\right) \times l \times w$$

$$= \frac{b^2\, l\, w\, \theta}{8} \qquad g\, g_1 = \left(\frac{2}{3} \text{ of } \frac{b}{2}\right) \times 2 = \frac{2\ b}{3}$$

$$\therefore \text{Moment due to transference} = \frac{b^2\, l\, w\, \theta}{8} \times \frac{2\ b}{3} = \frac{b^3\, l\, w\, \theta}{12}$$

but moment is also equal to W (the weight of the vessel) $\times$ B B_1 and W = V $\times$ w, where V is the volume of displacement,

$$B\,B_1 = \overline{BM}\,\theta \text{ for a small value of } \theta.$$

$$\therefore \overline{BM}\,\theta \times V \times w = \frac{b^3\, l\, w\, \theta}{12}$$

$$\therefore \overline{BM} = \frac{b^3\, l}{12\, V} = \frac{I}{V}.$$

Note that $\frac{\quad}{12}$ is the moment of inertia (I) of the water plane area about the longitudinal centre line of the vessel.

$\overline{BM}$ is thus determined.

Now, let K represent the keel of the vessel.

$$\overline{KM} = \overline{KB} + \overline{BM},$$

$$\text{and } \overline{GM} = \overline{KM} - \overline{KG} = \overline{KB} + \overline{BM} - \overline{KG}$$

$$\overline{BM} = \frac{I}{V} = \frac{\text{Moment of inertia of the water plane area}}{\text{Volume of water displaced}}$$

this is true for any vessel. If the water plane area is a rectangle, then $I = \frac{b^3 l}{12}$. If the water plane area is of the usual ship form and the co-efficient of water plane area is 0·8, then the value of I of the water plane area would be about 0·7 of $\frac{b^3 l}{12}$, or some fraction of $\frac{b^3 l}{12}$ which would be given.

Now consider the vessel being inclined in a fore and aft direction, that is, changing trim or "pitching," Again, B traces out a curve, and for small angles this curve is practically the arc of a circle whose centre is the **Longitudinal Metacentre,** M_1. $\overline{GM_1}$ is the longitudinal metacentric height and this has a value approximately equal to the length of the vessel. $\overline{BM_1}$ is calculated in a manner similar to the calculation to find the transverse $\overline{BM}$.

$$\overline{BM_1} = \frac{I}{V} = \frac{b\, l^3}{12\, V} \text{ for a rectangular water plane area.}$$

For the usual ship form of water plane area, I will have some value such as 0·55 of $\frac{b \cdot l^3}{12}$, but the value would be given in any problem where it is required.

Example. Calculate the transverse and longitudinal metacentric heights, of a box-shaped vessel 24 feet broad, 50 feet long, and 12 feet draught. The centre of gravity is 6 feet 9 inches above the keel.

Transverse B M =

$$\frac{}{12 \times l\,b\,d} \qquad \frac{}{12 \times d} \qquad \frac{24^2}{12 \times 12} = 4 \text{ feet.}$$

$\overline{BM}$ is 4 ft., i.e., M is 4 ft. above B.

$\therefore$ M is $4 + 6 = 10$ ft. above keel.

$\therefore$ M is $10 - 6{\cdot}75 = 3{\cdot}25$ ft. above G.

. Transverse $\overline{GM} = 3{\cdot}25$ feet. Ans.

Longitudinal B M

$$= \frac{I}{V} \qquad \frac{b\,l^3}{12 \times l\,b\,d} \qquad \frac{}{12 \times d} \qquad \frac{50^2}{12 \times 12} = 17{\cdot}36 \text{ ft.}$$

As G is $6{\cdot}75 - 6 = 0{\cdot}75$ ft. above B,

then longitudinal $\overline{GM} = 17{\cdot}36 - 0{\cdot}75 = 16{\cdot}61$ feet. Ans.

Inclining Experiment to determine the transverse metacentric height. A weight of w tons is placed on the centre line of the ship, and the total displacement W tons is noted. By shifting w transversely across the deck of the ship through a distance of d feet, the disturbing moment of $w\,d$ ft. tons causes the ship to heel. While the ship rests in this inclined position, the disturbing moment, $w\,d$ is equal to the restoring moment, $W\,\overline{GM}\,\theta$. The angle θ is found by means of a long plumb line hanging on a bulkhead, by dividing the deviation x by the length l. It is usual to incline the ship, first to one side, then to the other, and the mean deviation is taken.

Example. A ship of 6,230 tons displacement was inclined by moving a weight of 30 tons through 25 feet across the deck. The mean deviation of a 20 feet long plumb line was 7·5 inches. Determine the transverse metacentric height of the ship at the time of the experiment.

Disturbing moment = Restoring moment

$$w \times d = W\,\overline{GM}$$

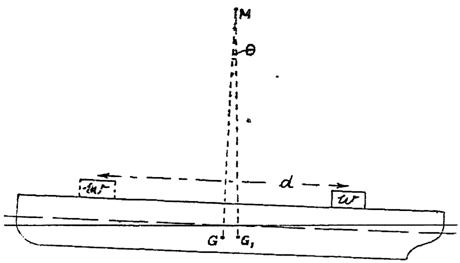
$$30 \times 25 = 6230 \times \overline{\mathrm{G\,M}} \times \frac{}{12 \times 20}$$

$$\therefore \overline{\mathrm{G\,M}} = \frac{30 \times 25 \times 12 \times 20}{6{,}230 \times 7{\cdot}5} = 3{\cdot}852 \text{ feet.} \quad \text{Ans.}$$

Change of Trim..

The centre of flotation is the centroid of the water plane area. When a vessel "tips" or changes trim, the change of trim takes place about this position.

Consider a weight of w tons which is already on board, shifted through a distance of d feet in a fore and aft direction. The change of moment is $w \times d$ ft. tons, and this is equal to $\mathrm{W} \times \overline{\mathrm{G\,G_1}}$ where W is the displacement and $\overline{\mathrm{G\,G_1}}$ is the shift of the centre of gravity.



Let θ = angle of tip, $\overline{\mathrm{G\,M_1}}$ = longitudinal metacentric height in feet, l = length of ship in feet, x = total change of trim in feet.

Then

$$x_1 = \overline{\mathrm{G\,M_1}}\,\theta, \text{ and } \theta = \frac{}{l}$$

$$\therefore w \times d = \mathrm{W}\,\overline{\mathrm{G\,M_1}} \times \frac{}{l}$$

$$\text{and change of trim} = x = \frac{w\;d\;l}{\overline{\mathrm{M}}} \text{ feet.}$$

Also, if we let x be one inch, i.e., $\frac{1}{12}$ foot, then

$$w \times d = W \times \overline{G\,M_1} \times \frac{\quad}{l}$$

$\therefore$ Moment to change trim one inch $= \dfrac{\quad}{12\ l}$ foot tons.

Example. A ship is floating at a draught of 20 feet forward and 21 feet aft. The following weights are now placed on board :

35 tons	at a	position	110 feet	forward	of the	centre	of	flotation.
42	,,	,,	45 ,,	,,	,,	,,		,,
10	,,	,,	85 ,,	abaft	,,	,,		,,
80	,,	,,	34 ,,	,,	,,	,,		,,

Find the new draughts, if the moment to change trim one inch is 920 ft. tons, the tons per inch immersion is 29, and the centre of flotation is at mid-length.

Total weight placed on board = 35 + 42 + 10 + 80 = 167 tons.

If this had been placed at the centre of flotation, the parallel sinkage would be $\frac{167}{29} = 5{\cdot}76$ inches.

The draughts would then be 20 ft. 5·76 ins. forward, and 21 ft. 5·76 ins. aft.

Moments forward of centre of flotation

$= (35 \times 110) + (42 \times 45)$

$= 5{,}740$ ft. tons.

Moments aft $= (10 \times 85) + (80 \times 34) = 3{,}570$ ft. tons.

$\therefore$ Moment causing alteration of trim $= 5{,}740 - 3{,}570$

$= 2{,}170$ ft. tons forward.

$\therefore$ Change of trim $= \frac{2170}{920} = 2{\cdot}36$ inches.

This causes a greater draught forward of $\dfrac{2{\cdot}36}{\quad} = 1{\cdot}18$ inches, and a less draught aft of the same amount.

$\therefore$ Draught forward = 20 ft. 5·76 ins. + 1·18 ins.
= 20 ft. 6·94 ins.

Draught aft = 21 ft. 5·76 ins. — 1·18 ins.
= 21 ft. 4·58 ins. Ans.

TEST EXAMPLES XXVII.

1. A vessel floats at a draught of 24 feet in sea water and the load water plane area is 13,400 sq. feet. The water plane areas below this, measured at regular intervals of 4 feet, are: 12,800, 11,940, 10,380, 8,210, 5,100, and 0 sq. feet respectively. Find the displacement in tons. If the length of the vessel is 350 feet, and breadth 40 feet, find its block co-efficient.

6,355 tons ; 0·662. Ans.

2. A ship 450 ft. long, 48 ft. broad, 21 ft. draught, has a displacement of 9,200 tons. The load water plane area is 17,280 sq. feet and the immersed 'midship section area is 911 sq. ft. Find (*a*) the block co-efficient, (*b*) prismatic co-efficient, (*c*) co-efficient of water plane area, (*d*) co-efficient of immersed 'midship section.

0·71 ; 0·7854 ; 0·8 ; 0·9038. Ans.

3. A ship displaces 10,000 tons and the area of the water plane is 13,000 sq. ft. The ship is loaded in water of 1,013 ozs. per cubic foot, and proceeds to sea where the water is 1,026 ozs. per cubic foot ; find the difference in the draught at sea.

4·135 inches. Ans.

4. The beam of a ship is one-eighth of its length, the co-efficient of water plane area is 0·75, and the tons per inch immersion is 28. Find (*a*) the area of the water plane, and (*b*) the length of the ship.

11,760 sq. ft. ; 354·2 ft. Ans.

5. A vessel 320 ft. long, 35 ft. beam, floats at a draught of 18 ft. 6 ins. forward and 20 feet aft. 300 tons of coal are put into a cross bunker which is situated at the centre of flotation ; if the co-efficient of water plane area is 0·8, find the new draughts.

19 ft. 8 ins. ford. ; 21 ft. 2 ins. aft. Ans.

6. A rectangular box barge 110 feet long, 20 feet broad, floats at a draught of 6 feet. Two transverse bulkheads divide the vessel to form a central watertight compartment 25 feet long. If this central compartment is holed and laid open to the sea, what would be the new draught ?

7 ft. 9·2 ins. Ans.

7. A rectangular box barge is 180 ft. long and 38 ft. broad, and draws 9 feet of water. A central compartment 28 feet long is formed by two transverse bulkheads. If this compartment has a permeability of 60%, find the draught of the barge when the compartment is holed to the sea.

9ft. 11·1 ins. Ans.

*8. A vessel of 10,000 tons displacement floats at a draught of 28 feet. The centre of buoyancy is 14·75 feet above the keel, and the transverse metacentre is 2 feet above the centre of gravity. Find the position of the centre of gravity relative to the keel, if the moment of inertia of the water plane area about the longitudinal axis is $4{\cdot}2 \times 10^6$ feet^4 units.

G is 24·75 ft. above keel. Ans.

9. A rectangular box barge is 250 ft. long and 35 ft. broad, and floats at 18 ft. draught. If the centre of gravity is $10\frac{1}{2}$ ft. above the keel, find the righting moment for an angle of heel of 4 degrees.

1,310 ft. tons. Ans.

10. During an inclining experiment, a weight of 30 tons is shifted 20 feet across the deck of a ship of 6,240 tons displacement, and caused a deviation of $12\frac{1}{2}$ inches on a pendulum 20 feet long. What was the transverse metacentric height at the time of the experiment?

1·846 feet. Ans.

*11. A ship 500 feet long has a displacement of 12,000 tons, and floats at draughts of 25 ft. 6 ins. forward and 25 ft. 7 ins. aft. The longitudinal metacentric height is 540 feet. Calculate the new draughts when a quantity of cargo weighing 35 tons which was stowed on the fore deck, is transferred to the after deck, through a distance of 280 feet. Centre of flotation is at mid-length.

25 ft. 1·46 ins. ford.; 25 ft. 11·54 ins. aft. Ans.

*12. A ship is floating on an even keel at a draught of 18 feet. The tons per inch immersion is 28, and the moment to change trim one inch is 720 ft. tons. What will be the draughts fore and aft after the following loads are placed on board?:—

50 tons at 120 feet forward of centre of flotation.
60 ,, 40 ,, abaft ,, ,, ,,
55 ,, 80 ,, ,, ,, ,, ,,
10 ,, 136 ,, ,, ,, ,, ,,

The centre of flotation is at mid-length.

18 ft. $4\frac{3}{4}$ ins. ford.; 18 ft. $7\frac{3}{4}$ ins. aft. Ans.

CHAPTER XXVIII.

ELECTRIC CIRCUITS.

Moving electricity is a form of energy. It is easily transmitted from a central supply and distributed to any number of places where it is to be used. No one knows the precise nature of electricity, but a great deal is known about its effects and the laws governing its use. The early workers on this subject thought it was an invisible fluid which could be made to flow through a path of suitable material. Modern knowledge indicates that there is, in fact, a kind of flow, although of a kind very different from that which was at first imagined. It is convenient to think of electricity as a fluid flowing—without loss—around a path, because many of the laws governing the flow of a fluid have a parallel in the laws of electrical circuits.

Imagine a complete water path consisting of a pump, pipes leading from the delivery of the pump to a hydraulic crane, and pipes leading back from the exhaust of the crane to the suction of the pump. When the pump is started, water flows to the crane at an increased pressure. The crane utilises the energy in the water which is then returned to the suction of the pump. Notice the following points about the system:—A complete path for the water is necessary and none is lost; the energy transmitted from the pump to the crane is by reason of the flow of water and its pressure; the water is made to flow by reason of its pressure; the flow is resisted by the friction of the passages and because of the work done by the crane.

These properties are similar to those of an electrical circuit. Imagine now an electrical circuit consisting of a dynamo and an electric motor. The two are connected by two wires. The dynamo is driven by some mechanical power such as a steam engine, and in the dynamo is set up a force or pressure which causes a current to flow from the dynamo to the motor. After passing through the windings of the motor the current, at a lower pressure, returns to the dynamo. As in the water circuit there must be a complete path through which the current can flow. Electrical energy is carried from the dynamo to the motor by reason of the flow and electrical pressure. There is a resistance to the flow in the wires, and, because of the work done by the motor, a pressure is set up which tends to impede the flow.

With these analogies in mind, the common electrical terms can be explained.

Electro-Motive-Force (E.M.F.) is that force or pressure which causes a flow of electricity in a circuit.

A difference of E.M.F. is called **Potential Difference (P.D.)** A difference of potential may be set up by chemical action, by heat or by mechanical methods. It is usual to regard one side of this difference as **Positive** + and the other as **Negative** — and to call the positive the high pressure and the negative the low pressure. The unit of electro-motive force is the **Volt.**

Current (I) is the rate of flow of electricity. The current is assumed to flow from the positive to the negative, and the unit of current is the **Ampere.** The quantity of electricity passing may be measured in Ampére-seconds, one ampére-second (sometimes termed one Coulomb) being a current of one ampére flowing for one second. Quantity may also be measured in ampére-hours.

One ampére will, in one second, deposit 0·001118 gram of silver from a solution of silver nitrate in water.

Electrolysis.

If two plates of platinum are immersed in slightly acidulated water, and a current of electricity is passed from one plate through the water to the other plate, hydrogen is liberated at one plate and oxygen at the other. If the water is weighed, it will be found that it has apparently lost weight ; actually, water being composed of the two elements hydrogen and oxygen, the water has been split up into its two component parts. It has therefore undergone a chemical change. Again, if two metal plates are immersed in a solution of copper sulphate, copper is deposited on one of the plates from the solution when an electric current is passed through. There are many other cases where a chemical change occurs in a liquid when electricity is passed through it and the process is called **electrolysis.**

The liquid is called the **electrolyte** and the two plates are the **electrodes ;** the electrode by which the current enters this electrolyte is called the **anode,** and the electrode by which the current leaves the electrolyte is the **cathode.** Deposited metal is always on the cathode.

The laws of electrolysis were stated by Faraday in 1834, and are as follows :—

(1) The weight of any substance deposited or liberated from an electrolyte is proportional to the quantity of electricity which flows through.

(2) The weights of different substances deposited or liberated by a given quantity of electricity, are proportional to their chemical equivalents.

Consider the first law. Quantity of electricity is the rate of flow of electric current multiplied by the time; thus, coulombs = ampéres × seconds, and the weight of substance deposited or liberated is directly proportional to this. The weight in grams of any substance deposited by one coulomb of electricity is called the **Electro-Chemical Equivalent** (E.C.E.) of the substance, and therefore :—

The total weight deposited when I ampéres flow for t seconds $= \text{I} \times t \times$ E.C.E. grams.

Example. Taking the electro-chemical equivalent of copper as 0·00033 gram per coulomb, calculate the weight of copper that will be deposited from a solution of copper sulphate in water, by a current of 12 amps. flowing for 48 minutes.

$$\begin{aligned} \text{Weight deposited} &= \text{I} \times t \times \text{E.C.E.} \\ &= 12 \times 48 \times 60 \times 0{\cdot}00033. \\ &= 11{\cdot}4 \text{ grams. Ans.} \end{aligned}$$

The current flowing through a circuit may be determined by this principle, by using an instrument called a **Voltameter.** A copper voltameter may consist of two plates, immersed in a solution of copper sulphate, and connected in a circuit through which a current of electricity flows. If the cathode plate is cleaned and carefully weighed, and after a steady current has passed for, say, one hour, the plate is again brushed and weighed, the difference in weights is the weight of deposit. The time, and the electro-chemical equivalent of copper, being known, then the current passing may be calculated. An ammeter may be connected in the circuit, and its accuracy tested. A variable resistance in the circuit will be necessary to maintain a steady current during the experiment.

The **Chemical-Equivalent** of a substance is the weight of that substance which will replace or combine with unit weight of hydrogen, through chemical action. In forming water, two atoms of hydrogen and one atom of oxygen chemically combine to form one molecule of water; the atomic weights of hydrogen and oxygen are 1 and 16 respectively, and so we have 2 atoms, each atom having a weight of 1, combining with 1 atom of weight 16. This is in the proportion by weight of 1 part hydrogen to 8 parts oxygen, and therefore the chemical equivalent of oxygen is 8.

From Faraday's second law, the weight of substance deposited or liberated is proportional to the chemical equivalent of the substance. If the chemical equivalents of hydrogen, copper,

and silver are 1, 31·8, and 107 respectively, the weights of hydrogen, copper, and silver deposited or liberated by a given quantity of electricity would be in the same proportion, and therefore the electro-chemical equivalent of a substance is the electro-chemical equivalent of hydrogen multiplied by the chemical equivalent of that substance. Taking the E.C.E. of hydrogen as 0·00001044, then the E.C.E. of copper is $0{\cdot}00001044 \times 31{\cdot}8 = 0{\cdot}00033$, and the E.C.E. of silver is $0{\cdot}00001044 \times 107 = 0{\cdot}001118$.

Example. When a current of 3·5 ampéres was passed through a solution of copper sulphate, 4·2 grams of copper were deposited. If the electro-chemical equivalent of copper is 0·00033 gram per coulomb and the chemical equivalent of copper is 31·8, find the time for which the current was passed through, and also the weight of hydrogen liberated from the solution.

$$\text{Weight deposited} = \mathrm{I} \times t \times \text{E.C.E.}$$

$$4{\cdot}2 = 3{\cdot}5 \times t \times 0{\cdot}00033$$

$$\therefore t = \frac{4{\cdot}2}{3{\cdot}5 \times 0{\cdot}00033}$$

$$= 3636 \text{ seconds} = 60{\cdot}6 \text{ minutes.}$$

Ans. (1)

By proportion :—

$$\frac{\text{wt. of hydrogen liberated}}{\text{wt. of copper deposited}} \qquad \frac{\text{chemical equivalent of hydrogen}}{\text{chemical equivalent of copper}}$$

$$\therefore \text{wt. of hydrogen} = \frac{1 \times 4{\cdot}2}{}$$

$$= 0{\cdot}1321 \text{ gram.} \quad \text{Ans. (2)}$$

Resistance (R) is offered by all substances to the flow of current through them. Some materials such as copper, and metals generally, have a very low resistance while others such as silk, glass, mica and rubber have an extremely high resistance. Resistance is measured in **Ohms.** When the resistance is very small, it may be measured in Microhms, one microhm being a millionth part of an ohm. When the resistance is great, it may be conveniently measured in Megohms, one megohm being a million ohms.

One ohm is the resistance of a column of mercury 106·3 cms. long and having a constant cross section of 1 sq. m.m. at 0°C. The weight of this column of mercury is 14·4521 grams.

Ohm's Law states that the current (I) in an electrical circuit is proportional to the electro-motive force (E) if the resistance remains constant.

One ampére is the current flowing in a circuit of one ohm resistance when the electro-motive force is one volt. Therefore Ohm's law may be written :—

$E = I\,R$ where E = electro-motive force in volts.
I = current in ampéres.
R = resistance in ohms.

Example. What is the resistance of a circuit in which the current is 28 ampéres, if the E.M.F. is 110 volts ?

$$R = \frac{E}{I} = \tfrac{110}{28} = 3{\cdot}928 \text{ ohms. Ans.}$$

The **Specific Resistance** (S) of a substance is the resistance between opposite faces of a cube of unit dimensions. This is sometimes called the Resistivity. The resistance of a conductor varies directly as its length and inversely as the area of its cross section.

$$\text{Therefore, Resistance} = \frac{\text{Specific resistance} \times \text{length}}{\text{Cross sectional area}}$$

The specific resistance of copper is 0·626 microhm per inch cube, or 1·59 microhms per centimetre cube, at a temperature of 0° Centigrade.

It should be noted particularly that the above is not per cubic inch or per cubic centimetre. A cubic inch of copper might consist of a very long wire of very small diameter, and obviously the resistance of such a conductor would be much greater than the resistance between opposite faces of an inch cube.

Conductance is the reciprocal of the resistance ; thus, if the resistance is R, the conductance is $\frac{1}{R}$. The unit of conductance has been called the "mho." This expression, however, is not often used and is unnecessary.

Example. If the specific resistance of copper is 1·7 microhms per centimetre cube at a temperature of 10°C., calculate the resistance in ohms, of a copper wire one millimetre diameter and one decimetre long, at the same temperature.

$$R = \frac{s\,l}{a} = \frac{1{\cdot}7 \times 10}{1{,}000{,}000 \times 0{\cdot}1 \times 0{\cdot}1 \times \frac{11}{14}}$$

$$= 0{\cdot}002163 \text{ ohm.} \quad \text{Ans.}$$

Temperature Co-efficient. The resistance of a material changes with change of temperature. A rise in temperature increases the resistance of metals and decreases the resistance of non-metallic solids and liquids. The change of resistance per ohm per degree change of temperature is the Temperature Co-efficent. If the resistance increases as the temperature rises the co-efficient is positive, and if the resistance decreases as the temperature rises the co-efficient is negative.

Suppose the resistance at 0°C. $= R_0$ and the temperature co-efficient per degree Centigrade $= a$. Then for one degree increase of temperature the increase in resistance is $R_0\,a$, and for t degrees rise of temperature the increase in resistance is $R_0\,a\,t$ ohms. The total resistance at $t°$ C. will therefore be R_0 + increase, which is $R_0 + R_0\,a\,t = R_0\,(1 + at)$. If the resistance at $t°$ C. be represented by R_t, then $R_t = R_0\,(1 + a\,t)$.

If the resistance at one temperature (t_1) is known, and the resistance at another temperature (t_2) is required, we have :—

$$R_1 = R_0\,(1 + at_1) \text{ and } R_2 = R_0\,(1 + at_2)$$

dividing the first expression by the second :—

$$R_1 \qquad R_0\,(1 + at_1)$$

$$R_0\,(1 + at_2)$$

$$\times\ (1 +$$

Therefore $R_2 =$

Example. A copper conductor carries a current of 6 ampéres when the temperature is 15°C. What will be the current when the temperature rises to 40°C. if the potential difference across the conductor remains unaltered ? The temperature co-efficient for copper is 0·00428 per degree centigrade.

at_1

R_2

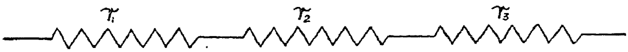
$$R_4 \qquad \frac{1 + 0{\cdot}00428 \times 40}{1 + 0{\cdot}00428 \times 15} = \frac{1{\cdot}1712}{1{\cdot}0642}$$
R_{15}

Since $E = I\ R$ and E is constant, then $I_1\ R_1 = I_2\ R_2$

$$R_{40} \times I_{40} = R_{15} \times 6 \qquad \frac{6 \times 1{\cdot}0642}{1{\cdot}1712}$$

$= 5{\cdot}45$ ampéres. Ans.

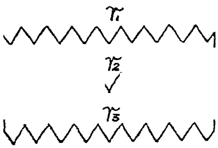
Resistances in Series. When resistances are connected in series the equivalent resistance is equal to the sum of the



resistances. Thus, if resistances r_1, r_2 and r_3 are connected in series, the equivalent resistance (R) $= r_1 + r_2 + r_3$.

Resistances in Parallel.

When resistances are connected in parallel, the reciprocal of the equivalent resistance is equal to the sum of the reciprocals of the resistances. If r_1, r_2 and r_3 are connected in parallel, then :—



$$\frac{1}{R} \qquad \frac{1}{r_1} \qquad \frac{1}{r_2}$$

Example. Three resistances, 10 ohms, 6 ohms and 12 ohms are connected in parallel, find the equivalent resistance.

$$\frac{1}{R} \qquad \frac{6 + 10 + 5}{60} =$$

$\therefore$ Equivalent resistance $R = \frac{60}{21} = 2\frac{6}{7}$ ohms. Ans.

Power. The unit of electrical power is the Watt, and:—

Watts (W) = current (I) × electro-motive force (E)

$\therefore$ W = I E

From Ohm's law, E = I R,

$$\text{therefore } W = I^2 R \text{ or } W = \frac{E^2}{R}$$

A watt is a very small unit and for this reason electrical power is often measured in kilowatts, one kilowatt being one thousand watts.

A horse-power is equal to 746 watts, therefore 1 kilowatt $= \frac{1000}{746} = 1\frac{1}{3}$ H.P. (approx.)

One kilowatt acting for one hour (or, one kilowatt-hour) is called a Board of Trade Unit.

Example. Calculate the brake horse power of an electric motor working off a 220 volt supply and taking a current of 30 ampéres. The efficiency of the motor is 0·9. Find also the energy consumed in B.O.T. units after working for 20 minutes.

$$\text{Efficiency} = \frac{\text{Power got out}}{\text{Power put in}}$$

$\therefore$ Brake Horse Power of motor

$$\frac{220 \times 30 \times 0{\cdot}9}{746} = 7{\cdot}962. \quad \text{Ans.}$$

$$220 \times 30 \text{ watts} = \frac{220 \times 30}{1{,}000} \text{ kilowatts.}$$

20 minutes $= \frac{1}{3}$ hour.

$$\therefore \text{Kilowatt-hours} = \frac{220 \times 30}{1{,}000} \times$$

= 2·2 B.O.T. units. Ans.

Electrical energy is sometimes measured in Joules, one Joule being equal to one watt acting for one second (or, one watt-second). The Joule is therefore the same *kind* of unit as the Board of Trade unit.

One B.O.T. unit = 1 kilowatt-hour = 1,000 watt-hours = 1,000 watts × 3,600 seconds.

$\therefore$ One B.O.T. unit = 3,600,000 Joules.

It has been found by experiment that 4·19 Joules is equal to one calorie. A calorie is the amount of heat required to raise the temperature of one gram of fresh water one degree Centigrade.

$$\text{One British Thermal unit} = \frac{1000}{2\cdot2046} \times \tfrac{5}{9} = 252 \text{ calories.}$$

Therefore one British Thermal unit $= 252 \times 4\cdot19 =$ 1,056 Joules.

And one Joule $= \frac{1}{1056} = 0\cdot000947$ B.T.U.

Also, one B.T.U. $= 778$ foot pounds.

Therefore, one Joule $= 0\cdot000947 \times 778 = 0\cdot7368$ foot pound.

One Board of Trade unit $= 1$ kilowatt hour

$$= \frac{1000 \times 3600}{1056} = 3410 \text{ B.T.U.}$$

Example. An electric heater has 5 resistance elements connected in parallel across a 110 volt supply. The resistance of each element is 20 ohms. Find the energy dissipated as heat in Joules per second, and in British thermal units per hour.

Current in each element $= \frac{110}{20} = 5\cdot5$ ampéres.

Total current $= 5\cdot5 \times 5 = 27\cdot5$ ampéres.

Energy dissipated $= I\ E = 27\cdot5 \times 110 = 3{,}025$ watts.

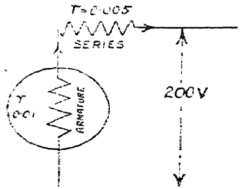
3,025 watts $\times$ 1 second $=$ 3,025 Joules. Ans.

$$\text{British thermal units per hour} = \frac{3{,}025 \times 60 \times 60}{1{,}056}$$

$$= 10{,}312. \quad \text{Ans.}$$

When mechanical energy is converted into electrical energy by a generator, or when electrical energy is converted into mechanical energy by a motor, some of the energy is lost. The losses are due to the resistance of the windings, to magnetisation and demagnetisation of the iron, eddy currents, and friction. The losses due to the resistance of the windings can be calculated and the efficiency found neglecting all the other losses. This is termed the electrical efficiency. The efficiency as found on test, that is, when all losses are taken into account, is called the commercial efficiency. The method of calculating the electrical efficiency is shown in the following examples.

1. A series wound dynamo has an output of 160 kilowatts at 200 volts. The resistance of the series winding is 0·005 ohm and the equivalent resistance of the armature is 0·01 ohm. Find the electrical efficiency and the internal voltage of the armature.



$$\text{Current } I = \frac{W}{E} = \frac{160 \times 1000}{200}$$

$= 800$ ampéres.

E.M.F. across series winding $= I\,R = 800 \times 0{\cdot}005 = 4$ volts.

Drop in E.M.F. across armature $= 800 \times 0{\cdot}01 = 8$ volts.

Internal voltage of armature $= 200 + 8 + 4 = 212$ volts. Ans.

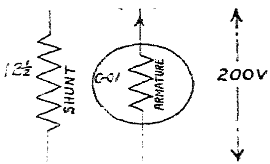
Watts lost in series winding $= I\,E = 800 \times 4 = 3{,}200$ watts $= 3{\cdot}2$ kilowatts.

Watts lost in armature $= 800 \times 8 = 6{,}400$ watts $= 6{\cdot}4$ kilowatts.

Total losses $= 3{\cdot}2 + 6{\cdot}4 = 9{\cdot}6$ kilowatts.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \quad \frac{160}{160 + 9{\cdot}6} = 0{\cdot}943. \quad \text{Ans.}$$

2. A shunt wound dynamo has an output of 160 kilowatts at 200 volts. The resistance of the shunt winding is 12·5 ohms, and the resistance of the armature is 0·01 ohm. Find the electrical efficiency.



$$\text{Output current } I = \frac{W}{E}$$

$$\frac{160 \times 1{,}000}{200} \quad 800 \text{ ampéres.}$$

E.M.F. applied to shunt winding is 200 volts.

$$\text{Current in shunt} = \frac{E}{R} = \frac{200}{12{\cdot}5} = 16 \text{ ampéres.}$$

Current in armature $= 800 + 16 = 816$ ampéres.

Drop in E.M.F. across armature $= 816 \times 0{\cdot}01 = 8{\cdot}16$ volts.

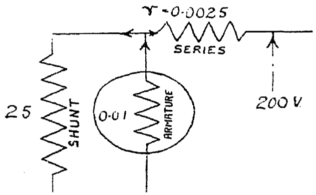
Watts lost in armature $= 816 \times 8{\cdot}16 = 6659$ watts $= 6{\cdot}659$ kilowatts.

Watts lost in shunt $= 16 \times 200 = 3{,}200$ watts $= 3{\cdot}2$ kilowatts.

Total loss $= 6{\cdot}659 + 3{\cdot}2 = 9{\cdot}859$ kilowatts.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \quad \frac{160}{160 + 9{\cdot}859} = 0{\cdot}942. \quad \text{Ans.}$$

3. A compound wound dynamo has an output of 160 kilowatts at 200 volts. The resistance of the series winding is 0·0025 ohm, the resistance of the shunt winding is 25 ohms and the resistance of the armature is 0·01 ohm. Find the electrical efficiency.



$$\text{Output current} = \frac{160 \times 1{,}000}{200}$$

$= 800$ ampéres.

E.M.F. across series winding

$= I\,R = 800 \times 0{\cdot}0025 = 2$ volts.

E.M.F. applied to shunt winding $= 200 + 2 = 202$ volts.

$$\text{Current in Shunt} = \frac{E}{R} = \frac{202}{25} = 8{\cdot}08 \text{ ampéres.}$$

Current in armature $= 800 + 8{\cdot}08 = 808{\cdot}08$ ampéres.

Drop in E.M.F. across armature $= I\,R = 808{\cdot}08 \times 0{\cdot}01 = 8{\cdot}0808$ volts.

Watts lost in series winding $= 800 \times 2 = 1{,}600$ watts $= 1{\cdot}6$ kilowatts.

Watts lost in shunt $= 8{\cdot}08 \times 202 = 1{,}632$ watts $= 1{\cdot}632$ kilowatts.

Watts lost in armature $= 808{\cdot}08 \times 8{\cdot}0808 = 6530$ watts $= 6{\cdot}53$ kilowatts.

Total loss $= 1{\cdot}6 + 1{\cdot}632 + 6{\cdot}53 = 9{\cdot}762$ kilowatts.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \quad \frac{160}{160 + 9{\cdot}762} = 0{\cdot}942. \quad \text{Ans.}$$

The **Centimetre-Gram-Second** (C.G.S.) System is that system which takes the unit length as one centimetre, the unit of mass as one gram, and the unit of time as one second.

The **Dyne** is the force which will produce an acceleration of one centimetre per second per second in a mass of one gram.

The **Erg** is the work done by one dyne acting through a distance of one centimetre. Therefore one erg = one dyne-centimetre. One Joule is equal to 10,000,000 Ergs.

TEST EXAMPLES XXVIII.

1. The weight of the cathode of a copper voltameter before deposit was 14·52 grams, and after a steady current was passed through the circuit for 50 minutes its weight was 19·34 grams. The reading of the ammeter was 5·1 amps. Find the error of the ammeter, taking the electro-chemical equivalent of copper as 0·00033. 0·231 ampére. Ans.

2. The resistance of a conducting wire is 20 ohms and the current passing through it 125 ampéres. Find the potential difference at its terminals. 2,500 volts. Ans.

3. An aluminium wire 10 yards long and $\frac{1}{10}$ inch diameter has a resistance of 0·0469 ohm. Calculate its specific resistance per centimetre cube. 2·6 microhms. Ans.

4. A $\frac{1}{4}$ inch diameter conducting wire 1,000 feet long has a resistance of 0·163 ohm. What will be the resistance of a wire 500 feet long and $\frac{1}{8}$ inch diameter, made of the same material and maintained at the same temperature. 0·326 ohm. Ans.

5. The resistance of a length of copper wire at 15° Centigrade is 25 ohms. Find its resistance at a temperature of 55° Centigrade, taking the temperature co-efficient as 0·00428 per degree centigrade. 29·02 ohms. Ans.

6. A copper conductor at a temperature of 10° Centigrade carries a current of 10 ampéres. Calculate the current carried when the temperature rises to 50° C. and the potential difference remains unaltered. The temp. co-efficient = 0·00428 per degree Centigrade. 8·59 ampéres. Ans.

7. Four resistances of 8, 10, 12 and 9 ohms respectively are connected in parallel. What is the equivalent resistance across the group? 2·384 ohms. Ans.

8. Three resistance of 6, 3, and 4 ohms respectively, connected in parallel form one group. Two resistances of 8 and 10 ohms respectively connected in parallel form another group. The two groups are connected in series, find the total resistance across the whole. $5\frac{7}{9}$ ohms. Ans.

9. A current of 36 ampéres flows through a circuit. At a certain point it is divided into three paths of 4 ohms, 3 ohms, and 0·6 ohm resistance respectively. Calculate the current flowing through each path.

4 amps. through 4 ohms resistance.
$5\frac{1}{3}$ amps. through 3 ohms resistance.
$26\frac{2}{3}$ amps. through 0·6 ohm resistance. Ans.

10. A pump delivers 2,800 gallons of water per hour into a boiler working at 220 lb. per square inch pressure. The pump, which is 82 per cent. efficient is driven by a 440 volt motor having an efficiency of 89 per cent. Calculate the current taken by the motor.

16·65 ampéres. Ans.

11. Calculate the quantity of electricity in ampére-hours and the energy consumed in B.O.T. units by a heating element having a resistance of 160 ohms working off a 200 volt supply system for 500 hours.

625 ampére-hours ; 125 B.O.T. units. Ans.

12. An electric kettle is fitted with a heater unit of 120 ohms resistance. The efficiency is 84 per cent. and the voltage 220. How long will it take to heat $1\frac{1}{2}$ pints of water from 42°F. to 212°F. ?

16 mins. 33 secs. Ans.

13. Four hundred 30-candle power lamps are supplied at 100 volts by a dynamo whose efficiency is 90 per cent., and the dynamo is driven by a 22 brake horse power oil engine. Calculate the consumption of the lamps in watts per candle power.

1·2309 watts per C.P. Ans.

14. A battery consists of 86 cells connected in series, each cell has an internal resistance of 0·3 ohm and an E.M.F. of 1·8 volts. The battery is connected to an electrical device of 64 ohms resistance by means of conductors having a total resistance of 0·2 ohm. Find the current flowing in the circuit, the terminal P.D. of the battery, and the horse power output.

1·72 ampéres ; 110·424 volts ; 0·2546 H.P. Ans.

15. The P.D. at the output terminals of a series wound dynamo is 220 volts. The equivalent resistance of the armature windings is 0·15 ohm, the resistance of the series winding is 0·082 ohm and the resistance of the external circuit is 5 ohms. Calculate (*a*) the output of the dynamo in electrical horse power ;

(*b*) the horse power to drive the dynamo neglecting all sources of loss except those due to the internal resistance ; and (*c*) the electrical efficiency.

(*a*) 12·97 H.P. ; (*b*) 13·577 H.P. ; (*c*) 0·955. Ans.

16. The output of a series wound dynamo is 50 kilowatts and the voltage 100. The resistance of the series winding is 0·005 ohm and the equivalent resistance of the armature windings is 0·007 ohm. Calculate the internal voltage of the machine and the electrical efficiency.

106 volts. ; 0·943. Ans.

17. A shunt wound dynamo has an output of 200 kilowatts at 200 volts. The resistance of the shunt winding is 20 ohms and the equivalent resistance of the armature winding is 0·008 ohm. Find the electrical efficiency.

0·951. Ans.

18. A compound wound dynamo has an output of 120 kilowatts at 110 volts. The resistance of the series winding is 0·002 ohm, the resistance of the shunt winding is 30 ohms and the equivalent resistance of the armature is 0·006 ohm. Find the electrical efficiency.

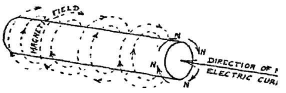
0·923. Ans.

CHAPTER XXIX.

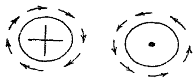
GENERATION AND DISTRIBUTION OF ELECTRICITY.

Magnetic Effects.

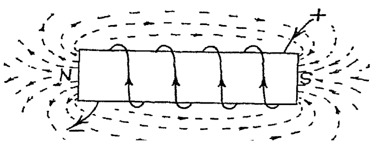
When an electric current flows through a conductor, a magnetic field is set up in and around the conductor. If a pocket compass is placed near the conductor, the North seeking pole of the compass arranges itself in a certain direction which indicates the magnetic field, this direction depends upon the direction of the flow of the electric current. These directions are shown in the sketch, and can easily be remembered by the right hand screw rule; the direction in which the screw is turned represents the direction of the magnetic field, and the axial direction in which the screw moves represents the direction of the flow of the electric current.



A sign + on the end view of a conductor is the conventional method of indicating that the electric current is flowing away from the reader, and a sign . that the current flows towards the reader.



The strength of the field is proportional to the current, and in order to express the strength of the field it is imagined as a number of magnetic lines in the direction of the field. Unit strength of field is one line per square centimetre, or one gauss. This unit is a very small one. In the air gap between the armature and poles of a generator or motor, the field strength is usually about 7,000 lines per square centimetre.



A solenoid is an arrangement of a number of turns of wire carrying electric current, and the strength of the field depends directly upon the product of the electric current and the number of turns. The product of the current in ampéres and the number of turns is given the name *Ampere-turns*. If an iron core is provided instead of an air core, the magnetic effect is increased, and this would be a simple bar magnet.

It can be shown experimentally that when a conductor, carrying a current, is placed at right angles to a magnetic field, a force is exerted tending to move the conductor across the field. The relative directions of the field and current, and the direction in which the conductor tends to move, can be found by Fleming's hand rule thus ;—*Place the thumb and first and second fingers each at right angles to each other, the thumb denotes the direction of motion of the conductor, the first finger denotes the direction of the field which is assumed to be from North to South, and the second finger the direction of flow of current ; the right hand must be used for a generator, and the left hand for a motor.* The magnitude of the force is also found experimentally. In a field of unit strength, a conductor one centimetre long carrying a current of 10 ampéres at right angles to the field, exerts a force of one dyne in a direction across the field. Let B = lines per square centimetre (this is sometimes referred to as the flux density), I = current flowing through conductor in ampéres, l = effective length of conductor in centimetres, then,

$$\text{Force on conductor} = \frac{\text{B I } l}{10} \text{ dynes.}$$

It has previously been shown (see page 160) that one dyne is that force which, acting on a mass which weighs one gram, produces an acceleration of one centimetre per sec. per sec., and also that 981 dynes = 1 gram, therefore,

$$1 \text{ lb.} = \frac{1{,}000 \times 981}{2{\cdot}2046} = 444{,}980 \text{ dynes, usually taken as } 445{,}000.$$

$$\therefore \text{ Force on conductor} = \frac{\text{B I } l}{10 \times 445{,}000} \text{ lb.}$$

Example. A conductor 20 centimetres long and carrying a current of 50 ampéres, is placed at right angles to a field of 7,000 lines per sq. centimetre. Find the force exerted on the conductor (*a*) in dynes, (*b*) in grams, and (*c*) in pounds.

$$\text{Force} = \frac{\text{B I } l}{10} \text{ dynes}$$

$$= \frac{7{,}000 \times 50 \times 20}{10} = 700{,}000 \text{ dynes. Ans } (a).$$

$$\frac{700{,}000}{981} = 713{\cdot}5 \text{ grams. Ans. } (b).$$

$$\frac{713{\cdot}5 \times 2{\cdot}2}{1{,}000} = 1{\cdot}57 \text{ lb.} \quad \text{Ans. } (c).$$

Example. The armature of a motor has 660 conductors whose effective length is 16 inches ; of these, only 0·7 are simultaneously in the magnetic field. The field strength is 6,500 lines per sq. centimetre, the effective diameter of the armature is 12 inches, and each conductor carries a current of 80 ampéres. If the armature runs at 800 revs. per minute, calculate the horse power of the motor.

$$\text{Force on one conductor} = \frac{6{,}500 \times 80 \times 16 \times 2{\cdot}54}{10 \times 445{,}000}$$

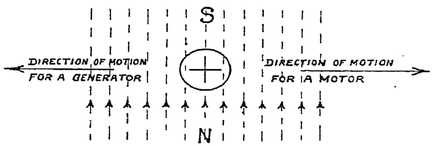
$$= 4{\cdot}747 \text{ lb.}$$

Number of conductors in field at any given instant $= 0{\cdot}7 \times 660$

$$\therefore \text{ Total force} = 4{\cdot}747 \times 0{\cdot}7 \times 660 = 2{,}194 \text{ lb.}$$

$$\therefore \text{ Horse power} = \frac{2{,}194 \times \pi \times 1 \times 800}{33{,}000} = 167{\cdot}1. \quad \text{Ans.}$$

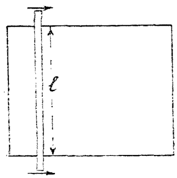
When relative movement takes place between a conductor and a magnetic field, an electro-motive-force is set up in the conductor. The magnitude of the E.M.F. depends upon the strength of the field and the rate of motion.



Consider a conductor moving across a magnetic field and let the direction of the field and that of the current be as shown. Fleming's left hand rule shows that a force would be exerted on the conductor tending to move it to the right. In moving, electrical energy would be converted into mechanical work. As the conductor moves to the right an E.M.F. is set up in it which opposes the current, this E.M.F. is called the back E.M.F. and for a steady current is exactly equal to the voltage applied to cause the current to flow minus the voltage drop due to the resistance. If E = applied voltage, I = current, and e = back E.M.F., then, $E = I\,R + e$,

I R being the voltage drop due to the resistance. The power supplied is I E, and of this, I^2 R is dissipated as heat due to the resistance, the remainder I e is converted into mechanical power; thus I E = I^2 R + I e.

Referring to the diagram on page 479, if the conductor is moved to the left, *i.e.*, against the force it exerts to the right, an E.M.F. will be set up causing a current to flow in the direction marked on the conductor. Work is thus done on the conductor and mechanical work is converted into electrical energy.



Now consider a conductor which has an active length of l centimetres, moving across the face of a pole. Let the density of the field be B lines per sq. centimetre, and let the conductor move x centimetres in t seconds.

Force on conductor × distance moved = work converted into electrical energy.

Force in dynes × distance moved in cms. = work done in ergs.

$$\frac{B\,I\,l}{10} \times x \text{ ergs.}$$

Electrical energy = E I t watt-seconds,
and 1 watt-second (or 1 joule) = 10^7 ergs.

$$\frac{B\,I\,l\,x}{10} = E\,I\,t \times 10^7,$$

$$\frac{B\,I\,l\,x}{10 \times 10^7 \times I \times t} \qquad \frac{B\,l\,x}{t \times 10^8}$$

$l\,x$ = Area covered per second.

$$\therefore \frac{B\,l\,x}{t} = \text{Rate of cutting lines per second,}$$

$$\therefore E = \frac{\text{Rate of cutting lines per second}}{10^8} \text{ volts}$$

or, $\frac{x}{t}$ being the velocity of the conductor in centimetres per second, this may be represented by v, then,

$$E = \frac{B\,l\,v}{10^8} \text{ or } B\,l\,v \times 10^{-8} \text{ volts.}$$

Example. A conductor 40 centimetres long moves across a field whose density is 7,000 lines per sq. centimetre, at a speed of 20 metres per second. Find the E.M.F. set up along the conductor.

$$\begin{aligned} E &= B\,l\,v \times 10^{-8} \\ &= 7{,}000 \times 40 \times 20 \times 100 \times 10^{-8} \\ &= 5{\cdot}6 \text{ volts. Ans.} \end{aligned}$$

Example. A magneto has two poles with a useful flux of 80,000 lines. The armature has a single winding consisting of 400 complete turns, and its speed is 900 revs. per minute. Find the average E.M.F. generated.

$$\begin{aligned} \text{Rate of cutting flux, per conductor} &= 80{,}000 \times 2 \times \frac{900}{60} \\ &= 2{\cdot}4 \times 10^6 \text{ lines per sec.} \end{aligned}$$

Total number of conductors $= 2 \times 400 = 800$, and all conductors are in series. The total average E.M.F. is therefore 800 times the E.M.F. generated in each conductor.

$$\begin{aligned} \text{Total average E.M.F.} &= 2{\cdot}4 \times 10^6 \times 800 \times 10^{-} \\ &= 19{\cdot}2 \text{ volts. Ans.} \end{aligned}$$

In direct current machines the armatures are of two types, lap wound and wave wound. Large machines, especially multipole, are almost always lap wound ; in this case the windings on the armature have a number of paths in parallel equal to the number of poles, and the number of conductors must be a multiple of the number of poles. In wave wound armatures there are only two parallel paths and the number of armature slots must not be a multiple of the number of poles. The voltage generated is the product of the E.M.F. per conductor and the number of conductors in series.

$$\text{The number of conductors in series, } Z_s = \frac{\text{Total conductors}}{\text{Paths in parallel}}$$

If there are *p pairs* of poles in a machine, the rate of cutting flux is p times the rate for a machine with one pair of poles. For a lap wound machine there are $2\,p$ parallel circuits in the armature, hence number of conductors in series :—

$$Z_s = \frac{\text{Total conductors}}{2\ p} = \frac{Z}{2\ p}$$

Let n represent the speed of the armature in revs. per second, and let the total flux per pole be denoted by Φ (phi) ; for a lap wound machine with one pair of poles :—

$$\text{Average E.M.F. generated} = 2\ \Phi \times \frac{Z}{2} \times n \times 10^{-8} \text{ volts.}$$

$$= \frac{\Phi\ Z\ n}{10^8}$$

$$\text{With } p \text{ pairs of poles, } Z_s = \frac{Z}{2\ p}$$

$$\text{and average E.M.F.} = p \times 2\ \Phi \times \frac{Z}{2\ p} \times n \times 10^{-8} \text{ volts.}$$

$$= \frac{\Phi\ Z\ n}{10^8}$$

which is, flux per pole $\times$ total conductors $\times$ revs. per sec. $\times\ 10^{-8}$

Now, for a wave wound machine with p pairs of poles, $Z_s = \dfrac{Z}{2}$

$\therefore$ Average E.M.F. generated

$$= p \times 2\ \Phi \times \frac{Z}{2} \times n \times 10^{-8} \text{ volts.}$$

$$= \frac{p\ \Phi\ Z\ n}{10^8}$$

which is, pairs of poles $\times$ flux per pole $\times$ total conductors $\times$ revs. per sec. $\times\ 10^{-8}$

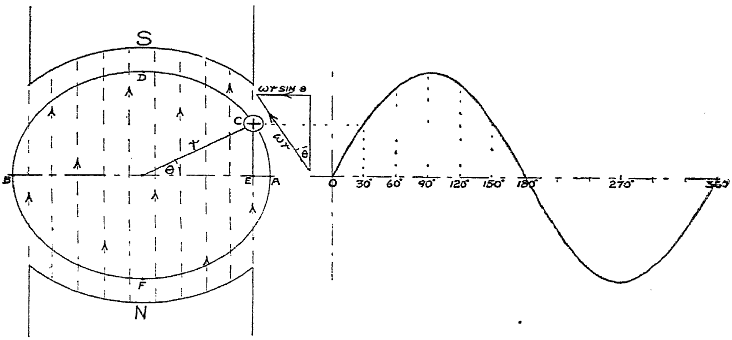
Example. A six pole generator has a useful flux of $6{\cdot}2 \times 10^6$ lines per pole. The armature is lap wound and has 612 conductors and runs at a speed of 900 revs. per minute. Find the E.M.F. generated.

$$\text{E.M.F.} = \frac{\Phi \ Z \ n}{10^8}$$

$$\frac{6{\cdot}2 \times 10^6 \times 612 \times 900}{10^8 \times 60} = 569 \text{ volts. Ans.}$$

Sinusoidal Curves of E.M.F. and Current.

Consider a conductor rotating in a uniform magnetic field at a constant angular velocity of ω radians per second, let the radius of its path be r feet, then the linear velocity of the conductor is $\omega \ r$ feet per second.



When the conductor passes the positions A and B, it is moving parallel to the field, and therefore no E.M.F. is induced in it. At positions D and F, the conductor moves at right angles to the field and maximum E.M.F. is set up. At positions D and F the velocity of the conductor across the field is $\omega \ r$, and at C its velocity across the field is $\omega \ r \sin \theta$ or $\omega \times$ C E. Since the E.M.F. generated is proportional to the velocity across the field, i.e., the rate of cutting the magnetic lines, then the E.M.F. must be proportional to sine θ or to the length C.E. Thus the curve on the right of the diagram shows how the voltage will vary throughout one revolution. If the radius of the circle is chosen to represent the maximum or *peak* value of the voltage to some suitable scale, then the height of the curve from the base line represents to the same scale the voltage at any given angle from the position A. This curve is called a sine curve, or the curve is said to be sinusoidal. If an alternating E.M.F. varies in this way it is said to have a sinusoidal wave form; this wave form is always adopted for power alternating current, and the current will vary in the same way as the voltage.

The time taken for the voltage or current to pass through a complete cycle of variation, is called a *period*, and the number of cycles per second is the *frequency*. Thus, if t = period in seconds, and f = frequency in cycles per second, then $f = \frac{1}{t}$

*Root Mean Square Values.

The unit of alternating current is the ampére, and is that value of alternating current which dissipates the same amount of energy in a resistance, or which would have the same heating effect, as an ampére of continuous current in the same time. The energy dissipated by a continuous current of I ampéres through a resistance of R ohms, is I^2 R watts, thus the energy dissipated varies as the square of the current flowing. As an alternating current varies in value throughout its cycle then the total energy dissipated in one cycle can be found by adding together all the instantaneous values of the $(\text{current})^2$ multiplied by the resistance, and therefore the value of a steady or continuous current which would dissipate the same amount of energy will be equal to the square root of $(\text{average value of alternating current})^2$.

This is termed the effective, or virtual, or root-mean-square (R.M.S.) value of the alternating current, and this is the value which is registered on an ammeter. For a sinusoidal alternating current :—

$$\text{R.M.S. value} = \sqrt{\text{Aver. val. of } \sin^2\theta} \times \text{peak value of current,}$$

$$= 0{\cdot}7071 \times \text{peak value,}$$

Peak value

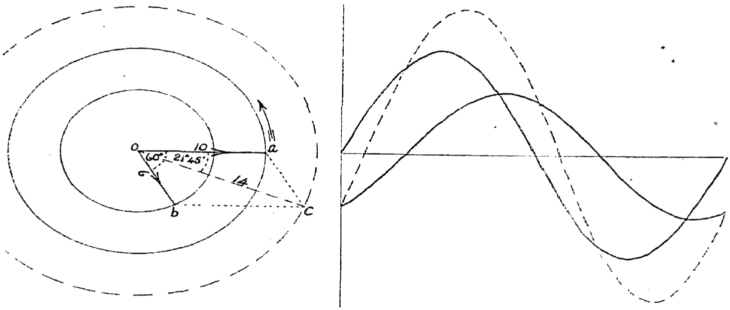
or

If the maximum or peak value of an alternating current was 5 ampéres, then the virtual, or effective value, is $5 \times 0{\cdot}7071 = 3{\cdot}5355$ ampéres.

Similarly, the virtual value of the voltage $= 0{\cdot}7071 \times$ peak value.

*Resultant of Two Alternating Currents.

Current and E.M.F. have magnitude and direction and are therefore vector quantities. Alternating sinusoidal current and E.M.F. have varying values which can be represented at any instant by considering them as revolving vectors, and the resultant of any two (or more) values of current or E.M.F. can be obtained in the same manner as for forces acting at a point.



Referring to the diagram, the two vectors $o\ a$ and $o\ b$ represent to scale currents of 10 amps. and 6 amps. maximum value respectively, and the two curves in full lines represent the manner in which each current varies during one cycle or period. The vectors are considered to revolve 360° for one period. Since the vector $o\ b$ reaches any position previously occupied by $o\ a$, one-sixth of a period later, the current represented by $o\ b$ is said to lag 60° behind the current represented by $o\ a$. The resultant of these two vectors is obtained by drawing $a\ c$ equal in length to, and parallel with, $o\ b$, and joining $o\ c$. The length of $o\ c$ to scale represents the resultant current of 14 amps. lagging behind $o\ a$ by an angle of 21° 45′. This is shown by the dotted curve on the diagram. The same procedure as to magnitude and angle of lag would give the resultant R.M.S. value if R.M.S. values were used instead of maximum values. The two currents of 10 amps. and 6 amps. have the same period, but are said to be 60°, or one-sixth of a period, out of phase.

***Inductance, Reactance and Impedance.**

It has been stated that when a current flows in a conductor, a magnetic field is set up around the conductor. This magnetic field represents energy which has to be supplied from the source of the current. Time is required to supply this energy; in the case of continuous current the time required to build up the magnetic field may be so short compared with the time the apparatus is in use, that it may be neglected, but even so, it is of great importance when the current is cut off quickly. In the case of alternating currents, which change in value very quickly, the time taken to build up the field when the current increases, and the time for the energy to be given up again when the current decreases, has a very important effect on the working of machines having such currents. It should be

emphasised that it is the time factor which is important in this case. In this country all alternating current supplies have a standard frequency of 50 cycles per second. Solenoids such as the field windings of motors and generators are run under such conditions that they have a strong magnetic field in them and therefore require considerable energy to build up these fields, and give out considerable energy when the field dies away. Such circuits are said to have *Inductance* (L). The unit of inductance is the *henry*. The rate at which the magnetic field builds up, depends upon the E.M.F. impressed on the circuit, upon the inductance, and upon the resistance. In a circuit whose inductance is one henry, and whose resistance is so low that it may be neglected, a constant E.M.F. of one volt causes the current to change at the rate of one ampére per second.

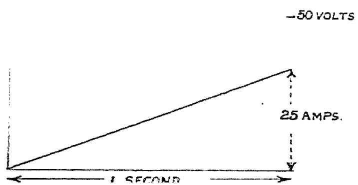
$$\text{Therefore, rate of change of current} = \frac{\text{Impressed E.M.F.}}{\text{Inductance}} = \frac{\text{E}}{\text{L}}$$

$\therefore$ Change of current $= \dfrac{\text{E}}{\text{L}} \times t$, where t is the time in seconds. In the case of varying E.M.F., the average E.M.F. over the period of time must be used for E.

Consider the case of a constant E.M.F. of 50 volts applied to a circuit whose inductance is 2 henrys, and whose resistance is negligible, and assuming that there is no current in the circuit when the E.M.F. is first applied :—

$$\text{Change of current} = \frac{\text{E}}{\text{L}} \times t$$

In $\frac{1}{4}$ second, I $=$ $\times \frac{1}{4} = 6{\cdot}25$ ampéres
,, $\frac{1}{2}$,, I $=$ $\times \frac{1}{2} = 12{\cdot}5$,,
$\times \frac{3}{4} = 18{\cdot}75$,,
$\times 1 = 25$,,

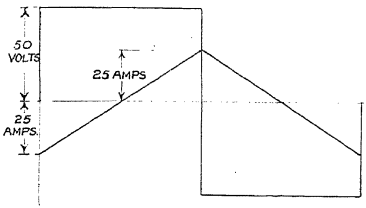


The relation between the voltage, current, and time, for the above values, is shown in the accompanying diagram.

Imagine an alternating voltage of rectangular wave form, having a period of one second. Let the E.M.F. be 50 volts in one direction for half the period, i.e., for $\frac{1}{2}$ second, and then reverse in direction for the other $\frac{1}{2}$ second of the period. Let the inductance be 0·5 henry and let the resistance be negligible.

$$\text{Change of current in } \tfrac{1}{2} \text{ second} = \frac{50}{0{\cdot}5} \times \tfrac{1}{2} = 50 \text{ ampéres,}$$

and this change takes place twice in one period. In the second half of the period the change of current is in the opposite direction because the direction of the E.M.F. is opposite. The current does not reach a maximum value of 50 ampéres because the *change* of current is 50 ampéres; the maximum value of the current is 25 ampéres first in one direction and then in the other direction.



The relation between the E.M.F. and the current is shown in the diagram, wherein it will be seen that the current lags a quarter of a period, or 90 degrees, behind the voltage.

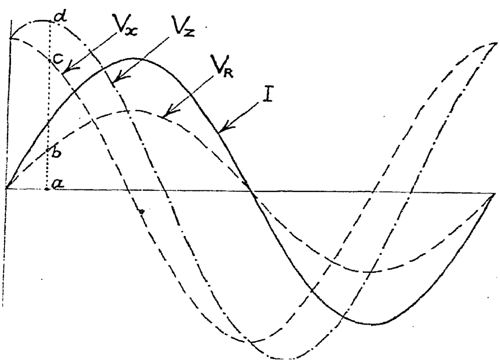
If the E.M.F. is sinusoidal the current will also be sinusoidal, and will also lag behind the voltage by a quarter of a period.

Since there is a voltage applied to the circuit and the resistance has been considered as negligible, there must be a reaction to oppose the current. This reaction is measured in ohms, as for resistance, and is called *Reactance* (x).

Reactance, $x = 2\,\pi \times \text{frequency} \times \text{inductance}.$

$x = 2\,\pi f\,L$

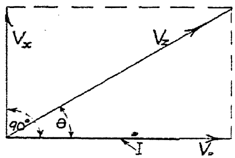
Also, if the resistance is negligible, the voltage drop is I x.

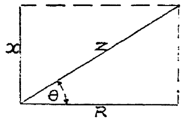


Any circuit must have resistance, and in practice the resistance cannot usually be regarded as negligible. For a single circuit the resistance may be regarded as being in series with the inductance; the voltage drop across the two, however, is not the sum of the volt drop across each taken separately. In this diagram a current is shown for such a circuit. Now, for the

inductance there is a drop in voltage V_x which leads the current by 90°, and for the resistance there is a voltage drop V_R in phase with the current. The resultant voltage drop V_Z, which is equal to the supply voltage, is the resultant, at any instant, of V_R and V_x.

Thus $a\ b + a\ c = a\ d$. These quantities can be treated in R.M.S. values by a vector diagram, and since V_x and V_R are at right angles, $V_Z^2 = V_x^2 + V_R^2$. $V_Z = E$, the supply voltage





and the current lags behind the voltage by an angle θ. This angle is very important. The effect which limits the current is the combined limiting effects of the resistance and the reactance, and is called the *Impedance* (z) and is measured in ohms. Thus, Ohm's law for an alternating current is,

$$I = \frac{E}{z}, \text{ also } z^2 = R^2 + x^2, \text{ and } \cos.\ \theta = \frac{R}{z}$$

A voltmeter across the circuit will give the R.M.S. value of E, and an ammeter will give the R.M.S. value of I, but the product of these two will not give the power taken by the circuit because the pressure is not in phase with the current. The product of E and I is called the apparent power. The true power is the sum of all the products of E and I at the same instant. This is equal to E I cos θ, that is, the product of E and the component of the current in phase with E.

The relation between the true power and the apparent power is called the *Power Factor*.

$$\text{Thus, power factor} = \frac{\text{True power}}{\text{Apparent power}} = \frac{\text{E I cos}}{\text{E I}} = \text{cos.}$$

$$\text{Power factor} = \cos.\ \theta = \frac{\text{Resistance}}{\text{Impedance}}$$

$$\text{and, true power} = \text{Apparent power} \times \cos.\ \theta$$
$$= \text{E I} \times \text{power factor.}$$

The product of E and I is expressed as volt-amps or, for large values, in kilovolt-amps. The true power is, of course, measured in watts or kilowatts.

Example. A circuit has a resistance of 10 ohms and an inductance of 0·045 henry. The potential difference is 120 volts and the frequency 50 cycles per second. Calculate (*a*) the reactance, (*b*) impedance, (*c*) power factor, (*d*) the power absorbed.

$$\text{Reactance } (x) = 2\,\pi\,f\,\text{L},$$

$$= 2 \times \tfrac{22}{7} \times 50 \times 0{\cdot}045 = 14{\cdot}14 \text{ ohms. Ans. } (a)$$

$$\therefore \text{z} =$$

$$\therefore \text{Impedance (z)} = \sqrt{10^2 + 14{\cdot}14^2} = \sqrt{100 + 200}$$

$$= \sqrt{300} = 17{\cdot}32 \text{ ohms. Ans. } (b).$$

$$\text{Power factor} = \frac{\text{Resistance}}{\text{Impedance}} \quad \frac{10}{17{\cdot}32} = 0{\cdot}5773. \text{ Ans. } (c).$$

$$\text{Now, I} = \frac{\text{E}}{\text{z}} = \frac{120}{17{\cdot}32}$$

$$\therefore \text{True power} = \text{E I} \times \text{power factor},$$

$$= 120 \times \frac{120}{17{\cdot}32} \times 0{\cdot}5773,$$

$$= 480 \text{ watts. Ans. } (d).$$

***Capacitance.**

If two plates, separated by an insulating medium, are each connected to the terminals of a battery, a current flows which charges up the space between them with an electric field. When the field is established, no more current flows unless the E.M.F. between the plates is altered. It is important to note that this is an electric field, not a magnetic field. A magnetic field is only set up while a current is flowing, whereas an electric field is set up wherever two parts are at a different electrical potential. The current which flows into the plates to set up the field, represents energy which is made up from the supply.

The unit of capacity or capacitance (C) is the *Farad* and is such that one ampére-second (one coulomb) of electricity charges the two parts to a potential difference of one volt.

When an arrangement is specially made to have capacity, it is called a *condenser*. The capacity of a condenser is usually stated in microfarads, that is, millionths of a farad, or one farad $\times$ 10^{-6}, since the farad is too large a unit to employ.

One ampére-second of electricity, raises the potential difference across a capacity of one farad, by one volt.

$$E = \frac{\text{Quantity of electricity}}{\text{Capacity}}$$

$$\frac{\text{Average current in amps} \times \text{time in seconds}}{\text{Capacity in farads}}$$

$$\therefore E = \frac{I\,t}{C} \text{ volts.}$$

For an alternating current, the E.M.F. reached during one quarter period will be

$$\frac{\text{Average current}}{C \times 4 \times \text{frequency}} \text{ because } t = \tfrac{1}{4} \text{ of } \frac{1}{f} \qquad t = \qquad f$$

For a sinusoidal current, the average value of the current for

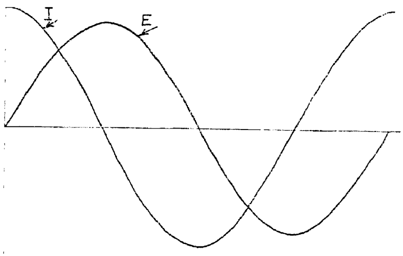
a quarter period is — of the peak value,

$\therefore$ Peak voltage reached in quarter period

$$\frac{\text{Maximum current}}{4\,f\,C} \qquad 2 \qquad \frac{\text{I max.}}{\pi\,f\,C}$$

or, taking R.M.S. values, E =

Thus the reactance of a capacity = $\dfrac{\quad}{f\,C}$ and therefore decreases with the frequency. Compare this with the reactance of an inductance, which is $2\,\pi\,f\,L$, this increases with the frequency.



The manner in which the current and the voltage vary over one period is shown in this diagram. At the point of switching in, the current is at its maximum value, and the voltage is rising at its maximum rate. As the voltage rises the current falls, and the *rate* at which the voltage rises decreases. When the current has decreased to zero, the voltage ceases to increase ; this occurs after a quarter period. During the next quarter period, the current rises to a maximum in the opposite direction, and the voltage therefore falls to zero again. The amount of electricity which flows into the capacity in one quarter of the period, flows out again during the next quarter. It will be seen from the diagram that the current *leads* the voltage by a quarter period, or 90°.

If a circuit has resistance and capacitance in series, the impedance is found in the same way as for a circuit with resistance and inductance in series. In this case, however, the current leads the E.M.F.

In practice, circuits may have inductance and capacitance, both of which may be important at the frequency of the current ; in addition, the resistance will usually not be low enough to be neglected. In such circuits, the E.M.F. to overcome the resistance is in phase with the current, the E.M.F. of induction leads the current by 90°, and the E.M.F. of the capacitance lags behind the current by 90°. Thus, the E.M.F. of the latter two are always opposite in direction, and the resultant of these can be obtained by finding the arithmetical difference.

The resultant reactance is thus found by taking the difference of $2 \pi f L$ and $\dfrac{1}{2 \pi f C}$.

The impedance of the circuit is then found in the usual way, viz., $z^2 = R^2 + x^2$. If the reactance of the inductance is greater than the reactance of the capacitance, then the current lags behind the voltage. If the reactance of the capacitance is the greater, the current leads the voltage.

Example. A circuit has a resistance of 12 ohms, an inductance of 0·015 henry, and a capacity of 200 microfarads, all in series. The frequency is 50 cycles per second. Find the impedance and the power factor.

$$\text{Inductive reactance} = 2\,\pi\,f\,\mathrm{L},$$
$$= 2 \times \pi \times 50 \times 0{\cdot}015 = 4{\cdot}71 \text{ ohms.}$$

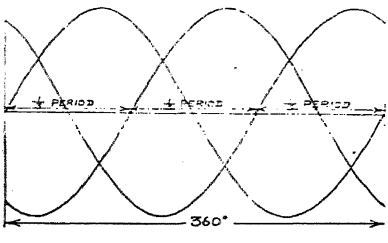
$$\text{Capacitive reactance} = \frac{1}{2\,\pi\,f\,\mathrm{C}}$$
$$\frac{1}{2 \times \pi \times 50 \times 200 \times 10^{-6}}$$
$$= 15{\cdot}9 \text{ ohms.}$$

$$\therefore \text{ Resultant reactance} = 15{\cdot}9 - 4{\cdot}71 = 11{\cdot}19 \text{ ohms.}$$

$$\therefore \text{ Impedance} = \sqrt{12^2 + 11{\cdot}19^2} = 16{\cdot}41 \text{ ohms. Ans.}$$

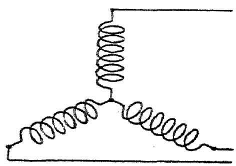
$$\text{and, power factor} = \frac{12}{16{\cdot}41} = 0{\cdot}731. \text{ Ans.}$$

***Polyphase Currents.**

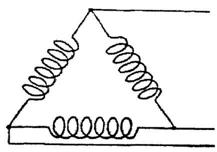


The diagram represents three alternating currents generated by separate windings on an alternator in such a way that there is one-third p

Such a machine would be termed a three phase alternator. The three separate coils could be connected with similar coils on a suitable motor; it is more economical in copper, however, to group the coils and use three leads only. The two methods of connecting them are shown here, and are said to be *Star* or *Y* connected



STAR CONNECTION



MESH CONNECTION

and *Mesh or Delta* connected. The phase voltage is the E.M.F. generated in each coil. The line voltage and line current is the E.M.F. and current respectively in the leads to the output.

It is sufficient for our present purpose to make the following statements.

$$\text{For star connection} \begin{cases} \text{line voltage} = \sqrt{3} \times \text{phase voltage.} \\ \text{line current} = \text{phase current.} \end{cases}$$

$$\text{For mesh connection} \begin{cases} \text{line voltage} = \text{phase voltage.} \\ \text{line current} = \sqrt{3} \times \text{phase current.} \end{cases}$$

The total power is the same in each case.

Total power

$$= 3 \times \text{phase voltage} \times \text{phase current} \times \text{power factor.}$$

$$= \sqrt{3} \times \text{line voltage} \times \text{line current} \times \text{power factor.}$$

Battery Efficiency.

Secondary cells are those which can be charged up from a source of supply, and then give out electrical energy on being discharged. The efficiency can be expressed in two ways, the ampére-hour efficiency and the watt-hour efficiency.

Ampére-hour efficiency

Quantity of electricity got out

Quantity of electricity put in

$$= \frac{\text{Discharging rate in amps} \times \text{time on discharge}}{\text{charging rate in amps} \times \text{time on charge}}$$

Watt-hour efficiency

Energy got out

Energy put in

$$\frac{\text{Power in watts during discharge} \times \text{time on discharge}}{\text{Power in watts during charge} \times \text{time on charge}}$$

TEST EXAMPLES XXIX.

1. Find the force (*a*) in dynes, (*b*) in lb. on a conductor 25 centimetres long when carrying a current of 75 ampéres at right angles to a magnetic field of 8,000 lines per sq. cm.

(*a*) 1,500,000 dynes ; (*b*) 3·37 lb. Ans.

2. The armature of an electric motor is 14 ins. diameter. It has 720 conductors, the effective length of each being 12 inches, and only two-thirds of the number of conductors are simultaneously in the magnetic field. The flux density in the air gap under the poles is 7,000 lines per sq. cm. and each conductor carries a current of 30 ampéres. If the armature runs at 680 revs. per minute, find (*a*) the torque in foot lb., (*b*) the horse power developed.

(*a*) 402·6 ft. lb. ; (*b*) 52·11 H.P. Ans.

3. A conductor, 15 centimetres long, moves across a magnetic field of 8,200 lines per sq. cm. at a speed of 32 metres per second. Calculate the value of the E.M.F. induced in the conductor.

3·936 volts. Ans.

4. A magneto dynamo has two poles with a useful flux of 90,000 lines. The armature has a single winding of 420 complete turns. Calculate the average E.M.F. generated when the armature is driven at 1,000 revs. per minute.

25·2 volts. Ans.

5. A six pole lap wound generator has a useful flux of 4·5 megalines per pole. It has 660 conductors and is driven at 920 revs. per minute. Find the E.M.F. generated.

455·4 volts. Ans.

6. An electric motor takes 180 amps. The supply voltage is 400. The resistance of the shunt field is 200 ohms, and the resistance of the armature is 0·02 ohm. There is a drop of 2 volts at the brushes. What is (*a*) the back E.M.F. of the motor ; (*b*) the horse power ; (*c*) the efficiency, neglecting all losses of which information is not given?

(*a*) 394·44 volts ; (*b*) 94·1 H.P. ; (*c*) 0·975. Ans.

*7. A circuit has a resistance of 3 ohms, and an inductance of 0·01 henry. The P.D. across its ends is 60 volts, and the frequency is 50 cycles per second. Calculate (*a*) the impedance ; (*b*) the power factor ; (*c*) the power absorbed.

(*a*) 4·344 ohms ; (*b*) 0·6906 ; (c) 572·3 watts. Ans.

*8. A 100 watt lamp for 100 volts supply is placed across a 220 volt supply. What value of resistance must be placed in series with it so that it will work under its proper conditions? If a coil is used instead of the resistance, and the resistance of the coil is small compared to its reactance, what is the inductance of the coil? The frequency is 50 cycles per sec. What is the total power absorbed in each case?

Series resistance = 120 ohms, and power absorbed = 220 watts.

Inductance = 0·624 henry, and power absorbed = 100 watts.

Ans.

*9. An alternating current, single phase motor, on a 440 volt supply has an output of 60 horse power. The efficiency is 0·86, and the power factor is 0·9. Calculate the current.

131·4 ampéres. Ans.

10. An accumulator is charged at the rate of 6 amps for 18 hours, and discharged at the rate of 3·5 amps for 28 hours. Find the ampére-hour efficiency.

90·7%. Ans.

SOLUTIONS TO TEST EXAMPLES I.

$\frac{7}{8} - \frac{1}{2} \quad \frac{47}{16} + \frac{1}{4} \times \frac{26}{5} = \frac{7}{8} - \frac{47}{32} + \frac{26}{20}$

L.C.M. = 160, therefore $\dfrac{140 - 235 + 208}{160}$ Ans.

2. Each piece, including the saw cut, is 1 ft. $3\frac{5}{16}$ inches long. This is $15\frac{5}{16}$ inches or 245, $\frac{1}{16}$ths of an inch.

17 ft. 3 inches is 207 inches or 3,312, $\frac{1}{16}$ths of an inch.
No. of pieces $= \frac{3312}{245} = 13$, and 127 sixteenths over.
Ans. 13 pieces and $7\frac{15}{16}$ inches left.

3. $8\frac{1}{4}$ knots is $\frac{33}{4} \times 6080$ feet per hour.
The speed of the train is $\frac{33}{4} \times 6080 \times \frac{25}{4}$ feet per hour.

or $\frac{}{4} \times 6080 \times \frac{25}{4} \times$ Statute miles per hour.

19 $\quad$ 480 16

$\times\ 25 \quad 19 \times 25$

$4 \times \quad 8$ $= 59{\cdot}375$ Statute miles per hour. Ans.

4. 25 minutes is $\frac{25}{60}$ or $\frac{5}{12}$ of an hour.

18 days, 12 hours, 25 minutes is $(18 \times 24) + 12 + \frac{5}{12}$ hours, this is $444\frac{5}{12}$ hours.

Speed in miles per hour $= \dfrac{\text{distance}}{\text{time}} \quad \dfrac{3500}{444\frac{5}{12}}$

Speed in miles per hour $= 3{,}500 \times \frac{12}{5333} = 7{\cdot}875$ miles per hour. Ans.

5. Time is 3 hrs. 45 mins., this is 225 minutes.
Total revs. turned in this time is $259898 - 246849 = 13049$.
Revs. per minute $= \frac{13049}{225} = 57{\cdot}995$. Ans.
Revs. in 15 minutes $= 15 \times 57{\cdot}995 = 869{\cdot}925$.
Counter reading at 4 p.m. $= 259898 + 870 = 260768$. Ans.

6. $\frac{12}{280}$ = day's run per ton of coal, and
$\frac{12}{280} \times 245$ = day's run on 245 tons of coal if quality was the same.

$\frac{12}{280} \times 245 \times \frac{21}{19}$ = days' run on 245 tons of Welsh coal.
$= 11{\cdot}6$ days' steaming. Ans.

7. Distance moved $= 9 \times 21{\cdot}47$ feet if no slip.
Actual distance moved $= 9 \times 21{\cdot}47 \times \frac{92}{100} = 177{\cdot}77$ feet. Ans.

8. $$\frac{15{\cdot}72 + 15{\cdot}59}{2} = 15{\cdot}655 \text{ lb. actual weight.}$$

Error in both cases is the same, $15{\cdot}72 - 15{\cdot}655 = 0{\cdot}065$ lb.

$$\% \text{ error is } \frac{0{\cdot}065}{15{\cdot}655} \times 100 = \frac{65}{15655} \times 100 = 0{\cdot}415 \text{ per cent. Ans.}$$

9. Length of arc of 1 degree, for a radius of 300 feet or 3,600 inches.

$$= \tfrac{22}{7} \times 3{,}600 \times 2 \times \tfrac{1}{360} = \tfrac{440}{7} \text{ inches.}$$

$$\text{Length of arc of 1 minute} = \frac{440}{7 \times 60} = \tfrac{440}{420} = \tfrac{22}{21} \text{ ins.}$$

Error $= \frac{22}{21} - 1 = \frac{1}{21}$ inch.

% error is $(\frac{1}{21} \div \frac{22}{21}) \times 100 = (\frac{1}{21} \times \frac{21}{22}) \times 100$
this is $\frac{100}{22} = \frac{50}{11} =$ %.

10. $825 \times 2\pi =$ radians per minute, since there are 2π radians in one revolution.

$$\frac{825 \times 2\pi}{60} = \text{radians per second} = 86{\cdot}43. \quad \text{Ans.}$$

11. If 27 cubic inches of copper weigh 125 ozs. then 1 cubic inch weighs $\frac{125}{27}$ ozs., and if 72 cubic inches of iron weigh 312·5 ozs., then 1 cubic inch weighs $\frac{312{\cdot}5}{72}$ ozs.

The weights of equal volumes then are as

$$\frac{125}{27} : \frac{312{\cdot}5}{72}, \text{ multiply both by } \tfrac{9}{25}.$$

$$\text{or as } \frac{5}{3} : \frac{12{\cdot}5}{8}$$

$$\text{or as } \frac{40}{24} \quad \frac{37{\cdot}5}{24}$$

or as $40 : 37{\cdot}5$. This is the same as 16 :

12. From January 5th to August 4th is 7 months, and we have also 6 days in August.

Total Time is $7\frac{6}{30}$ months.

From February 17th to July 16th is 5 months, there are 15 days left in July, and 12 in August.

Total Time $= 5\frac{27}{30}$ months.

From April 20th to August 19th is 4 months, and we have 5 days more in August.

Total Time $= 4\frac{5}{30}$ months.

$7\frac{1}{5}$	$\times$ £24	0	0	$=$	£172	16	0
$5\frac{9}{10}$	$\times$ £20	0	0	$=$	£118	0	0
$4\frac{1}{6}$	$\times$ £16	0	0	$=$	£66	13	4
		Total			£357	9	4

SOLUTIONS TO TEST EXAMPLES II.

1.
$$\begin{array}{l} 3\,x^2 + 5\,xy + 3\,y^2 \\ 4\,x^2 - 3\,xy + y^2 \\ \hline 7\,x^2 + 2\,xy + 4 \qquad \text{Ans.} \end{array}$$

$$\begin{array}{r} 5\,x^3 + 6\,x^2 - 12 \\ 3\,x^2 - 10 \end{array} \quad \text{change the signs and add.}$$

$$5\,x^3 + 3 \qquad \text{Ans.}$$

2.
$$(x + 5)^2 + 3\,(x^2 + 2\,x + 1) - x\,(x + 3)$$
$$= \cancel{x^2} + 10\,x + 25 + 3\,x^2 + 6\,x + 3 - \cancel{x^2} - 3\,x$$
$$= 3\,x^2 + 13\,x + 28. \quad \text{Ans.}$$

3. $x - 2)x^3 + x^2 + 2\,x - 16(x^2 + 3\,x + 8.$ Ans.

$$\begin{array}{l} x^3 - 2\,x^2 \\ \hline \qquad 3\,x^2 + 2\,x - 16 \\ \qquad 3\,x^2 - 6\,x \\ \hline \qquad\qquad 8\,x - 16 \\ \qquad\qquad 8\,x - 16 \end{array}$$

4. $3\ a^2 + 5$
$a - b$

$3\ a^3 + 5\ ab + 3\ a$
$- 3\ a^2b - 5\ b^2 - 3\ b$

$3\ a^3 + 5\ ab + 3\ a - 3\ a^2b - 5\ b^2 - 3\ b$ Ans.

This may be written $3\ a^3 - 3\ a^2b + 3\ a + 5\ ab - 3\ b - 5 \cdot b^2$, but the order does not affect the value.

5. By inspection, $x^2 + 13\ x + 30 = (x + 3)(x + 10)$. Ans.
$x^2 + 2\ x - 3 = (x + 3)(x - 1)$; $(x^2 - y^2) = (x + y)(x - y)$ Ans.

6. $a - b)a^3 - b^3\ (a^2 + ab + b^2$. Ans.

$+ a^2\ b -$

$+ a\ b^2 -$
$+ a\ b^2 -$

$b)^2 = a^2\ b^2$; $(a^2\ b)^2 = a^4\ b^2$; $(3\ a\ b\ c)^2 = 9\ a^2b^2c^2$;

$$\left(\frac{3\ a\ b}{4\ x^2\ y}\right)^2 = \frac{9\ a^2\ b^2}{16\ x^4\ y^2}$$

$(x + 1)\ [x + 3 - (x + 5)(2\ x + 3)]$
$= (x + 1)\ [x + 3 - (2\ x^2 + 13\ x + 15)]$
$= (x + 1)\ [x + 3 - 2\ x^2 - 13\ x - 15]$
$= (x + 1)\ (- 2\ x^2 - 12\ x - 12)$

Ans. $= - 2\ x^3 \quad 14\ x^2 - 24\ x - 12$, this could be written :—

$- 2\ [x^3 + 7\ x^2 + 12\ x + 6]$. Ans.

9. $3\sqrt{48} = 3\sqrt{3 \times 16} = 3 \times \sqrt{3} \times \sqrt{16} = 3 \times \sqrt{3} \times 4$
$= 12 \times 1 \cdot 732 = 20 \cdot 784$. Ans.
$\sqrt{98} = \sqrt{2 \times 49} = \sqrt{2} \times \sqrt{49} = \sqrt{2} \times 7$
$1 \cdot 414 \times 7 = 9 \cdot 898$. Ans.

$3\sqrt{54} \quad 3\sqrt{2} \times 27 \quad \sqrt{2}$

$3\sqrt{2} \times \sqrt{3} \times \sqrt{9} \times \sqrt{2} \quad 3 \times 2 \times \sqrt{3} \times 3$

$\sqrt{2} \times \sqrt{2}$

Ans. $= 9 \times 1{\cdot}732 = 15{\cdot}588$

$4 \times \sqrt{72} \quad 4 \times \sqrt{2 \times 36} \quad \sqrt{3}$

$$= \frac{4 \times \sqrt{2} \times 6 \times \sqrt{3}}{3} = 8\sqrt{2}\sqrt{3}$$

Note $\sqrt{6} = 2{\cdot}4495$ (nearly). Ans. $= 8\sqrt{6} = 19{\cdot}596$.

10. $3\,xy \quad \sqrt{a^2 b^2} \quad 3\,xy \quad a\,b$

$2\,ab \sqrt{4\,x^3 y^3}. \quad 2\,ab \quad \times \quad \times$

$3\,\cancel{xy}\,\cancel{ab}$

$2\,\cancel{ab} \times 2\,\cancel{xy}\,\sqrt{xy}$

Ans. is $\frac{3}{?} \times \frac{1}{2} \times \quad 1 \quad 3 \quad xy$

$= \frac{\;}{4} \times \quad \frac{\sqrt{18}}{\cancel{18}\;6} \quad \frac{3\sqrt{2}}{24} \quad 8 \quad 1{\cdot}414$

Ans. $= 0{\cdot}1767$.

SOLUTIONS TO TEST EXAMPLES III.

$3\ x - 21 = x + 5$; $3\ x - x = 21 + 5$; $2\ x = 26.$
$x = 13.$ Ans.

$3\ x + 9 = 7\ x - 3$; $3\ x - 7\ x = -9 - 3$;
$-4\ x = -12.$
$x = 3.$ Ans.

$$\frac{2\ x + 2}{} + \tfrac{4}{3}\ x = \frac{x}{} \qquad + 3,\ \text{L.C.M.} = 12.$$

$6\ x + 6 + 16\ x = 2\ x + 10 + 36$
$6\ x + 16\ x - 2\ x = 36 + 10 - 6$
$20\ x = 40,\ x = 2.$ Ans.

30

$5\ x - 3\ x = 30 - 10 -$
$2\ x = 14,\ x = 7.$ Ans.

$2\ (x^2 + 3) = 3(x^2 - 1)$; $2\ x^2 + 6 = 3\ x^2 - 3$
$2\ x^2 - 3\ x^2 = -3 - 6$, or $-x^2 = -9$
$x^2 = 9,\ x = 3$, by taking square root of both sides.
Ans. is $x = \pm\ 3.$

$$\frac{p - q}{(a + b)} = \frac{x\ y}{z};\ x = \frac{z\ (p - q)}{y\ (a + b)} \quad \text{Ans.}$$

$$x\ y\ (a + b) \qquad = \frac{xy}{}\ (a + b) +$$

$$d \times e \qquad \frac{k \times q}{a \times b},\ a = \frac{k \times q \times d \times e}{\times\ c \times b} \quad \text{Ans.}$$

9. $0{\cdot}3\ (x + 3) = 0{\cdot}5\ (x + 1)$; $0{\cdot}3\ x + 0{\cdot}9 = 0{\cdot}5\ x + 0{\cdot}5$
$0{\cdot}3\ x - 0{\cdot}5\ x = 0{\cdot}5 - 0{\cdot}9$
or $-0{\cdot}2\ x = -0{\cdot}4$
$0{\cdot}2\ x = 0{\cdot}4$
$x = 2.$ Ans.

EXAMPLES IN SOLUTION OF FORMULÆ.

1. $$H = \frac{2\,P\,A\,L\,N}{33{,}000}; \quad 33{,}000\,H = 2\,P\,A\,L\,N;$$

$$L = \frac{33{,}000\,H}{2\,P\,A\,N},$$

$$L = \frac{\overset{6600}{\cancel{33000}} \times 150\cancel{0}}{2 \times 28 \times 253\cancel{0} \times \underset{13}{\cancel{65}}} \qquad \frac{\overset{3300}{\cancel{6600}} \times 150}{\cancel{2} \times 28 \times 253 \times 13}$$

$$= 5{\cdot}37. \quad \text{Ans.}$$

$$H = \frac{\sqrt[3]{P}\,(3\,H + \sqrt[3]{S}\,D^2)}{700}, \text{ putting in the values :—}$$

$$450 = \frac{\sqrt[3]{216}\,(3 \times 13950 + \sqrt[3]{S} \times 6400)}{700}, \text{ now } \sqrt[3]{216} = 6$$

$$\frac{450 \times 700}{6} = 41{,}850 + \sqrt[3]{S} \times 6{,}400)$$

$$52{,}500 = 41{,}850 + \sqrt[3]{S} \times 6{,}400$$

$$\frac{52{,}500 - 41{,}850}{6{,}400} \qquad \frac{10{,}650}{6{,}400}$$

$$\sqrt[3]{S} = 1{\cdot}664; \quad S = (1{\cdot}664)^3$$

$$S = 4{\cdot}606. \quad \text{Ans.}$$

$$\frac{(D - d) \times T \times 28{,}000}{W \times D} = P$$

$$225 = \frac{(4{\cdot}5 - d) \times \frac{3}{4} \times 28{,}000}{25 \times 4{\cdot}5}$$

$$4{\cdot}5 - d = \frac{225 \times 25 \times 4{\cdot}5 \times 4}{3 \times 28{,}000}$$

$$4{\cdot}5 - d = 1{\cdot}205; \quad d = 4{\cdot}5 - 1{\cdot}205$$

$$d = 3{\cdot}295. \quad \text{Ans.}$$

4. $$P = \frac{99{,}000\ t^2}{(L+1)\ D}\ ;\ L + 1 = \frac{99{,}000\ t^2}{P\ D}\ ;\ L = \frac{99{,}000\ t^2}{P\ D} - 1$$

$$L = \left(\frac{\overset{110}{\cancel{99000}}}{\underset{2}{\cancel{180}} \times \cancel{40}} \times \frac{13}{16} \times \frac{13}{16}\right) - 1\ ;\ L = \frac{169 \times 110}{16 \times 128} - 1$$

$= 8{\cdot}077$. Ans.

5. $$P = \frac{9{,}900}{\cancel{3} \times 41} \times \frac{\cancel{3}}{4}\left[5 - \frac{29 + 12}{60 \times \frac{3}{4}}\right]$$

$$P = \frac{9{,}900}{41 \times 4}\left[5 - \frac{41}{45}\right]$$

$$P = \frac{9{,}900}{41 \times 4}\left[\frac{225 - 41}{45}\right] = \frac{\overset{3300}{\cancel{9900}}}{41 \times \cancel{4}} \times \frac{\overset{46}{\cancel{184}}}{\underset{15}{\cancel{45}}}$$

$= 246{\cdot}8$. Ans.

6. $$S^3 = \frac{C\ P\ D^2}{f\left(2 + \frac{D^2}{d^2}\right)}\ ;\ \left(2 + \frac{D^2}{d^2}\right) = \frac{C\ P\ D^2}{f\ S^3}$$

$$\frac{D^2}{d^2} = \frac{C\ P\ D^2}{f\ S^3} - 2\ ;\qquad d^2 = \frac{D^2}{\frac{C\ P\ D^2}{f\ S^3} - 2}$$

$$\text{or } d = \frac{D}{\sqrt{\frac{C\ P\ D^2}{f\ S^3} - 2}}$$

SOLUTIONS TO TEST EXAMPLES IV.

1. Let x = the number, then $\frac{x}{3} = \frac{x}{2} - 4$

or $2x = 3x - 24$, from which $x = 24$. Ans.

2. Let x = the unit digit, then $2x$ = the tens digit. The number is $(2x \times 10) + x = 21x$.

If the digits are reversed the number is $(10 \times x) + 2x = 12x$

Then $21x - 12x = 27$,

$9x = 27,\ x = 3$

$2x = 6$

Digits are 6 and 3, and the number is 63. Ans.

3. Let x = 1st man's age.
Then $2\ x = 3 \times$ other man's age.
other man's age $= \frac{2}{3}\ x$,
$\therefore\ x + \frac{2}{3}\ x = 73$, or $\frac{5}{3}\ x = 73$,
$x = 73 \times \frac{3}{5} = \frac{219}{5} = 43{\cdot}8$ years.
$73 - 43{\cdot}8 = 29{\cdot}2$ years.

Ages are 29·2 and 43·8 years. Ans.

4. Let x = smaller number, then $\frac{3}{2}\ x$ = larger.

If each increased by 15, they become, $\frac{3}{2}\ x + 15$ and $x + 15$
These will be equal if we now multiply the lesser by 6, and the greater by 5.

$$\therefore\ (x + 15)\ 6 = (\tfrac{3}{2}\ x + 15)\ 5$$
$$6\ x + 90 = \tfrac{15}{2}\ x + 75$$
$$-\ 1\tfrac{1}{2}\ x = -\ 15$$
$$x = 15 \times \tfrac{2}{3} = 10,$$
$$10 \times \tfrac{3}{2} = 15$$

The numbers are 10 and 15. Ans.

5. Let x = weight of copper to add.

Total weight of copper in new bronze $= (85 + x)$ lb.
But total weight of mixture $= (100 + x)$ lb.

$$\text{and } \frac{85 + x}{100 + x} = 0{\cdot}92$$

or $85 + x = 92 + 0{\cdot}92\ x$
$0{\cdot}08\ x = 7$
$x = \frac{700}{8} = 87{\cdot}5$ lb. Ans.

6. Let x = 1st year's salary.
Then $\frac{6}{5}\ x$ = 2nd year's salary,
and $\frac{6}{5} \times \frac{6}{5}\ x$ = 3rd year's salary,
$\therefore\ x + \frac{6}{5}\ x + \frac{36}{25}\ x = 728$
or, $25\ x + 30\ x + 36\ x = 728 \times 25$.
Then $91\ x = 728 \times 25$

$$x = \frac{728 \times 25}{91} = £200. \text{ Ans.}$$

7. Let x = first number, y = other. Then $x - \frac{y}{2} = 0{\cdot}6$

and $\frac{y}{4} - \frac{x}{5} = 0{\cdot}42.$

$x - \frac{x}{5} = .$

Multiply top equation

$$\frac{x}{2} - \cancel{\frac{y}{4}} = 0{\cdot}3$$

$$-\frac{x}{5} + \cancel{\frac{y}{4}} = 0{\cdot}42$$

Add

$$\frac{\quad}{10} \qquad = 0{\cdot}72$$

$$x = \frac{0{\cdot}72 \times 10}{3} = 2{\cdot}4$$

$2{\cdot}4 - \frac{y}{2} = 0{\cdot}6$, from which $y = 3{\cdot}6$.

Numbers are $2{\cdot}4$ and $3{\cdot}6$. Ans.

8. Let x = age of younger,
y = age of elder,
10 years ago, their ages were $x - 10$, and $y - 10$
and therefore $5\,(x - 10) = 3\,(y - 10)$
or $5\,x - 50 = 3\,y - 30$

5 years hence, their ages will be $x + 5$, and $y + 5$,
and therefore $4(x + 5) = 3\,(y + 5)$

$5\,x - 50 = 3\,y - 30$, or $5\,x - \cancel{3\,y}$
$4\,x + 20 = 3\,y + 15 \quad 4\,x - \cancel{3\,y} = -5$ s

$x = 25$ years

$(5 \times 25) - 3\,y = 20$

or $-3\,y = -125 + 20$, from which $y = 35$

Ages are 25 and 35 years. Ans.

SOLUTIONS TO TEST EXAMPLES V.

$$b = \sqrt{(\tfrac{17}{15}\,a)^2 - a^2} = \sqrt{}$$

$$b = \sqrt{\tfrac{64}{225}\,a^2} = \tfrac{8}{15}\,a$$

$$\text{Area} = a \times b$$

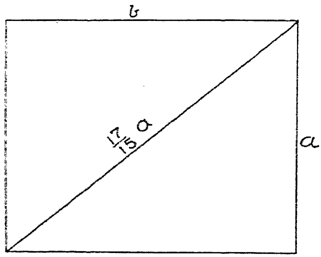
$$= a \times \tfrac{8}{15}\,a = \tfrac{8}{15}\,a^2$$

but area $= 30$ sq. feet.

$$\therefore \tfrac{8}{15}\,a^2 = 30$$

$$a = \sqrt{\frac{30 \times 15}{8}} = \sqrt{\frac{15 \times 15}{4}} = \tfrac{15}{2} = 7{\cdot}5.$$

The length is 7·5 feet and the breadth 4 feet. Ans.

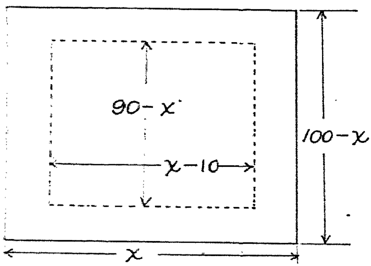


Original area $= x\,(100 - x) = 100\,x - x^2$

$\tfrac{5}{8}$ of this $= \tfrac{5}{8}\,(100\,x - x^2) = 62{\cdot}5\,x - \tfrac{5}{8}\,x^2$

Area after cutting $= (90 - x) \times (x - 10)$

Multiplying out, $= 100\,x - x^2 - 900$



Then $62 \cdot 5\ x - \frac{5}{8}\ x^2 = 100\ x - x^2 - 900$

or $\frac{3}{8}\ x^2 - 37 \cdot 5\ x = -900$, multiply across by $\frac{8}{3}$

or $x^2 - 100\ x = -2{,}400$

$x^2 - 100\ x + (50)^2 = -2{,}400 + (50)^2$ completing the square,

$x - 50 = \pm \sqrt{-2{,}400 + 2{,}500} = \pm \sqrt{100}$

$x = \pm 10 + 50$

$x = +60$ or $+40$, both values are positive and both are permissible.

The sides of the table then, are 60 and 40 inches. Ans.

3. Let x = smaller number, $(7 - x)$ = larger number.

Their squares are, x^2 and $(7 - x)^2$

or x^2 and $(49 - 14\ x + x^2)$

Then $x^2 \times \frac{81}{25} = 49 - 14\ x + x^2$

$\frac{81}{25}\ x^2 - x^2 + 14\ x = 49$

$\frac{56}{25}\ x^2 + 14\ x = 49$, or $\frac{56}{25}\ x^2 + 14\ x - 49 = 0$

Using the formula, $x = \dfrac{-b \pm \sqrt{b^2 - 4}}{2\ a}$

$$\frac{14 \pm \sqrt{14^2 - 4 \times \frac{56}{25} \times (-49)}}{2 \times \frac{56}{25}}$$

$$-14 \pm \sqrt{196 \times 439 \cdot 04} = -14 \pm \sqrt{635 \cdot 04}$$

$$x = \frac{-14 \pm 25 \cdot 2}{4 \cdot 48} = + \frac{11 \cdot 2}{4 \cdot 48} =$$

The minus value does not apply here.

The numbers are 2·5 and 4·5. Ans.

4. Let x = speed of slower vessel

and $x + 2$ = speed of faster vessel

Now $\dfrac{\text{distance}}{\text{speed}}$ = time

$\therefore\ \dfrac{160}{x}$ = time of slow ship, and $\dfrac{160}{x + 2}$ fast ship.

Now the time of the fast ship is less by 4 hours.

$\therefore \dfrac{160}{x} = \dfrac{160}{x + 2} + 4$, L.C.M. is $x\ (x + 2)$

$160\ (x + 2) = 160 \times x + [4 \times x\ (x + 2)]$

$+ 320 = \cancel{160\,x} + 4\,x^2 + 8\,x$, subtracting $160\,x$ from both sides.

$4\,x^2 + 8\,x = 320$, or $x^2 + 2\,x = 80$

$x^2 + 2\,x + 1^2 = 80 + 1^2$ completing the squares.

$x + 1 = \pm\ \sqrt{81}$, or $x = \pm\ 9 - 1$

$x = 8$ knots, and speed of faster ship is 10 knots. Ans.

x = lesser part, then $15 - x$ = greater part.

Their squares are, x^2 and $(15 - x)^2$

or x^2 and $225 - 30\,x + x^2$

By the conditions of the problem,

$225 - 30\,x + x^2 - 3\,x^2 = 73$

or $-\ 2\,x^2 - 30\,x + 225 - 73 = 0$

$2\,x^2 + 30\,x - 152 = 0$

$x = \quad \pm\ \sqrt{b^2 -}$

$= \dfrac{-\ 30 \pm \sqrt{30^2 - 4 \times 2\ (-\ 152)}}{2 \times 2}$

$-\ 30 \pm \sqrt{900 + 1216}$

$x = \dfrac{30 \pm 46}{4} = \frac{16}{4} = 4$

The other value is $\frac{76}{4} = 19$ which cannot apply.

The numbers are 4 and 11. Ans.

6. $S = u\,t + \dfrac{a\,t^2}{2}$, putting in the values,

$$125 = 10\,t + \frac{6\,t^2}{2}, \text{ or } 3\,t^2 + 10\,t - 125 = 0$$

$$\frac{-b \pm \sqrt{b^2 - 4\,ac}}{} \qquad -10 \pm$$

$$= \frac{-10 \pm 40}{6} = +\tfrac{30}{6} = 5 \text{ secs.}$$

SOLUTIONS TO TEST EXAMPLES VI.

1. 3,251 × 3·995 Logs. 3·512, 0·6015 } add

4·1135

4·1135 is log. of answer. Ans. = 12,990.

2. 2·718 × 0·00371 Logs. 0·4348, $\bar{3}$·5694 }

$\bar{2}$·0042

$\bar{2}$·0042 is log. of 0·01009. Ans.

3. 0·392 ÷ 0·895 Logs. $\bar{1}$·5933, $\bar{1}$·9518 } subtract

$\bar{1}$·6415

$\bar{1}$·6415 is log. of 0·438. Ans.

0·0356 ÷ 27·2 Logs. $\bar{2}$·5514, 1·4346 } subtract

$\bar{3}$·1168

$\bar{3}$·1168 is log. of 0·001308. Ans.

$\sqrt[2]{0{\cdot}0815}$ Logs. $\dfrac{\bar{2}{\cdot}9101}{2}$ or, 2)$\bar{2}$·9101

$\bar{1}$·45505

$\bar{1}$·455 is log. of 0·2851. Ans.

$\sqrt[3]{1{\cdot}235}$ Logs. 3)0·0917

0·03056

0·03056 is log. of 1·073. Ans.

$(27{\cdot}91)^{\frac{2}{5}}$ Logs. $1{\cdot}4458$

$$\begin{array}{r} 1{\cdot}4458 \\ 2 \\ \hline 5)\ 2{\cdot}8916 \\ \hline 0{\cdot}5783 \end{array}$$

And 0·5783 is log. of 3·787. Ans.

$\sqrt[3]{0{\cdot}0593}$ Logs. $3)\ \overline{2}{\cdot}7731$

$$\begin{array}{r} 3)\ \overline{2}{\cdot}7731 \\ \hline \overline{1}{\cdot}5910 \end{array}$$

And $\overline{1}{\cdot}591$ is log. of 0·3899. Ans.

Ans. 0·2851 ; 1·073 ; 3·787 ; 0·3899.

6. $\dfrac{\frac{11}{14}\left[(17{\cdot}5)^2 - (14{\cdot}75)^2\right] \times 5{,}600}{2{,}240} \qquad \frac{253}{113}$

$$\frac{11 \times 32{\cdot}25 \times 2{\cdot}75 \times \overset{40}{\cancel{5{,}600}}}{\cancel{14} \times 2{,}24\cancel{0}} + \frac{253}{113}$$

Logs. for 1st Fraction.

$$\left.\begin{array}{r} 1{\cdot}0414 \\ 1{\cdot}5086 \\ 0{\cdot}4393 \\ 1{\cdot}6021 \end{array}\right\} \text{add}$$

$$\begin{array}{r} 4{\cdot}5914 \\ 2{\cdot}3502 \\ \hline 2{\cdot}2412 \end{array} \quad \text{subtract}$$

Logs. for 2nd Fraction.

$$\left.\begin{array}{r} 2{\cdot}4031 \\ 2{\cdot}0531 \end{array}\right\} \text{subtract}$$
$$\begin{array}{r} \hline 0{\cdot}3500 \end{array}$$

2·2412 is log. of 174·3, and 0·35 is log. of 2·239

174·3 + 2·239 = 176·539. Ans.

$\frac{T_1}{T_2} = \left(\frac{v_2}{v_1}\right)^{0\cdot4}$, putting in the values,

$\frac{855}{505}$ $\quad ^{4}$, or $\frac{855}{505} =$

since $0\cdot4 = \frac{4}{10} = \frac{2}{5}$

By logarithms,

$$\text{Log. } 855 - \text{log. } 505 = \tfrac{2}{5} \text{ log. } 15\cdot2 - \tfrac{2}{5} \text{ log. } v_1$$

$$\tfrac{2}{5} \text{ log. } v_1 = \tfrac{2}{5} \text{ log. } 15\cdot2 - \text{log. } 855 + \text{log. } 505$$

Multiply each term of this equation by $\frac{5}{2}$

$$\text{Log. } v_1 = \text{log. } 15\cdot2 - \tfrac{5}{2} \text{ log. } 855 + \tfrac{5}{2} \text{ log. } 505$$

$$\text{Log. } v_1 = 1\cdot1818 - \tfrac{5}{2} \times 2\cdot9320 + \tfrac{5}{2} \times 2\cdot7033$$

$$\text{Log. } v_1 = 1\cdot1818 - 7\cdot3300 + 6\cdot7583$$

$$\text{Log. } v_1 = 0\cdot6101, \; v_1 = 4\cdot075. \quad \text{Ans.}$$

$$\frac{215}{13\cdot5}[1 + \log_e 13\cdot5] - 4$$

$$\frac{215}{13\cdot5}[1 + (1\cdot1303 \times 2\cdot3)] - 4$$

$$\begin{array}{r} 1\cdot1303 \\ 2\cdot3 \\ \hline 33909 \\ 22606 \\ \hline 2\cdot59969 \end{array}$$

Take this as $2\cdot599$

$$= \frac{215}{13\cdot5}[1 + 2\cdot599] - 4$$

Logs. $\left.\begin{array}{r} 2\cdot3324 \\ 0\cdot5562 \end{array}\right\}$ add

$$= \frac{215}{13\cdot5} \times 3\cdot599 - 4$$

$1\cdot1303$ subtract

$1\cdot7583$

$1\cdot7583$ is log. of $57\cdot32$

$$= 57\cdot32 - 4$$

Ans. $= 53\cdot32$.

9. $\dfrac{100\,[\mathrm{T} + 1]^2}{\mathrm{S} - 6}$ putting in the values,

$$\frac{100\,[13 + 1]^2}{165 - 6} = \frac{100 \times 14 \times 14}{159}$$

	Logs.
	2·0000
	1·1461
	1·1461
	4·2922
	2·2014
	2·0908

2·0908 is log. of 123·2

Ans. = 123·2.

SOLUTIONS TO TEST EXAMPLES VII.

Let p = pressure, t = thickness, d = diameter,

then $p \propto t$

and $p \propto \dfrac{1}{d}$ $\qquad \dfrac{p \times d}{}$ is constant,

or p_1 $\qquad p_1$

$t_1\ d_2$

$$\frac{150 \times 16\cancel{0} \times 1\frac{1}{2}}{1 \times 18\cancel{0}} \qquad \overset{25}{\cancel{15}\cancel{0}} \times \overset{8}{\cancel{16}} \times - = 200$$

p_2 = 200 lb. per sq. inch. Ans.

$\dfrac{p\,v}{\mathrm{T}}$ = 53·2 for one lb. of air.

$$v = \frac{53{\cdot}2 \times \mathrm{T}}{p} = \frac{53{\cdot}2 \times (460 + 300)}{200 \times 144}, \text{ cu. feet per lb.}$$

log. v = log. 53·2 + log. 760 — (log. 200 + log. 144)

Log. v = 0·1473

and v = 1·404 cu. ft. per lb.

Vol. of 10 lb. = 14·04 cu. feet. Ans.

Logs.	
1·7259	2·3010
2·8808	2·1584
4·6067	4·4594
4·4594	
0·1473	

Pressure varies as the absolute temperature if the volume is constant,

$= \text{constant}$

T

200

$300 + 460 \qquad 100 + 460$

$$P = \frac{200 \times 560}{760} = 147{\cdot}4 \text{ lb. per sq. inch.} \quad \text{Ans.}$$

Let p = pressure per sq. inch on thrust, k = knots, and H = horse power.

then $p \propto$ H

k

$= \text{constant}$

and $p \propto \dfrac{1}{k}$ H

or $p_1 \ k_1$ –, or $p_2 = p_1$

$$62 \times 10 \times \qquad \frac{4{,}340}{6 \times 9} = 80{\cdot}37 \text{ lb. per sq. inch.}$$

$p_2 = 80{\cdot}37$ lb. per sq. inch. Ans.

Linear Speed $\propto$ Diameter $\times$ revolutions

or $S \propto D \times R$, and $S^2 \propto D^2 R^2$

as Stress $\propto S^2$, it follows that Stress $\propto D^2 R^2$

$$\therefore \frac{\text{Stress}}{} = \text{cons.}$$

$2{,}200 \qquad \text{Stress}_2$

$(4{\cdot}5)^2 \times 1700^2 \qquad 10^2 \times 400^2$

$$\text{Stress}_2 = \frac{2{,}200 \times 10^2}{(4{\cdot}5)^2 \times 1{,}700^2}$$

Logs.	
3·3424	0·6532
2·0000	0·6532
2·6021	3·2304
2·6021	3·2304
10·5466	7·7672
7·7672	

2·7794 Log. of 601·7

$\text{Stress}_2 = 601{\cdot}7$ lb. sq. inch. Ans.

$$\frac{\quad}{16} d^3 \text{ S} = \frac{63{,}000 \text{ H.P.}}{\text{Revs.}}, \quad \text{or H.P.} = \frac{\pi\, d^3 \times \text{S} \times \text{R}}{16 \times 63{,}000}$$

We see that H.P. $\propto d^3 \times \text{S} \times \text{R}$, or $\dfrac{\text{H.P.}}{d^3 \text{ S R}}$ is constant.

$$\frac{\text{H P}_1}{d_1^3 \text{ S}_1 \text{ R}} \qquad \text{H P}_2 .$$

or $\text{H P}_2 =$

$$d_1^3 \times \text{S}_1 \times \text{R}_1$$

$$\text{H P}_2 = \frac{150 \times 4^3 \times 0{\cdot}8 \times 19\cancel{0}}{6^3 \times 1 \times 7\cancel{0}}$$

Note, let $1 = \text{S}_1$ \
then $0{\cdot}8 =$ } where S is the Stress.

$$\frac{\cancel{150} \times \cancel{4} \times \overset{2}{\cancel{4}} \times 4 \times 0{\cdot}8 \times 19}{6 \times \underset{3}{\cancel{6}} \times \underset{3}{\cancel{6}} \times 7}$$

$$\frac{110 \times 4 \times 0{\cdot}8 \times 19}{9 \times 7} = 96{\cdot}5$$

$\text{H P}_2 = 96{\cdot}5.$ Ans.

Let O = ohms, d = diameter, l = length

then $\text{O} \propto l$ \
and $\text{O} \propto \dfrac{1}{d^2}$ } $\therefore \dfrac{\text{O}\, d^2}{l} = \text{constant.}$

$$\frac{O_1}{l_1} \quad \frac{O_2\ d_2^2}{l_2}, \text{ or } d_2^2 = \frac{d_1^2 \times l_2}{l_1\ O_2}$$

Let 1 = length of first wire
and 4 = ,, ,, second ,,

$$d_2^2 = \frac{3{\cdot}2 \times 0{\cdot}1 \times 0{\cdot}1 \times 4}{1 \times 6{\cdot}8}$$

$$\sqrt{\frac{3{\cdot}2 \times 0{\cdot}1 \times 0{\cdot}1 \times 4}{1 \times 6{\cdot}8}} \qquad \sqrt{\frac{3{\cdot}2 \times 0{\cdot}04}{6{\cdot}8}}$$

d_2 = 0·1372 inch. Ans.

Logs.
0·5051
$\bar{2}$·6021

$\bar{1}$·1072
0·8325

2)$\bar{2}$·2747

$\bar{1}$·1373

$\bar{1}$·1373 is log. of 0·1372

7. m = men, y = yards, d = days, h = hours.

$m \propto y$

$$m \propto \frac{1}{d \times h} \qquad \frac{m\ d\ h}{} = \text{cons.}$$

$m_1\ d_1\ h_1$

y_2

$$\frac{m_1\ d_1\ h_1 \times y_2}{\times\ d_2 \times} \qquad \frac{10\!\!/ \times 5 \times 8 \times 25\!\!/0}{100\!\!/ \times 7 \times 1\!\!/0}$$

$$m_2 = \frac{100\!\!/0}{7\!\!/0} = \frac{100}{7} = 14\tfrac{2}{7} \text{ men. Ans.}$$

The weight will vary as the area.
,, ,, ,, thickness.
,, ,, ,, specific gravity.

$\therefore$ Wt. $\propto$ area $\times$ thickness $\times$ specific gravity.

$$\text{or} \quad \frac{\text{Wt.}}{a \times t \times \text{Sp. g.}} = \text{constant,}$$

$$\text{or} \quad \frac{\text{Wt.}_1}{a_1 \times t_1 \times \text{Sp.g.}_1} = \frac{\text{Wt.}_2}{a_2 \times t_2 \times \text{Sp.g.}_2}$$

$$\frac{5}{1 \times \frac{1}{8} \times 7{\cdot}7} \qquad \frac{\text{Wt.}_2}{(\frac{5}{4})^2 \times \frac{11}{14} \times \frac{5}{8} \times}$$

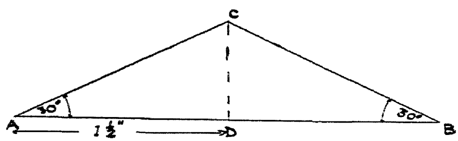
$$\text{Wt.}_2 = \frac{55 \times 25 \times \overset{5}{\cancel{40}} \times 11{\cdot}4}{7{\cdot}7 \times 14 \times \cancel{8} \times 16} = \frac{275 \times 11{\cdot}4 \times 25}{7{\cdot}7 \times 14 \times 16}$$

Logs.			
2·4393	0·8865		
1·0569	1·1461	and	4·8941
1·3979	1·2041		3·2367
4·8941	3·2367		1·6574 log. of 45·43

45·43 lb. Ans.

SOLUTIONS TO TEST EXAMPLES VIII.

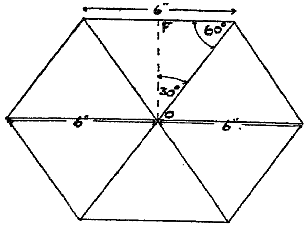
1. The half base is $1\frac{1}{2}$ inches long.



Now $C\,D = \dfrac{1\frac{1}{2}}{\quad}$ and $A\,C$ = twice $C\,D$

$$A\,C = \frac{1\frac{1}{2} \times 2}{\sqrt{3}} = \frac{3}{\sqrt{3}} = \sqrt{3} = 1{\cdot}732 \text{ inches.}$$

The equal sides are each 1·732 inches long. Ans.



The hexagon contains 6 equilateral triangles.

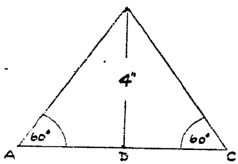
Length of O F $= 3 \times \sqrt{3}$

$\therefore$ Distance across flats $=$ $2 \times 3 \times \sqrt{3}$

$= 6 \times 1{\cdot}732 = 10{\cdot}392$ inches. Ans.

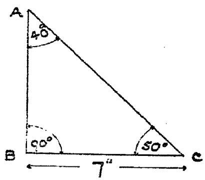
Distance across diagonal, or corners $= 6 + 6 = 12$ inches. Ans.

3.



$$A\,D = \frac{4}{\sqrt{3}} \text{ and } A\,C = \frac{2 \times 4}{\sqrt{3}} = \frac{8 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \quad 8 \times \sqrt{3}$$

$$\text{or } A\,C = \frac{8 \times 1{\cdot}732}{} \quad \frac{13{\cdot}856}{3} = 4{\cdot}618 \text{ inches. Ans.}$$



		Logs.
$\text{Sin. } 40° = \dfrac{}{A\,C}$		0·8451
	7	$\bar{1}$·8081
$\therefore A\,C = \dfrac{}{\text{Sin. } 40°}$		1·0370

log. A C = log. 7 — log. Sin. 40°

log. A C = 1·0370, and A C = 10·89 inches. Ans.

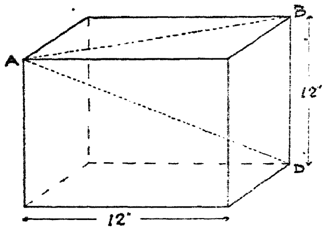
$$\text{Also Tan. } 40° = \frac{}{A\,B} \quad \text{or } A\,B = \frac{7}{\text{Tan. } 40°}$$

Logs.
0·8451
$\bar{1}$·9238
0·9213

log. A B = log. 7 — log. Tan. 40°

log. A B = 0·9213, and A B = 8·343. Ans.

5.



A B = 12 × $\sqrt{2}$ = 1·414 × 12 = 16·968 inches. Ans.

The diagonal A D is the hypotenuse of the right angled triangle A B D.

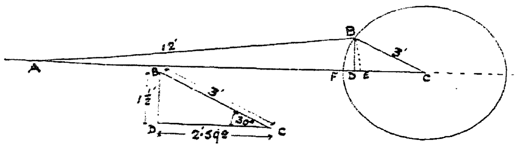
$$A D = \sqrt{(A B)^2 + (B D)^2}$$

$$A D = \sqrt{144 \times 2 + 144} = \sqrt{144\,(2 + 1)} = 12$$

A D = 12 × 1·732 = 20·784 inches. Ans.

6. With centre A and radius A B, describe arc B E. The piston is F E feet from the top centre or C E above half stroke, E in all cases giving the position of the piston.

Note that F G is the full piston stroke.



In the triangle B D C, B C = 3, ∴ B D = 1½ feet

D C = 1½ × $\sqrt{3}$ = 2·598 feet.

In the triangle A B D, A D = $\sqrt{}$

$= \sqrt{144 - 2{\cdot}25} = \sqrt{141{\cdot}75}$

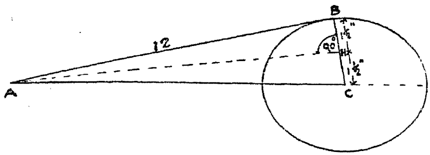
∴ A D = 11·91 feet. Now A E = A B

F D = F C — D C = 3 — 2·598
= 0·402 foot.

D E = A E — A D = 12 — 11·91
= 0·09 foot.

F E = F D + D E = 0·402 + 0·09 = 0·492 foot.

Piston is 0·492 foot or 5·904 inches from top centre. Ans. (*a*)



The piston here is at half stroke.

$\therefore$ A B = A C, and angle at B = angle at C.

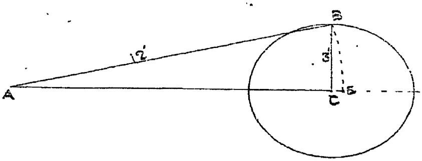
Drop a perpendicular from A bisecting B C at H.

Then B H = $1\frac{1}{2}$ feet = H C.

$$\text{Cos. } C = \frac{C\ H}{A\ C} = \frac{1\frac{1}{2}}{12} = \frac{3}{24} = \frac{1}{8} = 0{\cdot}125,$$

Cos. C = 0·125, and from the tables C = 82° 49′.

The crank is 82° 49′ from the top dead centre. Ans. (*b*).



The piston is here at E, when the crank is at 90° to A C.

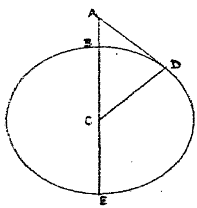
$$A\ C = \sqrt{(12)^2 - (3)^2}$$
$$= \sqrt{135} = 11{\cdot}62 \text{ feet.}$$

C E = 12   11·62 = 0·38 foot.

Piston is 3·38 feet below its top position.

Logs.
2)2·1305
1·06525
= log. of 11·62

or 0·38 foot or 4·56 inches below half stroke. Ans. (*c*).



As explained in the Text :—

E A × B A = $(A\ D)^2$

Now A B = $\frac{110}{5280}$

= 0·02084 mile.

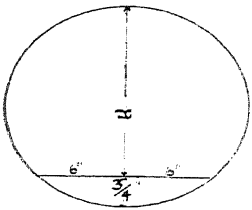
Logs.
1·0414
2·7226
$\overline{2}{\cdot}3188$
log. of 0·02084

$\therefore$ 8000·02084 × 0·02084 = $(A\ D)^2$

Now if we work with four figure logs. we cannot get 8000·02084, and as 0·02084 is small compared to 8000 we may neglect it.

	Logs.
$\therefore$ 8000 × 0·02084 = (A D)^2	3·9031
	$\bar{2}$·3188
A D = 12·91 miles. Ans.	2)2·2219
	1·1109
	log. of 12·91

8.



By the property of a circle :—

$x \times \frac{3}{4} = 6^2$, or $x = 36 \times \frac{4}{3}$

$x = 48$ inches.

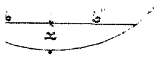
The furnace is $48\frac{3}{4}$ inches inside diameter. Ans.

$x\ (40 - x) = 6^2$

$40\ x - x^2 = 36$ (quadratic).

$x^2 - 40\ x = -\ 36$, changing the signs throughout.

$x^2 - 40\ x + (20)^2 = -\ 36 + (20)^2$



$x - 20 = \pm \sqrt{-36 + 400}$

$= \pm \sqrt{364}$

$x - 20 = \pm\ 19·07$

$x = 20 \pm 19·07 = 0·93$ or $39·07$ inches.

The segments are 0·93 and 39·07 inches high. Ans.

10. Every part of the rod has the same *angular* velocity, because when it makes a revolution, every point on it turns through 360 degrees.

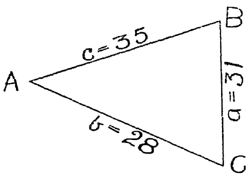
$$100 \text{ revs. per min.} = \frac{10\cancel{0} \times \cancel{2}\,\pi}{} \text{ radians per sec.} =$$

$$\omega = \tfrac{10}{3}\,\pi \text{ radians per sec.}$$

$$= \tfrac{10}{3} \times \tfrac{22}{7} = 10{\cdot}47 \text{ radians per sec.}$$

Linear speed of points $= 10{\cdot}47 \times 3 = 31{\cdot}41$ ft. per sec.
in feet per sec. $10{\cdot}47 \times 7 = 73{\cdot}29$ ft. per sec.
$10{\cdot}47 \times 10 = 104{\cdot}7$ ft. per sec.

11.



One ship is $3\frac{1}{2} \times 10 = 35$ miles from port.

The other ship is $7 \times 4 = 28$ miles from port.

Draw the triangle as shown, the angle A is the angle between the courses.

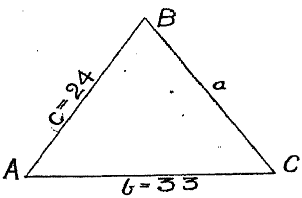
$$s = \frac{28 + 31 + 35}{} = 47$$

$$\frac{A}{2} : \frac{(47 - 31)}{} = \sqrt{\frac{47 \times 16}{28 \times 35}}$$

$$\text{Cos.} \frac{A}{2} = 0{\cdot}8758, \text{ from the tables } \frac{}{2} = 28^\circ\ 52'$$

A $= 57^\circ\ 44'$, this is the angle required. Ans.

12.



After 3 hours, one vessel is $3 \times 11 = 33$ miles from port.

After 3 hours, other vessel is $3 \times 8 = 24$ miles from port.

Then B C or a is the distance apart after 3 hours.

$a^2 = b^2 + c^2 - 2\ b\ c$ Cos. A,

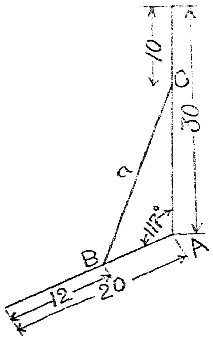
$a^2 = 33^2 + 24^2 - 2 \times 33 \times 24 \times$ Cos. 57°

$a^2 = 1{,}089 + 576 - 1{,}584 \times 0{\cdot}5446 = 802{\cdot}2$

$a = \sqrt{802{\cdot}2} = 28{\cdot}32.$

Ships are 28·32 miles apart. Ans.

13.



In quarter hour, slow train is $\frac{40}{4}$ = 10 miles from start.

In quarter hour, fast train is $\frac{48}{4}$ = 12 miles from start.

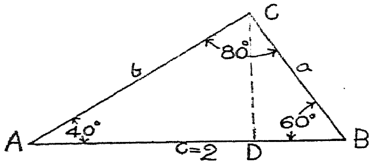
A C = 20, and A B = 8 miles.

$a^2 = 8^2 + 20^2 + 2 \times 8 \times 20 \times$ Cos. $(180^\circ - 117^\circ)$

$a^2 = 64 + 400 + 320 \times 0{\cdot}454 = 609{\cdot}28$

= 24·68 miles. Ans.

14.



$C = 180^\circ - 100^\circ = 80^\circ$

$$\frac{a}{\text{Sin. } 40^\circ} = \frac{2}{\text{Sin. } 80^\circ}$$

$$a = \frac{2 \times \text{Sin. } 40^\circ}{\text{Sin. } 80^\circ} = \frac{2 \times 0{\cdot}6428}{0{\cdot}9848}$$

$a = 1{\cdot}305$ feet, C D = a Sin. 60°.

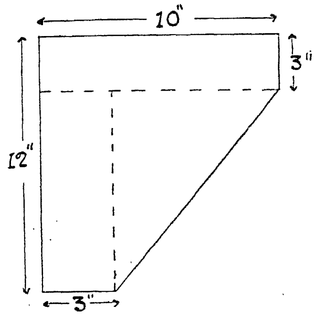
C D = 1·305 × 0·866 = 1·13 feet,

$$\text{Area} = \frac{\text{Base} \times \text{C D}}{2} = \frac{2 \times 1}{2} = 1{\cdot}13 \text{ sq. feet.}$$

Weight = $1{\cdot}13 \times \frac{1}{12} \times 0{\cdot}28 \times 1{,}728 = 45{\cdot}57$ lb. Ans.

SOLUTIONS TO TEST EXAMPLES IX.

1.



Area of knee =

$$(3 \times 10) + (9 \times 3) + \frac{9 \times 7}{2}$$

Area of knee $= 30 + 27 + 31{\cdot}5$
$= 88{\cdot}5$ sq. inches.

$$\text{Wt.} = \frac{88{\cdot}5}{144} \times 15$$

$= 9{\cdot}217$ lb. Ans.

No. of turns of packing $= \dfrac{8\frac{3}{4}}{1\frac{1}{4}} = 7$ turns.

Mean diameter of stuffing box $= 8\frac{1}{4}$ inches.

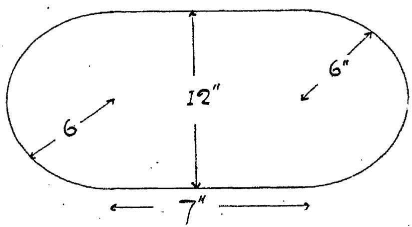
Length of packing $= [(8\frac{1}{4} \times \pi) - 1] \times 7 = 174{\cdot}4$ inches.

Length of packing $= 14$ feet $6\frac{1}{2}$ inches. Ans.

3. Area of plate cut out $= \frac{22}{7} \times 36 + 7 \times 12$

Area of plate $= 197{\cdot}14$ sq. inches.

$$\text{Wt.} = \frac{197{\cdot}14}{144} \times 18{\cdot}5 = 25{\cdot}32 \text{ lb. Ans.}$$



4. Area of pipe $= 8 \times 8 \times \frac{11}{14}$, and $1\frac{1}{2}$ times this is, $8 \times 8 \times \frac{11}{14} \times \frac{3}{2}$ sq. inches.

Area of one slot $= \frac{3}{10} \times 6 = \frac{18}{10} = \frac{9}{5}$ sq. inches.

$$\text{No. of slots} = \frac{\text{Area of pipe} \times 1\frac{1}{2}}{\text{Area of one slot}}$$

$$= \overset{4}{\cancel{8}} \times \overset{4}{\cancel{8}} \times \frac{11}{\cancel{14}} \times \frac{3}{2} \times \frac{5}{9} = \frac{16 \times 33 \times 5}{9 \times 7}$$

$$\text{No. of slots} = \frac{80 \times 11}{\underset{3}{\cancel{9}} \times 7} \quad \frac{880}{3 \times 7} = 41{\cdot}9 \text{ say } 42 \text{ slots.}$$

Ans.

Mean circumference of wheel $= 18{\cdot}75 \times \frac{22}{7}$

Speed of ship

$$= 18{\cdot}75 \times \tfrac{22}{7} \times \tfrac{85}{100} \times \frac{32}{608\cancel{0}} \times 6\cancel{0}$$

$$\frac{18{\cdot}75 \times 132 \times 85 \times 32}{700 \times 608}$$

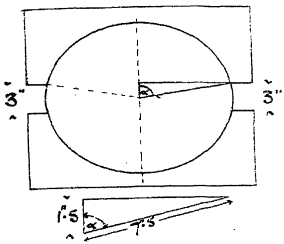
$= 15{\cdot}81$ knots. Ans.

6. Total taper $= 3\frac{10}{12} \times \frac{3}{4} = \frac{46}{\underset{4}{\cancel{12}}} \times \frac{\cancel{3}}{4} = 2{\cdot}875$ inches.

Diameter of small end $= 18 - 2{\cdot}875 = 15\frac{1}{8}$ inches. Ans.

7. $\text{Cos. } \alpha = \dfrac{1{\cdot}5}{7{\cdot}5} = 0{\cdot}2, \quad \alpha = 78^\circ\ 28'$

$3 \times (78^\circ\ 28') = 156^\circ\ 56'$, now $\frac{56}{60} = 0{\cdot}9333$.



Length of arc =

$$\frac{156\cdot9\dot{3}}{12} \times \frac{\cancel{22}^{\,11}}{7} \times \cancel{15}$$

= 20·55 inches. Ans.

Rubbing area = 16 × 20·55 × $\frac{93}{100}$
= 305·78 sq. inches. Ans.

8. Area of piston = 40 × 40 × $\frac{11}{14}$ sq. inches.

Area of shoe = 40 × $\frac{40}{4}$ × $\frac{11}{14}$

$$\frac{400 \times 11}{14} \text{ (sq. inches)}$$

$$\text{Length of shoe} = \frac{\text{Area}}{\text{Width}} \quad \frac{400 \times 11}{15 \times 14} \quad \frac{2{,}200}{105}$$

= 20·95.

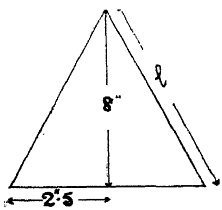
Length = 20·95 inches. Ans.

9. Circumference of valve × lift = area of escape.

$$\frac{\cancel{7}}{\cancel{2}} \times \frac{22}{7} \times \text{lift} = \frac{\cancel{7}}{\cancel{2}} \times \frac{7}{2} \times \frac{11}{14} \times \frac{1}{8}$$

Lift = $\frac{7}{2}$ × $\frac{11}{14}$ × $\frac{7}{22}$ × $\frac{1}{8}$ = $\frac{7}{64}$ inch. Ans.

10.



Curved surface of buoy = π R l,
$l = \sqrt{8^2 + (2\cdot5)^2}$ =
= 8·381 feet.

Curved surface =

$$\frac{\cancel{22}^{\,11}}{7} \times \frac{5}{\cancel{2}} \times 8\cdot381$$

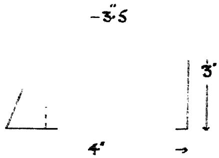
= 65·86 sq. feet.

Area of base $= \pi R^2 = \frac{22}{7} \times \frac{5}{2} \times \frac{5}{2}$

Area of base $= \dfrac{11 \times 25}{} = 19{\cdot}64$ sq. feet.

Wt. $= (65{\cdot}86 \div 19{\cdot}64) \times 6 = 513$ lb. Ans.

11.



Slant length $= \sqrt{3^2 + (\frac{1}{2})^2}$
$= \sqrt{9{\cdot}25} = 3{\cdot}041$ ft.

Surface of frustum
$= \pi\, l\, (R + r)$

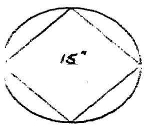
Surface of frustum
$= \frac{22}{7} \times 3{\cdot}041 \times 7{\cdot}5$

Wt. $= \frac{22}{7} \times 3{\cdot}041 \times 7{\cdot}5 \times 5$

$= \frac{110}{7} \times 3{\cdot}041 \times 7{\cdot}5$

Wt. $= 358{\cdot}4$ lb. Ans.

12.



Diagonal of square $= 15$ inches.

Side of square $= \dfrac{15}{\sqrt{2}} = \dfrac{15 \times \sqrt{2}}{\sqrt{2} \times \sqrt{2}}$

$\dfrac{15 \times \sqrt{2}}{2}$

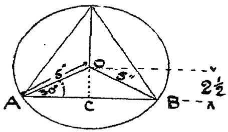
Side $= 10{\cdot}605$ inches long. Ans.

13. Surface of a sphere is the same as the curved surface of the circumscribing cylinder $= 2\,\pi\, r \times 2\, r = 4\, \pi\, r^2$

Surface of sphere $= 4 \times \frac{22}{7} \times 4 \times 4 = 201{\cdot}14$ sq. feet. Ans.

Surface of submerged part $= 2\,\pi\, r \times 5$
$= 2 \times \frac{22}{7} \times 4 \times 5 = 125{\cdot}7$ sq. feet. Ans.

14.



$O\,C = \frac{5}{2} = 2\frac{1}{2}$; $A\,B = 2 \times 2\frac{1}{2} \times \sqrt{3}$

Area of $\frac{1}{3}$ equilateral triangle

$$= 2\tfrac{1}{2} \times 2\tfrac{1}{2} \times \sqrt{3} \times \frac{2}{2}$$

Area of whole triangle

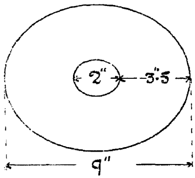
$$= 2\tfrac{1}{2} \times 2\tfrac{1}{2} \times \sqrt{3} \times 3$$

Area of whole $= 6{\cdot}25 \times 3 \times \sqrt{3} = 18{\cdot}75 \times \sqrt{3}$

$= 18{\cdot}75 \times 1{\cdot}732 = 32{\cdot}49$

32·49 sq. inches. Ans.

15.



$2{\cdot}5$ mm. $= 2{\cdot}5 \times 0{\cdot}03937$
$= 0{\cdot}09842$ inch.

Mean diameter of roll $= \quad + \; 2$

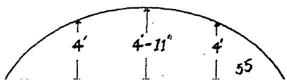
$= 5{\cdot}5$ inches.

No. turns on roll $= \dfrac{3}{0{\cdot}09842}$

$$\text{Length} = \frac{3{\cdot}5}{0{\cdot}09842} \times (5{\cdot}5 \times \tfrac{22}{7})$$

$= 614{\cdot}8$ inches or 51 feet 2·8 inches. Ans.

16.



Area $=$

$$\frac{3{\cdot}625}{3}\,[0 \times 1 + (4 \times 4) + (2 \times 4'\,11'') + (4 \times 4) + 0 \times 1]$$

This is Simpson's Rule.

$$\text{Area} = 1{\cdot}208\,[0 + 16 + 9\tfrac{5}{6} + 16 \cdot 0] = 1{\cdot}208 \times 41\tfrac{5}{6}$$

$$\text{Area} = 1{\cdot}208 \times \tfrac{251}{6}, \text{ or, } 1{\cdot}208 \times 41{\cdot}8\dot{3}$$

$$\text{Area} = 50{\cdot}53 \text{ sq. feet.} \quad \text{Ans.}$$

17. Area of similar figures $\propto$ as the squares of their corresponding dimensions.

$$\text{Area} = 25 \times \frac{(\text{hyp.})^2}{10^2}, \text{ and area} = 12 \text{ sq. inches.}$$

$$12 = 25 \times \frac{(\text{hyp.})^2}{100}$$

$$\frac{100 \times 12}{25} = 48$$

$$\text{hyp.} = \sqrt{48} = 6{\cdot}928 \text{ inches.} \quad \text{Ans.}$$

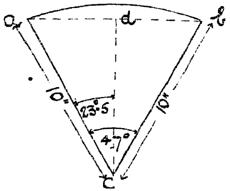
18. For a cylinder, in this case the tube :—

$$\frac{\text{Surface}}{\text{Area}} = \frac{\pi D L}{\frac{\pi}{4} D^2} = \frac{4 L}{D}, \text{ or } \frac{\text{Surface}}{\text{Area}} = \frac{4 L}{D}$$

$$\therefore \text{Area} = \frac{\text{Surface} \times D}{4 L},$$ and as there are 22 sq. feet per foot of grate, then area per foot of grate

$$\frac{22 \times 3}{4 \times (7 \times 12)} = 0{\cdot}196 \text{ sq. foot.} \quad \text{Ans.}$$

19.



Area of sector

$= \frac{47}{360} \times \frac{11}{14} \times (20)^2$

Area of sector

$$= \frac{47 \times 11 \times 40\cancel{0}}{36\cancel{0} \times 14}$$

$$\frac{470 \times 11}{126}$$

Area of sector $= 41{\cdot}03$ sq. inches. Ans.

Now area of triangle $a\ b\ c = a\ d \times d\ c$

$a\ d = 10$ Sin. $23{\cdot}5°$, $d\ c = 10$ Cos. $23{\cdot}5°$

Area triangle $a\ b\ c = 100$ Sin. $23{\cdot}5° \times$ Cos. $23{\cdot}5°$

Area triangle $= 100 \times 0{\cdot}3987 \times 0{\cdot}9171$

Area triangle $= 36{\cdot}57$ sq. inches.

Area of segment $= 41{\cdot}03 - 36{\cdot}57 = 4{\cdot}46$ sq. inches. Ans.

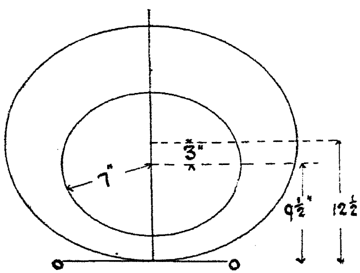
20. Surface of one tube $= \frac{3}{12} \times \frac{22}{7} \times 7 = \frac{11}{2}$ sq. feet.

Total surface of tubes per furnace $= \frac{11}{2} \times 74 = 407$ sq. feet.

Surface of one grate $= 3\frac{1}{2} \times 5\frac{1}{2} = \frac{77}{4}$ sq. feet.

Heating surface of tubes per sq. foot of grate $= 407 \times \frac{4}{77}$

Heating surface of tubes $= \frac{1628}{77} = 21{\cdot}14$ sq. feet. Ans.



21. Place the sheave symmetrically as shown and take moments about tangent $o\ o$.

C.G. above $o\ o =$

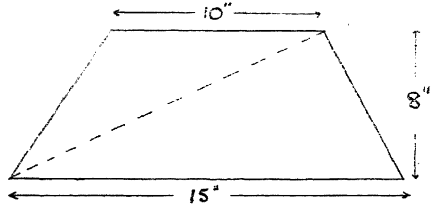
$$\frac{(12\frac{1}{2})^3 - 7^2 \times 9\frac{1}{2}}{(12\frac{1}{2} + 7)\ (12\frac{1}{2} - 7)}$$

$$\frac{(12{\cdot}5)^3 - 7^2 \times 9\frac{1}{2}}{19{\cdot}5 \times 5{\cdot}5}$$

$$\frac{1953 - 465{\cdot}5}{19{\cdot}5 \times 5{\cdot}5}$$

$$\frac{1487{\cdot}5}{19{\cdot}5 \times 5{\cdot}5} = 13$$

13·87 inches from $o\ o$, or 1·37 inches from centre of sheave. Ans.



Take moments about $o\ o$

$$\text{C.G. above } o\ o = \frac{\text{Sum of moments about } o\ o}{\text{Total area}}$$

$$8 \times 15 \qquad\qquad 10 \times 8$$

$$\times \tfrac{2}{3} \times$$

$$8 \times \frac{15 + 10}{2}$$

Note that the area of the triangles are $\frac{8 \times 15}{2}$ and $\frac{10 \times 8}{2}$ respectively, and their C.G.'s above $o\ o$ are $\frac{8}{3}$ and $\frac{2}{3} \times 8$ inches respectively.

Cancelling and reducing we get :—

$$\frac{160 + 213\frac{1}{3}}{8 \times 12\frac{1}{2}} \quad \frac{373\cdot\dot{3}}{100} = 3\cdot733 \text{ inches.}$$

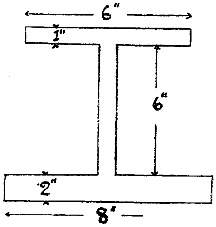
The C.G. is 3·733 inches above *o o*. Ans.

or the formula $\frac{h}{3}\left(\frac{2\,a + b}{a + b}\right)$ may be used,

$a = 10,\ b = 15,\ h = 8.$

$$\frac{8}{3}\left[\frac{20 + 15}{10 + 15}\right] = \frac{8}{3} \times \frac{35}{25} = 3\cdot733. \quad \text{Ans.}$$

23.



Take moments round *o o*

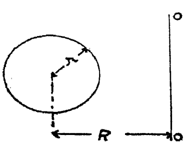
$$\text{Then C.G. above } o\ o = \frac{\text{Sum of moments about } o\ o}{\text{Total area}}$$

$$\text{Then C.G.} = \frac{(6 \times 1 \times 8\frac{1}{2}) + (6 \times 1 \times 5) + (2 \times 8 \times 1)}{(6 \times 1) + (6 \times 1) + (8 \times 2)}$$

$$\text{C.G. above } o\ o = \frac{51 + 30 + 16}{6 + 6 + 16} = \frac{97}{28} = 3\cdot464 \text{ inches.}$$

Ans.

24. By Theorem of Pappus
length of line × distance C.G. moves = Surface swept out

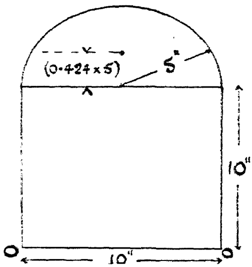


$2\,\pi \times 1 \times 2\,\pi \times 5 =$ surface area.

$$\text{Surface} = 20\,\pi^2 = \frac{20 \times 22 \times 22}{7 \times 7}$$

Surface = 197·55 square inches. Ans.

25. The C.G. of a semicircular *area* is at $0{\cdot}424 \times r$ from the diameter. Take moments round $o\ o$.



$$\text{C.G. above } o\ o = \frac{\text{Sum of moments about } o\ o}{\text{Total area}}$$

$$\text{C.G.} = \frac{(10 \times 10 \times 5) + \left(\frac{\pi\, 5^2}{2} \times (10 + 0{\cdot}424 \times 5)\right)}{(10 \times 10) + \frac{\pi\, 5^2}{2}}$$

$$\frac{500 + (39{\cdot}28 \times 12{\cdot}12)}{100 + 39{\cdot}28}$$

$$\frac{500 + 476{\cdot}1}{139{\cdot}28}$$

$$\text{C.G.} = \frac{976{\cdot}1}{139{\cdot}28} = 7{\cdot}004 \text{ above } o\ o$$

$7{\cdot}004$ inches above $o\ o$. Ans.

26. Area of 6 collars

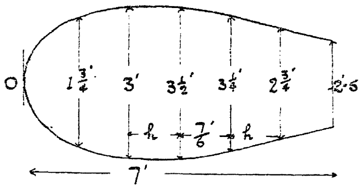
$= 6 \times \frac{11}{14}\,[18^2 - 12^2] = \frac{11}{14}\,(18 - 12)\,(18 + 12) \times 6$

Area $= 6 \times \frac{11}{14} \times 30 \times 6 = \frac{5940}{7}$ sq. inches.

Effective Area $= \frac{5940}{7} \times 0{\cdot}65$.

Total thrust in lb. $= \frac{5940}{7} \times 0{\cdot}65 \times 65 = 35{,}852$ lb. Ans.

27. By Simpson's Rule,



$$\frac{\quad}{3}[a + 4b + 2\,c + 4\,d + 2\,e + 4\,f + g] = \text{area}$$

$$h = \tfrac{7}{6}, \text{ and } \frac{h}{3} = \frac{7}{6 \times 3} = \tfrac{7}{18}$$

$$\text{Area} = \tfrac{7}{18}\,[2\tfrac{1}{2} + 4 \times 2\tfrac{3}{4} + 2 \times 3\tfrac{1}{4} + 4 \times 3\tfrac{1}{2} + 2 \times 3 + 4 \times 1\tfrac{3}{4} + 0 \times 1]$$

$= \tfrac{7}{18} \times 47 = 18{\cdot}27$ square feet. Ans.

By mid-ordinate method. First get the mean width of each strip of area. Thus :—

$2\frac{1}{2}$

$= 2{\cdot}625 =$ mean width of 1st strip.

$3\frac{1}{4}$

$= \quad 3 =$ 2nd strip

The mean widths are, 2·625, 3, 3·375, 3·25, 2·375, 0·875 feet.

Mean width of whole blade

$2{\cdot}625 + 3 + 3{\cdot}375 + 3{\cdot}25 + 2{\cdot}375 + 0{\cdot}875$

$$\frac{15{\cdot}5}{6} = 2{\cdot}58\dot{3} \text{ feet.}$$

Area = length × mean width

$= 7 \times 2{\cdot}58\dot{3} = 18{\cdot}08$ square feet. Ans.

The difference in the answers is due to the fact that both methods are only approximately true.

SOLUTIONS TO TEST EXAMPLES X.

1. Vol. of plain part of shaft $= \dfrac{12\cdot5 \times 12\cdot5}{12 \times 12} \times \frac{11}{14} \times 8$

$= 6\cdot82$ cubic feet.

Vol. of flanges $= 2 \times \frac{11}{14}\,[25^2 - 12\cdot5^2] \times \quad \times \dfrac{3\cdot5}{12}$

$= 1\cdot491$ cubic feet.

Vol. of 6 collars $= 6 \times \frac{11}{14}\,[25^2 - 12\cdot5^2] \times \frac{1}{144} \times \frac{2}{12}$
$= 2\cdot557$ cubic feet.

Total volume $= 6\cdot82 + 1\cdot491 + 2\cdot557 = 10\cdot868$ cu. feet.

Weight $= \dfrac{10\cdot868 \times 490}{2{,}240} = 2\cdot377$ tons. Ans.

$\frac{1}{4}$ pint $= \frac{1}{32}$ gallon $= \dfrac{1{,}728}{32 \times 6\cdot25} = 8\cdot64$ cubic inches.

Depth of oil $= \dfrac{\text{Volume}}{\text{Area}} \qquad \dfrac{8\cdot64}{[2^2 -} \qquad = 2\cdot89$ ins. Ans.

Area $= \dfrac{14\cdot5}{4} \times \frac{1}{3} \Big[(0 \times 1) + (4 \times 4) + (4\cdot916 \times 2)$

$+ (4 \times 4) + (0 \times 1)\Big]$ by Simpson's rule.

$\dfrac{14\cdot5}{12} \times 41\cdot833 = 50\cdot547$ square feet.

Vol. $= 50\cdot547 \times 16 = 808\cdot752$ cubic feet. Ans.

Vol. of liner $= \frac{11}{14}\,[16^2 - 14^2] \times 12 \times 12$ cubic inches.
$= 6788\cdot57$ cubic inches.

Weight $= \dfrac{6788\cdot57 \times 0\cdot3}{2{,}240} = 0\cdot909$ ton. Ans.

5. Vol. of shaft $= \frac{11}{14}\,[16^2 - 8^2] \times \frac{1}{144} \times 20 = 20{\cdot}94$ cubic feet.

Vol. of flanges $= \frac{11}{14}\,[32^2 - 16^2] \times \frac{1}{144} \times \frac{4}{12} \times 2 = 2{\cdot}793$ cubic feet.

Total $= 23{\cdot}733$ cubic feet.

$$\text{Weight} = \frac{23{\cdot}733 \times 490}{2{,}240}$$

Weight $= 5{\cdot}191$ tons. Ans.

6. Vol. of cylinder at cut off $= \frac{11}{14} \times (2{\cdot}5)^2 \times 5 \times 0{\cdot}6 = 14{\cdot}732$ cubic feet.

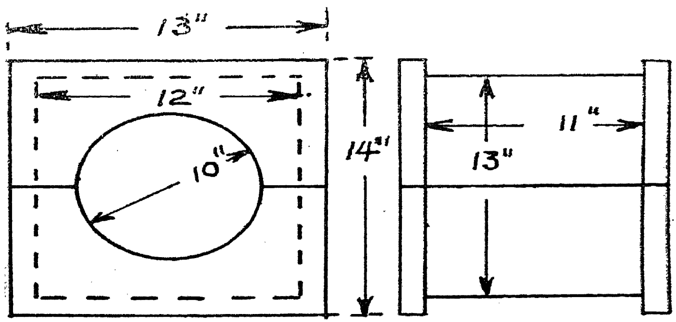
$$\text{No. of strokes} = \frac{1{,}850}{14{\cdot}732} = 125{\cdot}5 \text{ strokes. Ans.}$$

7. Volume of solid body $= 13 \times 11 \times 12 = 1{,}716$ cubic ins.

Volume of solid flanges $= 2 \times 14 \times 13 \times \frac{5}{8} = 227{\cdot}5$ cubic inches.

Volume of hole $= \frac{11}{14} \times 10^2 \times 12{\cdot}25 = 962{\cdot}5$ cubic inches.

Weight $= (1{,}716 + 227{\cdot}5 - 962{\cdot}5) \times 0{\cdot}3 = 294{\cdot}3$ lb. Ans.



8. $\text{Length} = \dfrac{\text{Volume}}{\text{Area}}$, area of wire $= \frac{11}{14} \times \frac{1}{16} \times \frac{1}{16} =$

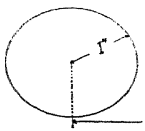
$\dfrac{11}{14 \times 256}$ square inch.

$\text{Length} = 2 \times 2 \times \frac{11}{14} \times 4 \times \dfrac{14 \times 256}{11}$ inches, cancelling $\frac{11}{14}$:—

$= 16 \times 256$ inches or $\dfrac{16 \times 256}{12}$ feet

Length $= 341\frac{1}{3}$ feet. Ans.

9. Vol. = area × distance C.G. moves.



$\text{Vol.} = \pi \times 1^2 \times 2\pi \times 6$

$= 12\,\pi^2$ cubic inches.

$\text{Weight} = \dfrac{22 \times 22}{7 \times 7} \times 12 \times 0{\cdot}28$

$= 33{\cdot}18$ lb. Ans.

10. $\text{Vol.} = \frac{4}{3}\,\pi\, r^3$, or $\dfrac{\pi\, d^3}{6}$ and $d^3 = \dfrac{\text{Vol.} \times 6}{\pi}$

Vol. of sphere $= \dfrac{156}{0{\cdot}26}$ cubic inches.

$\dfrac{156}{0{\cdot}26} \times \frac{7}{22} \qquad \dfrac{156 \times 21}{2{\cdot}86}$

$d = \sqrt[3]{\dfrac{156 \times 21}{2{\cdot}86}} = 10{\cdot}46$ inches diameter. Ans.

11.

Vol. of hollow sphere $= \dfrac{\quad}{6} [D^3 - d^3]$, or $\frac{4}{3}\ \pi\ [R^3 - r^3]$

Vol. $= \frac{4}{3} \times \frac{22}{7} [6^3 - 5^3] = \frac{88}{21} \times 91$ cubic inches.

Weight $= \frac{88}{21} \times 91 \times 0{\cdot}26 = 99{\cdot}14$ lb. Ans.

12. Volume $=$ area $\times$ thickness.

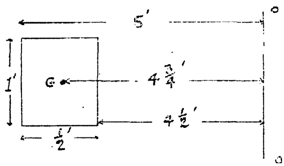
$= \frac{11}{14} [D^2 - d^2] \times t = \frac{11}{14} [10^2 - 9^2] \times 1$

$= \frac{11}{14} \times 19 \times 1 \times 1 = 14{\cdot}93$ cubic feet.

$$\text{Weight} = \frac{14{\cdot}93 \times 450}{2240} = 3 \text{ tons.}$$

Weight of boss $= \frac{11}{14} [12^2 - 6^2] \times \frac{1}{1728} \times$
$= 0{\cdot}117$ ton, total $3{\cdot}117$ tons. Ans.

Note that the volume might have been found by the theorem of Pappus as follows :—



Area of section $\times$ distance C.G. moves $=$ Vol. swept out.

$1 \times \frac{1}{2} \times 2\pi \times \frac{19}{4}$

$\pi \times 19$

This is the same as $\frac{11}{14} \times 19$ $= 14{\cdot}93$ cubic feet, as above.

13.

$$\text{Volume} = \frac{\quad}{12} h\ [D^2 + D\ d + d^2]$$

$= \frac{22}{7} \times \frac{4}{12} [5^2 + 5 \times 3{\cdot}5 + 3{\cdot}5^2]$

$= \frac{22}{21} [25 + 17{\cdot}5 + 12{\cdot}25] = \frac{22}{21} \times 54{\cdot}75 = 57{\cdot}35$ cubic feet. Ans.

14. Cos. $\theta = \frac{1}{6} = 0{\cdot}1666$, $\theta = 80° 24'$ nearly.

Whole angle of sector $= 160° 48' = 160{\cdot}8°$

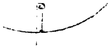
$x = 6 \times$ Sin. $80° 24'$

$= 6 \times 0{\cdot}986 = 5{\cdot}916$ feet.

Area of sector

$$= \frac{22}{7} \times 6 \times 6 \times \frac{160 \cdot 8}{360} = 50 \cdot 537$$

square feet.



Area of triangle $= 1 \times 5 \cdot 916 = 5 \cdot 916$ square feet.

Area of segment $= 50 \cdot 537 - 5 \cdot 916 = 44 \cdot 621$ square feet.

Volume of displacement $= 44 \cdot 621 \times 8$ cubic feet.

$$\text{Wt. of displaced water} = \frac{44 \cdot 621 \times 8}{35} = 10 \cdot 199 \text{ tons.}$$

Ans.

15.

$$\text{Vol. of sphere} = \frac{99 \cdot 15}{0 \cdot 26} = 381 \cdot 34 \text{ cu. ins.}$$

$$\text{Vol.} = \tfrac{4}{3}\,\pi\,[R^3 - r^3] = \tfrac{4}{3}\,\pi\,[6^3 - r^3] = 381 \cdot 34$$

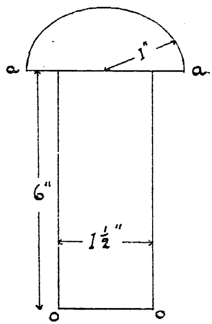
$$6^3 - r^3 = 381 \cdot 34 \times \frac{3}{4\,\pi} = 91$$

$$r^3 = 216 - 91 = 125$$

$$r = \sqrt[3]{125} = 5$$

Internal diameter $= 10$ inches. Ans.

16. Take moments about $o\ o$



C.G. above $o\ o$

$$\frac{\text{Sum of 1st moments about } o\ o}{\text{Total volume}}$$

The C.G. of plain part is 3" above $o\ o$

The C.G. of head is $6 + (\frac{3}{8} \times 1)$ above $o\ o$

Note that C.G. of head is $\frac{3}{8}\,r$ above $a\ a$, and $6 + (\frac{3}{8} \times 1)$ above $o\ o$.

C.G. above $o\ o$ =

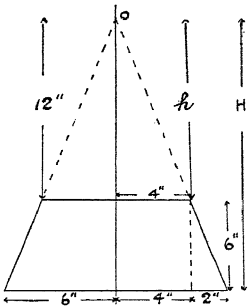
$$\frac{(1{\cdot}5 \times 1{\cdot}5 \times \frac{11}{14} \times 6 \times 3) + (\frac{2}{3} \times \frac{22}{7} \times 1^3 \times 6\frac{3}{8})}{(1{\cdot}5 \times 1{\cdot}5 \times \frac{11}{14} \times 6) + (\frac{2}{3} \times \frac{22}{7} \times 1^3)}$$

$$\text{C.G. above } o\ o = \frac{31{\cdot}82 + 13{\cdot}35}{10{\cdot}606 + 2{\cdot}095} \quad \frac{45{\cdot}17}{12{\cdot}701}$$

= 3·557 inches above $o\ o$. Ans.

17. H

$= \frac{6}{2}$, H = 18 inches, h = 18 — 6 = 12 inches.



Take moments about o,

C.G. below o =

$$\frac{\text{Moment of whole cone about } o - \text{Moment of small cone about } o}{\text{Volume of frustum}}$$

C.G. below o =

$$\left(\frac{\pi}{4} \times 12^2 \times \frac{18}{3} \times \frac{3}{4} \times 18\right) - \left(\frac{\pi}{4} \times \quad \times \frac{12}{3} \times \frac{3}{4} \times 12\right.$$

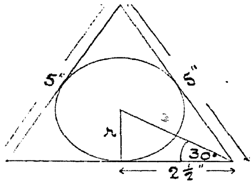
$$\left(\frac{\pi}{4} \times 12^2 \times \qquad - \left(\frac{\pi}{4} \times \qquad \times\right.\right.$$

$$\frac{\frac{12\pi}{4}(3 \times 18 \times 18) - \frac{12\pi}{4}(3 \times 64)}{\frac{12\pi}{4}(12 \times 6) - \frac{12\pi}{4}(\frac{64}{3})}, \text{ cancel } \frac{12\pi}{4}$$

$$\frac{972 - 192}{72 - 21{\cdot}33} \quad \frac{780}{50{\cdot}67} = 15{\cdot}39 \text{ inches below } o$$

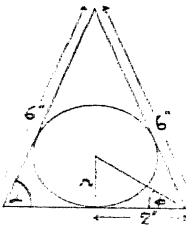
C.G. is 18 — 15·39 = 2·61 inches above base. Ans.

18. $r = \quad 2\frac{1}{2} \times \sqrt{3}$



$r = \dfrac{2\frac{1}{2}}{3} \times \sqrt{3} = 1{\cdot}443$ ins.

Wt. of cyl.
$= \frac{22}{7} \times (1{\cdot}443)^2 \times 10 \times 0{\cdot}3$
$= 19{\cdot}63$ lb. Ans. (a)



Cos. $\alpha = \frac{2}{6} = 0{\cdot}3333$; $\alpha = 70° \; 32'$
$\phi = 35° \; 16'$

$r = 2$ Tan. $\phi = 2 \times 0{\cdot}7072 = 1{\cdot}414$ ins.

or $r = \dfrac{\text{Area}}{\frac{1}{2}\text{ sum of sides}} \quad \dfrac{\sqrt{8 \times 2 \times 2 \times 4}}{8}$

$= 1{\cdot}413$ inches.

Wt. of cyl. $= \frac{22}{7} \times (1{\cdot}414)^2 \times 10 \times 0{\cdot}3 = 18{\cdot}85$ lb.
Ans. (b)

$$r = \frac{\sqrt{s\,(s - a)\,(s - b)\,(s - c)}}{s} \text{ as}$$

$$r = \frac{\sqrt{7 \times 4 \times 2 \times 1}}{} - \frac{7{\cdot}483}{} = 1{\cdot}069 \text{ inches.}$$

Wt. of cyl. $= \frac{22}{7} \times (1{\cdot}069)^2 \times 10 \times 0{\cdot}3 = 10{\cdot}76$ lb.
Ans. (c)

19. Volume of Buoy $= 2 \times 35 = 70$ cubic feet.

$$\text{Vol.} = \frac{\quad}{12} \qquad)^2 + D\,d + d^2] = 70$$

$$100 + 10\,d + d^2 = \frac{70 \times 12}{\pi \times 1} = 267{\cdot}27$$

$d^2 + 10\,d = 267{\cdot}27 - 100 = 167{\cdot}27$

Completing the squares and solving,

$d + 5 = \pm \sqrt{192{\cdot}27} = \pm 13{\cdot}86$

$d = \pm 13{\cdot}86 - 5 = + 8{\cdot}86$ feet. Ans.

20. Vol. of cone $= \frac{500}{450} = \frac{10}{9}$ cubic feet.

$$\text{Vol. of cone} = \tfrac{11}{14}\,d^2 \times \frac{25}{12 \times 3}, \text{ or,}$$

$$\text{of base} \times \frac{\text{height}}{\quad}$$

$\frac{11}{14}\,d^2 \times \frac{25}{36} = \frac{10}{9}$, note all sizes are in feet.

$d^2 = \frac{10}{9} \times \frac{36}{25} \times \frac{14}{11} = 2{\cdot}0363$ ft.^{2}

$d = \sqrt{2{\cdot}0363} = 1{\cdot}427$ feet or $17{\cdot}124$ inches. Ans.

21. Volume $= 30 \times 75 \times 140 = 315{,}000$ cubic centimetres.

$= \frac{315000}{1000} = 315$ litres. Ans.

Vol. in gallons $= 315 \times 0{\cdot}22 = 69{\cdot}3$ gallons. Ans.

22. Vol. $= 66 \times 66 \times \frac{11}{14} \times 127 = 434{,}666$ cubic centimetres.

$$\text{Vol.} = \frac{434{,}666}{(100)^3} = 0{\cdot}4346 \text{ cubic metre (taking 4 figures)}$$

$$1 \text{ cubic metre} = \frac{(39{\cdot}37)^3}{1{,}728} = 35{\cdot}33 \text{ cubic feet.}$$

Volume $= 35{\cdot}33 \times 0{\cdot}4346 = 15{\cdot}368$ cubic feet. Ans.

23. Area of a hexagon $= 2{\cdot}598\ (\text{side})^2$
Side $= 2{\cdot}6 \times 0{\cdot}3937 = 1{\cdot}0236$ inches.
Volume of bar $= 2{\cdot}598 \times (1{\cdot}0236)^2 \times 20 \times 12$
$= 653{\cdot}6$ cubic inches.
Wt. $= 653{\cdot}6 \times 0{\cdot}28 = 183$ lb. Ans.

$$\frac{183}{2{\cdot}2} = 83{\cdot}18 \text{ kilograms.}$$

24.
$$75 \text{ gallons} = \frac{}{0{\cdot}22} = 340{\cdot}9 \text{ litres.}$$

340·9 litres $= 340{\cdot}9 \times 1{,}000 = 340{,}900$ cubic centimetres.
$d^2 \times \frac{11}{14} \times 4 \times 100 = 340{,}900$

$$d^2 = \frac{340{,}900 \times 14}{11 \times 4 \times 100} = 1084{\cdot}68$$

$d = \sqrt{1085}$ say, $32{\cdot}94$ centimetres. Ans.
or $32{\cdot}94 \times 0{\cdot}3937 = 12{\cdot}97$ inches. Ans.

25. Let r = radius of the circle.
Then $(2\,r - 1) \times 1 = 2^2$
$2\,r - 1 = 4,\quad 2\,r = 5,\quad r = 2{\cdot}5$ inches.
The radius of the circle is 2·5 inches. Ans.
The ordinates measure :—
4 ins. ; 3·74 ins. ; 3·4 ins. ; 3 ins. ; 2·5 ins. ; 1·78 ins ; 0.
The common interval is $\frac{1}{6}$ inch.

Ordinate	Simpson's Multipliers	Functions of Ordinates	Distance of Ordinate from Chord	Functions for 1st Moments
4·0	1	4·0	0	0
3·74	4	14·96	$\frac{1}{6} \times 1$	$\frac{1}{6} \times 14{\cdot}96$
3·4	2	6·8	$\frac{1}{6} \times 2$	$\frac{1}{6} \times 13{\cdot}6$
3·0	4	12·0	$\frac{1}{6} \times 3$	$\frac{1}{6} \times 36{\cdot}0$
2·5	2	5·0	$\frac{1}{6} \times 4$	$\frac{1}{6} \times 20{\cdot}0$
1·78	4	7·12	$\frac{1}{6} \times 5$	$\frac{1}{6} \times 35{\cdot}6$
0	1	0	$\frac{1}{6} \times 6$	0

Sum $= 49{\cdot}88$ Sum $= \frac{1}{6} \times 120{\cdot}16$

Summation of 1st moments of areas

$$\frac{1}{6 \times 3} \times \tfrac{1}{6} \times 120{\cdot}16$$

$$\text{Summation of areas} = \frac{1}{6 \times 3} \times 49{\cdot}88$$

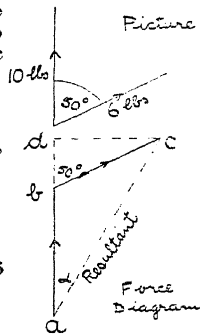
$$\text{C.G. from Chord} = \frac{\text{Summation of 1st moments}}{\text{Summation of Areas}}$$

$$\frac{120{\cdot}16}{18 \times 6} \times \frac{18}{49{\cdot}88} = 0{\cdot}401 \text{ inch}$$

Ans.

SOLUTIONS TO TEST EXAMPLES XI.

1. Set up $a\ b$ due North to represent the 10 lb. force. From b draw $b\ c$ parallel to the 6 lb. force, making the angle $d\ b\ c$ 50°. Join $a\ c$.



$$\frac{b\ d}{b\ c} = \text{Cos. } 50°, \ \therefore\ bd = b\ c \times \text{Cos. } 50°,$$

or $6 \times 0{\cdot}6428 = 3{\cdot}8568$ lb.

$$\frac{d\ c}{b\ c} = \text{Sin. } 50° \quad \therefore\ d\ c = 6 \times 0{\cdot}766$$

$= 4{\cdot}596$ lb.

$a\ d = 10 + 3{\cdot}8568$; $d\ c = 4{\cdot}596$

$a\ c = \sqrt{(13{\cdot}85)^2 + (4{\cdot}596)^2}$

$a\ c = \sqrt{191{\cdot}9 + 21{\cdot}15} = \sqrt{213{\cdot}05}$

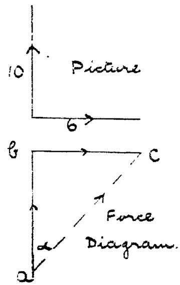
$a\ c = 14{\cdot}6$ lb. Ans.

$$\text{Tan. } \alpha = \frac{4 \cdot 596}{13 \cdot 85}$$

$\alpha = 18° \; 22'$. Ans.

Resultant 14·6 lb. at 18° 22′ East of North. Ans.

Set out the forces as shown.



Then $a\,c = \sqrt{10^2 + 6^2}$

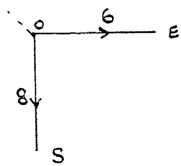
$= \sqrt{136}$

$a\,c = 11 \cdot 66$ lb. Ans.

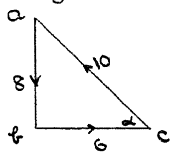
Tan. $\alpha = \frac{6}{10} = 0 \cdot 6$

$\alpha = 30° \; 58'$. Ans.

Draw $a\,b$ to represent the 8 lb. force. From b set out $b\,c$ equal to 6 lb., and at 90° to $a\,b$. Join $c\,a$. From o draw a line parallel to $c\,a$, this being the third force required.



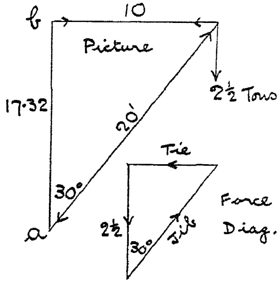
Note here for equilibrium the arrows must follow continuously round the triangle as shown.



Sin. $\alpha = \frac{8}{10} = 0 \cdot 8$, $\alpha = 53° \; 8'$.

The third force is at $90° - (53° 8')$ $= 36° \; 52'$ West of North.

4.



The tie will be 10 feet long with the given angle.

$a\,b = 10 \times \sqrt{3} = 17{\cdot}32$ feet.

Force in Tie

$$\frac{2{\cdot}5}{\sqrt{3}} \qquad \frac{2{\cdot}5 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}}$$

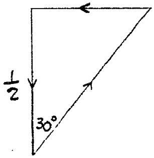
Force in Tie

$$= \frac{1{\cdot}732}{3} \times \tfrac{5}{2} = 1{\cdot}443 \text{ tons.}$$

Ans.

Force in derrick $= 1{\cdot}443 \times 2 = 2{\cdot}886$ tons. Ans.

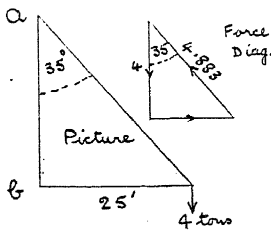
In the second case the weight of the derrick puts $\frac{1}{2}$ ton extra on derrick head.



Extra Force in Tie $= \frac{1}{2} \times \dfrac{}{\sqrt{3}}$

$$= \tfrac{1}{2} \times \frac{\sqrt{3}}{\sqrt{3} \times \sqrt{3}} = \frac{\sqrt{3}}{6} = 0{\cdot}2886 \text{ ton.} \quad \text{Ans.}$$

Extra Force in Derrick $= 0{\cdot}2886 \times 2 = 0{\cdot}5774$ ton. Ans.



If in addition to the right angle, one other angle is given, then the lengths of derrick and tie are not needed.

Force in Tie $= \dfrac{}{\text{Cos. } 35^\circ}$

$$0{\cdot}8192$$

Force in Tie $= 4{\cdot}883$ tons. Ans.

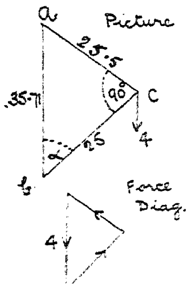
Force in Derrick $= 4{\cdot}883 \times \text{Sin. } 35^\circ$.

Force in Derrick $= 4{\cdot}883 \times 0{\cdot}5736 = 2{\cdot}801$ tons. Ans.

For the 2nd case, as we only know the angle 90°, the lengths or the angles may be found.

As $a\ b$ remains the same for any position of derrick, we find that first

$$\frac{25}{\text{Tan}.35^\circ} \quad \frac{25}{0{\cdot}7002} = 35{\cdot}71 \text{ feet.}$$



$$a\ c = \sqrt{35{\cdot}71^2 - 25^2}$$

$$= \sqrt{60{\cdot}71 \times 10{\cdot}71}$$

$$a\ c = 25{\cdot}5 \text{ feet.}$$

$$\frac{\text{Force in Tie}}{4} = \frac{25{\cdot}5}{35{\cdot}71}$$

$$\text{Force in Tie} = \frac{4 \times 25{\cdot}5}{35{\cdot}71}$$

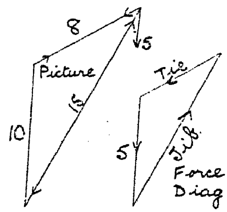
$$= 2{\cdot}857 \text{ tons.} \quad \text{Ans.}$$

$$\text{Force in} = \text{Derrick} - \frac{25}{35{\cdot}71} \times 4 = 2{\cdot}801 \text{ tons.} \quad \text{Ans.}$$

$$\text{Sin. } \alpha = \frac{25{\cdot}5}{35{\cdot}71} = 0{\cdot}7132$$

$$\alpha = 45^\circ\ 30'. \quad \text{Ans.}$$

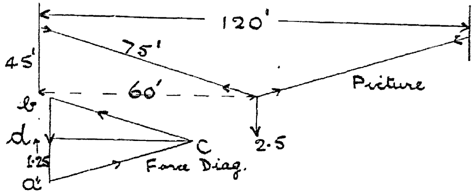
As all the lengths are given, this case is easy.
By similar triangles :—
Force in Tie



Force in Tie $= 4$ tons. Ans.

$$\frac{\text{Force in Jib}}{} = \tfrac{15}{10}, \text{ Force in Jib} = 7{\cdot}5 \text{ tons.} \quad \text{Ans.}$$

Draw $b\ a$ to represent 2·5 tons. Through b and a draw lines parallel to the arms of the wire sling. Then $b\ c$ is the force in each arm. Draw $c\ d$ horizontal. Then $b\ d$ = 1·25 tons.

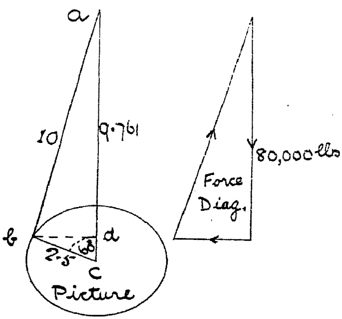


By similar triangles,

$$\frac{b\ c}{1{\cdot}25} = \frac{75}{45} \text{ or } b\ c = \frac{75}{45} \times \frac{5}{4} = 2{\cdot}08 \text{ tons. Ans.}$$

Note $c\ d$ is the horizontal component of $b\ c$ and represents the force pulling the two masts together.

8. $b\ d = 2{\cdot}5 \times 0{\cdot}866 = 2{\cdot}165$ feet, note $0{\cdot}866 = \text{Sin. } 60°$



$$c\ d = \frac{2{\cdot}5}{2} = 1{\cdot}25 \text{ feet.}$$

$$a\ d = \sqrt{10^2 - 2{\cdot}165^2}$$

$$= \sqrt{12{\cdot}165 \times 7{\cdot}835}$$

$$a\ d = 9{\cdot}761 \text{ feet.}$$

Force in rod

$$= \frac{10}{9{\cdot}761} \times 80{,}000$$

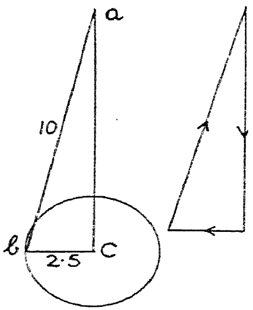
$$= 81{,}960 \text{ lb. Ans.}$$

$$\text{Force in Guide} = \frac{2\cdot165}{9\cdot761} \times 80{,}000$$

$= 17{,}740$ lb. Ans.

In 2nd position when crank is horizontal,

$a\ c = \quad \overline{-\ 2\cdot5^2} = \sqrt{12\cdot5 \times 7\cdot5} = 9\cdot68$



Force in Rod

$$= 80{,}000 \times \frac{10}{9\cdot68}$$

$= 82{,}620$ lb. Ans.

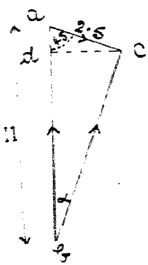
Force on guide

$$= 82{,}620 \times \frac{2\cdot5}{10}$$

$= 20{,}655$ lb. Ans.

9. $c\ d = 2\cdot5 \times 0\cdot707 = 1\cdot7675$, note Sine $45° = 0\cdot707$

$b\ d = 11 - 1\cdot767 = 9\cdot233$ knots.



$b\ c = \sqrt{9\cdot233^2 + 1\cdot767^2}$

$b\ c = \sqrt{85\cdot23 + 3\cdot123} = \sqrt{88\cdot353}$

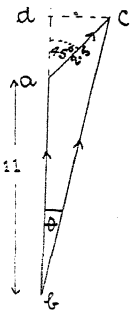
$b\ c = 9\cdot399$ knots. Ans.

$$\text{Cos. } \alpha = \frac{b\ d}{} = \frac{9\cdot233}{9\cdot399}$$

$= 0\cdot9822$

$\alpha = 10°\ 49'$.

Course is $10°\ 49'$ East of North. Ans.



$c\,d = 2{\cdot}5 \times 0{\cdot}707 = 1{\cdot}767 = a\,d$

$b\,c = \sqrt{12{\cdot}767^2 + 1{\cdot}767^2} = \sqrt{166{\cdot}12}$

$b\,c = 12{\cdot}88$ knots. Ans.

$$\text{Cos. } \theta = \frac{12{\cdot}767}{12{\cdot}88} = 0{\cdot}9906,$$

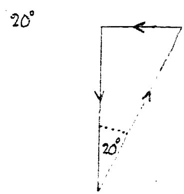
$\theta = 7° \ 53'$.

Course is 7° 53′ East of North. Ans.

10. Force in sling $= \dfrac{2{,}500}{\text{Cos. } 20°}$

Picture

$$\frac{2{,}500}{0{\cdot}9397} = 2{,}661 \text{ lb. Ans.}$$

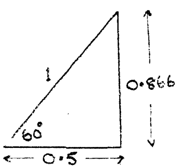


Side thrust $= 2{,}500 \times \text{Tan. } 20°$

$= 2{,}500 \times 0{\cdot}364$

$= 910$ lb. Ans.

11. Since the ships arrive simultaneously, the faster ship is twice as far from port as the slower ship. Because the angle between the courses is 60°, and the lengths of the sides formed by the courses are as 2 : 1, the triangle formed by the port and the positions of the ships is a right angled triangle.

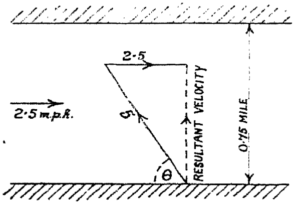


Sides are as $1 : 2 : \sqrt{3}$, or $1 : 2 : 1{\cdot}732$ or $0{\cdot}5 : 1 : 0{\cdot}866$.

When the distance apart is 86·6 miles

faster ship is $\dfrac{\quad}{0{\cdot}866} = 100$ miles from port, and after 3 hours steaming at 10 knots will be 70 miles from port, and as the triangle for the new position is similar, the distances apart is $70 \times 0{\cdot}866 = 60{\cdot}62$ miles. Ans.

12.



The resultant velocity must be at right angles to the river bank.

$$\text{Cos. } \theta = \frac{2 \cdot 5}{5} = 0 \cdot 5 \;\therefore\; \theta = 60^\circ$$

Resultant velocity
$= 5 \text{ sin. } 60^\circ = 4 \cdot 33$ miles per hr.
Time to cross

$$= \frac{0 \cdot 75}{4 \cdot 33} \times 60 = 10 \cdot 392 \text{ mins.}$$

The man must steer up stream at 60° to the river bank
He takes 10·392 minutes to cross. } Ans.

13.

$$\text{Turning moment} = \frac{\text{P} \times l}{\text{Cos.}^2 \;\theta}$$

In mid-position $\theta = 0^\circ$, and cos. $0^\circ = 1$, also cos.^{2} $0^\circ = 1$.
Turning moment when in mid-position

$$= 10^2 \times \frac{\pi}{4} \times 1{,}000 \times \frac{26}{12}$$

$$= 170{,}200 \text{ ft. lb.} \quad \text{Ans.}$$

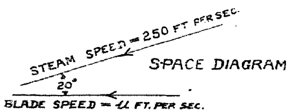
Turning moment at 35°

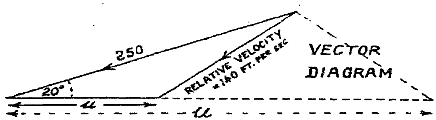
$$= \frac{170{,}200}{\text{Cos.}^2\; 35^\circ} = \frac{170{,}200}{(0 \cdot 8192)^2} = 253{,}600 \text{ ft. lb.}$$
Ans.

14. The velocity of the steam relative to the blades regarded as being fixed, is given as 140 feet per sec.

Bring the blades to rest by giving to steam and to blades a velocity of u feet per sec. in the opposite direction to the blade motion.

Draw a velocity vector diagram.





Then, $140^2 = u^2 + 250^2 - 2 \times u \times 250 . \text{Cos. } 20^\circ$

$$u^2 - 469{\cdot}85\, u = -42900$$

Solving the quadratic, $u = 124{\cdot}08$, or $345{\cdot}76$, both values being positive. The larger value is shown dotted on the vector diagram, but this value must be disregarded, because the relative velocity of the steam must be in the same direction as the steam jet.

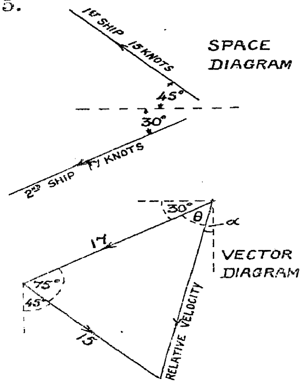
The blade speed is 124·08 feet per sec.

Let the mean diameter of the rotor be d feet.

$$d \times \pi \times \frac{500}{60} = 124{\cdot}08$$

$$d = \frac{124{\cdot}08 \times 60}{\pi \times 500} = 4{\cdot}739 \text{ feet. Ans.}$$

15.



The speed of the 2nd ship relative to the 1st is required, therefore bring the 1st ship to rest by applying to each a velocity of 15 knots in the direction S.E.

Draw the velocity vector diagram.

Relative Velocity $=$
$\sqrt{15^2 + 17^2 - 2 \times 15 \times 17 \text{ Cos. } 75^\circ}$
$= 19{\cdot}54$ knots.

$$\frac{15}{\text{Sin. } \theta} = \frac{19{\cdot}54}{\text{Sin. } 75^\circ}, \text{ from which } \theta = 47^\circ\ 52'$$

$$\propto\ = 90^\circ - 30^\circ - 47^\circ\ 52' = 12^\circ\ 8'$$

The time for the ships to become 100 miles apart is the time to do 100 miles at the relative velocity of 19·54 knots.

$$\text{Time} = \frac{100}{19{\cdot}54} = 5{\cdot}118 \text{ hours. Ans.}$$

The bearing of the 2nd ship, as seen from the 1st, is 12° 8′ West of South. Ans.

SOLUTIONS TO TEST EXAMPLES XII.

1. $v^2 = 2\,g\,s, \quad v^2 = 2 \times 32{\cdot}2 \times 600$

$v = \sqrt{64{\cdot}4 \times 600}$

$= 196{\cdot}5$ feet per sec. Ans.

also $v = g\,t$ or $t = \dfrac{v}{}$

$$t = \frac{196{\cdot}5}{32{\cdot}2} = 6{\cdot}103 \text{ secs.} \quad \text{Ans.}$$

$v^2 = u^2 \pm 2\,g\,s$, here $v = o$

$$\therefore u^2 = 2\,g\,s, \text{ or } s = \frac{u^2}{2\,g} = \frac{220^2}{64{\cdot}4}$$

$s = 751{\cdot}5$ ft. greatest height. Ans.

$$\text{also } v = g\,t, \therefore t = \frac{220}{32{\cdot}2} = 6{\cdot}831 \text{ secs. to rise.}$$

The time taken to fall is the same,

$\therefore$ total time $= 6{\cdot}831 \times 2 = 13{\cdot}662$ secs. Ans.

$$\text{or time to rise} = \frac{\text{Distance}}{\text{Mean velocity}} \quad \frac{751{\cdot}5}{110}$$

or time $= 6{\cdot}83$ secs.

Total time $= 13{\cdot}66$ secs. as above.

3. $v = g\,t, \quad v = 32{\cdot}2 \times 6 = 193{\cdot}2$ ft. per sec. Ans.

$= \dfrac{g}{2}, \quad s = 16{\cdot}1 \times 36 = 579{\cdot}6$ feet. Ans.

4. $v = u - g\,t, \; 120 = 250 - 32{\cdot}2\,t$

or $32{\cdot}2\,t = 130, \; t = \dfrac{130}{32{\cdot}2}$

$t = 4{\cdot}036$ secs. Ans.

$s = u\,t - \dfrac{g\,t^2}{2}, \quad 50 = 250\,t - 16{\cdot}1 \times t^2$

divide throughout by 16·1,

$\dfrac{50}{16{\cdot}1} = \dfrac{250\,t}{16{\cdot}1} - t^2, \text{ or } t^2 - \dfrac{250\,t}{16{\cdot}1} \qquad \dfrac{50}{16{\cdot}1}$

completing the squares we get

$t^2 - \dfrac{250\,t}{(16{\cdot}1)} \qquad \overline{32{\cdot}2}$

$- \dfrac{50}{16{\cdot}1} \qquad \left(\dfrac{250}{32{\cdot}2}\right)^2$

$\overline{32{\cdot}2} - \pm \sqrt{-\dfrac{50}{16{\cdot}1} + \left(\dfrac{250}{32{\cdot}2}\right)^2}$

$t - 7{\cdot}762 = \pm \sqrt{-3{\cdot}106 + 60{\cdot}25}$

$= \pm \sqrt{57{\cdot}144}$

$t = \pm \; 7{\cdot}559 + 7{\cdot}762$

$t = 15{\cdot}321$ secs. Ans.

If we took the other value given by the quadratic, it would give the time from the start when the stone is 50 feet above the ground, but going upwards.

5. $v = u - a\,t$, now v finally $= 0$

$\therefore \; u = a\,t \quad \text{or } t = \dfrac{\;}{a} = \frac{20}{2} = 10$ secs. Ans

also $s = u\,t - \dfrac{a\,t^2}{2}$

$$= 20 \times 10 - \frac{2 \times 10^2}{} = 100 \text{ feet. Ans.}$$

or average velocity $\times$ time $=$ distance covered.

$$\frac{20 + 0}{} \times 10 = 100 \text{ feet as before.}$$

$s = u\,t - \dfrac{g\,t^2}{2}$, here $a = g$, because once the stone leaves the balloon it is under the acceleration of g only.

$$s = 35 \times 15 - 16{\cdot}1 \times 15^2$$

$$s = 525 - 3{,}622$$

$s = -3{,}097$, the minus sign here, simply means that the balloon was 3,097 feet above the ground when the stone was thrown out. Note that this is not the greatest height of the stone above the ground; it ascends for a while after leaving the balloon.

Ans. = 3,097 feet above ground when stone was thrown out.

$$\frac{\text{Distance}}{\text{Time}} = \text{average velocity}, \therefore \frac{1 \text{ knot}}{\frac{12}{60} \text{ hour}} = 5 \text{ knots}$$

If the average speed during this time is 5 knots, then the speed at the end of this time is 10 knots. Ans.
1 nautical mile = 6,080 feet.

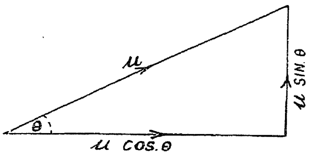
$$s = \frac{a\,t^2}{2} \text{ or } a = \frac{2\,s}{t^2}$$

$$a = \frac{2 \times 6{,}080}{(12 \times 60)^2} \quad \frac{2 \times 6{,}080}{144 \times 3{,}600}$$

$$a = \frac{608}{72 \times 360} = 0{\cdot}02349 \text{ ft. per sec.}^2$$

Acceleration $= 0{\cdot}02349$ ft. per sec.^{2} Ans.

8. Let t = time for shell to attain its maximum height, then $2\,t$ = time to hit.



$$u \text{ Sin. } \theta \qquad \times\ t = 6{,}000$$

$$\therefore\ u \text{ Sin. } \theta = \quad 12{,}000$$

also u Cos. $\theta \times 2\,t = 70{,}000$

$$\therefore\ u \text{ Cos. } \theta = \quad 70{,}000$$

$$\text{Now, } \frac{u \text{ Sin. } \theta}{u \text{ Cos. } \theta} = \text{Tan. } \theta = \frac{12{,}000}{t} \times \frac{2\,t}{70{,}000}$$

$$= \frac{24}{70} = 0{\cdot}3428 \quad \therefore\ \theta = 18^\circ\ 56'. \quad \text{Ans.}$$

The range is given by $\dfrac{2\,u^2 \text{ Sin. } \theta \text{ Cos. } \theta}{\qquad}$

$$\frac{2\,u^2 \text{ Sin. } 18^\circ\ 56' \text{ Cos. } 18^\circ\ 56'}{32{\cdot}2} = 70{,}000$$

$$\therefore\ u = \sqrt{\frac{70{,}000 \times 32{\cdot}2}{2 \times 0{\cdot}3245 \times 0{\cdot}946}} = 1{,}916 \text{ ft.}$$

Ans.

9. Change in velocity = 400 — 150 = 250 revolutions per minute.

$$\text{Change in velocity} = \frac{250 \times 2\,\pi}{60} = \tfrac{25}{3}\,\pi \text{ radians per second.}$$

$$\text{Retardation} = \frac{\text{Change in velocity}}{\text{Time to change}} = \frac{25\,\pi}{3 \times 8} \text{ radians per sec.}^2$$

$$\text{Retardation} = \frac{25 \times 22}{3 \times 8 \times 7} = 3{\cdot}273 \text{ radians per sec.}^2$$

Ans.

$$\text{No. of revs. turned} = \left(\frac{400}{\quad} + \qquad \right) \times \tfrac{8}{60} = 36\tfrac{2}{3} \text{ revs.}$$

$v = u - a\,t$, and as v the final velocity $= 0$,

$u = a\,t$, now substitute the angular notation,

$\omega = \phi\ t$, and $\phi = 3{\cdot}273$ radians per sec.^{2}

also $\omega = \dfrac{150 \times 2\ \pi}{60}$ radians per second,

$$t = \frac{\omega}{\phi} = \frac{150 \times 2 \times 22}{3{\cdot}273 \times 60 \times 7} \quad \frac{15 \times 44}{3{\cdot}273 \times 42}$$

$$t = \frac{15 \times 22}{3{\cdot}273 \times 21}$$

$= 4{\cdot}801$ secs. Ans.

10. Velocity (Linear) $= \omega\,r$

$$\frac{400 \times 2\ \pi}{60} \quad \frac{40\ \pi}{} \qquad \frac{22}{7} = 41{\cdot}9 \text{ radians per sec.}$$

Linear Velocity of a point 2 feet from centre $= 41{\cdot}9 \times 2$ $= 83{\cdot}8$ ft. per sec.

Linear Velocity of a point 3 feet from centre $= 41{\cdot}9 \times 3$ $= 125{\cdot}7$ ft. per sec.

$83{\cdot}8$ and $125{\cdot}7$ feet per sec. Ans.

SOLUTIONS TO TEST EXAMPLES XIII.

1. $$\text{Accel.} = \frac{\text{force} \times g}{\text{W}} \quad \frac{10 \times 32{\cdot}2}{400} = 0{\cdot}805 \text{ ft. per sec.}^2$$ Ans.

$v = a\ t$, $v = 0{\cdot}805 \times 12 = 9{\cdot}66$ feet per sec. Ans.

2. $$\text{Force} = \frac{\text{Change of Momentum}}{\text{Time to change}} \quad \frac{200\ (22 - 10)}{32{\cdot}2 \times 10}$$

$$\text{Force} = \frac{20 \times 12}{32\cdot2} \qquad \frac{240}{32\cdot2}$$

$$\text{Force} = 7\cdot452 \text{ lb.} \quad \text{Ans.}$$

$$\text{or Accel.} = \frac{\text{Change in vel.}}{\text{Time to change}} = \frac{22 - 10}{10} = 1\cdot2 \text{ feet per sec.}^2$$

$$\text{Force} = \frac{W}{-} \times a = \frac{200}{32\cdot2} \times 1\cdot2 = \frac{240}{32\cdot2} = 7\cdot452 \text{ lb.}$$

$$\text{Momentum destroyed} = \frac{W\ v}{g}$$

$$\text{Force} = \frac{\text{Momentum destroyed}}{\text{Time}} = \frac{W\ v}{g \times t}$$

$$\text{Force} = \frac{7 \times 30}{32\cdot2 \times \frac{1}{20}} = \frac{210 \times 20}{32\cdot2}$$

$$\text{Force} = \frac{4{,}200}{32\cdot2} = 130\cdot4 \text{ lb.} \quad \text{Ans.}$$

4.

$$\text{Force} = \frac{\text{Change of momentum}}{\text{Time to change}}$$

$$\text{or Time} = \frac{\text{Change of Momentum}}{\text{Force}}$$

$$\text{Time} = \frac{22\ (28 - 12)}{32\cdot2 \times 8} = \frac{11 \times 16}{32\cdot2 \times 4} = \frac{44}{32\cdot2}$$

$$\text{Time} = 1\cdot367 \text{ secs.}$$

Distance = average velocity × time.

$$= \left(\frac{28 + 12}{2}\right) \times 1{\cdot}367 = 20 \times 1{\cdot}367 = 27{\cdot}34 \text{ feet. Ans.}$$

or, Work done by force = Work stored in body.

$$8 \times \text{distance moved} = \frac{W}{} (28^2 - 12^2)$$

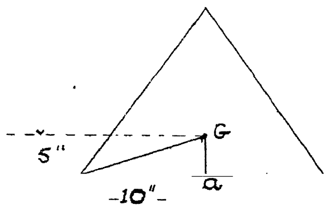
$$\text{Distance} = \frac{22}{64{\cdot}4} \times \frac{40 \times 16}{8} = \frac{22 \times 80}{64{\cdot}4}$$

$$\text{Distance} = \frac{1{,}760}{64{\cdot}4} = 27{\cdot}33 \text{ feet as above.}$$

$$\text{Note, original K.E.} = \frac{22}{64{\cdot}4} \times 12^2, \text{ final K.E.} = \frac{22}{64{\cdot}4} \times 28^2$$

$$\text{gain in K.E.} = \frac{22}{64{\cdot}4} (28^2 - 12^2)$$

When the C.G. is raised to its highest position, G is vertically above o, and the work done is the weight of the cone multiplied by the vertical distance the C.G. is raised.



C.G. $= \frac{20}{4} = 5$ inches above base.

OG $= \sqrt{10^2 + 5^2} = \sqrt{125} = 11{\cdot}17$ inches.

C.G. is raised $11{\cdot}17 - 5 = 6{\cdot}17$ inches.

Work done $= 25 \times 6{\cdot}17 = 154{\cdot}2$ inch lb.
or $12{\cdot}85$ foot lb. Ans.

6. A fathom is 6 feet.

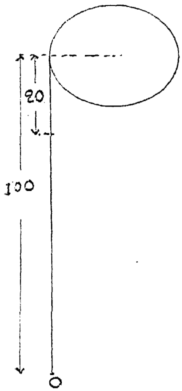
Weight of chain at start $= 100 \times 50 = 5{,}000$ lb.

Weight of chain at finish $= 20 \times 50 = 1{,}000$ lb.

$$\text{Mean weight} = \frac{5{,}000 + 1{,}000}{2} = 3{,}000 \text{ lb.}$$

Work done = mean weight × distance.

= 3,000 × 80 × 6 = 1,440,000 ft. lb. Ans.



or Work done = Total weight × rise of C.G.

Orig. C.G. is at 50 fathoms above *o*.

Finally when only 20 fathoms hang down, 80 fathoms have been heaved up, and the C.G. of this 80 fathoms is at 100 fathoms above *o*, but the C.G. of the 20 fathoms is at 90 fathoms from *o*.

Take moments about *o*, then finally

$$\frac{(20 \times 90) + (80 \times 100)}{100} = 98 \text{ fathoms}$$

position above *o* of the C.G. finally.

Rise in C.G. = 98 — 50 = 48 fathoms.

Work done = Total weight × 48 × 6 ft. lb.

= (100 × 50) × 48 × 6 = 1,440,000 ft. lb. as above.

The first method is the easier, and the second is merely given to show that we may use the principle of the vertical rise of the C.G. Note in taking moments round *o*, we may leave out 50, as it would come into the weights of the pieces of chain on both top and bottom of the equation.

7. When the speed is constant, the force in the wire is merely the weight of the cage. Ans.

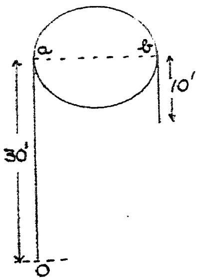
(*b*) The pull in the wire is the difference between the weight of the cage and the accelerating force.

$$\text{Accel. Force} = \frac{W}{g} \times a = \frac{1{,}500 \times 6}{32{\cdot}2} = \frac{9{,}000}{32{\cdot}2}$$

Accel. Force = 279·5 lb.
Pull in wire = 1,500 — 279·5 = 1,220·5 lb. Ans.

(*c*) The pull in the wire is the sum of the weight of the cage and the accelerating force.

Pull in wire = 1,500 + 279·5 = 1,779·5 lb. Ans.



Force at beginning = (30 × 10) — (10 × 10) = 200 lb.

Force when ends are equal = nil.

$$\text{Mean force} = \frac{200 + 0}{2} = 100 \text{ lb.}$$

Work = mean force × distance = 100 × 10 = 1,000 ft. lb. Ans.

Note to make the ends equal, the short length must be pulled down 10 feet.

or, Work = Total weight × vertical rise of C.G.

Take moments about *o*,

The original position of C.G. above *o*

$$\frac{(30 \times 15) + (10 \times 25)}{40} = 17·5 \text{ feet.}$$

C.G. above *o* finally = 20 feet.

Note that the piece *a b* on the pulley is in the same position at the end of the operation as at the beginning, and we may leave its moment out of both equations when finding the C.G. Of course, if we want the C.G. definitely for any one position, the size of the pulley should be given. As here we subtract the height of the original C.G. from its final height above *o*, and as the length *a b* is the same for both positions, we need not use it, and the rise of the C.G. is given accurately by 20 — 17·5 = 2·5 feet.

Work = 40 × 10 × 2·5 = 400 × 2·5 = 1,000 ft. lb. as above.

The first method is probably the easier to apply.

9. Vol. of wheel = area $\times$ thickness.

$= \frac{11}{14}\,[10^2 - 8^2] \times 1 = \frac{11}{14} \times 18 \times 2 = 28{\cdot}28$ cu. ft.

or by Pappus, Area of section $\times$ distance C.G. moves = Vol. swept out.

$1 \times 1 \times 2\,\pi \times 4{\cdot}5 = 2 \times \frac{22}{7} \times \frac{9}{2} = 28{\cdot}28$ cu. ft.

Wt. $= 28{\cdot}28 \times 450 = 12{,}726$, say 12,730 lb.

K.E. $= 0{\cdot}00017\ W\,R^2\,N^2 = 0{\cdot}00017 \times 12{,}730 \times (4{\cdot}5)^2 \times (100)^2$

Note, the radius of gyration has been taken as the mean radius.

K.E. $= 438{,}200$ ft. lb. of work at 100 revs. per min.

K.E. at 80 revs. per minute $= 438{,}200 \times (\frac{80}{100})^2 = 280{,}500$ ft. lb. Ans.

Work given out $= 438{,}200 - 280{,}500 = 157{,}700$ ft. lb. Ans.

10. Weight $= \frac{11}{14} \times 4 \times 4 \times \frac{1}{2} \times 450 = 2{,}828$ lb.

$$\text{Radius of gyration} = \frac{R}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2} \text{ feet.}$$

K.E. $= 0{\cdot}00017 \times 2{,}828 \times (\sqrt{2})^2 \times 80 \times 80$

$= 0{\cdot}00017 \times 2{,}828 \times 2 \times 6{,}400 = 6{,}152$ ft. lb. Ans.

11.

$$\text{Average revs.} = \frac{100 + 80}{2} = 90 \text{ revs. per min.}$$

Distance key moves in 5 secs. $= \frac{90}{60} \times 5 \times \frac{22}{7} \times \frac{8}{12} = \frac{110}{7}$ ft.

Work given out $= 157{,}700$ ft. lb. and this must equal force on key $\times \frac{110}{7}$ ft. lb.

Force on key $\times \frac{110}{7} = 157{,}700$

$$\text{Force on key} = \frac{157{,}700 \times 7}{110} = 10{,}035 \text{ lb.} \quad \text{Ans.}$$

12.

$$\text{C.F.} = \frac{W\,v^2}{g\;R} = 0{\cdot}00034\ W\ R\ N^2$$

C.F. $= 0{\cdot}00034 \times 0{\cdot}4 \times \frac{15}{12} \times (2{,}000)^2$

C.F. $= 680$ lb. pull. Ans.

13. C.F. $= 0{\cdot}00034 \times 50 \times \frac{1}{8} \times \frac{1}{12} \times (1{,}000)^2$
$= 177{\cdot}1$ lb. additional load. Ans.

14. K.E. of bullet $= \frac{\quad}{2\,g} = \frac{1{\cdot}5 \times 1{,}600^2}{16 \times 2 \times 32{\cdot}2}$
$= 3{,}727$ ft. lb. Ans. (*a*).

Let V = velocity at which wood starts to move.
Weight of wood and bullet = 161·5 ozs.

By conservation of momentum :—

$$\frac{1{\cdot}5 \times 1{,}600}{16 \times 32{\cdot}2} = \frac{161{\cdot}5 \times V}{16 \times 32{\cdot}2} \quad \therefore V = \frac{1{\cdot}5 \times 1{,}600}{161{\cdot}5}$$

$= 14{\cdot}86$ ft. per sec. Ans (*b*).

K.E. of wood and bullet

$$\frac{161{\cdot}5 \times 14{\cdot}}{16 \times 2 \times 32{\cdot}2} = 34{\cdot}61 \text{ ft. lb.}$$

Total kinetic energy before impact was 3,727 ft. lb., and after impact it was 34·61 ft. lb., hence
Loss of K.E. $= 3{,}727 - 34{\cdot}61 = 3{,}692{\cdot}4$ ft. lb. Ans. (*c*)

15.

At 50 R.P.M. height $= \frac{2{,}935}{50^2} = 1{\cdot}174$ feet, or 1 ft. 2·1 ins. Ans.

,, 75 ,, $= \frac{2{,}935}{75^2} = 0{\cdot}522$ foot, or 6·26 ins.

,, 100 ,, ,, $= \frac{2{,}935}{100^2} = 0{\cdot}2935$ foot, or 3·52 ins. Ans.

Note, as the height varies as $\frac{1}{N^2}$, the height at 100 R.P.M. is $\left(\frac{50}{100}\right)$ times the height at 50 R.P.M., that is, one quarter of the height.

16. At 200 R.P.M. height $= \left(1 + \frac{30}{3}\right) \frac{2,935}{200^2} = \frac{11 \times 2,935}{200^2}$ ft. $= \frac{11 \times 2,935 \times 12}{200^2}$ ins. $= 9{\cdot}686$ ins.

As the height varies as $\frac{1}{N^2}$, height at 250 R.P.M.

$= 9{\cdot}686 \times \left(\frac{200}{250}\right)^2$ ins. $= 9{\cdot}686 \times 0{\cdot}8^2 = 9{\cdot}686 \times 0{\cdot}64$
$= 6{\cdot}199$ ins.

$\therefore$ Difference in height $= 9{\cdot}686 - 6{\cdot}199 = 3{\cdot}487$ ($3\frac{1}{2}$) inches. Ans.

Velocity of piston $= \omega\, r \sin.\, \theta$, $\omega\, r = 12{\cdot}28 \times 2 = 24{\cdot}56$ ft. per sec.

Acceleration of piston $= \omega^2\, r \cos.\, \theta$, $\omega^2\, r = (12{\cdot}28)^2 \times 2 = 301{\cdot}5$ ft. per (sec.)^2

Velocities :—

At 30° and 150°, Sin. 30° and Sin. 150° $= \frac{1}{2}$

$\therefore$ Velocity of piston $= 24{\cdot}56 \times \frac{1}{2} = 12{\cdot}28$ ft. per sec. Ans.

At 60° and 120°, Sin. 60° and Sin. 120° $= \frac{\sqrt{3}}{2}$

$\therefore$ Velocity of piston $= 24{\cdot}56 \times \frac{\sqrt{3}}{2} = 21{\cdot}27$ ft. per sec. Ans.

At 90°, Sin. 90° $= 1$

$\therefore$ Velocity of piston $= 24{\cdot}56 \times 1 = 24{\cdot}56$ ft. per sec. Ans.

Accelerations :—

Cos. 30° $= \frac{\sqrt{3}}{2}$ Acceleration at 30° $= 301{\cdot}5 \times \frac{\sqrt{3}}{2}$
$= 261{\cdot}2$ ft. per (sec.)^2. Ans.

Cos. 60° $= \frac{1}{2}$, Acceleration at 60° $= 301{\cdot}5 \times \frac{1}{2}$
$= 150{\cdot}75$ ft. per (sec.)^2. Ans.

Cos. 90° $= 0$, Acceleration at 90° $= 0$.

$\text{Cos. } 120^\circ = -\text{ Cos. } 60^\circ = -\frac{1}{2}$

$\text{Acceleration at } 120^\circ = 301{\cdot}5 \times -\frac{1}{2}$

$= -150{\cdot}75$ ft. per (sec.)^2. Ans.

$\text{Cos. } 150^\circ = -\text{ Cos. } 30^\circ = -\frac{\sqrt{3}}{2}$

$\text{Acceleration at } 150^\circ = 301{\cdot}5 \times -\frac{\sqrt{3}}{2}$

$= -261{\cdot}2$ ft. per (sec.)^2. Ans.

18. Acceleration $= \omega^2 \times$ displacement from mid-travel

$\left(\frac{120 \times 2\pi}{60}\right)^2 \times 1{\cdot}125$ ft. per (sec.)^2

$= (4\pi)^2 \times 1{\cdot}125$

$\text{Accelerating force} = \frac{W \times a}{g} = \frac{310 \times 16 \times \pi^2 \times 1{\cdot}125}{32{\cdot}2}$

$= 1{,}710$ lb. Ans.

19. $T = 2\pi\sqrt{\frac{l}{g}} = 2\pi\sqrt{\frac{3}{32{\cdot}2}} = 2 \times \quad \times \quad 5{\cdot}675$

$= 1{\cdot}919$ seconds. Ans.

20. $= 2\pi\sqrt{\frac{l}{\;}}$ $\therefore T \propto \sqrt{l} \therefore \frac{\;}{\;} =$

$\frac{1}{\;} = \frac{\frac{1}{2}}{\;}$ $\therefore \sqrt{l_2} = \frac{1}{2} \times \sqrt{3} \therefore l_2 =$

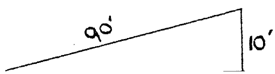
2

$\therefore l = \frac{3}{4}$ foot, or 9 inches. Ans.

SOLUTIONS TO TEST EXAMPLES XIV.

1. Total force $= 5 \times 15 + \frac{5 \times 2{,}240}{\;} = 1{,}319{\cdot}4$ lb.

Work done $= 1{,}319{\cdot}4 \times 90$ ft. lb.



Work per minute

$= 1{,}319{\cdot}4 \times 90 \times \frac{60}{30}$

$$\text{H.P.} = \frac{\text{Work per minute}}{33{,}000} \quad \frac{1{,}319{\cdot}4 \times 90 \times 2}{33{,}000}$$

H.P. = 7·196. Ans.

2. $$\text{Total force} = 5 \times 12 + \frac{5 \times 2240}{12} = 993{\cdot}3 \text{ lb.}$$

Work done = force × distance = 993·3 × 12.
Work done = 11,919·6 foot lb. Ans.

3. Total force $= \mu$ W Cos. α + W Sin. α, $\alpha = 16^\circ$
$= 0{\cdot}2 \times 5 \times 0{\cdot}9613 + 5 \times 0{\cdot}2756$
$= 0{\cdot}9613 + 1{\cdot}378 = 2{\cdot}3393$ tons. Ans.

The load would run down if unsupported, as the force due to the incline is greater than the frictional resistance. For the load to remain at rest unsupported, the plane must be at the friction angle to the horizontal.

Tan. $\phi = \mu$.

Tan. $\phi = 0{\cdot}2$, $\phi = 11^\circ\ 19'$. Ans.

4. Work absorbed by bearings per revolution
$= \pi \times \frac{8}{12} \times 10 \times 0{\cdot}03$ ft. ton.

$$\text{Work stored in wheel} = \text{K.E.} = \frac{1}{5{,}870} \times n^2 \text{ ft. tons.}$$

$$\text{K.E.} = \frac{10 \times 3 \times 3}{5{,}870} \times 200 \times 200$$

$$\text{No. of revs. to absorb K.E.} = \frac{\text{K.E.}}{\text{Work per rev.}}$$

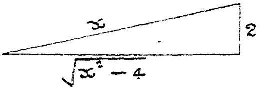
$$\text{No. of revs.} = \frac{10 \times 9 \times 200 \times 200 \times 12}{5{,}870 \times \pi \times 8 \times 10 \times 0{\cdot}03}$$

= 976 revolutions. Ans.

$$\frac{\text{No. of revs.}}{\text{Mean speed}} \quad \frac{976}{200 + 0}$$

$= \frac{976}{100} = 9{\cdot}76$ minutes. Ans.

5. Total force $= \mu$ W Cos. $\propto$ + W Sin. $\propto$



$$= 0{\cdot}2 \times \frac{\sqrt{x^2 - 4}}{} \times 10 + 10 \times \frac{2}{}$$

Work = force × distance.

$60 =$

$60 = 2\sqrt{x^2 - 4} + 20$, since x cancels

$2\sqrt{x^2 - 4} = 40$

$\sqrt{x^2 - 4} = 20$, square both sides

$x^2 - 4 = 400$, $x^2 = 404$

$x = 20{\cdot}1$ feet.

$$\text{Sin. } \propto \; = \frac{2}{20{\cdot}1} = 0{\cdot}0995$$

$\propto \; = 5^\circ\ 43'$. Ans.

Total force $= \mu$ W Cos. $\propto$ — W Sin. $\propto$, $b = \sqrt{399} = 19{\cdot}97$

$$= 0{\cdot}18 \times 10 \times \frac{19{\cdot}97}{20} - 10 \times \tfrac{1}{20}$$

$$= 1{\cdot}797 - 0{\cdot}5 = 1{\cdot}297 \text{ tons. Ans.}$$

Friction force $= 120 \times 10 = 1{,}200$ lb.

$$\text{Force due to incline} = \frac{120 \times 2{,}240}{80} = 3{,}360 \text{ lb.}$$

Total pull on incline = 4,560 lb.

Of the 3 tons force, 4,560 lb. must go to support train on incline, the remainder, 6,720 — 4,560 is available for acceleration.

$$\text{Acceleration} = \frac{\text{Accel. force}}{\text{Mass}} = \frac{2{,}160 \times 32{\cdot}2}{120 \times 2{,}240}$$

Acceleration $= 0{\cdot}258$ ft. sec.^{2}

30 miles per hour, is 44 feet per second.

$v = a\,t$, $44 = 0{\cdot}258 \times t$

$$\text{Time} = \frac{44}{0{\cdot}258} = 170{\cdot}5 \text{ seconds}$$

or 2 mins. 50·5 secs. Ans.

As the body needs a force to prevent it from moving down, it follows that W Sin. α is greater than μ W Cos. α, or that the force acting down due to the incline is greater than the friction force.

Force up $=$ W Sin. α $+ \mu$ W Cos. α

Force to hold $=$ W Sin. α $- \mu$ W Cos. α
putting in the given values

$$\left.\begin{array}{lrl} (1) & 142 & = 400 \text{ Sin. } \alpha + \mu\, 400 \text{ Cos. } \alpha \\ (2) & 25 & = 400 \text{ Sin. } \alpha - \mu\, 400 \text{ Cos. } \alpha \end{array}\right\}$$

$$167 = 800 \text{ Sin. } \alpha$$

$\therefore$ Sin. $\alpha = \frac{167}{800} = 0{\cdot}2087$
and $\therefore$ $\alpha = 12° \; 3'$

From (2) we get, μ 400 Cos. α $= 400$ Sin. α $- 25$
$\mu \times 400 \times 0{\cdot}9779 = 400 \times 0{\cdot}2087 - 25$

$$\mu = \frac{58{\cdot}48}{391{\cdot}16} = 0{\cdot}15 \text{ nearly.}$$

9. Here the friction force is greater than the force acting down due to the plane, and the body needs to be pulled down.

Force up $= \mu$ W Cos. α $+$ W Sin. α

Force down $= \mu$ W Cos. α $-$ W Sin. α

$$\left.\begin{array}{lrl} (1) & 152 & = \mu\, 400 \text{ Cos. } \alpha + 400 \text{ Sin. } \alpha \\ (2) & 5 & = \mu\, 400 \text{ Cos. } \alpha - 400 \text{ Sin. } \alpha \end{array}\right\}$$

$$147 = 800 \text{ Sin. } \alpha$$

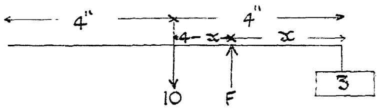
Sin. $\alpha = \frac{147}{800} = 0{\cdot}1837$
$\alpha = 10° \; 35'$.

From (2) $\mu \times 400 \times 0{\cdot}983 = 400 \times 0{\cdot}1837 + 5$
$393{\cdot}2 \; \mu = 78{\cdot}48$

$$\mu = \frac{78{\cdot}48}{393{\cdot}2} = 0{\cdot}199 \text{ say } 0{\cdot}2$$

SOLUTIONS TO TEST EXAMPLES XV.

1. Take moments about the fulcrum.



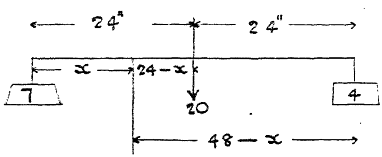
Let the fulcrum be at x inches from the 3 lb. weight.

Then, $3 \times x = 10 \times (4 - x)$

$3\,x = 40 - 10\,x$, or $13\,x \doteq 40$

$x = 3{\cdot}077$ inches from 3 lb. Ans.

2. Take moments round the fulcrum.

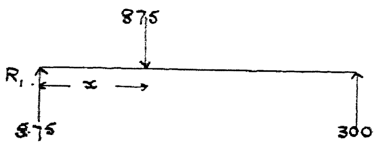


Then $7 \times x = 20\,(24 - x) + 4\,(48 - x)$

$7\,x = 480 - 20\,x + 192 - 4\,x$

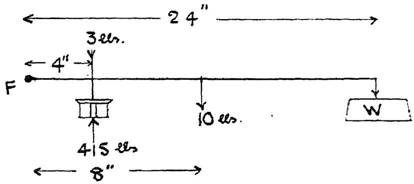
$31\,x = 672$, $x = 21{\cdot}68$ inches. Ans.

3. Total weight of rod $= 575 + 300 = 875$ lb. Ans.
That is, the sum of the end reactions is equal to the total downward weight.



Let the C.G. be at x feet from the heavy end.
Take moments round R_1
Then, $300 \times 6 = 875 \times x$
$x = 2{\cdot}057$ feet. Ans.

4. Steam load on valve $= \frac{13}{4} \times \frac{13}{4} \times \frac{11}{14} \times 50 = 415$ lb.



Take moments about F.

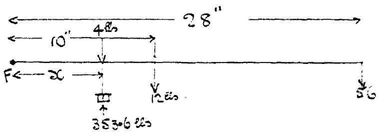
$(415 - 3) \times 4 = (10 \times 8) + W \times 24$

$412 \times 4 = 80 + 24\ W$

$24\ W = 1{,}648 - 80,\ W = \frac{1{,}568}{24}$

$W = 65{\cdot}34$ lb. Ans.

5. Steam load on valve $= 3 \times 3 \times \frac{11}{14} \times 50 = 353{\cdot}6$ lb.



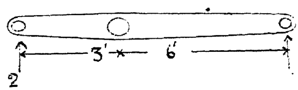
Take moments round F.

Then $(353{\cdot}6 - 4) \times x = (12 \times 10) + (56 \times 28)$

$349{\cdot}6\ x = 120 + 1{,}568$

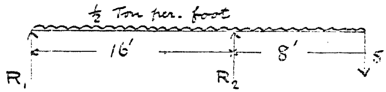
$$\frac{1{,}688}{349{\cdot}6} = 4{\cdot}829 \text{ inches. Ans.}$$

6. Force at engine end $= 2 \times \frac{3}{6} = 1$ ton.



Force on rocking shaft $= 2 + 1 = 3$ tons. Ans.

Take moments round R_1.



Then $R_2 \times 16 = (5 \times 24) + (12 \times 12)$

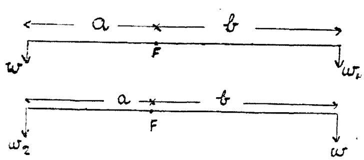
$$\frac{120 + 144}{16}$$

$R_2 = 16{\cdot}5$ tons. $R_1 = 12 + 5 - 16{\cdot}5 = 0{\cdot}5$ ton. Ans.

Note the uniform load is 12 tons, and it acts through its C.G. at 12 feet from R_1.

8. Moments round F.

$w\ a = w_1\ b$ from the first figure and $w_2\ a = w\ b$ from the second figure.



Divide, $\dfrac{w\ a}{w_2\ a} = \dfrac{w_1\ b}{w\ b}$

a and b cancel, and we g

w_2 w

or $w^2 = w_1\ w_2$ or $w = \sqrt{w_1 \times w_2}$

Correct weight $= \sqrt{50 \times 45} = \sqrt{2250}$

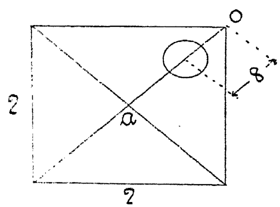
$= 47{\cdot}43$ lb. Ans.

9. Diagonal $= 2 \times \sqrt{2} = 2{\cdot}828$ feet.

Half Diagonal $= o\ a = 1{\cdot}414$ ft. or $16{\cdot}96$ inches.

Area of hole $= 8 \times 8 \times \frac{11}{14}$ $= 50{\cdot}3$ sq. inches.

Area of solid plate $= 2 \times 2 \times 144 = 576$ sq. inches.



Area of plate remaining $= 576 - 50{\cdot}3 = 525{\cdot}7$ sq. inches.

Moments round o, $576 \times 16\cdot96 - 50\cdot3 \times 8 = 525\cdot7 \times x$

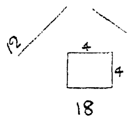
$9{,}770 - 402\cdot4 = 525\cdot7\, x$

$$x = \frac{9367\cdot6}{525\cdot7} = 17\cdot81 \text{ inches.}$$

C.G. is on the diagonal 17·81 inches from o or 17·81 — 16·96, or 0·85 inch from the centre of the square plate.
Ans.

10.

$$\text{Vertical height} = \frac{\text{Area of triangle}}{\text{base}}$$



$$\text{Area} = \sqrt{s\,(s-a)\,(s-b)\,(s-c)},\ s = \frac{18 + 12 + 15}{\ }$$

$$\text{Area} = \sqrt{22\cdot5 \times 4\cdot5 \times 10\cdot5 \times 7\cdot5} = 89\cdot3 \text{ sq. inches.}$$

Area of figure remaining $= 89\cdot3 - 16 = 73\cdot3$ sq. inches.

$$\text{Vertical height} = \frac{89\cdot3}{\ } = 9\cdot922 \text{ inches.}$$

Position of C.G. is at one-third of vertical height above

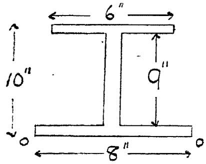
$$\text{the base} = \frac{9\cdot922}{3}$$

$$\text{Moments round } o\,o,\ 89\cdot3 \times \frac{9\cdot922}{3} - 16 \times 3 = 73\cdot3 \times x$$

$295\cdot3 - 48 = 73\cdot3\, x$

$$x = \frac{247\cdot3}{73\cdot3} = 3\cdot373 \text{ inches.} \quad \text{Ans.}$$

11. Moments round *o o*



C.G. above *o o*

$$= \frac{\text{Sum of 1st Moments about } o\ o}{\text{Total area}}$$

$$\frac{(6 \times 0{\cdot}5 \times 9{\cdot}75) + (9 \times 0{\cdot}5 \times 5) + (8 \times 0{\cdot}5 \times 0{\cdot}25)}{(6 \times 0{\cdot}5) + (9 \times 0{\cdot}5) + (8 \times 0{\cdot}5)}$$

$$\frac{29{\cdot}25 + 22{\cdot}5 + 1}{11{\cdot}5} \cdot \frac{52{\cdot}75}{11{\cdot}5} = 4{\cdot}587$$

C.G. is 4·587 inches above *o o*. Ans.

12. Vol. of cylindrical part $= \frac{11}{14}(8^2 - 6{\cdot}5^2) \times 19 = 324{\cdot}7$ cu. ins.

Vol. of flat end $= \frac{11}{14} \times 8^2 \times 1 = 50{\cdot}28$ cu. ins.

Vol. of bosses $= 2 \times 4^2 \times \frac{11}{14} \times 2 = 50{\cdot}28$ cu. ins.

Total Vol. $= 425{\cdot}26$ cu. ins.

Take moments about closed end

$$\text{C.G. from } o\ o = \frac{\text{Sum of 1st moments about closed end}}{\text{Total volume}}$$

$$\frac{(324{\cdot}7 \times 10{\cdot}5) + (50{\cdot}28 \times 0{\cdot}5) + (50{\cdot}28 \times 8)}{425{\cdot}26}$$

$$= \frac{3409{\cdot}35 + 25{\cdot}14 + 402{\cdot}06}{425{\cdot}26} = \frac{3836{\cdot}55}{425{\cdot}26}$$

= 9·02 inches from closed end.

13. V.R. = 5. Theoretical load $= 230 \times 5 = 1{,}150$ lb. Ans.

Actual load lifted $= 230 \times 5 \times \frac{62}{100} = 713$ lb. Ans.

14. $$\text{V.R.} = \frac{2\text{ D}}{(\text{D} - d)} \quad \frac{2 \times 10}{10 - 9} = 20$$

$20 \times \text{pull} \times \frac{35}{100} = 3$

Pull $= \frac{15}{35}$ or $\frac{3}{7}$ ton $= 960$ lb. Ans.

15. It takes $\frac{80}{2}$ or 40 pulls to turn the ratchet one revolution, and to turn the engine half a revolution takes $\frac{128}{2}$ or 64 turns of the ratchet.

No. of pulls $= 64 \times 40 = 2{,}560$.
Time $= \frac{2560}{20} = 128$ minutes. Ans.

16. $$\text{V.R.} = \frac{}{(\text{R} + r)} \times \frac{\text{Teeth in driven wheel}}{\text{Teeth in driver}}$$

$$\frac{20}{(5 + 0{\cdot}75)} \times \frac{82}{15} = 19{\cdot}01$$

Load lifted $= 60 \times 19{\cdot}01 = 1{,}140{\cdot}6$ lb. Ans.

17. $$\text{V.R.} = \frac{24}{(4{\cdot}5 + 1)} \times \frac{40}{15} \times \frac{65}{18} = 42{\cdot}02$$

Load lifted $= 42{\cdot}02 \times 100 = 4{,}202$ lb. Ans.

18. $$\text{V.R.} = \frac{30}{} = 502{\cdot}8$$

$502{\cdot}8 \times \text{force applied} \times \frac{36}{100} = 2 \times 2{,}240$

$$\text{Force applied} = \frac{2 \times 2{,}240 \times 100}{36 \times 502{\cdot}8}$$

$= 24{\cdot}75$ lb. Ans.

19. V.R. $= 2 \times \frac{22}{7} \times 25 \times 4 = 628{\cdot}5$

$628{\cdot}5 \times 60 \times \text{Efficiency} = 5 \times 2{,}240$

$$\text{Efficiency} = \frac{186{\cdot}6}{628{\cdot}5} = 0{\cdot}296. \quad \text{Ans.}$$

$$\text{M.A.} = \frac{\text{Load lifted}}{\text{Force applied}} \quad \frac{2{,}240 \times 5}{60} = 186{\cdot}6. \quad \text{Ans.}$$

20. Let w = wt. of valve, and x = distance from F to valve.

By moments about fulcrum

$$o \times (21 + x) + 6 \times (7{\cdot}5 + x) + w\,x = 8 \times 3 \times x$$
$$62 \times (21 + x) + 6 \times (7{\cdot}5 + x) + w\,x = 8 \times 65 \times x$$

$$62 \times (21 + x) \qquad = 62 \times 8 \times x$$
$$21 + x = 8\,x$$

x = 3 inches. Ans.

Substitute in the first equation

$$o\,(21 + 3) + 6\,(7{\cdot}5 + 3) + 3\,w = 8 \times 3 \times 3$$
w = 3 lb. Ans.

21. $10 \times 8 \times \frac{13}{4} = L \times 60$

$$\frac{10 \times 8 \times 13}{60 \times 4} = 4\tfrac{1}{3} \text{ inches.} \quad \text{Ans.}$$

$$\text{V.R.} = \frac{2\text{ D}}{} \quad \frac{2 \times 32}{} \quad \frac{64}{2{\cdot}5} = 25{\cdot}6. \quad \text{Ans.}$$

$$\text{M.A.} = \frac{\text{W}}{\text{P}} = \frac{300}{25} = 12. \quad \text{Ans.}$$

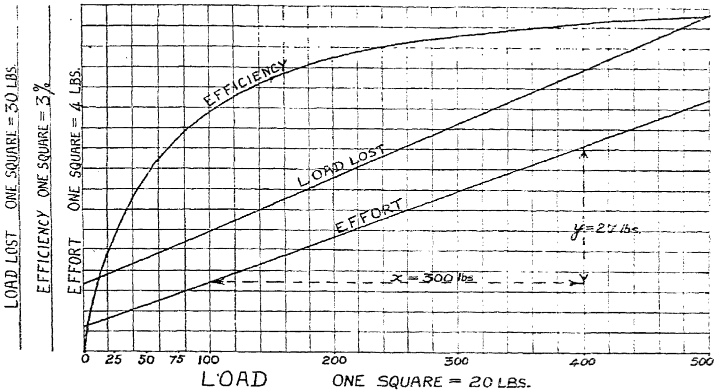
$$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}} = \frac{12}{25{\cdot}6} = 0{\cdot}468 \text{ or } 46{\cdot}8\%. \quad \text{Ans.}$$

Load lost due to friction = P × V.R. — W = 25 × 25·6 — 300 = 340 lb. Ans.

23.

$$\text{V.R.} = \frac{2\text{ D}}{\text{D} - d} \quad \frac{2 \times 10}{10 - 9} = 20$$

Load lb.	25	50	75	100	200	300	400	500
Effort lb.	7·25	9·5	11·8	14	22·9	32	41	50
M.A.	3·45	5·26	6·36	7·14	8·73	9·38	9·76	10
Efficiency %	17·25	26·3	31·8	35·7	43·65	46·9	48·8	50
Load lost, lb	120	140	161	180	258	340	420	500



$$a = 5, \ b = \frac{y}{x} = \frac{27}{300} = 0{\cdot}09$$

$\therefore$ Linear law is $P = 5 + 0{\cdot}09\ W$. Ans.

When $W = 240$ lb., $P = 5 + 0{\cdot}09 \times 240 = 5 + 21{\cdot}6$
$= 26{\cdot}6$ lb. Ans.

$$\left.\begin{aligned} 80 &= a + b \times 1{,}200 \\ 20 &= a + b \times 210 \end{aligned}\right\} \text{subtracting}$$

$$60 = 990\,b$$

$$\therefore b = \frac{60}{990} = 0{\cdot}0606$$

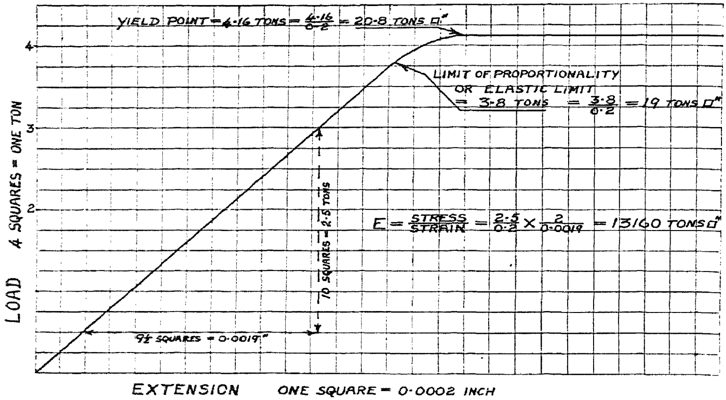
$$20 = a + \frac{60}{990} \times 210 \quad \therefore 20 = a + 12{\cdot}73$$

$\therefore a = 20 - 12{\cdot}73 = 7{\cdot}27$

$\therefore$ Linear law is $P = 7{\cdot}27 + 0{\cdot}0606\ W$.

When $W = 1{,}000$ lb., $P = 7{\cdot}27 + 0{\cdot}0606 \times 1{,}000$
$= 7{\cdot}27 + 60{\cdot}6 = 67{\cdot}87$ lb. Ans.

SOLUTIONS TO TEST EXAMPLES XVI.



1. For a given material, the limit of stress, within which the resulting strain entirely disappears after the removal of the stress, is called the elastic limit. If the material is subjected to a stress greater than the elastic limit, however little above this value, then part of the strain remains after the removal of the stress, and this part is "permanent set." Evidently the determination of the elastic limit depends upon the ability of the extensometer used to detect the smallest possible permanent set, and it may be a matter of great difficulty. For wrought iron and mild steel, the elastic limit and the limit of proportionality are practically the same, the yield point is somewhat higher. In commercial testing it is usual to assume the yield point and the elastic limit to be one and the same. When the values of loads and extensions, given in the example, are plotted, it is observed that a straight line will join all the plotted points up to about load 3·8 tons and extension 0·00285 inch, showing that up to this point the extension is directly proportional to the load. But the remaining points lie on a curve, indicating that the extension is no longer directly proportional to the load. Therefore the elastic limit, and we may take this as the limit of proportionality, is reached when the graph deviates from a straight line. It may be difficult to ascertain exactly where this does occur, in fact we can only approximate to the elastic limit. The yield point is very definitely marked by a rapid increase of the extension to many times the elastic extension.

Area of specimen $= 0{\cdot}7 \times 0{\cdot}7 \times \frac{11}{14} = 0{\cdot}385$ sq. inch.
Final area $= 0{\cdot}48 \times 0{\cdot}48 \times \frac{11}{14} = 0{\cdot}181$ sq. inch.

$$\text{Stress at yield point} = \frac{6{\cdot}93}{0{\cdot}385} = 18 \text{ tons per sq. inch. Ans.}$$

$$\text{Ultimate stress} = \frac{11{\cdot}55}{0{\cdot}385} = 30 \text{ tons per sq. inch. Ans.}$$

$$\%\ \text{Elongation} = \frac{2{\cdot}25}{} \times 100 = 28{\cdot}12 \text{ per cent. Ans.}$$

$$\%\ \text{Reduction of area} = \frac{0{\cdot}385 - 0{\cdot}181}{0{\cdot}385} \times 100 = 53 \text{ per cent. Ans.}$$

$$\text{Stress for } 0{\cdot}77 \text{ ton load} = \frac{0{\cdot}77}{0{\cdot}385} = 2 \text{ tons per sq. in.}$$

$$\text{Strain} = \frac{\text{Extension}}{\text{Orig. length}} = \frac{0{\cdot}00128}{8} = 0{\cdot}00016$$

$$\frac{\text{Stress}}{\text{Strain}} = \text{E, or } \frac{2 \times 2{,}240}{0{\cdot}00016} = 28{,}000{,}000 \text{ lb. per sq. inch. Ans.}$$

The other set of values could also be used to find E, as within the elastic limit, stress is proportional to strain. If there were slight errors in the observed extensions, two values of E might be found, and their mean taken.

3. Area of bottom of thread $\times$ Stress $=$ Area of cylinder $\times$ Steam pressure.

$$\frac{11}{14} d^2 \times 5{,}000 = 24 \times 24 \times \frac{11}{14} \times 220$$

$$\frac{24 \times 24 \times 220}{5{,}000} = 25{\cdot}34$$

$d = \sqrt{25{\cdot}34} = 5{\cdot}034$ inches. Ans.

4. Back pressure on H.P. piston $= 52 + 3 = 55$ lb. sq. inch.

$5{\cdot}4 \times 5{\cdot}4 \times \frac{11}{14} \times \text{stress} = 27 \times 27 \times \frac{11}{14} \times (200 - 55)$

$$\text{Tensile stress} = \frac{27 \times 27 \times 145}{5{\cdot}4 \times 5{\cdot}4} = 3{,}625 \text{ lb. sq. inch. Ans.}$$

$$\text{Compressive stress} = \frac{3{,}625 \times 5{\cdot}4 \times 5{\cdot}4}{7 \times 7} = 2{,}158 \text{ lb. per sq. inch. Ans.}$$

The difference in load due to the area of the piston rod on the bottom side of the piston is neglected here.

5.
$$\text{Area of one stud} = \frac{11 \times (11 - 1)}{100} = 1{\cdot}1 \text{ sq. inches.}$$

Load on cover $= 28 \times 28 \times \frac{11}{14} \times 200 = 123{,}200$ lb.

Load per stud $= 5{,}000 \times 1{\cdot}1 = 5{,}500$ lb.

No. of studs $= \frac{123200}{5500} = 22{\cdot}4$, say 23. Ans.

6. Area of cover $= 66 \times 66 \times \frac{11}{14} = 3{,}422$ sq. centimetres.

Load on cover $= 3{,}422 \times 35 = 119{,}800$ kilograms.

$$\text{Area of one stud} = \frac{12 \times (12 - 1)}{100} = 1{\cdot}32 \text{ sq. inches.}$$

$$\text{Area of 34 studs} = \frac{1{\cdot}32 \times 34}{(0{\cdot}3937)^2} = 289{\cdot}5 \text{ sq. centimetres.}$$

Then $289{\cdot}5 \times \text{Stress} = 119{,}800$

$$\text{Stress} = \frac{119{,}800}{289{\cdot}5} = 413{\cdot}8 \text{ kilograms per sq. centimetre.}$$

$$\text{Or Stress} = \frac{413{\cdot}8 \times 2{\cdot}2}{(0{\cdot}3937)^2} = 5{,}868 \text{ lb. per sq. inch. Ans.}$$

Area of stay × Stress = Area supported × Boiler pressure

$$\tfrac{11}{14}\, d^2 \times 9{,}000 = 8 \times 8 \times 165$$

$$= \sqrt{\frac{64 \times 165 \times 14}{9{,}000 \times 11}} = 1{\cdot}222 \text{ inches.}$$

$d = 1{\cdot}222$ inches. Ans.

The material must be mild steel, since 9,000 lb. per sq. inch is the stress allowed.

$$\text{Factor of safety} = \frac{29 \times 2{,}240}{9{,}000} = 7{\cdot}217. \quad \text{Ans.}$$

8. $3 \times 3 \times \tfrac{11}{14} \times 9{,}000 = 18 \times 18 \times$ Boiler pressure

$$\text{Boiler pressure} = \frac{99 \times 9{,}000}{14 \times 18 \times 18}$$

$= 196{\cdot}4$ lb. per sq. inch. Ans.

9. Load on bolts = Load on piston.

$$2 \times d^2 \times \tfrac{11}{14} \times 6{,}000 = 42 \times 42 \times \quad \times 65$$

$$d^2 = \frac{42 \times 42 \times 11 \times 65 \times 14}{14 \times 11 \times 6{,}000 \times 2}$$

$$d = \frac{\overline{42 \times 65}}{\times 2}$$

$d = 3{\cdot}09$ inches if two bolts. Ans.

$d = \sqrt{4{\cdot}775} = 2{\cdot}185$ inches if four bolts. Ans.

10. Area of stay = Area of cotter in double shear.

$3 \times 3 \times \tfrac{11}{14} = \tfrac{15}{8} \times \tfrac{7}{8} \times$ Depth of cotter.

Depth of cotter $= \tfrac{99}{14} \times \tfrac{64}{105} = 4{\cdot}31$ inches. Ans.

11. Let x = increase in diameter of hole in web after shrinkage.

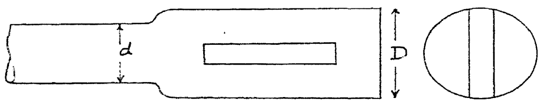
$$\text{Strain} = \frac{x}{12}. \quad \text{But strain} = \frac{\text{Stress}}{\text{E}}$$

$$\text{Therefore } \frac{x}{12} = \frac{12 \times 2{,}240}{30{,}000{,}000}$$

$$\frac{12 \times 12 \times 2{,}240}{30{,}000{,}000} = 0{\cdot}010752 \text{ inch.}$$

Original diam. of hole $= 12 - 0{\cdot}010752 = 11{\cdot}9892$ inches. Ans.

12. Let D = diam. of swelled part.



Then Area of large part — Area of cotter = Area of small part.

$$D^2 \times \tfrac{11}{14} - D \times \tfrac{5}{8} = \tfrac{5}{2} \times \tfrac{5}{2} \times \quad = 4{\cdot}91$$

$$D^2 - D \times \tfrac{5}{8} \times \tfrac{14}{11} = 4{\cdot}91 \times \tfrac{14}{11}$$

$$D^2 - 0{\cdot}795\ D = 6{\cdot}25$$

$$D - 0{\cdot}3975 = \pm \sqrt{6{\cdot}25 + 0{\cdot}158}$$

$$D = \pm\ 2{\cdot}531 + 0{\cdot}3975$$

$D = 2{\cdot}929$ inches. Ans.

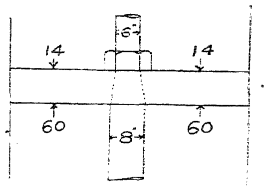
13.
$$170 = \frac{100\,(9 + 1)^2}{S - 6} \qquad \frac{10{,}000}{S - 6}$$

$$S - 6 = \tfrac{10000}{170} = 58{\cdot}83$$

$$S = 58{\cdot}83 + 6 = 64{\cdot}83$$

Pitch $= \sqrt{64{\cdot}83} = 8{\cdot}052$ inches. Ans.

14. The small part of the rod can be in tension only on the up stroke as shown.



Total load acting up
$= 60 \times \frac{11}{14} [52^2 - 8^2]$

Total load acting down
$= 14 \times \frac{11}{14} [52^2 - 6^2]$

Effective load acting up
$= 60 \times \frac{11}{14} [52^2 - 8^2] - 14 \times \frac{11}{14} [52^2 - 6^2]$
$= \frac{11}{14} [60 (52^2 - 8^2) - 14 (52^2 - 6^2)] = \frac{11}{14} \times 121{,}048$

$$\text{Stress} = \frac{\text{Effective load}}{\text{Area}}$$

$$\text{Stress} = \frac{11}{14} \times 121{,}048 \times \frac{14}{11 \times 36} \quad \frac{121{,}048}{36}$$

Stress 3,362 lb. per sq. inch. Ans.

15. Diam. at bottom of thread $= 3 \cdot 5 - (2 \times 0 \cdot 06) = 3 \cdot 38$ ins.
Area of tube $= \frac{11}{14} (3 \cdot 38^2 - 2 \cdot 75^2) = 3 \cdot 034$ sq. inches.
$3 \cdot 034 \times \text{Stress} = 97 \times 165$

$$\text{Stress} = \frac{97 \times 165}{3 \cdot 034} = 5{,}276 \text{ lb. per sq. inch. Ans.}$$

16.
$$\text{Strain} = \frac{10{,}000}{30{,}000{,}000}$$

$$\text{Elongation} = \frac{10{,}000}{30{,}000{,}000} \times 12 \times 12 \text{ inches.}$$

Load = Stress × Area $= 10{,}000 \times \frac{3}{2} \times \frac{3}{2} \times \frac{11}{14}$ lb.

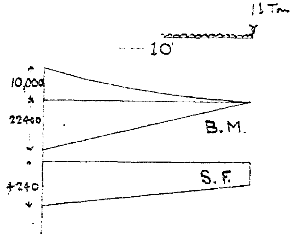
$$\text{Work} = \frac{\text{Load} \times \text{Extension}}{2}$$

$$10{,}000 \times \frac{3}{2} \times \frac{3}{2} \times \frac{11}{14} \times \frac{10{,}000}{30{,}000{,}000} \times 12 \times 12$$

$= 424\frac{2}{7}$ inch lb. Ans.

SOLUTIONS TO TEST EXAMPLES XVII.

1. $M_{\text{at support}} = 1 \times 2{,}240 \times 10 + 200 \times 10 \times 5$
$= 32{,}400$ ft. lb.

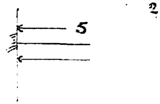


Stress $= \quad 6\,M$

$$\frac{6 \times 32{,}400 \times 12}{3 \times 10 \times 10} = 7{,}776$$

Stress $= 7{,}776$ lb. per sq. inch. Ans.

$M_{\text{at support}} = 8 \times 1 + 5 \times 4 = 18$ foot tons.
$= 18 \times 2{,}240 \times 12$ inch lb.



$$\text{Stress} = \frac{6\ M}{b\ d^2};\ d^2 = \frac{6\ M}{b \times \text{stress}}$$

$$d^2 = \frac{6 \times 18 \times 2{,}240 \times 12}{3 \times 6{,}000} = 161{\cdot}3$$

$d = \sqrt{161{\cdot}3} = 12{\cdot}7$ inches. Ans.

3. Total load $= 4 \times 112 \times 8 = 3{,}584$ lb.

$_{\text{at support}} = 3{,}584 \times 4$ or $\quad W\ l$

$= 3{,}584 \times 4 \times 12 = 172{,}032$ inch lb.

Let b = breadth of beam, then $4\,b$ = depth.

$$\text{Stress} = \frac{6\ M}{b\ d^2},\quad 3 \times 2{,}240 = \frac{6 \times 172{,}032}{b \times (4\ b)^2}$$

$$16\ b^3 = \frac{6 \times 172{,}032}{3 \times 2{,}240},\ b^3 = 9{\cdot}6$$

$b = \sqrt[3]{9{\cdot}6} = 2{\cdot}125$ inches. Ans.

$d = 2{\cdot}125 \times 4 = 8{\cdot}5$ inches. Ans.

4. $M = \frac{3}{2} \times 5 \times 12$ inch tons.
$= 90$ inch tons.

$$M = \frac{\pi D^3}{32} \times \text{stress, for round section.}$$

$$\text{or } M = \frac{D^3}{10{\cdot}2} \times \text{stress}$$

$$\text{Stress} = \frac{10{\cdot}2 \times 90}{6 \times 6 \times 6} = 4{\cdot}25 \text{ tons per sq. inch.}$$

5. $$M_{\text{centre}} = 2 \times 8, \text{ or } \frac{W\,l}{4} = \frac{4 \times 16}{4} = 16 \text{ foot tons.}$$

$M = 16 \times 12 = 192$ inch tons. Ans.

Take moments round R_1 then

$R_2 \times 16 = 4 \times 4,\ R_2 = 1$ ton

$R_1 = 4 - 1 = 3$ tons.

M is maximum at load.

$M_{\text{load}} = 3 \times 4 = 12$ foot tons or 144 inch tons. Ans.

$$p = \frac{6\ M}{b\ d^2},\ \text{Stress} = \frac{6 \times 192}{3 \times 8 \times 8} = 6 \text{ tons per sq. inch}$$

Ans.

$$\text{Stress} = \frac{6 \times 144}{3 \times 8 \times 8} = 4{\cdot}5 \text{ tons per sq. inch.} \quad \text{Ans.}$$

6 tons per sq. inch when the load is at the centre. Ans.

4·5 tons per sq. inch when the load is 4 feet from centre. Ans.

6. $\text{Strength} \propto \dfrac{b\ d^2}{\text{L}}, \quad \therefore \dfrac{\text{Strength} \times \text{L}}{b\ d^2}$ is constant,

this is the same as $\text{Load}_1 \times \qquad \text{Load}_2 \times \text{L}_2$

$$\text{Load}_2 = \frac{\text{Load}_1 \times \text{L}_1 \times b_2 \times d_2^2}{b_1\ d_1^2 \times \text{L}_2}$$

$$= \frac{3 \times 10 \times 3 \times 8 \times 8}{4 \times 12 \times 6 \times 6} = \tfrac{10}{3} = 3\tfrac{1}{3} \text{ tons.} \quad \text{Ans.}$$

Total load on beam $= 6 \times \frac{1}{4} = 1\frac{1}{2}$ tons.

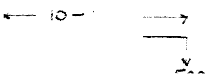
$\text{M}_{\text{at support}} = \frac{3}{2} \times \frac{6}{2} = 4{\cdot}5$ ft. tons.

Area of flange × stress × depth of girder = B.M.

$\frac{11}{2} \times \frac{1}{2} \times 6{,}000 \times x = \frac{9}{2} \times 2{,}240 \times 12$

$$x = \frac{9 \times 2{,}240 \times 12 \times 4}{11 \times 6{,}000 \times 2} = 7{\cdot}33 \text{ inches.} \quad \text{Ans.}$$

8. Take moments round the support.



For equilibrium,

$680 \times x = 500\ (10 -$

$680\ x = 5{,}000 - 500\ x$

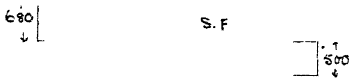
$1{,}180\ x = 5{,}000$

Maximum B.M. occurs at support and is $680 \times 4{\cdot}237$ foot lb.

$\text{B.M.}_{\text{max.}} = 680 \times 4{\cdot}237 \times 12$ inch lb.

$$\text{Stress} = \frac{6\ M}{b\ d^2}$$

$$5{,}000 = \frac{6 \times 680 \times 4{\cdot}237 \times 12}{b \times (3\ b)^2}$$

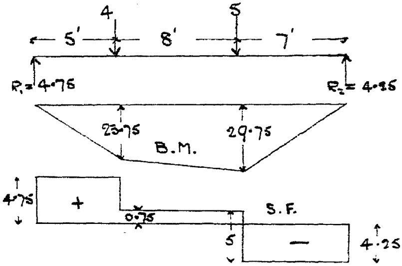


$$\frac{6 \times 680 \times 4{\cdot}237 \times 12}{9 \times 5{,}000}$$

$b = 1{\cdot}664$ inches. Ans.

$d = 4{\cdot}992$ inches. Ans.

9. Take moments round R_1 then :—



$R_2 \times 20 = 4 \times 5 + 5 \times 13$

$R_2 = 4{\cdot}25$ tons, $R_1 = 9 - 4{\cdot}25 = 4{\cdot}75$ tons.

M at 4 tons load $= 4{\cdot}75 \times 5 = 23{\cdot}75$ ft. tons.

M at 5 tons load $= 4{\cdot}25 \times 7 = 29{\cdot}75$ ft. tons.

The maximum B.M. occurs at the 5 tons load and is $29{\cdot}75$ ft. tons. Ans.

$$= 4{\cdot}75 \times 10 - 4 \times 5 = 27{\cdot}5 \text{ ft. tons. Ans.}$$

$$\text{Stress} = \frac{M}{b\ d^2}, \text{ Stress (maximum)} = \frac{6 \times 29{\cdot}75 \times 12}{6 \times 12 \times 12}$$

Maximum stress $= 2{\cdot}48$ tons per sq. inch. Ans.

$$\text{Stress at centre} = \frac{6 \times 27{\cdot}5 \times 12}{6 \times 12 \times 12} = 2{\cdot}3 \text{ tons per sq.in. (nearly). Ans.}$$

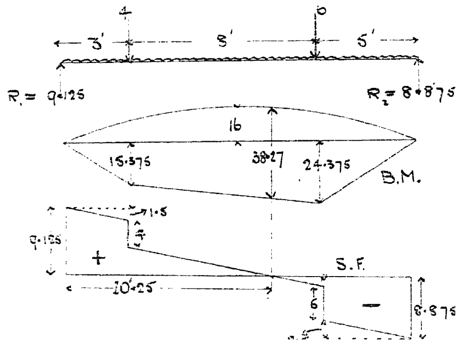
10. Take moments round R_1 then :—

$R_2 \times 16 = 4 \times 3 + 6 \times 11 + 8 \times 8$

$R_2 = 8{\cdot}875$ tons. $R_1 = 9{\cdot}125$ tons.

The maximum ordinate for the uniform load acting alone

is $\dfrac{8 \times 16}{} = 16$ ft. tons.



For the 4 and 6 ton loads acting alone, $R_1 = 5{\cdot}125$ and $R_2 = 4{\cdot}875$.

For the ordinates of B.M. for these concentrated loads alone, $M_{3\text{ feet}} = 5{\cdot}125 \times 3 = 15{\cdot}375$ ft. tons, $M_{5'} = 4{\cdot}875 \times 5 = 24{\cdot}375$ ft. tons.

The B.M. diagram may now be sketched as shown.

The figures on the S.F. diagram explain themselves.

Now in this question, it is seen from the S.F. diagram, that where the S.F. changes sign, the S.F. = zero. Taking x from R_1 into the mid span we have :—

$$\text{S.F.}_x = 0 = 9{\cdot}125 - 4 - \frac{x}{2}, \qquad x = 10{\cdot}25 \text{ feet from } R_1$$

and this is where the maximum M occurs.

$$M \text{ at } 10{\cdot}25 \text{ ft.} = 9{\cdot}125 \times 10{\cdot}25 - 4\,(10{\cdot}25 - 3) - \frac{10{\cdot}25}{2} \times \frac{10{\cdot}25}{2}$$

$$= 93{\cdot}53 - 29 - 26{\cdot}26 = 38{\cdot}27 \text{ ft. tons.} \quad \textbf{Ans.}$$

$M_{\text{centre}} = 9{\cdot}125 \times 8 - 4 \times 5 - 4 \times 4 = 37$ ft. tons. Ans.

$S.F._{\text{centre}} = 9{\cdot}125 - 4 - 4 = 1{\cdot}125$ tons. Ans.

Maximum B.M. $= 38{\cdot}27$ ft. tons. Ans.

B.M. at centre $= 37$ ft. tons. Ans.

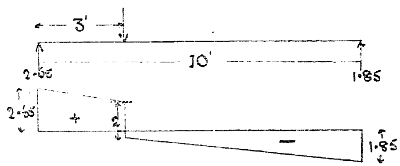
S.F. at centre $= 1{\cdot}125$ tons. Ans.

Take moments round R_1.

Then $10 \times R_2 = 2 \times 3 + 10 \times 0{\cdot}25 \times \frac{10}{2}$

$R_2 = 1{\cdot}85$ tons. $R_1 = 4{\cdot}5 - 1{\cdot}85 = 2{\cdot}65$ tons.

Shear changes sign under the concentrated load, and the Max. B.M. occurs there.

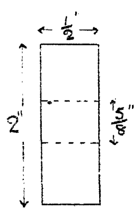


$M_{\max} = 2{\cdot}65 \times 3 - 3 \times \frac{1}{4} \times \frac{3}{2} = 6{\cdot}825$ ft. tons. Ans.

$$\frac{D^3}{10{\cdot}2} \times 3{\cdot}5 = 6{\cdot}825 \times 12$$

$$D^3 = \frac{6{\cdot}825 \times 12 \times 10{\cdot}2}{3{\cdot}5}, \quad D = 6{\cdot}203 \text{ inches. Ans.}$$

12. B.M. at pin $= 64 \times 20 = 1{,}280$ inch lb.



I of lever at pin

$$\frac{B\,(D^3 - d^3)}{12} = \frac{0{\cdot}5\,(2^3 - 0{\cdot}625^3)}{12}$$

I of lever at pin $= 0{\cdot}3231$ inch^4 units.

$$\text{Stress} = \frac{M\,y}{I} = \frac{1{,}280 \times 1}{0{\cdot}3231} = 3{,}961 \text{ lb. per sq. inch.}$$

13. Area of section $= 2 \times 9 \times \frac{1}{2} + 11 \times \frac{1}{2} = 14{\cdot}5$ sq. inches.

$$\text{Wt. of beam} = \frac{14{\cdot}5}{144} \times 12 \times 490 = 592 \text{ lb.}$$

B.M. due to load at end $= 2{\cdot}5 \times 12 \times 12 = 360$ inch tons or $360 \times 2{,}240 = 806{,}400$ inch lb.

$$\text{B.M. due to weight of beam} = 592 \times \frac{12 \times 12}{2} = 42{,}624 \text{ inch lb.}$$

Total B.M. $= 849{,}024$ inch lb.

$$\text{Stress} = \frac{M\,y}{I}, \text{ and } I = \frac{9 \times 12^3 - 8{\cdot}5 \times 11^3}{12} = 353{\cdot}2 \text{ inch}^4$$

$$\text{Stress} = \frac{806{,}400 \times 6}{353{\cdot}2} = 13{,}700 \text{ lb. sq. inch neglecting wt. of joist. Ans.}$$

$$\text{Stress} = \frac{849{,}024 \times 6}{353{\cdot}2} = 14{,}420 \text{ lb. sq. inch with weight of joist. Ans.}$$

14. Take moments round R_1, then :—

$R_2 \times 16 = 3 \times 4 + 4 \times 11, \quad R_2 = 3{\cdot}5$ tons.

$R_1 = 7 - 3{\cdot}5 = 3{\cdot}5$ tons.

$M_{max} = 5 \times 3{\cdot}5 \times 12 = 210$ inch tons.

Area of flange $\times$ depth $\times$ stress $= 210$

$$\text{Depth} = \frac{210}{8 \times 0{\cdot}5 \times 3{\cdot}5} = 15 \text{ inches. Ans.}$$

15.

$$\text{Strength} \propto \frac{I}{y}$$

$$I \text{ as an I section} = \frac{8 \times 10^3 - 7\tfrac{1}{2} \times 9^3}{12} = 211 \text{ inch}^4.$$

as an I section $= 5$ inches, $\frac{\bar{\ }}{y} = \frac{211}{5} = 42.2$

I as an H section $= \dfrac{0.5 \times 8^3 \times 2 + 9 \times 0.5^3}{12}$

$= 42.76$ inch^4.

y as an H section $= 4$ inches, $\dfrac{I}{y} = \dfrac{42.76}{4} = 10.69$

Strengths as $42.2 : 10.69$ or as $1 : 0.2533$. Ans.

or as $3.95 : 1$.

16. $D^2 - 1^2 = 2^2$, $D = \sqrt{5} = 2.236$ inches.

Strengths vary as $D^3 : \dfrac{D^4 - d^4}{D}$

or as $2^3 : \quad 2.236^4 - 1^4$

or as $8 : 10.73$

or $1 : 1.34$

or hollow bar is 34 per cent. stronger.

17. Stress $= \quad 6\ M$

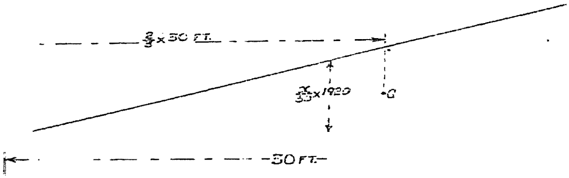
$$4 = \frac{6 \times 38.27 \times 12}{b \times (3b)^2}$$

$$b^3 = \frac{6 \times 38.27 \times 12}{4 \times 9}$$

$b = 4.245$ inches. Ans.

$d = 12.735$ inches. Ans.

18.



$$\text{Total load} = \frac{1{,}920 \times 30}{} = 28{,}800 \text{ lb.} \quad \text{Ans. } (a).$$

Taking moments about R_1 :—

$R_2 \times 30 = 28{,}000 \times \frac{2}{3} \times 30$

$\therefore R_2 = \frac{2}{3} \times 28{,}800 = 19{,}200$ lb. Ans. (*b*).

$\therefore R_1 = 28{,}800 - 19{,}200 = 9{,}600$ lb. Ans. (*b*).

$$\text{Shearing force at Z} = R_1 - \frac{x \times 1{,}920}{30} \times \frac{x}{2} = 0$$

$$\therefore 9{,}600 - \frac{x^2 \times 1{,}920}{60} = 0$$

$$\therefore 9{,}600 = 32\,x^2 \qquad \therefore x = \sqrt{\frac{9{,}600}{32}}$$

$\therefore x = 17{\cdot}32$ feet from R_1. Ans. (*c*).

$$\text{M at Z} = 9600 \times 17{\cdot}32 - \frac{17{\cdot}32 \times 1{,}920}{30} \times \frac{17{\cdot}32}{2} \times \frac{17{\cdot}32}{3}$$

$$= 166{,}300 - 55{,}400$$

$$= 110{,}900 \text{ foot lb.} \quad \text{Ans. } (d)$$

SOLUTIONS TO TEST EXAMPLES XVIII.

Force to break 1 inch bar at 20 inches leverage $= 800 \times \frac{12}{20}$ lb.

Force to break $\frac{3}{4}$ inch bar at 20 inches leverage

$$= 800 \times \tfrac{12}{20} \times (\tfrac{3}{4})^3 \times \frac{1}{1^3}$$

$= 480 \times \frac{27}{64} = 202\cdot5$ lb. Ans.

$$\text{Stress} = \frac{5\cdot1 \times T}{D^3} \qquad \frac{5\cdot1 \times 202\cdot5 \times 20 \times 64}{27}$$

Stress $= 48{,}960$ lb. per square inch. Ans.

Strength varies as D^3

Strengths as $(13\frac{1}{2})^3$:

or as 2,460 : 3049 or as 1 : 1·24

The $14\frac{1}{2}$ inch shaft is 24 per cent. stronger. Ans.

Weight varies as D^2 for the same length.

Weights as $(13\frac{1}{2})^2 : (14\frac{1}{2})^2$

or as 182·3 : 210·3 or as 1 : 1·154

The $14\frac{1}{2}$ inch shaft is 15·4 per cent. heavier. Ans.

T.M. $= \quad \times \quad \times \frac{11}{14} \times 75 \times 28$ inch lb.

$$\text{Stress} = \frac{5\cdot1 \times 46 \times 46 \times 11}{14 \times 14 \times 14 \times 14} \times 75 \times 28$$

$= 6{,}489$ lb. per sq. inch. Ans.

The stress allowed on crank shafts is 7,700 lb. per sq inch.

4.

Strengths as $\dfrac{}{D} : D^3$

or as $\dfrac{14^4 - 6^4}{14} : 14^3$ or as 2,651 : 2,744

or as 1 : 1·035

The solid shaft is 3·5 per cent. stronger than the hollow one. Ans.

5. Moment of bolts = moment of shaft.

$$d^2 \times \frac{\pi}{4} \times \text{stress} \times 10 \times 8 = \frac{\pi}{16} \times 14^3 \times \text{stress}$$

$$d^2 = \frac{14^3 \times 4}{16 \times 10 \times 8}, \ \pi \text{ cancels and stress is the same}$$

$d^2 = 8{\cdot}574$, $d = 2{\cdot}928$ inches. Ans.

6.
$$\frac{\text{H.P.}}{\text{Stress} \times \text{Revs.} \times d^3} = \text{constant.}$$

$$\frac{95}{70 \times 6^3} = \frac{\text{H.P.}_2}{90 \times \,}, \text{ the stress being the same.}$$

$$\frac{95 \times 90 \times 81 \times 9}{70 \times 36 \times 6} = 412{\cdot}2 \text{ H.P.} \quad \text{Ans.}$$

The stresses allowed are in the ratio of 100 : 70

$$\frac{\text{H.P.}}{\text{Stress} \times \text{Revs.} \times d^3} = \text{constant.}$$

$$\frac{270}{100 \times 2{,}500 \times (1{\cdot}75)^3} = \frac{\text{H.P.}_2}{70 \times 125 \times 9^3}$$

$$\text{H.P.}_2 = \frac{270 \times 70 \times 125 \times 81 \times 9}{100 \times 2{,}500 \times 1{\cdot}75 \times 1{\cdot}75 \times 1{\cdot}75}$$

$= 1{,}286$. Ans.

8.
$$\text{T.M.} = \frac{63{,}000 \times \text{H.P.}}{\,} \times \text{ratio of maximum to mean twist.}$$

$$\frac{63{,}000 \times 3{,}000}{80} \times 1{\cdot}3 \text{ inch lb.}$$

$$\frac{D^3}{5 \cdot 1} \times 7{,}000 = \frac{63{,}000 \times 3{,}000 \times 1 \cdot 3}{80}$$

$$D^3 = \frac{9 \times 3{,}000 \times 1 \cdot 3 \times 5 \cdot 1}{80} = 2{,}238.$$

$$D = \sqrt[3]{2{,}238} = 13 \cdot 08 \text{ inches diameter.}$$

$$(14 \cdot 5^2 - d^2) \times \tfrac{11}{14} \times \tfrac{1}{144} \times 490 = 410$$

$$14 \cdot 5^2 - d^2 = \frac{410 \times 14 \times 144}{11 \times 490} = 153 \cdot 3$$

$$d^2 = 210 \cdot 25 - 153 \cdot 3 = 56 \cdot 95$$

$$d = 7 \cdot 546 \text{ inches.}$$

The twisting moment is the same for both shafts.

$$\frac{(14 \cdot 5^4 - 7 \cdot 546^4)}{14 \cdot 5 \times 5 \cdot 1} \times 7{,}000 = \frac{14^3 \times \text{Stress}}{5 \cdot 1}$$

$$\text{Stress} = \frac{(14 \cdot 5^4 - 7 \cdot 546^4) \times 7{,}000}{14 \cdot 5 \times 14^3} = 7{,}211 \text{ lb. per sq. inch.} \quad \text{Ans.}$$

10. $$\text{T.M.} = \frac{D^3}{5 \cdot 1} \times \text{stress} = \frac{1{,}000}{5 \cdot 1} \times 8{,}000$$

$$\frac{63{,}000 \times \text{H.P.}}{100} = \frac{1{,}000}{5 \cdot 1} \times 8{,}000$$

$$\text{H.P.} = \frac{8{,}000 \times 100}{5 \cdot 1 \times 63} = 2{,}490. \quad \text{Ans.}$$

$$\frac{q}{r} = \frac{C\,i}{l}, \text{ or } i = \frac{q\,l}{C\,r} \text{ radians.}$$

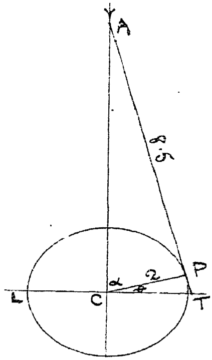
$$i = \frac{8{,}000 \times 15 \times 12}{12{,}000{,}000 \times 5} = 0 \cdot 024 \text{ radian.}$$

or $0 \cdot 024 \times 57 \cdot 3 = 1 \cdot 375^\circ$. Ans.

11. Draw L T at right angles to line of stroke, produce A P cutting C T at T.

Then torque = load on piston × C T

$$\text{Tan.}\ \alpha = \frac{8{\cdot}5}{\ } = 4{\cdot}25.$$



From the tables $\alpha = 76^\circ\ 46'$

$\theta = 90^\circ - 76^\circ\ 46' = 13^\circ\ 14'$

$$\text{Cos.}\ \theta = \frac{C\ P}{C\ T}$$

$$\therefore C\ T = \frac{C\ P}{\text{Cos.}\ 13^\circ\ 14'}$$

and Cos. $13^\circ\ 14' = 0{\cdot}9735$

$$C\ T = \frac{2}{0{\cdot}9735}$$

$= 2{\cdot}054$ feet.

T.M. $= 28 \times 28 \times \frac{11}{14} \times 180 \times 2{\cdot}054 \times 12$ inch lb.

$= 2{,}733{,}000$ inch lb. Ans.

T.M. for all engines $= 2{,}733{,}000 \times 2 = 5{,}466{,}000$ inch lb.

$$\frac{D^3}{5{\cdot}1} \times 7{,}500 = 5{,}466{,}000$$

$$\frac{00}{7{,}500} = 15{\cdot}5 \text{ inches.}$$

12. $$S^3 = \frac{C\ P\ D^2}{\ } \qquad \frac{26 \times 195 \times 73^2}{\ } + \frac{73 \times 73}{27 \times 27}$$

$$S = \sqrt[3]{\frac{26 \times 195 \times 73 \times 73}{1{,}110 \times 9{\cdot}31}} = 13{\cdot}78 \text{ inches.}$$

13. $S^3 = \quad C\ P\ D^2$

$$\left(2 + \frac{D^2}{d^2}\right) = \frac{C\ P\ D^2}{1,.}$$

$D^2 \qquad C\ P\ D^2$

$d^2 \qquad 1{,}110\ S^3$

$75 \times 75 \qquad 25 \times 195 \times 75 \times 75 \quad - 2$

$d^2 \qquad 1{,}110 \times 14 \times 14 \times 14$

$$\frac{5{,}626}{d^2} = 9{\cdot}005 - 2 = 7{\cdot}005$$

$5{,}626$

$7{\cdot}005$

$d = 28{\cdot}34$ inches.

$$\text{Number of Expansions} = \frac{\text{Final volume}}{\text{Initial volume}}$$

$(L.P.)^2 \qquad 75 \times 75$

$(H.P.)^2 \times$ cut off in H.P. $\qquad 803{\cdot}2 \times 0{\cdot}6$

No. of Expansions $= 11{\cdot}68$. Ans.

14. $$\frac{H.P.}{D^3 \times \text{revs.}} = \text{constant.}$$

$$\frac{100}{(2{\cdot}75)^3 \times 300} = \frac{1{,}000}{D^3 \times 100}$$

$D = 2{\cdot}75\ \sqrt[3]{30}$

$D = 2{\cdot}75 \times 3{\cdot}107 = 8{\cdot}54$ inches. Ans.

15. $$\frac{T}{J} = \frac{q}{r} \quad \therefore \frac{W \times R}{J} = \frac{q}{r} \quad \therefore W = \frac{J \times q}{R \times r}$$

$$W = \frac{\pi \times d^4 \times q \times 2}{32 \times R \times d} = \frac{\pi\ d^3\ q}{16\ R}$$

$$\frac{22 \times (\frac{5}{8})^3 \times 25{,}000}{7 \times 16 \times 1{\cdot}75} = 684{\cdot}8 \text{ lb.} \quad \text{Ans.}$$

$$\delta \text{ (deflection)} = \frac{64 \text{ W R}^3 \text{ N}}{} = \frac{64 \times 684{\cdot}8 \times 1{\cdot}75^3 \times 10}{12 \times 10^6 \times 0{\cdot}625^4}$$

$= 1{\cdot}28$ inches. Ans.

SOLUTIONS TO TEST EXAMPLES XIX.

1.

$$\text{Plate Strength} = \frac{2\frac{1}{8} - \frac{7}{8}}{2\frac{1}{8}} \times 100 = 58{\cdot}8 \text{ per cent.}$$

Rivet Strength $= \frac{7}{8} \times \frac{7}{8} \times \frac{11}{14} \times \frac{23}{28} \times \frac{8}{17} \times \frac{8}{5} \times 100$
$= 37{\cdot}2$ per cent.

Strength of the Joint is $37{\cdot}2$ per cent. Ans.

2. Strength of Plate = Strength of Rivet.

$$\frac{p -}{p} \quad \times \frac{11}{14} \times \frac{23}{28} \times \frac{4 \times 2}{p \times 3}$$

p cancels from the denominators.
$p - \frac{7}{8} = 1{\cdot}3176.$
$p = 1{\cdot}3176 + 0{\cdot}875 = 2{\cdot}1926$ inches. Ans.

$$\text{Efficiency of Joint} = \frac{1{\cdot}3176}{2{\cdot}1926} \times 100 = 60{\cdot}1 \text{ per cent.}$$

$$\text{Plate Strength} = \frac{5 -}{} \times 100 = 81{\cdot}25 \text{ per cent.}$$

Rivet Strength

$$= \frac{15}{16} \times \frac{15}{16} \times \quad \frac{23}{28} \times \quad \times 3 \times \frac{}{5 \times 3} \times 100$$

$= 85{\cdot}07$ per cent.
Strength of Joint is $81{\cdot}25$ per cent. Ans.
Apply the formula,

$$\text{W.P.} = \frac{2\,t\,s}{\text{D} \times \text{F}} \times \frac{\text{per cent. strength of joint}}{100} \quad \text{Ans.}$$

Plate Strength $= \dfrac{\frac{1}{4}}{8\frac{1}{2}} \times 100 = 85{\cdot}3$ per cent. nearly.

Rivet Strength $= \frac{5}{4} \times \frac{4}{4} \times \frac{11}{14} \times \frac{23}{28} \times 5 \times \frac{15}{8} \times \frac{2}{17} \times \frac{4}{5} \times 100$
$= 88{\cdot}98$ per cent.

Combined strength of plate in middle row and rivet in

outer row $= \dfrac{}{p} \times 100 \quad 88{\cdot}98$

$$= \frac{8{\cdot}5 - 2{\cdot}5}{8{\cdot}5} \times 100 + 17{\cdot}79 = 88{\cdot}37 \text{ per cent.}$$

Strength of Joint $= 85{\cdot}3$ per cent. of solid plate.

$$\text{W.P.} = \frac{2\ t\ s}{\text{D} \times \text{F}} \times \frac{\text{per cent. strength of joint}}{100}$$

$$\text{W.P.} = \frac{2 \times 1\frac{1}{4} \times 28 \times 2{,}240}{13 \times 12 \times 4{\cdot}5} \quad \frac{85{\cdot}3}{100}$$

$= 190{\cdot}5$ lb. square inch. Ans.

$$\text{W.P.} = \frac{2\ t\ s}{\text{D} \times \text{F}} \text{ or } \text{F} = \frac{2\ t\ s}{\text{D} \times \text{P}}$$

$$\text{F} = 2 \times \tfrac{5}{16} \times \frac{22 \times 2{,}240}{8 \times 210} = 81{\cdot}\dot{3}. \quad \text{Ans.}$$

$$195 = \frac{2 \times 1\frac{1}{2} \times 28 \times 2{,}240 \times 85}{15 \times 12 \times \text{F} \times 100}$$

$$\text{F} = \frac{3 \times 28 \times 2{,}240 \times 85}{15 \times 12 \times 195 \times 100} = 4{\cdot}556. \quad \text{Ans.}$$

Equating the strength of plate at outer row to the total rivet strength :

p

$\times \frac{11}{14} \times 4 \times \frac{23}{28} \times$

$\times$

$$p - \tfrac{13}{16} = \tfrac{13}{16} \times \tfrac{13}{16} \times \tfrac{11}{14} \times 4 \times \tfrac{23}{28} \times \tfrac{16}{11}$$

$$p = 2{\cdot}48 + 0{\cdot}8125 = 3{\cdot}2925 \text{ inches.}$$

$$\text{Strength at outer row} = \frac{3{\cdot}2925 - 0{\cdot}8125}{3{\cdot}2925} \times 100$$

$= 75{\cdot}3$ per cent.

Strength of rivets $= 75{\cdot}3$ per cent.

Strength at middle row

$$= \left(\frac{3{\cdot}2925 - 1{\cdot}625}{3{\cdot}2925}\right) \times \qquad \frac{75{\cdot}3}{4} \qquad \text{per}$$

The joint is weakest at the middle row, and a better solution is given by equating the strength through the middle row to the total rivet strength as follows :—

$$\left(\frac{p - 2\,d}{p}\right) 100 + \text{per cent. strength 1 rivet}$$

$=$ per cent. strength of 4 rivets.

or $\left(\dfrac{p - 2\,d}{p}\right) 100 =$ per cent. strength of 3 rivets.

$$p - 1{\cdot}625 = \tfrac{13}{16} \times \tfrac{13}{16} \times \tfrac{11}{14} \times 3 \times \tfrac{23}{28} \times \tfrac{16}{11} = 1{\cdot}859$$

$$p = 3{\cdot}484 \text{ inches.}$$

Strength of rivets

$$= \tfrac{13}{16} \times \tfrac{13}{16} \times \tfrac{11}{14} \times 4 \times \qquad \times \qquad \times \frac{100}{3{\cdot}484}$$

$= 71{\cdot}12$ per cent.

Strength at outer row

$$= \left(\frac{3{\cdot}484 - 0{\cdot}8125}{3{\cdot}48}\right) 100 = \qquad \text{cent.}$$

Strength at middle row

$$= \left(\frac{3{\cdot}484 - 1{\cdot}625}{3{\cdot}484}\right) 100 \qquad \frac{71{\cdot}12}{4} = 71{\cdot}12 \text{ per cent.}$$

The best pitch is 3·484 inches, and the per cent. strength is 71·12. Ans.

8. Strength of Plate $= \dfrac{2\frac{3}{4} - \frac{7}{8}}{2\frac{3}{4}} \times 100 = 68 \cdot 18$ per cent.

Strength of rivet in thick plate
$= \frac{7}{8} \times \frac{7}{8} \times \frac{11}{14} \times \frac{23}{28} \times 2 \times \frac{4}{11} \times \frac{8}{5} \times 100$
$= 57 \cdot 5$ per cent.

Strength of Joint $= 57 \cdot 5$ per cent.

Strength of rivet in thin plate $=$
$\frac{7}{8} \times \frac{7}{8} \times \frac{11}{14} \times \frac{23}{28} \times 2 \times \frac{4}{11} \times \frac{2}{1} \times 100$
$= 71 \cdot 87$ per cent.

Strength of Joint $= 68 \cdot 18$, the plate per cent. being the same for both.

$$\text{W.P. for } \tfrac{5}{8} \text{ inch plate} = \frac{2 \times \frac{5}{8} \times 28 \times 2{,}240}{5 \times 7 \times 12} \times \frac{57 \cdot 5}{100}$$

$= 107 \cdot 4$ lb. per square inch.

$$\text{W.P. for } \tfrac{1}{2} \text{ inch plate} = \frac{2 \times \frac{1}{2} \times 28 \times 2{,}240}{5 \times 7 \times 12} \times \frac{68 \cdot 18}{100}$$

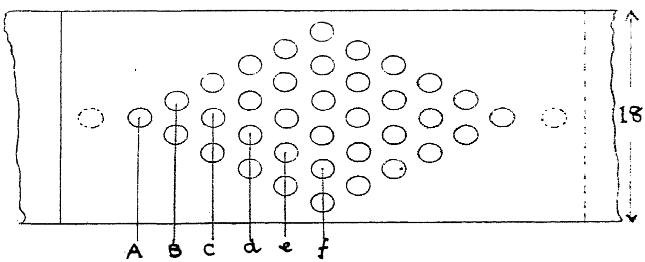
$= 101 \cdot 8$ lb. per square inch.

$\therefore$ W.P. of boiler $= 101 \cdot 8$ lb. per square inch. Ans.

$(18 - \frac{7}{8}) \times 1 \times 30 = \frac{7}{8} \times \quad \times \frac{11}{14} \times 24 \times n$

$n = 35 \cdot 5$ rivets, say 36.

Arrange as shown.



Strength at A $= \dfrac{(18}{18}$

Strength at B = $\qquad\qquad \times\ 100 + \dfrac{95{\cdot}13}{36}$

$= 92{\cdot}9$ per cent.

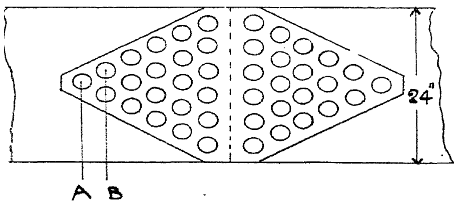
Strength at C = $\qquad 18 \qquad \times\ 100 + \qquad \times\ 95{\cdot}13$

$= 93\dot{3}{\cdot}33$ per cent.

The strength at *d* will be found to be greater than at B, and the strength increases and is greatest at row *f*. Strength of joint is 92·9 per cent. Note by putting two extra rivets in as shown dotted in sketch, the strength at B is increased by the shearing strength of another rivet, and this improves the joint.

10. $(24 - \frac{3}{4}) \times \frac{5}{8} \times 28 = \frac{3}{4} \times \frac{3}{4} \times \frac{11}{14} \times 24 \times \frac{15}{8} \times n.$

$n = 20{\cdot}4$, say 21 rivets.



There must be 21 rivets on *each* side of the butt.

Strength at A = $\dfrac{24 - \frac{3}{4}}{24} \times 100 = 96{\cdot}87$ per cent.

Strength at B = $\qquad\qquad \times\ 100 + \dfrac{96{\cdot}87}{21}$

$= 98{\cdot}36$ per cent.

The strength of the joint is 96·87 per cent. Ans.

Thickness of straps $= \frac{5}{8} \times \frac{5}{8} \times \left(\dfrac{24 - \frac{3}{4}}{24 - 6 \times \frac{3}{4}}\right)$

$= 0{\cdot}466$ inch. Ans.

11. The load on the spring varies directly as the compression. Load on spring $= \frac{1800}{1} \times \frac{7}{8} = 1{,}575$ lb.

$$\text{Boiler Pressure} = \frac{\text{Load on valve}}{\text{Area of valve}} = \frac{1{,}575 \times 14}{3{\cdot}5 \times 3{\cdot}5 \times 11}$$

$= 163{\cdot}6$ lb. per square inch. Ans.

12. Diameter of coil $= 4 - \frac{3}{4} = 3\frac{1}{4}$ inches centre to centre. Total weight on valve $= \frac{11000}{13} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} \times 4$ $= 1{,}428$ lb. nearly.

$$\text{Boiler Pressure} = \frac{1{,}428 \times 14}{3{\cdot}5 \times 3{\cdot}5 \times 11} = 148{\cdot}4 \text{ lb. per sq. inch. Ans.}$$

13. Load on valve $= 3\frac{1}{4} \times 3\frac{1}{4} \times \frac{11}{14} \times 180$ lb.

$$\frac{11{,}000}{D} \times \frac{3}{4} \times \frac{3}{4} \times \frac{3}{4} = \frac{13}{4} \times \frac{13}{4} \times \frac{11}{14} \times 180$$

$$D = \frac{11{,}000 \times 27 \times}{26 \times 13 \times 11 \times 180} = 3{\cdot}107 \text{ inches.}$$

Outside Diameter $= 3{\cdot}107 + 0{\cdot}75 = 3{\cdot}857$ inches. Ans

14. Area of Valves $= \left(\frac{75}{\text{gross press.}} \times \frac{1}{2}\right) \times$ grate area in sq. ft.

$= \frac{75}{215} \times \frac{1}{2} \times 6 \times 6 \times \frac{13}{4} = 20{\cdot}41$ square inches nearly.

If two valves are fitted each has $10{\cdot}205$ square inches.

$d^2 \times \frac{11}{14} = 10{\cdot}205, \quad d = \sqrt{\frac{10{\cdot}205 \times 14}{11}}$

$d = 3{\cdot}602$ inches. Ans.

If three valves are fitted each has $6{\cdot}8$ square inches.

$$d = \sqrt{\frac{6{\cdot}8 \times 14}{11}} = 2{\cdot}942 \text{ inches. Ans.}$$

15. As the allowance for area of valves found from the formula is based upon a consumption of 20 lb. per square foot of grate with natural draught, we must multiply the area previously found by $\frac{27}{20}$.

Total area of valves $= 20{\cdot}41 \times \frac{27}{20} = 27{\cdot}55$ square inches.

If two valves fitted, $d = \sqrt{\dfrac{13{\cdot}75 \times 14}{11}} = 4{\cdot}183$ inches. Ans.

If three valves fitted, $d = \sqrt{\dfrac{9{\cdot}183 \times 14}{11}} = 3{\cdot}42$ inches. Ans.

16. Load on valve $= 3{\cdot}5 \times 3{\cdot}5 \times 0{\cdot}7854 \times 170 = 1{,}635$ lb.

$\therefore$ Load on spring $= 1{,}635 - 35 = 1{,}600$ lb.

$$\text{Compression} = \frac{3^3 \times 1{,}600 \times 14}{12^4 \times 30} = 0{\cdot}972 \text{ inch.}$$

Pressure when blowing $= 170 \times 1{\cdot}1$ lb. square inch.
$\therefore$ Load on valve $= 1{,}635 \times 1{\cdot}1 = 1{,}798{\cdot}5$ lb.
and load on spring $= 1{,}798{\cdot}5 - 35 = 1{,}763{\cdot}5$ lb.
Compression $\propto$ load

$$\therefore \text{Compression} = 0{\cdot}972 \times \frac{1{,}763{\cdot}5}{1{,}600} = 1{\cdot}071 \text{ inches.}$$

Lift of valve $= 1{\cdot}071 - 0{\cdot}972 = 0{\cdot}099$ inch. Ans.

17.
$$\text{W.P.} = \frac{99{,}000 \times \frac{7}{16} \times}{(6{\cdot}25 + 1) \times 36} \qquad \frac{99000 \times 49}{7{\cdot}25 \times 36 \times 256}$$
$= 72{\cdot}61.$

W.P. $= 72{\cdot}61$ lb. per square inch.

Also $\frac{9900}{36} \times \frac{7}{16} = 120$ lb. per square inch.

The working pressure is $72{\cdot}61$ lb. per square inch. Ans.

The working pressure varies as the (thickness)^2,

$\therefore$ New pressure $= 72{\cdot}61 \times (\frac{3}{8})^2 \times (\frac{16}{7})^2 = 53{\cdot}33$ lb. sq. inch.

The pressure must be reduced by $72{\cdot}61 - 53{\cdot}33 = 19{\cdot}28$ lb. per sq. inch. Ans.

18. Working pressure = $\qquad \times \frac{9}{16}$ = 175·7 lb. per sq. inch. Ans.

$$175{\cdot}7 = \frac{9900\ T}{3 \times 48}\left[5 - \frac{36 + 12}{60\ T}\right]$$

$$175{\cdot}7 = \quad 16 \quad \left[5 - \frac{48}{60\ T}\right.$$

$$\frac{175{\cdot}7}{1100} \qquad\qquad \frac{4}{5\ T}$$

$2{\cdot}555 = 5\ T - \frac{4}{5}$

$5\ T = 3{\cdot}355$, $T = 0{\cdot}671$ inch. Ans.

19.
$$P = \frac{6{,}000\ (T - \frac{1}{16})}{D}$$

$$200 \qquad \frac{6{,}000\ (T - \frac{1}{16})}{8}$$

$16 = 60\ (T - \frac{1}{16})$

$T - \frac{1}{16} = \frac{16}{60} = 0{\cdot}2666.$

$T = 0{\cdot}2666 + 0{\cdot}0625 = 0{\cdot}3291$ inch. Ans.

SOLUTIONS TO TEST EXAMPLES XX.

1. $C = (-\ 20 - 32) \times \frac{5}{9} = -\ 28°{\cdot}\dot{8}$ C. Ans.
$C = (229 - 32) \times \frac{5}{9} = 109°{\cdot}4$ C. Ans.
$F = (-\ 20 \times \frac{9}{5}) + 32 = -\ 4°$ F. Ans.
$F = (112 \times \frac{9}{5}) + 32 = 233°{\cdot}6$ F. Ans.

2.
$$\frac{p\ v}{T} = 53{\cdot}2 \text{ for one lb. of air.}$$

$$v = \frac{53{\cdot}2 \times T}{p} = \text{vol. per lb.}$$

$$v = \frac{53{\cdot}2 \times (460 + 120)}{250 \times 144} = 0{\cdot}857 \text{ cu. foot per lb.}$$

Vol. of 6 lb. $= 0{\cdot}857 \times 6 = 5{\cdot}142$ cu. feet. Ans.

3. If the volume remains constant, then the pressure varies as the absolute temperature.

$$\text{Final pressure} = 35 \times \frac{(380 +}{(100 + 460)}$$

$$= 35 \times \frac{840}{560} = 52{\cdot}5 \text{ lb. per sq. inch absolute.} \quad \text{Ans.}$$

4. If the pressure remains constant, the volume varies as the absolute temperature.

$$\text{Final volume} = 2{\cdot}5 \times \frac{(460 + 500)}{(130 + 460)}$$

$$= 2{\cdot}5 \times \frac{960}{590} = 4{\cdot}067 \text{ cubic feet.} \quad \text{Ans.}$$

5. The velocity will vary as the volume.

$$\text{Velocity} = 1{,}300 \times \frac{675 + 460}{1{,}200 + 460}$$

$$= 1{,}300 \times \frac{1135}{1660} = 888{\cdot}8 \text{ feet per sec.} \quad \text{Ans.}$$

Tubes choke at front ends due to reduced velocity of the gases.

6.

$$75 \text{ kilos per sq. cm.} = \frac{75 \times 2{\cdot}2}{(0{\cdot}3937)^2}$$

$= 1{,}064$ lb. per sq. inch gauge.

or $1{,}064 + 14{\cdot}7 = 1{,}078{\cdot}7$ lb. per sq. inch absolute.

45° C. $= (45 \times \frac{9}{5}) + 32 = 113^\circ$ F.

$p_1 \quad v_1 \qquad p_2 \quad v_2$

$$\frac{14{\cdot}7 \times 144 \times 12{\cdot}4}{(32 + 460)} = \frac{1{,}079 \times 144 \times v_2}{(113 + 460)}, \text{ 144 cancels.}$$

$$v_2 = \frac{14{\cdot}7 \times 12{\cdot}4 \times 573}{1{,}079 \times 492} = 0{\cdot}1967 \text{ cu. ft. per lb.}$$

One litre $= (0{\cdot}3937)^3 \times 1{,}000$ cu. ins.

$= (0{\cdot}3937)^3 \times \frac{1000}{1728}$ cu. foot.

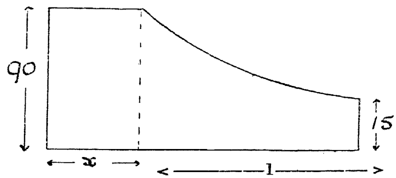
$= 0{\cdot}03533$ cu. foot.

lb. in one litre $= \dfrac{0{\cdot}03533}{0{\cdot}1967} = 0{\cdot}1796$ lb. Ans.

$75 + 15 = 90$ lb. per sq. inch absolute.

Let the stroke $= 1$.

Let $x =$ clearance as a fraction of stroke.



$p_1 \; v_1 = p_2 \; v_2$

$15 \times (1 + x) = 90 \times x$

$15 + 15\,x = 90\,x$; $75\,x = 15$

$x = \frac{15}{75} = \frac{1}{5}$ of stroke. Ans.

8. $\dfrac{22 \times 0{\cdot}3937}{12} = 0{\cdot}7218$ foot diam. nearly.

$2 \times \dfrac{39{\cdot}37}{12} = 6{\cdot}561$ feet long.

$\dfrac{100 \times 2{\cdot}2}{(0{\cdot}3937)^2} = 1{,}419$ lb. per sq. inch.

$(30 \times \frac{9}{5}) + 32 = 86°$ F.

$p_1 \cdot v_1$ $\dfrac{p_2 \quad {}_2}{T_2}$, or $\dfrac{p}{T}$ $= 53{\cdot}2$ for air per lb.

$$\text{Vol. of 1 lb.} = \frac{(86 + 460) \times 53{\cdot}2}{144 \times (1{,}419 + 14{\cdot}7)} = 0{\cdot}1407 \text{ cu. ft.}$$

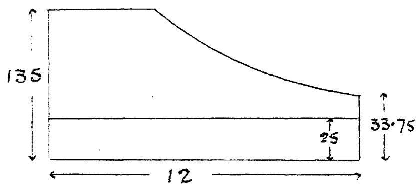
Capacity of bottle $= (0{\cdot}7218)^2 \times \frac{11}{14} \times 6{\cdot}561 = 2{\cdot}686$ cu. feet.

$$\text{lb. weight in bottle} = \frac{2{\cdot}686}{0{\cdot}1407} = 19{\cdot}09 \text{ lb.}$$

19·09 lb. of air. Ans.

9. $r = \frac{12}{3} = 4.$

Final pressure $= \frac{135}{4} = 33{\cdot}75$ lb. sq. inch absolute.



Work $= (135 \times 3 + 135 \times 3 \times \log. 4 \times 2{\cdot}3 - 12 \times 25)\ 144$

Work $= (405 + 560{\cdot}8 - 300) \times 144 = 95{,}880$ ft. lb. Ans.

$$\text{Mean effective pressure} = \frac{95{,}880}{12 \times 144} = 55{\cdot}49 \text{ lb. sq. inch.} \quad \text{Ans.}$$

Heat units put in $= \frac{95880}{778} = 123{\cdot}2$ B.T.U. Ans.

10. B.T.U. per lb. $= 0{\cdot}169\ (T_2 - T_1)$

$= 0{\cdot}169\ (760 - 500) = 0{\cdot}169 \times 260$

$= 43{\cdot}94$ B.T.U. per lb.

B.T.U. for 10 lb. $= 43{\cdot}94 \times 10 = 439{\cdot}4$. Ans.

Ft. lb. of work given $= 439{\cdot}4 \times 778 = 341{,}800$ ft. lb. Ans.

If volume constant :—

$$\therefore P_2 = \frac{P_1 T_2}{T_1} = \frac{15 \times 760}{500}$$

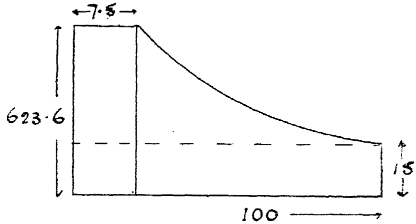
$= 22{\cdot}8$ lb. square inch. Ans.

11.

$$15 \times 107{\cdot}5^{1{\cdot}4} = p_2 \times 7{\cdot}5^{1{\cdot}4}$$

$$p_2 = 15 \times \left(\frac{107{\cdot}5}{7{\cdot}5}\right)^{1{\cdot}4}$$

$$= 623{\cdot}6 \text{ lb. sq. inch absolute. Ans.}$$



$$\frac{T_2}{545} = \left(\frac{623{\cdot}6}{15}\right)^{\frac{2}{7}}$$

$$T_2 = 545 \times \left(\frac{623{\cdot}6}{15}\right.$$

$$T_2 = 1{,}581^\circ\text{F. absolute.}$$

$$t_2 = 1{,}121^\circ\text{F.} \quad \text{Ans.}$$

Mean pressure

$$\frac{\text{Work done during compression}}{\text{Stroke}} - \text{back pressure.}$$

$$= \left(\frac{p_1 v_1 - p_2 v}{n - 1}\right. \times \qquad \text{pressure.}$$

$$\frac{15 \times 107{\cdot}5 - 623{\cdot}6 \times}{0{\cdot}4 \times 100} - 15$$

$$= 76{\cdot}6 - 15 = 61{\cdot}6 \text{ lb. square inch.} \quad \text{Ans.}$$

SOLUTIONS TO TEST EXAMPLES XXI.

1. Heat to bring ice to 32°F. = 0·5 × 12 B.T.U. per lb.
Heat to convert ice to water at 32°F. = 143 + 6 = 149 B.T.U. per lb.

$$\text{Final Temp.} = \frac{\text{Total Heat in B.T.U.}}{\text{Total weight in lb.}}$$

$$= \frac{(100 \times 70) + (112 \times 80) + (15 \times 32) - (149 \times 15)}{100 + 112 + 15}$$

$$= \frac{14205}{227} = 62^\circ{\cdot}57 \text{ F.} \quad \text{Ans.}$$

Heat to bring ice to 32°F. = 0·5 × 2 = 1 B.T.U. per lb.

Heat to convert ice to water at 32°F. = 143 + 1 = 144 B.T.U. per lb.

Total heat = 320 — 32 + 966 — 0·7 (320 — 212) + 144 B.T.U. per lb.

= 288 + 890·4 + 144 = 1,322·4 B.T.U. per lb.

Heat needed for 20 lb. of ice = 1,322·4 × 20 = 26,448 B.T.U. Ans.

Water equivalent of cast iron = 0·13 × 5 = 0·65 lb.

Water equivalent of copper = 0·095 × 3 = 0·285 lb.

$$\text{Final Temp.} = \frac{(0{\cdot}65 \times 500) + (0{\cdot}285 \times 800) + (20 \times 41)}{20 + 0{\cdot}65 \quad 0{\cdot}285}$$

$$= \frac{1{,}373}{20{\cdot}935} = 65{\cdot}58^\circ\text{F.} \quad \text{Ans.}$$

Latent heat at 365°F. = 966 — 0·7 (365 — 212) = 858·9 B.T.U. per lb.

$$\text{Final Temp.} = \frac{\text{Total B.T.U.}}{\text{Total weight}}$$

$$= \frac{0{\cdot}08 \times (365 - 200 + 858{\cdot}9) + (1 \times 125)}{1} = 206{\cdot}9^\circ\text{F. Ans.}$$

Note that the heating steam does not mix with the feed.

Latent heat at 390°F. = 966 — 0·7 (390 — 212) = 841·4 B.T.U. per lb.

$$\text{Final Temp.} = \frac{\text{Total B.T.U.}}{\text{Total weight}}$$

$$\frac{20 \times (390 + 841{\cdot}4) + (2{,}000 \times 100)}{2{,}000 + 20}$$

Ans.

6. $F = \frac{9}{5} \times (-5) + 32 = 23°F.$

$F = \frac{9}{5} \times 195 + 32 = 383°F.$

10 kilos. = 2·2 × 10 = 22 lb.

Heat to bring ice to 32°F. = 0·5 × 9 B.T.U. per lb.

Heat to bring ice to water at 32° = 0·5 × 9 + 143 B.T.U. per lb.

= 147·5 B.T.U. per lb.

Latent heat at 383°F. = 966 — 0·7 (383 — 212) = 846·3 B.T.U. per lb.

Total heat = 383 — 32 + 846·3 + 147·5 = 1,344·8 B.T.U. per lb.

Total heat = 1,344·8 × 22 = 29,585·6 B.T.U. Ans.

Latent heat at 380°F. = 966 — 0·7 (380 — 212) = 848·4 B.T.U. per lb.

Total heat = 380 — 160 + (0·9 × 848·4) = 983·56 B.T.U. per lb. Ans.

Heat to dry the steam = 0·1 × 848·4 = 84·84 B.T.U. per lb.

Heat to superheat = 0·48 (580 — 380) = 96 B.T.U. per lb.

Extra heat = 84·84 + 96 = 180·84 B.T.U. per lb. Ans.

8. Latent heat at 300°F. = 966 — 0·7 (300 — 212) = 904·4 B.T.U. per lb.

Let x = dryness fraction.

$$\text{Final Temp.} = \frac{\text{Total B.T.U.}}{\text{Total weight}}$$

$$101 = \frac{0\cdot5\,(300 + x \times 904\cdot4) + (10 \times 50)}{10 + 0\cdot5}$$

$$1{,}060\cdot5 = 150 + 452\cdot2\,x + 500$$

$$452\cdot2\,x = 410\cdot5, \quad x = \frac{410\cdot5}{452\cdot2}$$

$x = 0\cdot907$. Ans.

1 gall. = 10 lb. 1 pint = 1·25 lb.

Let x = dryness fraction.

$$185 = \frac{(10 \times 65) + 1\cdot25\,(212 + 966 \times x)}{10 + 1\cdot25}$$

$$2{,}081\cdot25 = 650 + 265 + 1{,}207\cdot5\,x$$

$$1{,}207\cdot5\,x = 1{,}166\cdot25$$

$$x = \frac{1{,}166\cdot25}{1{,}207\cdot5} = 0\cdot965. \quad \text{Ans.}$$

10. Latent heat at 325°F. = 966 — 0·7 (325 — 212) = 886·9 B.T.U. per lb.

Total heat per lb. of steam = 325 100 + 886·9 = 1,111·9 B.T.U.

Boiler efficiency

$$= \frac{\text{Total heat per lb. steam} \times \text{lb. evaporated}}{\text{Total heat per lb. coal}}$$

$$\frac{1{,}111\cdot9 \times 8\cdot9}{13{,}000} \quad \frac{9{,}895\cdot91}{13{,}000} = 0\cdot761. \quad \text{Ans.}$$

Equivalent evaporation from and at 212°F.

$$\frac{1{,}111\cdot9 \times 8\cdot9}{966} = 10\cdot24 \text{ lb.} \quad \text{Ans.}$$

11. Heat available per lb. of steam $= 1{,}197{\cdot}5 + 32 - 130 = 1{,}099{\cdot}5$ B.T.U.

B.T.U. per horse power per hour $= 1{,}099{\cdot}5 \times 15$

One horse power per hour $= \dfrac{33{,}000 \times 60}{778} = 2{,}545$ B.T.U.

$$\text{Efficiency} = \frac{\text{Heat converted into work}}{\text{Heat supplied}}$$

$$\frac{2{,}545}{1{,}099{\cdot}5 \times 15} = 0{\cdot}1549$$

or $0{\cdot}1549 \times 100 = 15{\cdot}49$ per cent. Ans.

12. Latent heat at 373°F. $= 966 - 0{\cdot}7\ (373 - 212) = 853{\cdot}3$ B.T.U. per lb.

Let the total steam be 100 lb. then 4 lb. go to the feed and 96 lb. go to the engines.

$$\text{Final Temp.} = \frac{\text{Total B.T.U.}}{\text{Total weight}}$$

$$= \frac{4 \times (373 + 853{\cdot}3) + (96 \times 120)}{100} = 164{\cdot}25^\circ\text{F.} \quad \text{Ans.}$$

13. Let 100 = original quantity of circulating water and 115 = quantity after temp. of sea rises.

B.T.U. absorbed $(a) = 100 \times (98 - 60) = 3{,}800$

B.T.U. absorbed $(b) = 115 \times (T - 70)$

$\therefore\ 115 \times (T - 70) = 3{,}800$

$T - 70 = \frac{3800}{115}$

$T = \frac{3800}{115} + 70 = 103^\circ{\cdot}04$ F. Ans.

14. Latent heat at 120°F. $= 966 - 0{\cdot}7\ (120 - 212) = 1{,}030{\cdot}4$ B.T.U. per lb.

Heat lost per lb. by steam $= 120 + 1{,}030{\cdot}4 - 101 = 1{,}049{\cdot}4$ B.T.U.

Circulating water rises 101 — 60 = 41 F°.

$\therefore$ 41 B.T.U. are given per lb. to the sea water.

$$\text{B.T.U. lost by steam per min.} = \frac{4{,}000 \times 16 \times 1{,}049{\cdot}4}{60}$$

$$\text{Tons of water} = \frac{4{,}000 \times 16 \times 1{,}049{\cdot}4}{60 \times 41 \times 2{,}240} = 12{\cdot}13. \quad \text{Ans.}$$

15. $T - 160 + 0{\cdot}9\,[966 - 0{\cdot}7\,(T - 212)\,] = 961$

$T - 160 + 869{\cdot}4 - 0{\cdot}63\,(T - 212) = 961$

$T - 160 + 869{\cdot}4 - 0{\cdot}63\,T + 133{\cdot}56 = 961$

$0{\cdot}37\,T = 118{\cdot}04$

$T = 319°F.$ Ans.

16. Let x lb. of steam per 100 lb. be taken to the heater, then $(100 - x)$ lb. go to the engines.

Latent heat at 380°F. $= 966 - 0{\cdot}7\,(380 - 212)$
$= 848{\cdot}4$ B.T.U. per lb.

$$\text{Final Temp.} = \frac{\text{Total B.T.U.}}{\text{Total weight}}$$

$$200 = \frac{(100 - x) \times 120 + x\,(380 + 848{\cdot}4)}{100}$$

$20{,}000 = 12{,}000 - 120\,x + 1228{\cdot}4\,x.$

$1{,}108{\cdot}4\,x = 8{,}000$

$$x = \frac{8{,}000}{1{,}108{\cdot}4} = 7{\cdot}21 \text{ lb.}$$

Steam which goes to heater is 7·21 per cent.

To engines 100 — 7·21 = 92·79 per cent. Ans.

17. $2 \times 2{,}240$

$= 746\frac{2}{3}$ lb. per watch.

B.T.U. taken from 1 lb. water at 32°F. = 143 (latent heat).

B.T.U. to cool from 57° to 32° = 25.

B.T.U. to cool from 32° to 24° = $0{\cdot}5 \times 8 = 4$.

Total per lb. = $143 + 25 + 4 = 172$ B.T.U.

To make ice from water at 32° takes $746\frac{2}{3} \times 143$ B.T.U.

$$\text{Pounds of ice in 4 hours} = \frac{746\frac{2}{3} \times 143}{172} = 620{\cdot}7 \text{ lb.} \quad \text{Ans.}$$

18. Let W = weight of feed water heated per lb. of steam.

Heat lost by steam = Heat gained by the water.

$(379{\cdot}2 - 208) + 966 - 0{\cdot}7\,(379{\cdot}2 - 212) = W\,(178 - 126)$
$171{\cdot}2 + 848{\cdot}96 = 52\ W$

$$\therefore\ W = \frac{171{\cdot}2 + 848{\cdot}96}{52} = 19{\cdot}62 \text{ lb.} \quad \text{Ans.}$$

SOLUTIONS TO TEST EXAMPLES XXII.

1. (*a*) $$r = \frac{\text{Final vol.}}{\text{Initial vol.}} = \tfrac{36}{12} = 3$$

$p\ v$ = constant.

$175 \times 12 = p_2 \times 36$, $p_2 = 58{\cdot}33$ lb. sq. inch absolute.

Terminal pressure $= 58{\cdot}33 - 15 = 43{\cdot}33$ lb. sq. inch gauge. Ans.

$$p_m = \frac{p_1}{r}\left[1 + \log_e r\right] - \text{back pressure}$$

$$p_m = \frac{175}{3}\left[1 + \log.\ 3 \times 2{\cdot}3\right] - 26 = 96{\cdot}3 \text{ lb. sq. inch.}$$ Ans.

(*b*) $v_1 = 12 + \frac{36}{10} = 15{\cdot}6$ inches.

$v_2 = 36 + \frac{36}{10} = 39{\cdot}6$ inches.

$$\frac{39{\cdot}6}{15{\cdot}6} = 2{\cdot}539$$

$175 \times 15{\cdot}6 = p_2 \times 39{\cdot}6$, $p_2 = 68{\cdot}93$ lb. sq. inch absolute.

Terminal pressure $= 68{\cdot}93 - 15 = 53{\cdot}93$ lb. sq. inch gauge. Ans.

$$p_m = \quad \frac{+ \log_e}{v_2 - c} \qquad p_2$$

$$p_m = \frac{175 \times 15{\cdot}6\ (1 + \log.\ 2{\cdot}539 \times 2{\cdot}3) - 175 \times 3{\cdot}6}{36} - 26$$

$p_m = 128{\cdot}7 - 26 = 102{\cdot}7$ lb. per sq. inch. Ans.

Let the stroke $= 1$, then clearance $= 0{\cdot}08$

$v_1 = 0{\cdot}32 + 0{\cdot}08 = 0{\cdot}4$, $v_2 = 1 + 0{\cdot}08 = 1{\cdot}08$

$$r = \frac{1{\cdot}08}{0{\cdot}4} = 2{\cdot}7$$

$215 \times 0{\cdot}4 = p_2 \times 1{\cdot}08$, $p_2 = 79{\cdot}62$ lb. sq. inch absolute.

Terminal pressure $= 79{\cdot}62 - 15 = 64{\cdot}62$ lb. sq. inch gauge. Ans.

$$p_m = \frac{p_1 v_1 (1 + \log_e r) - p_1 c}{} - c$$

$$p_m = \frac{215 \times 0{\cdot}4\,(1 + \log.\ 2{\cdot}7 \times 2{\cdot}3) - 215 \times 0{\cdot}08}{1} - 75$$

$p_m = 154{\cdot}1 - 75 = 79{\cdot}1$ lb. per sq. inch. Ans.

Piston speed $= 2 \times$ stroke $\times$ revs. per minute.

$480 = 2 \times$ stroke $\times 80$

Stroke $= 3$ feet.

Work per stroke $= 3 \times 1{,}600 \times 24 = 115{,}200$ foot lb. Ans.

$$\text{Horse power} = \frac{1600 \times 24 \times 480}{33{,}000.} = 558{\cdot}5. \quad \text{Ans.}$$

$20 + 15 = 35$ lb. per sq. inch absolute.

$75 + 15 = 90$ lb. per sq. inch absolute.

Let the stroke $= 100$, then clearance $= 10$

$35 \times v_1 = 90 \times 10$, $v_1 = 25{\cdot}71$

Exhaust must close at $25{\cdot}71 - 10 = 15{\cdot}71$.

Since the stroke was 100, then exhaust must close at 15·71 per cent. of the stroke from the end. Ans.

5. $v_1 = 0{\cdot}4 \times 42 + \quad \times 42 = 20{\cdot}16$ inches.

$v_2 = 42 + \quad \times 42 = 45{\cdot}36$ inches.

$$r = \frac{45{\cdot}36}{20{\cdot}16} = 2{\cdot}25.$$

p_m during expansion alone $= \dfrac{p_1 \; v_1 \; \log_e r}{}$ absolute,

neglecting back pressure.

p_m during expansion alone $= \dfrac{200 \times 20{\cdot}16 \times \log.2{\cdot}25 \times 2{\cdot}3}{25{\cdot}2}$

$p_m = 129{\cdot}6$ lb. sq. inch absolute, neglecting back pressure.

p_m during expansion $= 129{\cdot}6 - 60 = 69{\cdot}6$ lb. sq. inch. Ans.

Gross pressure during whole stroke

$$\frac{p_1\,(v_1 - c) + (129{\cdot}6 \times 25{\cdot}2)}{v_2 - c}$$

$$\frac{200 \times 16{\cdot}8 + 129{\cdot}6 \times 25{\cdot}2}{42} = 157{\cdot}7 \text{ lb. sq. inch.}$$

Mean effective pressure $= 157{\cdot}7 - (45 + 15) = 97{\cdot}7$ lb. per sq. inch.

This is the theoretical p_m, actually it is less than this.

$p_m = 97{\cdot}7 \times 0{\cdot}7 = 68{\cdot}39$ lb. per sq. inch. Ans.

$$\text{I.H.P.} = 24 \times 24 \times \tfrac{11}{14} \times \frac{68{\cdot}39 \times 3{\cdot}5 \times 2 \times 80}{33{,}000}$$

$= 525{\cdot}3$. Ans.

6. Let x = clearance and let stroke = 1

then $v_1 = (\frac{1}{3} + x)$

and $v_2 = (\frac{3}{4} + x)$ where v_2 = vol. at $\frac{3}{4}$ stroke.

$p_1 \; v_1 = p_2 \; v_2$

$195\;(\frac{1}{3} + x) = 97{\cdot}5\;(\frac{3}{4} + x)$

$2\;(\frac{1}{3} + x) = \frac{3}{4} + x$

$\frac{2}{3} + 2\,x = \frac{3}{4} + x$

and $\frac{1}{12} \times 100 = 8{\cdot}33$ per cent. Ans.

$$\text{Vol. per lb.} = \frac{410 + \frac{175}{4}}{175 + 1} = 2\cdot578 \text{ cu. feet.}$$

$$\text{Vol. up to cut off} = 20 \times 20 \times \tfrac{11}{14} \times \tfrac{1}{1728} = 2\cdot181 \text{ cu. feet.}$$

$$\text{Weight admitted} = \frac{2\cdot181}{2\cdot578} = 0\cdot8461 \text{ lb.} \quad \text{Ans.}$$

$$\frac{\text{L.P. Volume}}{\text{H.P. Volume}} \quad \frac{}{26^2} = 6\cdot84$$

$$\frac{\text{L.P. Volume}}{\text{M.P. Volume}} = \frac{68^2}{42^2} = 2\cdot62$$

Mean pressure in L.P. $= \frac{30}{3} =$ 10 lb. sq. inch.

Mean pressure in H.P. $= 10 \times 6\cdot84 = 68\cdot4$ lb. sq inch. Ans.

Mean pressure in M.P. $= 10 \times 2\cdot62 = 26\cdot2$ lb. sq. inch. Ans.

$$\frac{80^2}{25^2} = 10\cdot24. \qquad \frac{80^2}{36^2} = 4\cdot938. \qquad \frac{80^2}{54^2} = 2\cdot194.$$

Mean pressure in H.P. referred to L.P. $= \frac{}{10\cdot24}$

$= 7\cdot225$ lb. per sq. inch.

Mean pressure in 1st M.P. referred to L.P. $= \frac{38}{4\cdot938}$

$= 7\cdot696$ lb. sq. inch.

Mean pressure in 2nd M.P. referred to L.P. $= \frac{17}{2\cdot194}$

$= 7\cdot749$ lb. sq. inch.

Mean pressure in L.P. $= 8$ lb. per sq. inch.

Mean pressure referred all to L.P.

$= 7\cdot225 + 7\cdot696 + 7\cdot749 + 8$

$= 30\cdot67$ lb. per sq. inch.

10.

$$\text{Mean height of card} = \frac{\text{Area}}{\text{Length}} = \frac{4{\cdot}75}{4} \text{ inches.}$$

$$\text{Note average area} = \frac{\quad}{2} = 4{\cdot}75 \text{ sq. inches.}$$

$$\text{Mean pressure} = \frac{4{\cdot}75}{4} \times 80 = 95 \text{ lb. per sq. inch.}$$

Ans.

$$\text{I.H.P.} = 25 \times 25 \times \tfrac{11}{14} \times \frac{3{\cdot}5 \times 2 \times 75}{33{,}000} \times 95$$

$= 742{\cdot}2$. Ans.

11.

$$p_m = \left[\frac{p_1}{r}\,(1 + \log_e r) - \text{B.P.}\right] \times \text{diagram factor.}$$

$$p_m = [\tfrac{195}{12}\,(1 + \log.\, 12 \times 2{\cdot}3) - 4] \times 0{\cdot}7 = 36{\cdot}82 \text{ lb. per sq. inch.}$$

$$2{,}200 \times 33{,}000 = \text{D}^2 \times \tfrac{11}{14} \times 36{\cdot}82 \times 3{\cdot}5 \times 2 \times 72$$

$$\text{D} = \sqrt{\frac{2{,}200 \times 33{,}000 \times 14}{11 \times 36{\cdot}82 \times 7 \times 72}} = 70{\cdot}57 \text{ inches.}$$

$$\frac{\text{Final Vol.}}{\text{Initial Vol.}} \qquad \frac{\text{D}^2}{\text{d}^2 \times \text{cut off}}$$

$$12 = \frac{(70{\cdot}57)^2}{d^2 \times 0{\cdot}6} \qquad d = \sqrt{\frac{(70{\cdot}57)^2}{\quad}}$$

$70{\cdot}57$

$= 26{\cdot}3$ inches.

$$\text{M.P.} = \sqrt{70{\cdot}57 \times 26{\cdot}3} = 43{\cdot}08 \text{ inches.}$$

Cylinders are: 26·5, 43 and 71 inches diam. (say).
Ans.

12. By measuring and taking the mean of both ends, $p_m = 18{\cdot}4$ lb. per sq. inch.

$$\text{Horse power} = 42 \times 42 \times \tfrac{11}{14} \times \frac{18{\cdot}4 \times 4 \times 2 \times 70}{33{,}000}$$

H.P. $= 432{\cdot}7$. Ans.

13. By measuring as suggested in the text, the cubic feet of steam per stroke is about 10·14 cubic feet.

$$\text{Wt. of one cubic foot} = \frac{P + 1}{410 + \dfrac{P}{4}}$$

$$\text{Wt.} = \frac{103 + 1}{410 + {}^{1}} \quad \frac{104}{435{\cdot}75} \text{ lb. per cubic foot.}$$

$$\text{Wt. per revolution} = \frac{104}{435{\cdot}75} \times 2 \times 10{\cdot}14$$

$= 4{\cdot}841$ lb.

$$\text{Wt. per day} = \frac{4{\cdot}841 \times 60 \times 60 \times 24}{2{,}240} = 186{\cdot}7 \text{ tons.}$$

Ans.

SOLUTIONS TO TEST EXAMPLES XXIII.

(Lap at bottom + lead at bottom — exhaust lap at top) = opening to exhaust at top.

In this position the engine is on bottom centre, and the valve has opened the bottom port to lead.

Lap at bottom + 0·37 — (— 0·2) = 2·38 inches.
Lap at bottom = 2·38 — 0·37 — 0·2
Lap at bottom = 1·81 inches.

$$\text{Sin. of angle of advance} = \frac{\text{lap} + \text{lead}}{\frac{1}{2}\text{ travel}}$$

$$\frac{1{\cdot}81 + 0{\cdot}37}{3{\cdot}5} \quad \frac{2{\cdot}18}{3{\cdot}5} = 0{\cdot}6228$$

Angle of advance = 38° 33′. Ans.

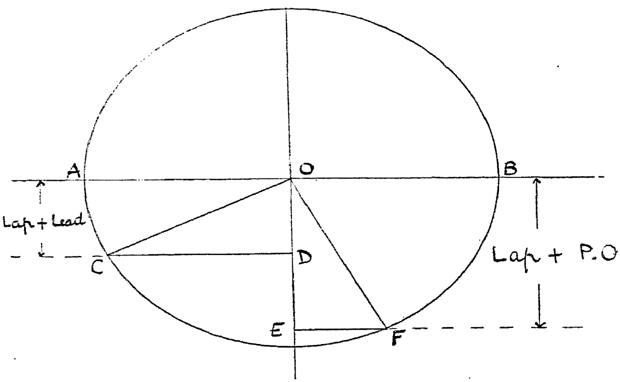
$$\text{Sin. of angle of advance} = \frac{\text{lap} +}{\text{travel}}$$

$$\frac{2 + 0{\cdot}25}{3{\cdot}5} = 0{\cdot}6428$$

Angle of advance = 40°.

Distance from mid-position is given by :—
$\frac{1}{2}$ travel $\times$ Sin (angle of advance + crank angle from centre).
$= \frac{7}{2} \times \text{Sin}\ (40° + 30°) = \frac{7}{2} . \text{Sin}\ 70°$
Distance from mid-position $= \frac{7}{2} \times 0{\cdot}9397$
$= 3{\cdot}289$ inches. Ans.

3. Let A B = valve travel.
Make O D = lap + lead, and this is the distance the valve is from mid-position when the engine is on the top centre.
When the engine turns through 90°, the eccentric also moves through 90°, from O C to O F.
Now O E = displacement from mid-position when the crank is 90° from top centre.

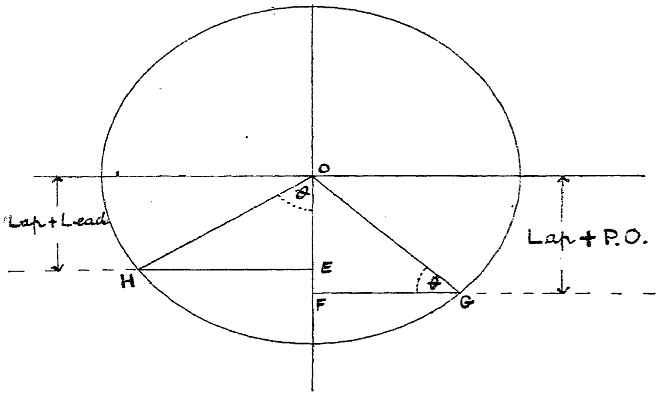


$\therefore$ O E = lap + port opening.
O D $= 1\frac{1}{4} + \frac{1}{4} = 1\frac{1}{2}$ ins.
O E $= 1\frac{1}{4} + 2\frac{1}{4} = 3\frac{1}{2}$ ins.
The triangles O D C and O E F are similar and have equal sides.

$\left.\begin{array}{l} OC = OF \\ OD = EF \end{array}\right\}$ EF =

$(OF)^2 = (OE)^2 + (EF)^2$
$OF = \sqrt{(3\frac{1}{2})^2 + (1\frac{1}{2})^2} = 3{\cdot}808$ ins.
half travel $= 3{\cdot}808$ ins.
Travel $= 7{\cdot}616$ ins. Ans.

O H E and O F G are similar triangles.
O H = O G, and O E = F G.
O F = lap + P.O.
O F = 0·7 + 0·7 = 1·4
O E = F G
O G = 1·6 inches.
F G
F G = 0·7746 inch.
O E = lap + lead.
O E = 0·7746 inch.
Lead = 0·7746 — 0·7
= 0·0746 inch. Ans.



5. Area of Cyl. × max. piston speed = area of port × speed of steam

$30 \times 30 \times \frac{11}{14} \times 2\,\pi \times 1{\cdot}5 \times 75$
$= \text{area of port} \times 6{,}000$

$$\text{Area of port} = \frac{900 \times 22 \times 22 \times 1{\cdot}5 \times 75}{14 \times 7 \times 6{,}000}$$

= 83·34 square inches.

$$\text{Opening to steam} = \frac{83{\cdot}34}{25} = 3{\cdot}334 \text{ inches. Ans.}$$

Depth of port = $3{\cdot}334 \times \frac{4}{3}$ = 4·445 inches. Ans.

SOLUTIONS TO TEST EXAMPLES XXIV.

1. Calorific value =

$$14{,}500 \times 0{\cdot}83 + 62{,}000\left\{0{\cdot}04 - \frac{0{\cdot}056}{}\right\}$$

$$= 12{,}035 + 62{,}000 \times 0{\cdot}033$$
$$= 14{,}081 \text{ B.T.U. per lb.} \quad \text{Ans.}$$

$$\text{Thermal efficiency} = \frac{\text{Heat turned into work in cylinders}}{\text{Heat supplied}}$$

$$\frac{2545}{1{\cdot}45 \times 14{,}081} = 0{\cdot}1247,$$

or $12{\cdot}47\%$. Ans.

Note, the heat equivalent of one horse power hour is 2,545 B.T.U.

Weight of oxygen to burn the Carbon $= 2\frac{2}{3} \times 0{\cdot}85$
$= 2{\cdot}233$ lb.

Weight of Oxygen for available Hydrogen

$$= 0{\cdot}945 \text{ lb.}$$

Total weight of Oxygen required $= 2{\cdot}233 + 0{\cdot}945$
$= 3{\cdot}178$ lb.

Air required $= 3{\cdot}178 \times \frac{100}{23} = 13{\cdot}82$ lb. per lb. of fuel.
Ans.

3. Theoretical weight of air required per 1 lb. of oil

$$= \left\{2\frac{2}{3} \times 0{\cdot}84 + \left(0{\cdot}13 - \frac{0{\cdot}02}{8}\right)\right. \times \frac{100}{23}$$

$= 14{\cdot}18$ lb.

Actual weight supplied $= 14{\cdot}18 \times \frac{170}{100} = 24{\cdot}09$ lb.

Weight of constituents of the oil which form gas
$= 1 - 0{\cdot}01 = 0{\cdot}99$ lb.

Weight of gases passing up funnel for each 1 lb. of oil burnt
$= 24{\cdot}09 + 0{\cdot}99 = 25{\cdot}08$ lb.

$\therefore$ weight of gases per hour $= 1{,}400 \times 25{\cdot}08$
$= 35{,}112$ lb. Ans.

4. Combustibles present in 1 lb. of oil $= 1 - 0{\cdot}014 = 0{\cdot}986$ lb.

Weight of gases passing up funnel for each 1 lb. of oil supplied to the furnace $= 24 + 0{\cdot}986 = 24{\cdot}986$ lb.

Weight of Nitrogen in 24 lb. of air $= 0{\cdot}77 \times 24 = 18{\cdot}48$ lb.

Weight of Oxygen in 24 lb. of air $= 0{\cdot}23 \times 24 = 5{\cdot}52$ lb.

Weight of Oxygen to burn 0·86 lb. of Carbon $= 2\frac{2}{3} \times 0{\cdot}86$
$= 2{\cdot}293$ lb.

,, Carbon dioxide formed $= 0{\cdot}86 + 2{\cdot}293$
$= 3{\cdot}153$ lb.

,, available Hydrogen $= \left(0{\cdot}11 - \dfrac{0{\cdot}016}{8}\right)$
$= 0{\cdot}108$ lb.

,, Oxygen to burn 0·108 lb. of Hydrogen
$= 8 \times 0{\cdot}108 = 0{\cdot}864$ lb.

,, Water in gases $= 0{\cdot}11 \times 9 = 0{\cdot}99$ lb.

,, Oxygen used $= 2{\cdot}293 + 0{\cdot}864 = 3{\cdot}157$ lb.

,, surplus Oxygen $= 5{\cdot}52 - 3{\cdot}157$
$= 2{\cdot}363$ lb.

Composition of the gases is :—

$$\left.\begin{aligned}
CO_2 &= \frac{3{\cdot}153}{24{\cdot}986} \times 100 = 12{\cdot}62\% \\
N &= \frac{18{\cdot}48}{24{\cdot}986} \times 100 = 73{\cdot}97\% \\
\text{Water} &= \frac{0{\cdot}99}{24{\cdot}986} \times 100 = 3{\cdot}96\% \\
\text{Free O} &= \frac{2{\cdot}363}{24{\cdot}986} \times 100 = 9{\cdot}45\%
\end{aligned}\right\} \text{Ans.}$$

5. $0{\cdot}8 \times \text{Calorific value} = (1800 + 450) \times (30{\cdot}8 - 27)$

$$0{\cdot}8 \times \text{C.V.} = 2250 \times 3{\cdot}8$$

$$\text{C.V.} = \frac{2250 \times 3{\cdot}8}{0{\cdot}8} = 10{,}688 \text{ gram calories per gram.}$$

$$\text{C.V.} = \frac{10688 \times 1000}{252 \times 2{\cdot}2} = 19{,}460 \text{ B.T.U. per lb. Ans.}$$

6. Theoretical weight of air required

$$= \left\{2\tfrac{2}{3} \times 0{\cdot}85 + 8\left(0{\cdot}13 - \frac{0{\cdot}01}{8}\right)\right\} \times \frac{100}{23}$$

$$= 14{\cdot}33 \text{ lb. per 1 lb. of oil}$$

Weight of oil used per cylinder per

$$\frac{0{\cdot}36 \times 4000}{92 \times 60 \times 6} \quad \frac{1}{23}$$

$\therefore$ Theoretical weight of air required per cylinder per cycle.

$$= 14{\cdot}33 \times \tfrac{1}{23} = 0{\cdot}623 \text{ lb. Ans. } (a).$$

$$\frac{p\,v}{} = 53{\cdot}2, \quad \frac{(14{\cdot}7 + 3) \times 144 \times v}{(90 + 460)} = 53{\cdot}2$$

$$v = \frac{53{\cdot}2 \times 550}{17{\cdot}7 \times 144} = 11{\cdot}48 \text{ cu. feet per lb. of air}$$

Weight of air actually supplied per cylinder per cycle

$$= \frac{20}{11{\cdot}48} = 1{\cdot}742 \text{ lb. Ans. } (b)$$

$$\text{Excess air} = 1{\cdot}742 - 0{\cdot}623 = 1{\cdot}119 \text{ lb.}$$

$$\text{Excess \%} = \frac{1{\cdot}119}{0{\cdot}623} \times 100 = 179{\cdot}5\%. \text{ Ans. } (c).$$

SOLUTIONS TO TEST EXAMPLES XXV.

Friction Force $\propto V^2$, 10 knots is $\dfrac{10 \times 6080}{60}$

$= 1{,}013$ ft. per min.

Friction force per sq. foot at 10 knots

$= \frac{1}{4} \times \left(\dfrac{1013}{600}\right)^2$ lb.

Total friction force $= \frac{1}{4} \times 15000 \times \left(\dfrac{1013}{600}\right.$ lb.

$$\text{H.P.} = \frac{1}{4} \times \frac{15000}{33000} \times \left(\frac{1013}{600}\right)^2 \times \frac{10 \times 6080}{60}$$

$= 328{\cdot}1.$ Ans.

Let V = velocity in feet per minute.

Friction force per sq. foot at V

$$= 0{\cdot}3 \times \left(\frac{V}{600}\right)^2 \text{ lb.}$$

$$\text{H.P.} = \frac{\text{Force in lb.} \times \text{ft. per min.}}{33{,}000}$$

$$2{,}000 = \frac{0{\cdot}3 \times \dfrac{V^2}{600^2} \times 18{,}000 \times V}{33{,}000}$$

$$2{,}000 = \frac{0{\cdot}3 \times 18{,}000 \times V^3}{600 \times 600 \times 33{,}000}$$

$$V^3 = \frac{2{,}000 \times 600 \times 600 \times 33{,}000}{0{\cdot}3 \times 18{,}000}$$

V = 1,639 ft. per min. or 16·18 knots. Ans.

3. $10\cdot5$ knots $= 10\cdot5 \times \frac{6080}{60} = 1064$ feet per min.

Frictional resistance per sq. foot

$= 1\cdot085$ lb. per sq. foot. Ans. (*a*)

Total resistance $= 25{,}000 \times 1\cdot085$ lb.

Work done per min. against resistance $=$ $25{,}000 \times 1\cdot085 \times 1{,}064$ ft. lb.

$$\text{Total I.H.P.} = \frac{25{,}000 \times 1\cdot085 \times 1{,}064}{33{,}000} \times \frac{100}{70} = 1{,}250.$$

Ans. (*b*).

4.

$$\text{Friction Force} = 35{,}970 \times \frac{1}{4} \times \left(\frac{12\cdot5}{\quad}\right)^2 \text{ lb. total.}$$

Tow Rope H.P.

$$\frac{35970}{4} \times \left(\frac{12\cdot5}{6}\right)^2 \times \frac{12\cdot5 \times 6{,}080}{60 \times 33{,}000} \times \frac{100}{80}$$

$= 1{,}873.$ Ans.

I.H.P. $= 1{,}873 \times \frac{100}{55} = 3{,}404.$ Ans.

$$\text{Force on thrust} = \frac{\text{Work per minute}}{\text{Speed in feet per minute}}$$

$$\frac{5{,}000 \times 33{,}000}{\dfrac{6080 \times 12}{60}} \times \frac{45}{100}$$

$$\text{Force} = \frac{5{,}000 \times 33{,}000 \times 45 \times 60}{6{,}080 \times 12 \times 100} = 61{,}060 \text{ lb.}$$

Ans.

6.

$$\text{H.P. lost} = 14{,}000 \times \mu \times \tfrac{13}{2} \times \tfrac{1}{12} \times 100 \times \frac{2\,\pi}{33{,}000} = 15$$

$$\mu = \frac{2 \times 12 \times 33{,}000 \times 15}{14{,}000 \times 13 \times 200 \times \pi} = 0\cdot1039. \quad \text{Ans.}$$

$$\text{I.H.P.} = \frac{(2{,}880)^{\frac{2}{3}} \times 9^3}{250} = 590{\cdot}3. \quad \text{Ans.}$$

The consumption will be two-thirds after shutting off one boiler. Consumption $\propto$ V^3

$$\frac{\text{Consumption}}{V^3} = \text{constant}$$

$$\text{———} = \text{———}, \quad \text{or } V_2^3 = 12^3 \times$$

$$V_2 = 12\sqrt[3]{\tfrac{2}{3}} = 10{\cdot}48 \text{ knots.} \quad \text{Ans.}$$

The power is half in the second case, and
∴ the consumption is half.

1

$$= 22 \times$$

$$V_2 = 17{\cdot}46 \text{ knots.} \quad \text{Ans.}$$

10. $$\frac{\text{Pressure on thrust} \times V}{\text{H.P.}} = \text{constant}$$

$$\frac{40 \times 12}{2{,}000} \qquad \frac{\text{Pressure on thrust} \times 10}{1{,}800}$$

Pressure on thrust $=$ 43·2 lb. per sq. inch. Ans.

11. Here the weather conditions are assumed the same.

$$\frac{p_m}{R^2} = \text{constant}, \qquad \frac{31}{62^2} \qquad 56^2$$

$$p_m = 31 \times \frac{}{62^2} = 25{\cdot}29 \text{ lb. per square inch.}$$

H.P. $\propto$ $p_m \times R$, now $p_m \propto R^2$.
or H.P. $\propto$ $R^2 \times R$, or H.P. $\propto$ R^3

$$\frac{\text{H.P.}}{R^3} = \text{cons.}$$

$$\frac{2500}{62^3} = \frac{\text{H.P.}_2}{56^3} \qquad \text{H.P.}_2 = 1{,}842. \quad \text{Ans.}$$

12. $60 \times 30 = 56 \times p_m$

$$p_m = \frac{30 \times 60}{56} = 32{\cdot}14 \text{ lb. per sq. inch.} \quad \text{Ans.}$$

13. Part circum. $= \sqrt{4^2 - 2^2} = 3{\cdot}464$ feet

P.C. : W.C. :: P.P. : W.P.

$$\text{Pitch} = \frac{2\,\pi \times 6 \times 2}{3{\cdot}464} = 21{\cdot}81 \text{ feet.} \quad \text{Ans.}$$

14. $$\frac{18 \times 70}{6{,}080} \times \frac{90}{100} \times 60 = 11{\cdot}19 \text{ knots.} \quad \text{Ans.}$$

$$\text{Propeller speed} = \frac{11{\cdot}19}{0{\cdot}9} = 12{\cdot}43 \text{ knots.}$$

Speed of slip $= 12{\cdot}43 - 11{\cdot}19 = 1{\cdot}24$ knots.

Area of stream $= \frac{11}{14}\,[16^2 - 2\tfrac{1}{2}^2] = 196{\cdot}2$ sq. feet.

Thrust $= 5{\cdot}67 \times 196{\cdot}2 \times 12{\cdot}43 \times 1{\cdot}24 = 17{,}150$ lb. Ans.

15. Let K = co-efficient of fineness.

$$1{\cdot}7\,L(D \quad \cdot\ \cdot \quad (D + 1{\cdot}5) \quad -\left[1{\cdot}7LD + \frac{L \times B \times D \times K}{D}\right] = 1150$$

cancelling and changing signs, this becomes :—

$1{\cdot}7\,L\,(D + 1{\cdot}5) + L \times B \times K - 1{\cdot}7\,L\,D - L \times B \times K = 1{,}150$

$1{\cdot}7\,L\,(D + 1{\cdot}5) - 1{\cdot}7\,L\,D = 1{,}150$, since $L \times B \times K$ cancels.

$1{\cdot}7\,L\,(D + 1{\cdot}5 - D) = 1{,}150,$

$1{\cdot}7\,L \times 1{\cdot}5 = 1{,}150$, since D cancels.

$$L = \frac{1{,}150}{2{\cdot}55} = 451 \text{ feet.} \quad \text{Ans.}$$ (see next page)

This type of question has been given, but it is not possible to determine the length of a vessel from the empirical formula, wetted surface $= 1{\cdot}7\ L\ D + \dfrac{V}{D}$.

The solution is the only possible one for the question given, but the answer is absurd. If the beam of the ship approached zero then the distance round the ship would be 2×451 feet, and for 1·5 feet change of draught the change of wetted surface would be

$$2 \times 451 \times 1{\cdot}5 = 1{,}353 \text{ sq. feet.}$$

1,353 sq. feet is the least possible change of wetted surface for a ship 451 feet long and for a change of draught of 1·5 feet. Yet the question gives the change as 1,150 sq. feet.

16. Thermal efficiency of engine and boiler together $= \frac{65}{100} \times 17$
$= 11{\cdot}05$ per cent.

Mechanical efficiency of engine and propeller combined
$= \frac{70}{100} \times 88 = 61{\cdot}6$ per cent.

Percentage of heat in coal used

$$= \frac{61{\cdot}6}{100} \times 11{\cdot}05 = 6{\cdot}806 \text{ per cent.} \quad \text{Ans.}$$

17.
$$\text{Efficiency} = \frac{\text{Heat given out}}{\text{Heat put in}}$$

Let x lb. coal per I.H.P. per hour.

$$\frac{11{\cdot}4}{100} = \frac{\text{B.T.U. per horse power hour}}{14{,}000 \times x}$$

$$\text{B.T.U. per horse power hour} = \frac{33{,}000 \times 60}{778} = 2{,}545$$

$$\frac{11{\cdot}4}{100} = \frac{2{,}545}{14{,}000 \times x}, \qquad x = \frac{254{,}500}{14{,}000 \times 11{\cdot}4}$$

$x = 1{\cdot}595$ lb. per I.H.P. per hour. Ans.

SOLUTIONS TO TEST EXAMPLES XXVI.

1. Vol. of 1 lb. of fresh water $= \dfrac{1}{62 \cdot 5} \times 1{,}728$ cu. ins.

Vol. of 1 lb. of mercury $= \dfrac{1}{62 \cdot 5} \times \dfrac{1728}{13 \cdot 6}$

$= 2 \cdot 032$ cu. ins. Ans.

If the plank floated just submerged its Sp. G. would be 1.
As it floats at $\frac{4}{6}$ of its depth its Sp. G. is $\frac{4}{6}$. Ans.
Wt. of plank = Wt. of displaced water
$= 12 \times 1 \times \frac{4}{12} \times 62 \cdot 5$
$= 250$ lb. Ans.

Draught in salt water $= 4 \times \dfrac{62 \cdot 5}{64} = 3 \cdot 906$ inches. Ans.

Wt. of drum = surface of drum $\times 1 \cdot 75$
$= \left[(2 \times \frac{11}{14} \times \frac{5}{2} \times \frac{5}{2}) + (\frac{22}{7} \times \frac{5}{2} \times 4) \right] \times 1 \cdot 75 = 72 \cdot 17$ lb.
Wt. of oil $= \frac{11}{14} \times \frac{5}{2} \times \frac{5}{2} \times 4 \times 62 \cdot 5 \times 0 \cdot 88 = 1{,}080$ lb.
Wt. of drum and oil $= 72 \cdot 17 + 1{,}080 = 1152 \cdot 17$ lb.
Wt. of displaced fresh water also must equal $1152 \cdot 17$ lb.
Vol. of displaced water = draught $\times$ area of drum.

Draught $\times$ Area of drum $= \dfrac{1152 \cdot 17}{62 \cdot 5}$ cu. feet.

Draught $= \dfrac{1152 \cdot 17 \times 14 \times 2 \times 2}{62 \cdot 5 \times 11 \times 5 \times 5} = 3 \cdot 753$ feet. Ans.

4. Weight of displaced sea water $= 25 + 8 = 33$ lb.
Weight of equal vol. of fresh water

$= 33 \times \dfrac{62 \cdot 5}{64}$ lb.

$$\text{p} \quad \frac{\text{Wt. of plank}}{\text{Wt. of equal vol. fresh water}} \qquad \frac{25 \times 64}{33 \times 62{\cdot}5}$$

Sp. G. $= 0{\cdot}7756$. Ans.

5.

$$\text{Wt. of cast iron in water} = \frac{7{\cdot}2 - 1}{7{\cdot}2} \times \tfrac{1}{2} \text{ ton.}$$

$$\frac{3{\cdot}1}{7{\cdot}2} \text{ ton.}$$

Then since draught $\propto$ weight

$$\therefore \text{ Increase in draught} = \frac{3{\cdot}1}{7{\cdot}2 \times 2} \times 4 = \frac{6{\cdot}2}{7{\cdot}2}$$

$$= 0{\cdot}8611 \text{ foot. Ans.}$$

$$\text{Wt. in air} - \text{Wt. in water} = 3 - 2{\cdot}75 = 0{\cdot}25 \text{ lb.}$$

$$\text{Sp. G} = \frac{\text{Wt. in air}}{\text{Wt. in air} - \text{Wt. in water}} \quad \frac{3}{0{\cdot}25} = 12.$$

$$\therefore \text{ Wt. per cubic foot} = 62{\cdot}5 \times 12 = 750 \text{ lb. Ans.}$$

Pressure due to head of liquid $= W\,H$

$\therefore W_1 H_1 = W_2 H_2$

1 cubic foot of mercury weighs 849 lb.

$\therefore 849 \times \frac{30}{12} = 54{\cdot}8 \times H_2$

$$\therefore \text{ Head of oil} = \frac{849 \times 30}{54{\cdot}8 \times 12} = 38{\cdot}73 \text{ ft. Ans.}$$

8.

$$\text{Wt. of displaced water} = \frac{\;}{8{\cdot}4} \text{ lb.} = 0{\cdot}5952 \text{ lb.}$$

Wt. of displaced oil $= 0{\cdot}5952 \times 0{\cdot}82 = 0{\cdot}488$ lb.

Pull on string $= 5 - 0{\cdot}488 = 4{\cdot}512$ lb. Ans.

9. Wt. of chain $= 15 \times 12 = 180$ lb.

Loss of weight $= \dfrac{180}{7 \cdot 6} = 23 \cdot 69$ lb.

Wt. of chain in water $= 180 - 23 \cdot 69 = 156 \cdot 31$ lb.

Work to bring chain to surface $= 156 \cdot 31 \times 10 = 1563 \cdot 1$ ft. lb.

Work to lift clear of surface

$180 + 0$

$\times 15 = 1{,}350$ ft. lb.

Total work $= 1563 \cdot 1 + 1{,}350 = 2913 \cdot 1$ ft. lb. Ans.

10. Work per cu. foot = press. sq. foot $\times$ volume in cu. ft. $= 220 \times 144 \times 1 = 31{,}680$ ft. lb.

Cu. feet per min. $= \dfrac{1{,}000 \times 18}{60 \times 62 \cdot 5} = 4 \cdot 8.$

H.P. $= \dfrac{4 \cdot 8 \times 31{,}680}{33{,}000} = 4 \cdot 608.$ Ans.

11. Quantity pumped per hour $\frac{1}{15}$ by feed pump.
Quantity pumped per hour $\frac{1}{6}$ by ballast pump.
Total per hour $= \frac{1}{15} + \frac{1}{6} = \frac{7}{30}$ of tank.

Time together $= \dfrac{1}{\;} = \ldots = 4\frac{2}{7}$ hours. Ans.

12 (*a*) Pressure on top $= 0$ because the head is nothing. Ans.

Pressure on bottom $=$ H.A. $w = \dfrac{4 \times 60 \times 40 \times 64}{2{,}240}$

$= 274\frac{2}{7}$ tons. Ans.

Pressure on long side $=$ H.A. $w = \dfrac{2 \times 60 \times 4 \times 64}{2{,}240}$

$= 13\frac{5}{7}$ tons. Ans.

(*b*)

Pressure on top $= \text{H.A. } w = \dfrac{10 \times 60 \times 40 \times 64}{2{,}240}$

$= 685 \cdot 7$ tons. Ans.

Pressure on bottom $= \text{H.A. } w = \dfrac{14 \times 60 \times 40 \times 64}{2{,}240}$

$= 960$ tons. Ans.

Pressure on long side $= \text{H.A. } w = \dfrac{12 \times 60 \times 4 \times 64}{2{,}240}$

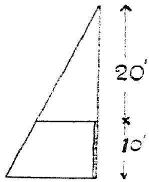
$= 82\frac{2}{7}$ tons. Ans.

13. Pressure $= \text{H.A. } w = \dfrac{5 \times 20 \times 10 \times 62 \cdot 5}{2{,}240}$

$= 27 \cdot 9$ tons. Ans.

Centre of pressure $= \dfrac{\quad}{3} = \frac{10}{3} = 3\frac{1}{3}$ feet from bottom. Ans.

14. Pressure $= \text{H.A. } w = \dfrac{25 \times 10 \times 5 \times 64}{2{,}240}$



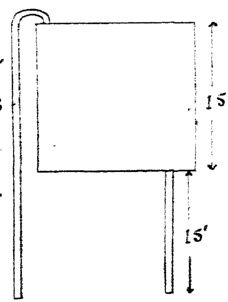
$= 35 \cdot 71$ tons. Ans.

Centre of pressure $= \frac{10}{3}\left[\dfrac{2 \times 20 + 30}{20 + 30}\right]$

$= 4\frac{2}{3}$ feet.

Centre of pressure $= 4\frac{2}{3}$ feet above bottom. Ans

15. If filled from the top, the head is constant.



Available head to cause velocity

$= 50 \times 2 \cdot 3 - 30 = 85$ feet.

$v = \sqrt{2\,g\,h} = 73 \cdot 98$ feet per sec.

Cu. ft. per sec. $= \frac{1}{3} \times \frac{1}{3} \times \frac{11}{14} \times 73 \cdot 98$

$= 6 \cdot 46$ nearly.

Cu. feet to pump in

$= 15 \times 15 \times \frac{11}{14} \times 15 = 2{,}652$ cu. ft.

Time $= \dfrac{2{,}652}{6 \cdot 46}$ secs. $= 410 \cdot 5$.

$= 6 \cdot 84$ minutes. Ans.

If filled from the bottom the head varies.

Initial head $= 50 \times 2{\cdot}3 - 15 = 100$ feet.

Final head $= 50 \times 2{\cdot}3 - 30 = 85$ feet.

$$\text{Mean head} = \frac{100 + 85}{} = 92{\cdot}5 \text{ feet.}$$

Cu. ft. per sec. $= \frac{1}{3} \times \frac{1}{3} \times \frac{11}{14} \times \sqrt{64{\cdot}4 \times 92{\cdot}5}$
$= 6{\cdot}737$ cu. feet.

$$\frac{2{,}652}{6{\cdot}737} = 393 \text{ secs.}$$

$= 6{\cdot}56$ mins. Ans.

Heat per lb. steam before blowing
$= (390 - 190) + 966 - 0{\cdot}7\ (390 - 212)$
$= 1041{\cdot}4$ B.T.U. per lb.

Heat per lb. steam when blowing $= 1041{\cdot}4 \times \frac{103}{100}$
$= 1072{\cdot}642$ B.T.U.

Additional heat per lb. steam $= 1072{\cdot}642 - 1041{\cdot}4$
$= 31{\cdot}242$ B.T.U.

This additional heat is the sensible heat which has been given to that fraction of the feed blown out. The sensible heat for one lb. of feed $= 390 - 190 = 200$ B.T.U.

For each lb. of steam formed $\dfrac{31{\cdot}242}{200} = 0{\cdot}1562$ lb. of boiler water has been blown out, therefore for each lb. of steam formed $1{\cdot}1562$ lb. of feed water must be pumped in and $0{\cdot}1562$ lb. is blown out.

$$\text{But } \frac{\text{Feed density}}{\text{Boiler density}} = \text{fraction of feed blown out.}$$

$$\frac{0{\cdot}25}{\text{Boiler density}} \qquad \frac{0{\cdot}1562}{1{\cdot}1562}$$

$$\text{Boiler density} = \frac{1{\cdot}1562 \times 0{\cdot}25}{0{\cdot}1562} = 1{\cdot}85$$

Boiler density is $1{\cdot}85$ times the sea water density. Ans.

17. Extra feed per watch $= \frac{12}{6} = 2$ tons.

For each time the water is changed in the evaporator the weight of fresh water made

$$= 0\cdot75 \times \frac{62\cdot5}{64} = 0\cdot733 \text{ ton.}$$

$$\therefore \text{ times water is changed per watch } = \frac{2}{0\cdot733} = 2\cdot73$$

Increase in density $= 2\cdot73 \times 5 = 13\cdot65$ ozs. per gall.

Final density $= 13\cdot65 + 5 = 18\cdot65$ ozs. per gall. Ans.

18. Rise in density $= 19 - 5 = 14$ ozs. per gall.

Now $0\cdot75$ ton is evaporated for every 5 ozs. rise in density

$\therefore$ Tons evaporated $= \frac{14}{5} \times 0\cdot75 = 2\cdot1$ tons. Ans.

19. In 10 lb. of boiler water there are $\frac{18}{16}$ lb. of sodium chloride.

$$\text{Total sodium chloride} = \frac{18}{16} \times \frac{25 \times 2{,}240}{10} = 6{,}300 \text{ lb.}$$

As only the sulphate and carbonate of lime is given, we assume that these only are deposited.

$$\text{Scale deposited} = \frac{1\cdot4 + 0\cdot33}{\quad} \times 6{,}300 = 474 \text{ lb. Ans.}$$

20. $(50 \times 20) - (x \times 20) + (x \times 5) = 50 \times 17$

$15\,x = 150$

$x = 10$ tons. Ans.

$(50 \times 20) - (x \times 20) + (x \times 0) = 50 \times 17$

$1{,}000 - 20\,x + 0 = 850$

$20\,x = 150$

$x = 7\cdot5$ tons. Ans.

SOLUTIONS TO TEST EXAMPLES XXVII.

1. By Simpson's rule :—

Areas	Simpson's Multiplier	Functions of areas
13400	1	13400
12800	4	51200
11940	2	23880
10380	4	41520
8210	2	16420
5100	4	20400
0	1	0

Sum = 166820

Common interval between areas = 4 feet

∴ Volume
$= \frac{4}{3} \times 166{,}820$ cubic feet.
$= 222{,}430$ cu. feet.

∴ Displacement
$$\frac{222{,}430}{35}$$
$= 6{,}355$ tons. Ans.

Block co-efficient
$$= \frac{222{,}430}{350 \times 40 \times 24}$$
$= 0{\cdot}662$. Ans.

$$\text{Block co-efficient} = \frac{9{,}200 \times 35}{450 \times 48 \times 21} = 0{\cdot}71. \text{ Ans.}(a)$$

$$\text{Prismatic co-efficient} = \frac{9{,}200 \times 35}{450 \times 911} = 0{\cdot}7854. \text{ Ans.}(b)$$

$$\text{Co-efficient of W.P.A.} = \frac{17{,}280}{450 \times 48} = 0{\cdot}8. \text{ Ans.}(c)$$

$$\text{Co-eff. of immersed midship section} = \frac{911}{48 \times 21}$$
$= 0{\cdot}9038$. Ans. (d)

Difference in draught

$$\frac{\text{Displ. in tons} \times 2{,}240 \times 16 \times 12}{13{,}000}\left(\frac{1}{1013} - \frac{1}{1026}\right)$$

$$\frac{10{,}000 \times 2{,}240 \times 16 \times 12}{13{,}000} \times \frac{1{,}026 - 1{,}013}{1{,}013 \times 1{,}026}$$

$= 4{\cdot}135$ inches. Ans.

$$\text{Tons per inch immersion} = \frac{\text{W.P.A.}}{12 \times 35} \qquad \frac{\text{W.P.A.}}{420}$$

$$\therefore \text{W.P.A.} = 420 \times 28 = 11{,}760 \text{ sq. ft.} \quad \text{Ans. } (a)$$

$$\text{Co-efficient of W.P.A.} = \frac{11{,}760}{l \times b} \text{ and } b = \frac{l}{8}$$

$$\therefore 0{\cdot}75 \times l \times \frac{l}{8} = 11{,}760$$

$$\therefore l = \sqrt{\frac{\quad}{0{\cdot}75}} = 354{\cdot}2 \text{ feet.} \quad \text{Ans. } (b)$$

5. Area of water plane $= 320 \times 35 \times 0{\cdot}8 = 8{,}960$ sq. ft.

$$\text{Tons per inch immersion} = \frac{8{,}960}{420} = 21{\cdot}33$$

$$\therefore \text{increase in draught} = \frac{300}{21{\cdot}33} = 14{\cdot}06 \text{ (say 14 ins)}$$

then draught forward $= 18' \, 6'' + 1' \, 2'' = 19$ ft. 8 ins.
and draught aft $= 20' \, 0'' + 1' \, 2'' = 21$ ft. 2 ins. Ans.

6. Water plane area before bilging $= 110 \times 20$ sq. ft.

Water plane area after bilging
$= (110 - 25) \times 20$ sq. ft.

Since the weight of the barge does not change, the volume displaced must be the same.

Let x feet = new draught

Volume before bilging = volume after

$$110 \times 20 \times 6 = (110 - 25) \times 20 \times x$$

$$\frac{110 \times 6}{85} = 7{\cdot}765 \text{ ft. or 7 ft. } 9{\cdot}2 \text{ ins.} \quad \text{Ans.}$$

Water plane area before bilging $= 180 \times 38$ sq. ft.
Water plane area after bilging
$= (180 - 0{\cdot}6 \times 28) \times 38$ sq. ft.

Note that, since the permeability is 60%, the loss of water plane area is $0{\cdot}6 \times (28 \times 38)$ sq. ft., therefore the water plane area after bilging is

$(180 \times 38) - (0{\cdot}6 \times 28 \times 38)$
$= (180 - 0{\cdot}6 \times 28) \times 38$ sq. ft.

Let x feet $=$ new draught,

$180 \times 38 \times 9 = (180 - 16{\cdot}8) \times 38 \times x$

$$\therefore x = \frac{180 \times 9}{163{\cdot}2} = 9{\cdot}926 \text{ ft. or } 9 \text{ ft. } 11{\cdot}1 \text{ ins.} \quad \text{Ans.}$$

$$\overline{BM} = \frac{I}{V} = \frac{4{,}200{,}000}{10{,}000 \times 35} = 12 \text{ feet.}$$

$\overline{KM} = \overline{KB} + \overline{BM} = 14{\cdot}75 + 12 = 26{\cdot}75$ feet.
$\overline{KG} = \overline{KM} - \overline{GM} = 26{\cdot}75 - 2 = 24{\cdot}75$ feet.
$\therefore$ the centre of gravity is $24{\cdot}75$ feet above the keel.
Ans.

$$\overline{BM} = \frac{I}{} \qquad \frac{l \times b^3}{12 \times l \times b \times d} \qquad \frac{b^2}{12\,d}$$

$$\frac{35^2}{12 \times 18} = 5{\cdot}67 \text{ feet.}$$

$\overline{KG} = 10{\cdot}5$ ft. $\therefore \overline{BG} = 1{\cdot}5$ ft.
$\therefore \overline{GM} = \overline{BM} - \overline{BG} = 5{\cdot}67 - 1{\cdot}5 = 4{\cdot}17$ ft.

$$\text{Displacement} = \frac{250 \times 35 \times 18}{35} = 250 \times 18 \text{ tons.}$$

$$\theta = \frac{4}{360} \times 2\pi = \frac{\pi}{45} \text{ radian.}$$

Righting moment $= W\,\overline{GM}\,\theta$

$= 250 \times 18 \times 4{\cdot}17 \times \dfrac{\quad}{45}$

$= 1{,}310$ ft. tons. Ans.

10. Disturbing moment = Restoring moment

$$w \times d = W\ \overline{GM}\ \theta$$

$$30 \times 20 = 6{,}240 \times \overline{GM} \times \frac{12 \cdot 5}{20 \times 12}$$

$$\therefore \overline{GM} = \frac{30 \times 20 \times 20 \times 12}{6{,}240 \times 12 \cdot 5}$$

$= 1 \cdot 846$ feet. Ans.

11. Change of trim $= \dfrac{w\ d\ l}{W\ \overline{GM}} = \dfrac{35 \times 280 \times 500 \times 12}{12{,}000 \times 540}$

$= 9 \cdot 08$ inches.

decrease in draught forward, and increase in draught aft

$9 \cdot 08$
$= 4 \cdot 54$ ins.

$\therefore$ draught forward $= 25'\ 6'' - 4 \cdot 54'' = 25$ ft. $1 \cdot 46$ ins. Ans.

draught aft $= 25'\ 7'' + 4 \cdot 54'' = 25$ ft. $11 \cdot 54$ ins. Ans.

12. Total load placed on board $= 50 + 60 + 55 + 10 = 175$ tons.

Parallel sinkage $= \frac{175}{28} = 6 \cdot 25$ ins.

Moments forward $= 50 \times 120 = 6{,}000$ ft. tons.

Moments aft $= (60 \times 40) + (55 \times 80) + (10 \times 136) = 8160$ ft. tons.

Nett change of moment $= 8{,}160 - 6{,}000 = 2{,}160$ ft. tons (aft).

$\therefore$ change of trim $= \frac{2160}{720} = 3$ inches.

Draught forward $= 18'\ 0'' + 6 \cdot 25'' - 1 \cdot 5'' = 18$ ft. $4\frac{3}{4}$ ins.

and draught aft $= 18'\ 0'' + 6 \cdot 25'' + 1 \cdot 5'' = 18$ ft. $7\frac{3}{4}$ ins. Ans.

SOLUTIONS TO TEST EXAMPLES XXVIII.

1. Weight of deposit $= \mathrm{I} \times t \times$ E.C.E.

$$(19{\cdot}34 - 14{\cdot}52) = \mathrm{I} \times 50 \times 60 \times 0{\cdot}00033$$

$$\therefore \mathrm{I} = \frac{4{\cdot}82}{50 \times 60 \times 0{\cdot}00033} = 4{\cdot}869 \text{ amps.}$$

Error of reading $= 5{\cdot}1 - 4{\cdot}869 = 0{\cdot}231$ ampére. Ans.

2. $\mathrm{E} = \mathrm{I\,R} = 125 \times 20 = 2{,}500$ volts. Ans.

3. $$\mathrm{R} = \frac{s\,\mathrm{L}}{\mathrm{A}} \quad \therefore s = \frac{\mathrm{R\,A}}{\mathrm{L}}$$

$$s = \frac{0{\cdot}0469 \times \frac{1}{1}\frac{1}{4} \times (0{\cdot}1 \times 2{\cdot}54)^2}{10 \times 36 \times 2{\cdot}54}$$

$= 0{\cdot}0000026$ ohm per cm. cube

$= 2{\cdot}6$ microhms per cm. cube. Ans.

4. As the material is the same, then the specific resistance is the same.

Then · $\quad . \ \mathrm{R}_2 =$

$\mathrm{L}_1 \qquad \mathrm{L}_2 \qquad \mathrm{A}_2\,\mathrm{L}_1$

Therefore $$\frac{0{\cdot}163 \times \frac{1}{1}\frac{1}{4} \times (\frac{1}{4})^2 \times 500 \times 12}{\frac{1}{1}\frac{1}{4} \times (\frac{1}{8})^2 \times 1{,}000 \times 12}$$

$0{\cdot}326$ ohm. Ans.

5. $\mathrm{R}_0 \qquad [1$

$\quad . \ \mathrm{R}_2 =$

$\mathrm{R}_0\,[1 + a\,t_2] \qquad [1 + a\,t_1]$

$$= \frac{25\,[1 + 0{\cdot}00428 \times 55]}{1 + 0{\cdot}00428 \times 15}$$

$= 29{\cdot}02$ ohms. Ans.

6. $$\mathrm{R}_2 = \frac{\mathrm{R}_1\,[1 + a\,t_2]}{[1 + a\,t_1]} \qquad \frac{\mathrm{R}_1\,[1 + 0{\cdot}00428 \times 50]}{1 + 0{\cdot}00428 \times 10}$$

$$\therefore R_2 = \frac{R_1 \times 1{\cdot}214}{1{\cdot}0428}$$

$E = I\,R$, E is constant, $\therefore I_1\,R_1 = I_2\,R_2$

$$\text{then } 10 \times R_1 = I_2 \times \frac{R_1 \times 1{\cdot}214}{1{\cdot}0428}$$

$$\therefore I_2 = \frac{10 \times R_1 \times 1{\cdot}0428}{R_1 \times 1{\cdot}214} = 8{\cdot}59 \text{ ampéres. Ans.}$$

7. $$\frac{1}{R} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} + \frac{1}{r_4} = \tfrac{1}{8} + \tfrac{1}{10} + \tfrac{1}{12} + \tfrac{1}{9}$$

$$\therefore \frac{1}{R} = \frac{135 + 108 + 90 + 120}{1{,}080} = \frac{453}{1{,}080}$$

$\therefore$ Equivalent resistance $(R) = \frac{1080}{453} = 2{\cdot}384$ ohms. Ans.

In the first group :—

$$\frac{1}{R_1} = \tfrac{1}{6} + \tfrac{1}{3} + \tfrac{1}{4} = \frac{2 + 4 + 3}{12} = \tfrac{9}{12}$$

$\therefore$ Equivalent resistance (R_1) across group $= \frac{12}{9} = 1\frac{1}{3}$ ohms.

In the second group :—

$$\frac{1}{R_2} = \tfrac{1}{8} + \tfrac{1}{10} = \frac{10 + 8}{80} = \tfrac{18}{80}$$

$\therefore$ Equivalent resistance (R_2) across group $= \frac{80}{18} = 4\frac{4}{9}$ ohms.

R_1 and R_2 are in series, therefore, resistance across the whole system $= R_1 + R_2 = 1\frac{1}{3} + 4\frac{4}{9} = 5\frac{7}{9}$ ohms. Ans.

Let equivalent resistance $= R$

$$\text{Then } \frac{1}{R} = \tfrac{1}{4} + \tfrac{1}{3} + \tfrac{}{o}$$

$$\therefore R = \tfrac{4}{9} \text{ ohm.}$$

P.D. across system $= \text{I R} = 36 \times \frac{4}{9} = 16$ volts.

Current in 4 ohm resistance $= \dfrac{\text{E}}{\text{R}} = \frac{16}{4} = 4$ ampéres $\Big\}$ Ans.

2 —16— 81

" 0·6 "

10. Work done in pumping 2,800 gallons of water against a pressure of 220 lb. per sq. inch $= 2{,}800 \times 10 \times 220 \times \dfrac{144}{62{\cdot}5}$ foot pounds per hour.

This is equivalent to a horse power of

$$\frac{2{,}800 \times 10 \times 220 \times 144}{60 \times 33{,}000 \times 62{\cdot}5} = 7{\cdot}168$$

Combined efficiency of pump and motor $= 0{\cdot}82 \times 0{\cdot}$

$\therefore$ Horse power input of motor

$$\frac{7{\cdot}168}{0{\cdot}82 \times 0{\cdot}89} \qquad \frac{7{\cdot}168 \times 746}{0{\cdot}82 \times 0{\cdot}89} \text{ watts.}$$

$\therefore$ Current taken by motor

$$\frac{7{\cdot}168 \times 746}{0{\cdot}82 \times 0{\cdot}89 \times 440} = 16{\cdot}65 \text{ ampéres.} \quad \text{Ans.}$$

11. Watts supplied $= \dfrac{\text{E}^2}{\text{R}} = \dfrac{200^2}{160} = 250$ watts $= 0{\cdot}25$ kilowatt.

$0{\cdot}25$ kw. for 500 hours $= 0{\cdot}25 \times 500 = 125$ B.O.T. units. Ans. (*a*)

Current flowing $= \dfrac{\text{E}}{\text{R}} = \frac{200}{160}$ ampéres.

$\therefore$ Quantity of electricity $= \frac{200}{160} \times 500 = 625$ ampére-hours. Ans. (*b*)

12. Weight of $1\frac{1}{2}$ pints of water $= \frac{3}{2} \times \frac{10}{8} = \frac{15}{8}$ lb.

Heat required $= \frac{15}{8}(212 - 42) = \dfrac{15 \times 85}{\quad}$ B.T.U.

$$\dfrac{15 \times 85}{\quad} \times 1{,}056 \text{ watt-seconds.}$$

Heater output $= \dfrac{220^2}{120} \times 0{\cdot}84$ watts.

$\therefore$ Time in seconds $= \dfrac{15 \times 85 \times 1{,}056 \times 120}{4 \times 220 \times 220 \times 0{\cdot}84} = 993$

$= 16$ minutes 33 seconds. Ans.

13. Output of dynamo $= 22 \times 746 \times 0{\cdot}9$ watts.

$\therefore$ each lamp must take $\dfrac{22 \times 746 \times 0{\cdot}9}{400}$

$= 36{\cdot}927$ watts.

Consumption per candle power $= \dfrac{36{\cdot}927}{30}$

$= 1{\cdot}2309$ watts per c.p. Ans.

14. Total E.M.F. $= 86 \times 1{\cdot}8 = 154{\cdot}8$ volts.

Total resistance $= (86 \times 0{\cdot}3) + 64 + 0{\cdot}2$
$= 25{\cdot}8 + 64 + 0{\cdot}2 = 90$ ohms.

$\therefore$ current flowing through circuit

$\dfrac{154{\cdot}8}{90} = 1{\cdot}72$ ampéres. Ans. (*a*).

Volt drop in battery $= IR = 1{\cdot}72 \times 25{\cdot}8 = 44{\cdot}376$ volts.

$\therefore$ terminal p.d. of battery $= 154{\cdot}8 - 44{\cdot}376 = 110{\cdot}424$ volts. Ans. (*b*).

Horse power output

$\dfrac{110{\cdot}424 \times 1{\cdot}72}{746}$ $0{\cdot}2546$ H.P. Ans. (*c*).

15.

$$\text{Current flowing} = \frac{E}{R} = \tfrac{220}{5} = 44 \text{ ampéres.}$$

$$\text{Output} = \frac{220 \times 44}{746} = 12{\cdot}97 \text{ horse power. Ans. } (a)$$

Total internal resistance $= 0{\cdot}15 + 0{\cdot}082 = 0{\cdot}232$ ohm.

Volts lost internally $= 44 \times 0{\cdot}232 = 10{\cdot}208$ volts.

$\therefore$ internal voltage $= 220 + 10{\cdot}208 = 230{\cdot}208$ volts.

$$\text{Input} = \frac{230{\cdot}208 \times 44}{746} = 13{\cdot}577 \text{ horse power. Ans. } (b)$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \quad \frac{12{\cdot}97}{13{\cdot}577} = 0{\cdot}955. \text{ Ans. } (c)$$

16.

$$\text{Current flowing} = \frac{W}{E} = \frac{50 \times 1{,}000}{100} = 500 \text{ ampéres.}$$

E.M.F. across series winding $= 500 \times 0{\cdot}005 = 2{\cdot}5$ volts.

E.M.F. across armature $= 500 \times 0{\cdot}007 = 3{\cdot}5$ volts.

$\therefore$ internal voltage of armature $= 100 + 2{\cdot}5 + 3{\cdot}5 = 106$ volts. Ans. (a)

Watts lost in series winding $= 2{\cdot}5 \times 500 = 1{,}250 = 1{\cdot}25$ kilowatts.

Watts lost in armature $= 3{\cdot}5 \times 500 = 1{,}750 = 1{\cdot}75$ kilowatts.

Total loss $= 1{\cdot}25 + 1{\cdot}75 = 3$ kilowatts.

$$\text{Electrical efficiency} = \frac{\text{Output}}{\text{Input}} \quad \frac{50}{50 + 3} = 0{\cdot}943. \text{ Ans. } (b)$$

17.

$$\text{Output current} = \frac{W}{E} = \frac{200 \times 1{,}000}{200} = 1{,}000 \text{ ampéres.}$$

E.M.F. applied to shunt winding is 200 volts.

Current passing through shunt $= \tfrac{200}{20} = 10$ ampéres.

$\therefore$ current passing through armature $= 1{,}000 + 10 = 1{,}010$ ampéres.

E.M.F. across armature $= I\,R = 1{,}010 \times 0{\cdot}008 = 8{\cdot}08$ volts.

Watts lost in armature $= E\,I = 8{\cdot}08 \times 1{,}010 = 8{,}160{\cdot}8 = 8{\cdot}16$ kilowatts.

Watts lost in shunt $= 200 \times 10 = 2{,}000 = 2$ kilowatts.

Total loss $= 8{\cdot}16 + 2 = 10{\cdot}16$ kilowatts.

$$\text{Electrical efficiency} = \frac{\text{Output}}{\text{Input}} \quad \frac{200}{200 + 10{\cdot}16}$$

$= 0{\cdot}951$. Ans.

18 $$\text{Output current} = \frac{W}{E} = \frac{120 \times 1{,}000}{110} = 1{,}091 \text{ ampéres.}$$

E.M.F. across series winding $= 1{,}091 \times 0{\cdot}002 = 2{\cdot}182$ volts.

E.M.F. applied to shunt $= 110 + 2{\cdot}182 = 112{\cdot}182$ volts.

$$\text{Current passing through shunt} = \frac{112{\cdot}182}{30} = 3{\cdot}739 \text{ ampéres.}$$

$\therefore$ current passing through armature $= 1{,}091 + 3{\cdot}739 = 1{,}094{\cdot}739$ ampéres.

E.M.F. across armature $= 1{,}094{\cdot}739 \times 0{\cdot}006 = 6{\cdot}568$ volts.

Watts lost in armature $= 6{\cdot}568 \times 1{,}094{\cdot}739 = 7{,}190 = 7{\cdot}19$ kilowatts.

Watts lost in shunt $= 112{\cdot}182 \times 3{\cdot}739 = 419{\cdot}5 = 0{\cdot}4195$ kilowatt.

Watts lost in series winding $= 2{\cdot}182 \times 1{,}091 = 2{,}380 = 2{\cdot}38$ killwatts.

Total loss $= 7{\cdot}19 + 0{\cdot}4195 + 2{\cdot}38 = 9{\cdot}9895$ kilowatts.

$$\text{Electrical efficiency} = \frac{\text{Output}}{\text{Input}} \quad \frac{120}{120 + 9{\cdot}9895}$$

$= 0{\cdot}923$. Ans.

SOLUTIONS TO TEST EXAMPLES XXIX.

1.

$$\text{Force in dynes} = \frac{B\,I\,l}{10} = \frac{8{,}000 \times 75 \times 25}{10}$$

$$= 1{,}500{,}000 \text{ dynes. Ans. } (a)$$

$$\text{Force in lb.} = \frac{1{,}500{,}000}{445{,}000} = 3 \cdot 37 \text{ lb. Ans. } (b)$$

2.

$$\text{Force on one conductor} = \frac{7{,}000 \times 30 \times 12 \times 2 \cdot 54}{10 \times 445000}$$

$$= 1 \cdot 438 \text{ lb.}$$

Number of conductors in field $= \frac{2}{3}$ of $720 = 480$

$$\text{Torque} = 480 \times 1 \cdot 438 \times \frac{14}{12 \times 2} = 402 \cdot 6 \text{ ft. lb.}$$

Ans. (a)

$$\frac{\text{Torque (ft. lb.)} \times \text{radians turned per minute}}{33{,}000}$$

$$\text{H.P. developed} = \frac{402 \cdot 6 \times 2\,\pi \times}{33{,}000} = 52 \cdot 11 \text{ H.P.}$$

Ans. (b)

3. E.M.F. $= B\,l\,v \times 10^{-8}$

,, $= 8{,}200 \times 15 \times 32 \times 100 \times 10^{-8}$

,, $= 3 \cdot 936$ volts. Ans.

4.

$$\text{Rate of cutting flux} = 90{,}000 \times 2 \times \frac{1{,}000}{60}$$

$= 3 \times 10^6$ lines per second.

Total number of conductors, which are all in series $= 2 \times 420 = 840$

Total average E.M.F. $= 840 \times 3 \times 10^6 \times 10^{-8}$

,, $= 25 \cdot 2$ volts. Ans.

5. E.M.F. $= \Phi\,Z\,n \times 10^{-8}$

$$\frac{4 \cdot 5 \times 10^6 \times 660 \times 920}{10^8 \times 60} = 455 \cdot 4 \text{ volts. Ans.}$$

$$\text{Current in shunt} = \frac{400}{200} = 2 \text{ amps.}$$

$\therefore$ Current in armature $= 180 - 2 = 178$ amps.

Voltage drop in armature $= 178 \times 0{\cdot}02 = 3{\cdot}56$ volts.

Voltage drop in armature and brushes $=$
$3{\cdot}56 + 2 = 5{\cdot}56$ volts.

$\therefore$ Back E.M.F. $= 400 - 5{\cdot}56 = 394{\cdot}44$ volts. Ans. (*a*)

$$\text{Horse power} = \frac{394{\cdot}44 \times 178}{746} = 94{\cdot}1. \quad \text{Ans. } (b)$$

Output $= 394{\cdot}44 \times 178 = 70200$ watts.

Input $= 400 \times 180 = 72{,}000$ watts.

$$\therefore \text{ Efficiency} = \frac{70{,}200}{72{,}000} = 0{\cdot}975. \quad \text{Ans. } (c)$$

The losses not taken into account are those due to friction, windage, and what are termed "iron losses."

Reactance $= 2\,\pi f l = 2 \times \pi \times 50 \times 0{\cdot}01 = 3{\cdot}1416$ ohms

Impedance $= \sqrt{3^2 + (3{\cdot}1416)^2} = \sqrt{9 + 9{\cdot}867}$
$= 4{\cdot}344$ ohms. Ans. (*a*)

$$\text{Power factor} = \frac{\text{Resistance}}{\text{Impedance}} \qquad 4{\cdot}344 \qquad = 0{\cdot}6906$$
Ans. (*b*)

$$\text{Power absorbed} = \frac{E^2}{Z} \times \text{power factor}$$

$$\text{,,} \qquad = \frac{60^2}{4{\cdot}344} \times 0{\cdot}6906 = 572{\cdot}3 \text{ watts.}$$
Ans. (*c*)

8.

$$\text{Current in lamp} = \frac{W}{E} = \frac{100}{100} = 1 \text{ ampére.}$$

$$\text{Resistance of lamp} = \frac{E}{I} = \frac{100}{1} = 100 \text{ ohms.}$$

Total resistance to give 1 amp. across 220 volt mains

$$= \frac{220}{1} = 220 \text{ ohms.}$$

$\therefore$ Series resistance $= 220 - 100 = 120$ ohms. Ans.

Total power absorbed $= 220 \times 1 = 220$ watts. Ans.

In the 2nd case, with inductance and resistance of lamp in series,

$$\text{Impedance} = \frac{220}{1} = 220 \text{ ohms.}$$

$$\text{Reactance} = \sqrt{(220)^2 - (100)^2} = 196 \text{ ohms.}$$

$$\text{Reactance} = 2\,\pi\,f\,l$$

$$\therefore 196 = 2\,\pi \times 50 \times l$$

$$l = \frac{196}{2\,\pi \times 50} = 0{\cdot}624 \text{ henry.} \quad \text{Ans.}$$

The inductance does not absorb power, therefore total power absorbed $= 100$ watts. Ans.

Or alternatively,

$$\text{Power factor} = \frac{R}{Z} = \frac{100}{220} = \frac{5}{11}$$

$\therefore$ Power absorbed $=$ E I $\times$ power factor

$$= 220 \times 1 \times \tfrac{5}{11} = 100 \text{ watts.} \quad \text{Ans.}$$

9. Output $= 60 \times 746 = 44{,}760$ watts.

E I $\times$ efficiency $\times$ power factor $= 44{,}760$

$$\therefore I = \frac{44{,}760}{440 \times 0{\cdot}86 \times 0{\cdot}9} = 131{\cdot}4 \text{ amps.} \quad \text{Ans.}$$

10 Input $= 6 \times 18$ amp. hours.

Output $= 3{\cdot}5 \times 28$ amp. hours.

$$\text{Amp. hour efficiency} = \frac{3{\cdot}5 \times 28}{6 \times 18} = 0{\cdot}907,$$

or $90{\cdot}7\%$. Ans.

DATA GIVEN AT THE EXAMINATION

Material	Weight in pounds		Specific Gravity	Specific Heat
	per cu. in.	per cu. ft.		
Cast Iron ...	0·26	450	7·21	0·13
Wrought Iron	0·281	486	7·78	0·113
Steel	0·283	490	7·86	0·116
Copper ...	0·317	548	8·77	0·095
Brass	0·303	524	8·4	0·094
Lead	0·412	712	11·4	0·029
Mercury ...	0·491	849	13·6	0·033

One cubic foot of fresh water weighs 62·5 pounds.

One cubic foot of sea water weighs 64 pounds.

One cubic foot contains 6·25 gallons.

1 Metre = 39·37 inches.

1 Litre = 0·22 gallon.

1 Kilogram = 2·2 pounds.

Latent heat of steam = 966 — 0·7 (T — 212) B.T.U. per pound.

Latent heat of water = 143 B.T.U. per lb.

Absolute temperature = 460 + degrees Fah.

Joule's equivalent = 778 foot pounds = 1 B.T.U.

Simpson's Rule.—To the sum of the first and last ordinates add four times the sum of the even ordinates and two times the sum of the odd ordinates. Multiply the result by one-third the common interval between the ordinates. This gives the area of the figure.

SECOND-CLASS EXAMINATION QUESTIONS.

GENERAL ENGINEERING SCIENCE

1. In a lever safety valve, the valve diameter is 3 inches, and the boiler pressure is 90 pounds per square inch. The length of the lever between the centre of the valve and the weight is 12 inches, and the weight on the end of the lever is 90 pounds. Find the distance between the fulcrum and the centre of the valve.

2. A barrel is 2 feet 6 inches diameter at its ends, 2 feet 10 inches at quarter length from each end, and 3 feet at the centre. Its length is 4 feet. How many gallons will it hold ?

3. The stays on the flat surface of a boiler are $1\frac{5}{8}$ ins. diam. and are pitched 8 inches apart. The working pressure of the boiler is 160 pounds per square inch. If one stay breaks what is then the stress on the adjacent stays ?

Note.—When a stay breaks, the additional stress on each of the near stays may be taken as one-third of the stress on the broken stay.

4. A tank is 4 feet 3 inches deep. It can be run up in 45 minutes and pumped out in $1\frac{1}{4}$ hours. It is filled to a depth of 1 foot 6 inches, and then the injection is opened and the pump started. How long will it take to fill the tank ?

5. A boiler is being tested to 430 pounds per square inch by a hand pump. The plunger of the pump is 2 inches diameter, the distance from the fulcrum to the centre of the plunger is $3\frac{1}{4}$ inches and from the centre of the plunger to the end of the handle is $36\frac{3}{4}$ inches. Find the force required at the end of the handle.

6. A solid drawn steam pipe is 3 inches internal diameter, and carries a pressure of 180 lb. per square inch.

$$\text{Given working pressure} = \frac{' - 10}{D} \times 120$$

the thickness in inches,

if T is the thickness in hundredths,

and D is the internal diameter in inches.

7. A tank, 124 feet long, 42 feet wide, and 3 feet deep, is filled with sea water. The discharge is 19·5 feet above the top of the tank. An 18 horse power pump empties the tank in 45 minutes. What is the percentage loss of work in the pump ?

8. The longitudinal seams of a boiler are treble riveted double butt strap joints with alternate rivets omitted in the outer row. The rivets are $1\frac{1}{4}$ inches diameter and the plate is $1\frac{1}{8}$ inches thick. The pitch is $8\frac{1}{2}$ inches and the factor for double shear is $1\frac{3}{4}$. The tensile strength of the plate is 28 tons per square inch, and the shear strength of the rivets is 23 tons per square inch. What is the percentage strength of the seam ?

9. The sectional area of a Whitworth bolt at the bottom of the thread is given by the formula :—

$$\frac{8\,D\,(8\,D - 1)}{100},$$ where D is the diameter of the bolt in inches.

A cylinder is 36 inches diameter inside the joint. The steam pressure is 160 pounds per square inch. There are 25 bolts, $1\frac{1}{2}$ inches diameter in the cover. Find the stress in the bolts due to the steam pressure.

10. A cylindrical buoy is 3 tons weight and floats in fresh water at 5 feet draught with its axis vertical. A cast iron weight of three-quarters ton is suspended from the bottom. Find the increase in draught.

11. A rectangular beam is 3 feet long and $2\frac{3}{4}$ inches broad. It is fixed at one end and a weight of 2,800 pounds is suspended from the other. The maximum stress produced is 6,000 pounds per square inch. Find the depth of the beam.

12. A derrick is 20 feet long. It is suspended from a vertical post by a topping lift, 10 feet long, at right angles to the vertical post. The weight of the derrick is 750 lb. and its centre of gravity is at mid length. Find the tension in the topping lift and the force in the derrick.

13. A bearing is 12 inches diameter and 13 inches long. The lower brass is lined with white metal for 40 per cent. of the circumference, and 9 per cent. of the metal is cut away for clearance at the sides. What is the area of bearing surface of the lower brass ?

14. A boiler is 12 feet diameter. The rivets in the longitudinal seam are 1·625 inches diameter and $7\frac{1}{2}$ inches pitch. The shell plate is $1\frac{1}{2}$ inches thick and the working pressure of the boiler is 180 pounds per square inch. What is the stress produced in the plate between the rivet holes ?

15. In a wheel and axle the drum is 12 inches diameter, the rope is $1\frac{1}{4}$ inches diameter, and the length of handle 18 inches. The force applied on the handle is 90 lb., and the efficiency 79 per cent. What weight will be lifted ?

16. The diameter of a piston is 72 inches. The effective pressure on the piston when the crank is horizontal is 20 pounds per square inch. The connecting rod is 4 times as long as the crank. The diameter of the crank pin is 14 inches, and the pressure per square inch on the pin is 400 pounds. Find the length of the crank pin.

17. A 'thwartship bunker is 48 feet long. The widths at the top, $\frac{3}{4}$ height, $\frac{1}{2}$ height, $\frac{1}{4}$ height and bottom are 42·5 feet, 41·5 feet, 40 feet, 36 feet and 25 feet respectively. The height is 26 feet. How many tons of coal will it hold at 46 cubic feet to the ton ?

18. An engine indicates 30 horse power and its mechanical efficiency is 83·33 per cent. How long will it take to pump 2,500 cubic feet of sea water to an average height of 22 feet, neglecting losses in the pump itself ?

19. A shaft is subjected to torsion by a crank acted upon by a piston. The shaft is $3\frac{7}{8}$ inches diameter, the crank 11 inches long, and the piston 14 inches diameter. The effective pressure on the piston is 32 pounds per square inch. Is the shaft strong enough and what stress is produced in it ?

Note. D =

$$q$$

Where D = diameter of shaft in inches.
T = twisting moment in inch pounds.
q = stress produced in pounds per sq. inch.

20. A roll of jointing material is 10 inches diameter, and the roller on which it is wrapped is 2 inches diameter. The thickness of the jointing is 2·5 millimetres. Find its length.

21. An eccentric sheave is 8 inches diameter, and the shaft hole is $3\frac{1}{2}$ inches diameter. The travel of the valve is $3\frac{1}{4}$ inches. How far is the centre of gravity of the sheave from its geometrical centre ?

22. The water level of a boiler is 264 square feet in area. The pressure is 180 pounds per square inch. A rivet $1\frac{1}{4}$ inches in diameter is blown out below the furnaces. There are 12 inches of water over the combustion chamber top. How long will it take to expose that plate ?

Note.—The quantity of water that will flow through a rivet hole per minute $= 2\frac{1}{2}\ d^2\ \sqrt{P}$ cubic feet.

Where d = diameter of the hole in inches,
and P = head of pressure in pounds per square inch.

23. A weight of 2 tons is hung at the centre point of a beam that is supported at both ends. The distance between the supports is 8 feet, and the beam is 6 inches broad and 9 inches deep. Find the maximum stress.

24. A safety valve is 4 inches diameter. The spring is made of $\frac{3}{4}$ inch square steel, and the boiler pressure is 180 pounds per square inch. What is the outside diameter of the spring ?

Note.—The Board of Trade formula for the size of square steel for safety valve springs is :—

$$S = \sqrt[3]{\frac{D\,W}{11{,}000}}$$

Where S = size of steel in inches.
D = mean diameter of spring.
W = load on spring in pounds.

25. A truck of coal, weighing 5 tons, is drawn up an incline 30 yards long and 10 feet high in 30 secs. The tractive resistance on the level is 10 per cent. of the load. What is the horse-power exerted while doing this work ?

26. A thrust shaft is $12\frac{1}{2}$ inches diameter. There are 8 collars 20 inches diameter and $2\frac{1}{2}$ inches thick. The couplings are 25 inches diameter and $3\frac{1}{2}$ inches thick. The overall length is 8 feet. Find the weight of the shaft.

27. A stuffing box is 10 inches diameter and $8\frac{3}{4}$ inches deep, and the packing is $1\frac{1}{4}$ inches square. What length of packing will be required to fill the stuffing box allowing $\frac{1}{2}$ inch between the ends of each turn ?

28. The width of a propeller blade at 6 feet radius is 4 feet. The distance the forward edge leads the after edge is 2 feet. What is the pitch of the propeller ?

29. A boiler shell plate is $\frac{5}{8}$ inch thick. The longitudinal seam is a double riveted lap joint with $\frac{7}{8}$ inch diameter rivets. The tensile strength of the plate is 28 tons per square inch, and the shearing strength of the rivets 26 tons per square inch. What pitch will give the highest efficiency of joint ?

30. A pump delivers 1,000 gallons of water per minute against a pressure of 160 lb. per square inch. The efficiency of the pump is 60 per cent. Find the horse power of the pump.

31. The load on a fork-ended stay is 20 tons, and the stress allowed on the pin is 5,000 lb. per square inch. Allowing a factor of $1\frac{1}{2}$ for double shear, find the diameter of the pin.

32. A furnace is 42 inches diameter and 5 feet 6 inches long. The working pressure is 135 lb. per square inch. Find the thickness.

Note.—Working pressure

$$\frac{99{,}000\ T^2}{(L + 1) \text{ in feet } \times \text{ diameter in inches}}$$

providing that it is does not exceed that given by

$$\text{Working pressure} = \frac{9{,}900\ T}{\text{Diameter in inches}}$$

33. A parallelogram has sides 50 feet and 30 feet long. The angle between adjacent sides is 60°. Find the area.

34. A cubical block of stone rests on one face. The length of each side is 3 feet and the weight is 4,455 lb. Find the work done in tilting the block on one edge until the diagonal is at 60°.

35. In a water cistern the spherical float is 6 inches diameter and floats half immersed when the valve is closed. The distance from the fulcrum to the centre of the float is 15 inches and from the fulcrum to the centre of the valve spindle is $1\frac{1}{2}$ inches. The valve is $\frac{1}{2}$ inch diameter. What is the pressure per square inch on the valve ?

36. A stone, 2 feet wide, 2 feet deep, projects 3 feet from a dock wall. The top face is level with the surface of the water. What is the pressure on the end, each side, and bottom of the stone ?

37. The beam of a weighing machine is 24 inches long. The knife edge on which it balances is $\frac{1}{32}$ inch out of the centre. What is the error in weighing 18 pounds ?

38. A pulley makes 180 revolutions per minute. It is 3 feet diameter and the difference in tension between the slack side and the tight side of the belt is 120 lb. Find the horse power transmitted.

39. A steel shaft, 10 inches diameter, transmits 1,000 H.P. at 72 revolutions per minute. What horse power will a wrought iron shaft 8 inches diameter transmit at 80 revolutions per minute if the stress in the steel shaft is 40 per cent. greater than that in the iron one ?

40. A cubic foot of oil weighs 55 pounds. If the barometer stands at 29 inches, what would be the height of a column of oil to balance the atmospheric pressure ?

41. A solid shaft is 12 inches diameter. A hollow shaft is 12 inches diameter outside and 5 inches diameter inside. By what percentage is one shaft stronger than the other ?

42. The stays of a combustion chamber are pitched $9\frac{1}{4}$ inches in both the vertical and horizontal direction, and the plate is $\frac{11}{16}$ inch thick. Find the working pressure.

Note.—Working pressure $= \dfrac{100\ (T + 1)^2}{S - 6}$

Where T = Thickness of plate in sixteenths of an inch.
S = Surface supported by each stay in square inches.

43. A twisting moment of 9,600 inch pounds just breaks an inch diameter bar. What would be the diameter of a bar broken by a force of 270 pounds acting at a radius of 15 inches ?

44. The deck beams of a ship weigh 32 lb. per foot run and are placed 3 feet apart. The plating weighs 10 lb. per square foot. The deck is covered with teak, 3 inches thick and weighing 50 lb. per cubic foot. Find the weight of a portion of the deck 24 feet long fore and aft, and 15 feet wide.

45. A fathom of rope weighs 20 pounds. The rope is hanging down a mine to a depth of 80 feet, and there is a 60 lb. weight at the end. Find the work done in drawing up the rope.

46. A tank, 12 feet long, 4 feet wide and 3 feet deep, is filled through a 2 inch diameter pipe. The rate of flow in the pipe is 80 feet per minute. In what time will the tank be filled ?

47. A lever safety valve is loaded with a spring balance, the load on the balance being 85 lb. The centre of the valve is 3 inches from the fulcrum and the length of the lever is 13 inches. The combined weight of the lever and valve is equal to a pressure of 5 lb. per square inch on the valve. The boiler pressure is 90 lb. to the square inch. What is the diameter of the valve ?

48. The longitudinal seam of a boiler is a treble riveted lap joint. The rivets are 1 inch diameter and the pitch is $3\frac{7}{8}$ inches. The plate is $\frac{11}{16}$ inch thick, and its tensile strength is 28 tons per square inch. The shearing strength of the rivets is 23 tons per square inch. The diameter of the boiler is 6 feet 6 inches and the factor of safety 5·5. What is the working pressure of the boiler ?

49. A beam, fixed at one end, is loaded with a weight of 9,000 lb. at the other end. The length of the beam is 2 feet 9 inches and the depth 6 inches. If the maximum stress produced is 22,000 lb. per square inch, find the width of the beam.

50. A 10 knot ship steamed up a river for 4 hours, and did the same distance down stream in $2\frac{1}{2}$ hours. What was the speed of the current ?

51. Find the average velocity of a body, falling freely from rest, during the 5th, 6th and 7th seconds ; find also the distance passed through in that time.

52. A ballast pump runs at 110 revolutions per minute. The water is discharged through a 5 inch diameter pipe at 230 feet per minute, to a height of 19 feet. The efficiency of the pump is 62 per cent. Find the horse power exerted.

53. The body of a crosshead of an engine is 12 inches by 12 inches by 12 inches. The hole for the piston rod tapers from $6\frac{1}{2}$ inches to 6 inches. The gudgeon pins are $6\frac{1}{2}$ inches diameter and 7 inches long. It is made of mild steel, find its weight.

54. A cylindrical tank is 25 inches diameter and 5 feet 2 inches high, and contains 90 gallons of oil. After 6 days the depth of oil in the tank is 3 feet 3 inches. How many gallons have been used per day ?

55. A ship leaves a port in 5° 30′ East longitude and later arrives at another port situated in 63° 17′ East longitude. By what amount and in what direction has the clock been altered ?

56. Two weights are suspended at the ends of a bar. The weight on end A is 1·73 times the weight on end B. The bar is 8 feet long. At what point in the bar will it balance ?

57. A body is projected upwards with an initial velocity of 1,000 feet per second. What height will it attain and what is the time of ascent ? What is meant by acceleration ?

58. The effective diameter of a paddle wheel is 18 feet, and the slip is 10 per cent. Find the distance moved forward by the ship when the crankshaft turns through an angle of 21·75 radians.

59. A cantilever has a load uniformly distributed over its length. Give the expression for the maximum bending moment, and show how it is obtained. Give also the expression for the maximum bending moment in a beam supported at the ends and loaded at the centre, and show how it is obtained.

A cast iron beam 10 feet long, 3 inches wide and 6 inches deep, is supported at both ends. Find the maximum stress in the material due to its own weight.

60. A pulley block is used to pull a weight of 2 tons up an incline of 1 foot in 18. There are 3 sheaves in the block at the fixed point and 2 sheaves in the block attached to the weight. Find the force exerted, neglecting friction. If the blocks were reversed what would then be the force necessary ?

61. A uniform beam is 30 feet long and weighs $2\frac{1}{2}$ tons. A weight of 5 tons is suspended at 8 feet from the support at one end. How far from the support at the other end must a 7 tons weight be suspended in order that the load on each support may be equal ?

62. A circle has a right angled triangle inscribed within its circumference. The perpendicular is 7·9 inches long, and the base 9·2 inches long. What is the area of the circle ?

63. How does the strength of a beam vary with respect to its length, width and depth ?

A beam is $1\frac{1}{4}$ inches wide, $2\frac{1}{2}$ inches deep and 4 feet long. It can carry safely a load of 500 lb. Another beam loaded and supported in the same manner, is 3 inches wide, $6\frac{1}{2}$ inches deep and $6\frac{1}{4}$ feet long. If the second beam is of the same material as the first, what load can it carry safely ?

64. The angular velocity of an engine shaft is 8·5 radians per second. The piston speed is 500 feet per minute. What is the stroke of the engine ?

65. What is meant by Moment of Resistance ?

The Moment of Resistance to bending of a certain solid round shaft is :—

$$\frac{p \times 1{,}000}{10 \cdot 2}, \quad = \text{stress in lb. per square inch.}$$

What would be the side of a square beam to give the same stress ?

66. The load on a piston is 36 tons. The stroke is 4 feet and the length of the connecting rod is 8 feet 4 inches. Find the load on the crank pin and on the guide, when the crank has moved through an angle of 60°. from the top centre.

67. A beam is subjected to a bending moment of 324 inch tons. The stress allowed on the material is 8 tons per square inch. Find the cross sectional area of the beam which is rectangular, the depth being 3 times the width.

68. The diameter of a cylinder is 72 inches and the effective pressure is 20 pounds per square inch. The maximum bearing pressure allowed on the crank pin is 500 pounds per square inch, and the length of the pin is 1·2 times its diameter. What is the diameter of the crank pin ?

69. Two ships have speeds of 10 and 12 knots respectively. Each ship is bound for the port left by the other one, and the ports are 200 miles apart. The faster ship leaves 3 hours before the slower one. Where do they pass each other ?

70. The diameter of a piston rod at the bottom of the thread is 7 inches. The stress allowed is 7,000 lb. per square inch. What is the effective diameter of the top end bolts, if the stress is not to exceed 6,500 pounds per square inch ?

71. A cube of 1 decimetre side is sawn into 27 smaller cubes. The saw cut is $\frac{3}{64}$ inch wide. Find the volume of a small cube in cubic inches.

72. The connecting rod of an engine is 9 feet 9 inches long, and the stroke is 4 feet 6 inches. How far is the piston from the top of its stroke, when the crank is at 90° from the top centre ?

73. A ship leaves port 12 hours before another ship which has a speed of 14 knots. The second one overtakes the first one in 36 hours. What is the speed of the first ship ?

74. What is the area of the largest hexagon which can be cut out of a circular plate 15 inches diameter ?

75. The area of the safety valve of an evaporator is 7·5 square inches, and it is loaded with weights of 155 lb. What pressure will the valve blow at when the ship heels over 30° ?

76. A hollow cast iron sphere is $\frac{3}{8}$ inch thick and weighs 32 pounds. Find the outside diameter.

77. A truck is drawn up an incline of 4° 30′ by a force of 9,300 pounds. The friction on the level is 11 per cent. of the weight. What is the weight ?

78. A connecting rod is 8′ 8″ long and weighs $1\frac{1}{2}$ tons. Its centre of gravity is at mid-length. Find the horizontal force, which, if applied at the end of the rod, will draw the end 2′ 6″ out of the centre line of the engine.

79. An isosceles triangle is inscribed in a circle. The base of the triangle is 1 inch long and the angle at the apex is 30 degrees. What is the diameter of the circle ?

80. The perimeter of a right-angled triangle is 30 inches and the base is 10 inches. Find the lengths of the other two sides.

81. The difference in area of two circular plates is 28 square feet, and the difference of their circumference is 8 feet. Find the diameter of the large plate.

82. Two ships are bound for the same port at which they will arrive simultaneously. The speeds of the ships are 8 knots and 10 knots respectively. When they are 4 hours from port their distance apart is 44 miles. Find the angle between their courses.

83. The area of a rectangle is 42 square inches, and the diagonal is $12\frac{1}{2}$ inches. What are the lengths of the sides ?

84. An I beam has a top flange 6 inches wide, and 1 inch thick. The bottom flange is 8 inches wide and $1\frac{1}{4}$ inches thick. The web is 1 inch thick and the total depth of the beam is 10 inches. Find the centre of gravity of the section.

85. An hexagonal steel plate of uniform thickness is cut 1·05% too small across the flats. By what percentage is the weight too small ?

86. Define "centrifugal force." An unbalanced mass in a flywheel rotating at 240 revolutions per minute weighs 220 pounds and acts at a radius of 10 inches from the centre of the wheel. Find the pull on the bearings due to centrifugal force.

87. A Diesel cylinder tie-bar, 26 feet 6 inches long, is heated until it extends $\frac{1}{16}$ inch. The nut is put on while the bar is hot. Find the tensile stress in the bar when it has cooled down to its original temperature. $E = 14{,}000$ tons per sq. inch.

88. A weight of $5\frac{1}{2}$ pounds on the end of a string is rotated in a vertical plane at 70 revolutions per minute. The string is 3 feet 6 inches long. Find the greatest and least tension in the string.

89. A beam of regular section is 29 feet in length and is supported at a point 3 feet $1\frac{1}{2}$ inches out of centre. A man weighing 12 stone 4 pounds stands on the beam at 1 foot 6 inches from the short end, and another man weighing 10 stone 7 pounds stands at 3 feet 3 inches from the same end. Under these conditions the beam is balanced. Calculate the weight of the beam.

90. Fresh water is pumped from a double-bottom tank to a present-use tank which is 10 feet by 6 feet by 5 feet deep. The bottom of this tank is 50 feet above the water level in the double-bottom tank. The area of the double-bottom tank is 600 square feet. How many foot-pounds of work are done in filling the present-use tank ?

91. The speed of a vessel of 5,000 tons displacement is uniformly increased from 10 to 12 knots in 15 minutes. Find the additional propeller thrust required to increase the ship's speed neglecting the augmentation of resistance to its motion as its speed increases.

92. An ingot of brass is in the form of a triangular prism 6 inches long. The angles of the triangle are 30°, 60°, and 90° and the short side is $1\frac{1}{4}$ inches long. Find the weight of the largest isosceles triangular prism that can be cut out of this.

93. The stroke of an internal combustion engine is 1,040 millimetres, and the length of the connecting rod 1,960 millimetres. When the crank is at an angle of 45° off the top centre the torque in the shaft is 8,750 kilogram-metres. Find the load on the guide in this position, in pounds.

94. A hollow shaft transmits 5,000 horse power at 90 revolutions per minute. The internal diameter of the shaft is half the external diameter. If the ratio of maximum turning moment to mean turning moment is 1·5 to 1, find the external diameter of the shaft allowing a stress of 8,000 lb. per square inch.

95. A raft is composed of fir logs, the specific gravity of which is 0·56. It is 9 feet 9 inches wide by 9 inches deep, and floats in fresh water at a draught of 8 inches when it has a weight of $2\frac{1}{2}$ tons placed on the top. What is the length of the raft ?

96. A winch lifts a weight of 13 cwts. through a distance of 77 feet from a ship's hold. The efficiency of the winch is 87 per cent. The diameter of the winch drum is 31 inches and the diameter of the rope is 1 inch. If the winch drum makes 72 revolutions per minute, calculate the horse power of the winch.

97. An air container is 5 feet 9 inches internal diameter and 11 feet long. The main seam is a double-riveted butt joint with double straps, having three rivets in a pitch. The thickness of the plates is $1\frac{3}{32}$ inches, diameter of rivet holes $1\frac{3}{16}$ inches, pitch $5\frac{7}{8}$ inches. The tensile strength of the plates is 28 tons per square inch and shearing strength of rivets 23 tons per square inch. Allowing a factor of safety of 4·5, and 1·875 for double shear, find the working pressure.

98. A screw jack is used to lift a weight of 6·75 tons. The pitch of the screw is 0·475 inch, the length of the toggle bar 135 centimetres, and a force of 28 kilograms is applied at the end of the bar. Find the percentage work lost in friction.

99. A steel ingot weighing one hundredweight is drawn along a level track at a uniform speed by means of a rope inclined at 30 degrees to the track. If the pull in the rope is 16 lb., find the co-efficient of friction.

100. A plank of wood 12 feet long, 11 inches broad and 4 inches deep just floats awash in sea water when a weight of 51 lb. is placed on top. The hydrometer reading of the sea water is 1,022. Find the weight of the plank and its specific gravity.

101. Define acceleration and state the units in which it is measured. A ship travelling at 19 knots reduces its speed to 13 knots in 5 minutes, find the retardation in feet per second per second and the space passed over during that time.

102. A piece of metal is 13 inches square. An equilateral triangle is cut out of this and its area is two-fifths of the remaining portion. Find the length of the side of the triangle Find also the area of the largest equilateral triangle that could be cut from the square plate

103. Define shear stress. A beam 35 feet long and weighing 2 tons is of uniform section and is supported at each end. A load of 8 tons is placed at 7 feet from one end and a load of 6 tons at 10 feet from the other end. Find the shearing force at each end.

104. A cast iron prism lies lengthwise on a horizontal table. The section of the prism is 1 foot 6 inches square and its length is 3 feet. Find the work done in inch tons to lift it around one of its short edges until its base makes an angle of 17° with the horizontal.

105. A shaft transmits 4,000 horse power at 125 revolutions per minute. There are 6 bolts in the coupling at a radius of 12 inches from the centre of the shaft. The stress in the bolts is 5,000 lb. per square inch, find their diameters.

106. A rectangular plate has an area of 1,307 square centimetres, the ratio of the sides being as 1 : 2·5. Find the area of the largest hexagon in square inches that can be cut from this plate.

107. A body requires a force equal to 0·7 per cent. of the weight to draw it along the level, and a force of 8·4 per cent. of the weight to draw it up. an incline. What is the rise of of the incline ?

108. The weight of 1 lb. of cast iron in oil is 0·89 lb. Express the specific gravity of the oil in degrees Beaume.

109. An oil engine cylinder is offset 4 inches. The length of the connecting rod is 6 ft. and the radius of the crank pin circle is 2 ft. Find the angle turned through by the crank while the piston passes from the top of its stroke to the bottom. Find also the stroke of the piston, and make a sketch to show the direction of rotation of the crank.

110. A steam engine governor consists of two 4 lb. bobs, connected to an extension on the crank shaft by means of springs. The weights are held at an effective radius of $3\frac{1}{2}$ inches, and in this position the control valve is full open. The valve is set to close when a speed of 300 revs. per min, is reached. What will then be the force in the springs ?

111. The fuel valve of an internal combustion engine is open for 43° of the crank circle and closes when the piston is one-eighth of its stroke from top centre. The connecting rod is twice the length of the stroke. Find the position of the crank when the fuel valve opens.

112. The spring of a safety valve can be compressed 0·625 inch by a load of 0·714 ton. When it is in its working position, the compression of the spring is 0·8125 inch. The combined weight of the valve, spindle, and spring is 37 lb., and the diameter of the valve is 3·25 inches. What will be the boiler pressure when the valve lifts ?

113. A double bottom tank 3 ft. 9 ins. deep contains oil of specific gravity 0·89, and the oil rises to a height of 35 feet up the sounding pipe. Find the pressure in lb. per square foot on the outer bottom. If the floors are spaced 30 inches apart, and the rivets in the reverse frames are $\frac{7}{8}$ inch diameter pitched at 7 diameters apart, find the additional stress in the rivets on the inner bottom due to the oil pressure.

114. A flat compensating ring is to be fitted around a manhole 16 ins. by 12 ins. cut in the shell of a boiler. The thickness of the ring is the same as the boiler shell plates, that is $1\frac{5}{8}$ ins. The material used for the ring has a tensile strength of 26 tons per sq. inch, and that for the shell plates 29 tons per sq. inch. Rivets $1\frac{1}{16}$ ins. diameter are to be used, having a shearing strength of 23 tons per sq. inch. Allowing two rivets to be placed on the minor axis, calculate the minimum width of the ring and the minimum number of rivets.

115. The stroke of an engine is 4 feet and its speed is 12·28 radians per second. Find the piston speed as the crank passes a point 45° from top centre, neglecting the obliquity of the connecting rod.

116. A simply supported beam 32 feet long carries a uniformly distributed load of 56 lb. per foot run over its entire length and a concentrated load of 2 tons at 8 ft. from one end. Calculate the shearing force and bending moment at every 4 feet of length along the beam and draw to scale the shearing force and bending moment diagrams.

117. A ship steaming on a straight course sights a lighthouse 54° on the port bow at 10 a.m. At 10-27 a.m. the light was 45° abaft the beam. Find how near the ship passed the lighthouse and at what time, if the ship's speed is 20 knots.

118. A turbine casing is lifted horizontally by three chains connected to a common ring. Two of the chains are fastened to side brackets ; the third chain, which is 10 feet long, is fastened to a bracket on the centre line of the casing at the end. Axially, from the side attachments to the end attachment is 12 feet. The centre of gravity of the casing is, axially, 3·34 feet from the side attachments. The distance transversely from the centre line of the casing to each side bracket is 2 ft. 6 ins. Find the lengths of the two front chains, and also the tension in the back chain if the weight of the casing is 3 tons.

119. Two trains, each 150 feet long, approach each other on parallel tracks. The speed of one is $1\frac{1}{2}$ times the speed of the other and they pass each other in 3 seconds. Find the speeds of the two trains in miles per hour.

120. Find the weight of fresh water contained in a boiler which is 11 ft. 6 ins. long, 16 feet diameter, and area of water level 146 square feet, if the combustion chambers, furnaces, etc., take up 32% of the internal volume.

121. A rectangular beam of regular cross section is simply supported at each end. It is subjected to a maximum bending moment of 53,120 ft. lb., and a maximum stress of 3·65 tons per sq. inch, when it supports a concentrated load of 8·25 tons at the centre of its length. Calculate the length and depth of the beam, if its breadth is 4 inches. Neglect the weight of the beam.

122. In a wheel and differential axle, the diameter of the big axle is $7\frac{1}{2}$ inches and that of the small axle 5 inches, and the lifting rope is $\frac{1}{2}$ inch diameter. An effort of 25 lb. is applied to the end of the handle which is 16 inches long, and the load lifted is 300 lb. Find the velocity ratio, mechanical advantage, and efficiency.

123. Find the greatest weight that a man whose weight is 175 lb., can pull in a horizontal direction along a horizontal plane, if the co-efficient of friction between the soles of his boots and the plane is 0·42, and the co-efficient of friction between the weight and plane is 0·36.

124. State the law of Archimedes. A tank 5 feet long, 2 feet wide, and 6 feet deep, contains fresh water to a depth of 4 feet. A block of wood weighing 250 lb., and having a specific gravity of 0·8 is now put into the tank and allowed to float in the water. Find the water pressure on the bottom, sides, and ends of the tank.

125. Two ships travel on the same course between the same ports. The speed of one is 14·25 knots and that of the other 15·75 knots. The second ship sets out 6 hours after the first and arrives at her destination 15·25 hours sooner. Find the distance between the two ports.

126. A cubical tank of 3 feet sides contains fresh water to a depth of 2 feet. Find the total water pressure on the base and sides of the tank after 1 cubic foot of iron weighing 450 lb. is suspended by means of a rope and wholly immersed in the water. Find also the tension in the rope.

127. A ship leaves port and after travelling 490 nautical miles on a straight course, is overtaken by another ship which left the same port 9 hours later. If the second ship is 5 knots faster than the first, what was the speed of the first ship ?

128. The mean diameter of a cast iron flywheel rim is 8 feet. What rotational speed would set up a stress of 1,000 lb. per sq. inch in the rim due to centrifugal force ?

129. In a differential pulley block the mechanical advantage is 12 and the efficiency is 0·42 when lifting a certain load. If the diameter of the small pulley on the compound sheave is 9 inches, find the diameter of the large pulley.

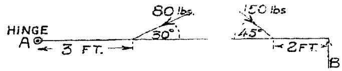
130. The roof frame shown in the diagram supports a load of 4 tons on the apex. Find the load carried by each support and the magnitude and nature of the forces in each member.

131. The width of the water level of a boiler is the same as its length. The height from the water level to the crown is 3 ft. 9 ins., the height to the boiler shell at quarter and three-quarter widths is 3 feet. If the volume of the steam space is 390 cubic feet, find the length of the boiler.

132. A length of wire 5 feet long, and 0·1 inch diameter, is heated from 60°F. to 236°F. and its ends are securely fastened while the wire is hot. Find the stress in the wire when it cools down to its original temperature assuming the elastic limit of the wire is not passed. Take the co-efficient of linear expansion as 0·0000067 per degree F., and modulus of elasticity as 30×10^6 lb. per sq. inch. What load, hanging freely from the end of the wire, would cause the same stress ?

133. 25 Horse power is transmitted by two spur wheels in gear. The driver has 80 teeth of 1 inch pitch, and the pressure between the teeth is 250 lb., find its speed in revs. per min.

134. A uniform bar 10 feet long, and weighing 50 lb., is freely hinged at A and supported at B. The reaction at B is vertical and the bar is horizontal. Forces in the plane of the bar, act as shown on diagram. Find the reactions at A and B, and the direction of the reaction at A.



135. The diameter of the ram of a hydraulic jack is 3 inches, and the plunger is $\frac{3}{4}$ inch diameter. The mechanical advantage of the lever is 10, and the efficiency of the jack is 0·8. Calculate the load lifted by a force of 140 lb. on the end of the lever.

136. A motor car, of weight W tons, is turning from the straight at a radius of 80 feet, the road being horizontal. If the co-efficient of friction between the tyres and the road is 0·63, find the maximum speed of the car, in miles per hour, so that skidding shall not occur.

137. What is meant by the "Moment of a Force," and by the "Moment of a Couple"? A plate is mounted on a horizontal axle, and a force of 6 lb. acts perpendicular to, and at the end of a radius of 3 feet from the axle. Find the forces of the couples which would separately balance this force, if the arms of the couples are (*a*) 3 feet, (*b*) 9 feet, (*c*) 5 inches.

138. A simple jib crane has a vertical post 7 feet high. The jib is 12 feet long and the tie 9 feet long. There is a load of 4 tons hanging from the jib head. Find by graphical construction, the forces in jib and tie, when the load is pulled directly away from the crane, until it makes an angle of 60° to the vertical.

139. A tank 3 feet 5 inches deep is pumped out by a pump in 1 hour 55 minutes, and runs up from the sea in 1 hour 15 minutes when the inlet is submerged to a depth of 16 feet. If the inlet is now submerged to a depth of 25 feet, and the tank contains 9 inches depth of water when the inlet is opened and the pump is started up, find the time to fill the tank.

140. Three ships A, B and C have speeds of 10, 12 and 16 knots respectively. B leaves a port 2 hours after A, steering in the same direction. How long after B started should C leave order that the three ships shall pass simultaneously ?

141. A manhole, elliptical in shape, is 12 inches by 16 inches. It is arranged with the minor axis vertical, and there is a head of fresh water of 20 feet above the centre of the door. The door is $14\frac{1}{2}$ inches by $18\frac{1}{2}$ inches, and the door joint is 1 inch wide. Find the load on the door due to water pressure, and the load per square inch on the joint.

$$\text{Area of an ellipse} = \frac{\quad}{4} \times \mathrm{D} \times d \text{ (approximately).}$$

142. A simple jib crane has a vertical post 7 feet high. The jib is 11 feet long and the tie 8 feet long. A rope, passing over a pulley at the jib head, leads backwards in the plane of the frame to a winch on the ground, and the rope makes an angle of 45° to the vertical. Find, by graphical construction, the forces in jib and tie when 600 lb. is being lifted at a uniform speed. Neglect frictional resistances.

143. A block of cast iron weighing 60 lb. in air is completely immersed in water of specific gravity 1·05. Find the tension in the wire from which the block is hung, its diameter being 0·0625 inch.

144. A cylindrical vessel 3·5 ft. diameter and 2·5 ft. long, is entirely enclosed and is full of oil. The sheet metal of the tank weighs 3·8 lb. per sq. foot. Find the specific gravity of the oil if the vessel floats awash in water of density 1024 ozs. per cu. ft.

145. A flywheel rim 49 inches diameter has a shroud of steel 1 inch thick shrunk on. Neglecting the stress due to shrinking, find the stress in the shroud due to centrifugal force action when the flywheel runs at 450 revs. per minute.

146. From the deck of a ship steaming due north at 12 knots, the wind appears to come from 65° on the port bow. Its apparent velocity is 65 ft. per sec. Find its true velocity in magnitude and direction.

147. A plank of wood weighing 140 lb. was placed in a tank of oil of specific gravity 0·91. When a weight of 60 lb. was placed on the plank it was just awash. Find the specific gravity of the wood.

148. A square plate of 5 inches sides has a circle, whose area is one quarter of the area of the square, cut out of it. The centre of the circle is at one third of the diagonal from one corner of the square. Find the position of the centre of gravity of the plate remaining, from the sides of the square.

149. A weight of 2 lb. is fastened to the end of a wire 3 feet long, and is rotated in a vertical circle at 200 revs. per minute.

Find the tension in the wire when the weight is at its highest position ; when it is at its lowest position, and when at 45° from its highest position.

150. Find the area, the lengths of sides and base of the largest isosceles triangle, having angles at the base of 75°, that can be cut from a circular plate whose area is 804·25 sq. inches.

151. A cast iron pyramid has a base 6 inches square and a perpendicular height of 4 feet. It is lying on its side. Find the work done in raising it to a position of unstable equilibrium (*a*) about one edge of its base, (*b*) about its apex.

152. A feed heater cylinder is 23 inches internal diameter and is made of plates $\frac{1}{2}$ inch thick. It is welded by an electric process, and the thickness of the weld is the same as the plate. Their respective strengths in tension are 23 and 25 tons per sq. inch. If the working pressure of the cylinder is 150 lb. per sq. inch, find the factor of safety and the stress in the weld.

153. A cylindrical vessel 5·5 feet diameter, and 16 feet long, is placed with its longitudinal axis horizontal. It is less than half full of fresh water, and the breadth of the water level is 3 feet. Find the weight of water contained in the vessel.

154. A tank 3 feet 9 inches deep, 30 inches wide and 60 inches long, is fitted with a vertical pipe 2 inches diameter leading from the top. The tank and the pipe are filled with oil of specific gravity 0·92 to a height of 35 feet above the tank top. The tank is supported on trestles. Find the load in pounds per sq. foot on the bottom and sides of the tank, and the stress in the rivets holding the top to the sides. The rivets are $\frac{7}{8}$ inch diameter and are pitched 7 diameters apart. Find also the load on the trestles, neglecting the weight of the tank and pipe.

155. Two ports are 420 nautical miles apart. A ship leaves each port at the same time, bound for the other port. After they pass, one ship takes 9 hours to reach its destination, the other takes 16 hours. Find the speeds of the ships.

156. An aeroplane is flown off an aircraft carrier at 6 a.m. for scouting duty directly ahead, and it can remain in the air for 2 hours. The speed of the carrier is 20 knots and that of the aeroplane 150 knots. At what time must the aeroplane turn back in order to land on the carrier ?

157. A solid cone stands on an inclined plane. The vertical height of the cone is four times the diameter of its base, and the co-efficient of friction between cone and incline is 0·3. If the

angle of the incline is gradually increased, determine the state of equilibrium of the cone when it is about to slide down the incline. If the height had been eight times the diameter of the base, would the cone slide, or overturn first, as the inclination is increased ?

158. A uniform ladder weighs 70 lb. and is 20 feet long. It stands against a smooth vertical bulkhead and makes an angle of 60° to the horizontal deck. Find, by means of a vector diagram or by calculation, the vertical reaction of the deck and the reaction of the bulkhead.

159. Define specific gravity. Find the weight of one cubic foot of oil if a column of the oil 10·89 metres high balances a column of mercury 747 millimetres high.

160. A piston is 95 inches diameter, the piston rod 10 inches diameter and the tail rod $6\frac{1}{2}$ inches. At a certain point in the stroke, the pressures are 7 lb. per square inch gauge on one side of the piston and 25 inches of vacuum on the other side. At another point the pressure conditions are exactly reversed. What is the effective load on the piston in tons in each case ? The mercury barometer stands at 30 inches.

161. A ship is being towed by two tugs, the tow-ropes making angles of 15° on each bow. The tow-ropes themselves make angles of 20° to the surface of the water, and the force acting in each is 4 tons. Find the force urging the ship through the water, and if its speed is 3 knots what is the horse power developed ?

162. In a treble riveted butt joint with double straps, the pitch of the rivets is 6 inches and the plate thickness is $1\frac{1}{8}$ inches. The tensile strength of the plate is 28 tons per sq. inch, and the shearing strength of the rivets 23 tons per sq. inch. Find the diameter of rivet that will give equal plate and rivet efficiencies. Allow $1\frac{7}{8}$ for rivets in double shear.

163. A shaft $11\frac{1}{4}$ inches diameter and 10 feet long weighs 1·5 tons and is supported in bearings, one at each end of the shaft. An unbalanced flywheel weighing 0·75 ton is mounted on the shaft at 4 feet from one bearing, the plane of the wheel being perpendicular to the axis of the shaft. When the shaft is rotated at 100 revs. per min. it is found that the load on the bearing nearer to the flywheel varies between 0·7 and 1·7 tons. Determine the distance of the centre of gravity of the flywheel from the shaft axis.

164. Construct a formula to give the weight of fresh water passing through a mushroom valve per minute. The diameter

of the valve is d inches and the velocity of the water is v feet per second.

Use the formula to find the weight of water passing per minute when $d = 6\frac{1}{2}$ inches and $v = 8{\cdot}5$ ft. per sec.

165. In order to determine the position of the centre of gravity of a homogeneous hemisphere weighing 69·28 lb., it was placed on a horizontal table on its curved surface, and when 15 lb. is hung from a point on the periphery of the flat surface, the flat surface is inclined at 30° to the horizontal. Determine the distance of the C.G. from the flat surface in terms of the radius of the hemisphere.

166. Two rods of equal diameter and equal length, but of different materials, are welded together at their ends to form one long straight rod. This rod is found to balance on a knife-edge at five-eighths of the total length from one end. Determine the ratio of the weights per unit volume of the two materials.

167. A worn valve seat 6 ins. diameter and 5 ins. deep is being extracted from a valve chest. The mating pressure between seat and chest is 400 lb. per sq. inch, and the co-efficient of friction is 0·4. Find the work done in extracting the seat.

168. A wire 0·06 inch diameter and 5 feet long was loaded in tension and the observations taken were :—

Load (pounds)	20	40	60	80	100	120	140	150	160
Extension (inches)	0·013	0·027	0·038	0·052	0·064	0·077	0·091	0·1	0·114

Plot the load-extension diagram, on a base of extension, to the scales 1 inch = 20 lb. load, 1 inch = 0·02 inch extension.

Mark on the diagram the approximate elastic limit of the wire.

Calculate the stress at the elastic limit, and the modulus of elasticity of the wire.

169. A solid round shaft $6\frac{1}{2}$ ins. diameter transmits a certain horse power when running at 162 revs. per min., and the shear stress induced is 7,300 lb. per sq. inch. What would be the stress induced in a solid shaft $4\frac{1}{2}$ ins. diameter, running at 388 revs. per min., when transmitting an equal horse power ?

170. A treble riveted lap joint has rivets pitched at $4\frac{1}{2}$ inches in the outer rows and half that in the inner row. The plates are 1 inch thick ; the tensile strength is 28 tons per sq. inch, and the shearing strength of the rivets 24 tons per sq. inch. Find the diameter of the rivets in order that the strength of the plate section shall be 10 per cent. greater than that of the rivet section. What is then the efficiency of the joint ?

SECOND-CLASS EXAMINATION QUESTIONS.

HEAT AND HEAT ENGINES

1. There are 39 tons of sea water in a boiler and the evaporation is 125 tons per day. If the boiler is fed with sea water, what will be the density after 9 hours' steaming if there has been none blown out ?

2. A certain weight of ice at 32°F. is placed in a tank containing 200 pounds of water at 80°F. The temperature of the mixture is found to be 50°F. What was the weight of ice ?

3. A pound of coal evaporates 8·2 pounds of water into steam at 367°F. from water at 126°F. Find the equivalent evaporation from and at 212°F.

4. If the temperature of the funnel rises it is found that the new consumption is $(1 + 0{\cdot}000455\ T)$ times the first consumption, where T is the rise in temperature in degrees Fahrenheit. At the beginning of a voyage the temperature of the funnel was 576°F. At the end of the voyage the funnel temperature was 731°F., and the consumption 60 tons per day. What was the consumption at the beginning of the voyage ?

5. A slide valve has 7 inches travel. The opening to exhaust when the crank is on the top centre is $2\frac{1}{4}$ inches. There is $\frac{3}{32}$ inch negative exhaust lap and $\frac{1}{16}$ inch lead top and bottom. Find the steam lap of the valve.

6. The boilers contain 95 tons of fresh water at the commencement of a voyage. Through a leak in the condenser the water in the hotwell was uniformly 0·4 ounce to the gallon. At the end of the voyage, the density of the boilers was $3\frac{1}{2}$ times the sea density, and there has been no water blown out. How many tons of water have been evaporated ?

7. If 18 pounds of steam at 180 pounds pressure per square inch and at a temperature of 377·7° Fahr. are blown into a tank containing 1,900 pounds of water at 95° Fahr., what is the resulting temperature ?

8. An engine when running at full speed indicates 1,250 horse power. The boilers are fed by a double acting feed pump with piston 6 inches diameter, stroke 15 inches, efficiency 75 per cent., making 16 double strokes per minute. What weight of steam is used per horse power per hour ?

9. Steam is taken from the low pressure valve chest and is used to warm the feed water in a heater. The temperature of the feed water is 215°F., and the hotwell is 120°F. The temperature of the heating steam is 275°F. How many pounds of steam are used per pound of feed water ?

10. A slide valve has $8\frac{1}{2}$ inches travel. The exhaust laps are $\frac{1}{4}$ inch at top and $\frac{1}{8}$ inch at bottom. The steam laps are $2\frac{3}{4}$ inches at top and $2\frac{5}{8}$ inches at bottom. The top lead is $\frac{1}{8}$ inch and the bottom lead $\frac{1}{4}$ inch. What is the width of the exhaust port opening when the crank is passing (*a*) the top centre, (*b*) the bottom centre ?

11. In 2·2 pounds of water there are 1·406 grams of sulphate of lime and 0·033 gram of carbonate of lime. How many pounds of solid matter are left in the boiler for each ton of water evaporated ?

12. A compound engine has cylinders 26 inches and 69 inches diameter. The mean effective pressures are 67 and 14·75 pounds respectively. The stroke is 4 feet and the revolutions 60 per minute. Find the combined horse power.

13. A boiler contains 29 tons of water at 18 ounces per gallon. If 4 tons are blown out and replaced with fresh water, what is now the density of the water in the boiler ?

14. An engine uses $14\frac{1}{2}$ pounds of steam per I.H.P. hour. In condensing, each pound of steam gives out 965 B.T.U. The temperature of the sea is 67°F., and the discharge is 86°F. 1,300 tons of cooling water are used per hour. What is the I.H.P. of the engine ?

15. At the beginning of a voyage the temperature of the funnel was 600°F. and the consumption was 34 tons per day. After several days' steaming the funnel temperature had risen to 827°F., and the consumption to 37·5 tons per day. What is the percentage increase in consumption for each degree rise in funnel temperature ? What is the probable reason for the increase of temperature in the funnel ?

16. A ship's boilers at the beginning of a voyage contained 119 tons of fresh water. At the end of the voyage the density of the boilers was $3\frac{1}{2}$ times the sea density and the total water evaporated was 4,500 tons. What has been the average hotwell density in ounces per gallon ?

17. A triple expansion engine has cylinders 34, 51 and 96 inches diameter. The stroke is $4\frac{1}{2}$ feet ; revolutions per minute

90·6 and the mean pressure referred to L.P. 42 pounds per square inch. The nominal horse power is 960. How many times is the nominal horse power contained in the indicated horse power?

18. The steam lap of a slide valve is $2\frac{1}{4}$ inches; the lead is $\frac{3}{32}$ inch and the maximum steam port opening is 2 inches. Find the travel and the distance the valve is from its mid position when the engine is on the top centre.

19. An engine of 4,000 indicated horse power uses 16 pounds of steam per I.H.P. hour. Each pound of steam in condensing gives out 1,000 B.T.U. The circulating water is raised in temperature from 51°F. to 82°F. What weight of cooling water is used per day and what is the equivalent horse power in the heat given to the cooling water?

20. A high pressure cylinder is 25 inches diameter, and the boiler pressure is 160 pounds per square inch. The initial pressure in the next engine is 49 pounds per square inch, and there is a drop in pressure between the two engines of 3 pounds per square inch. The piston rod is $6\frac{1}{2}$ inches diameter at the bottom of the thread. Find the maximum stress on the rod when the engine is running, and when just starting the engine.

21. An engine has a piston 20 inches diameter. The mean effective pressure is 28 pounds per square inch. The brake horse power is 120 and 22·8 horse power is lost in friction Find the mechanical efficiency of the engine and the mean piston speed.

22. The area of the water level of a boiler is 280 square feet, and the area of firegrate 120 square feet. Each pound of coal evaporates $8\frac{1}{2}$ pounds of water, and 16 pounds of coal are burnt per square foot of firegrate per hour. If there are 9 inches of water in the glass and the feed pump stops, in how many minutes will the water be out of sight?

23. In a certain steamer the boiler efficiency is 68 per cent., the thermal efficiency of the engine is 17·5 per cent., the mechanical efficiency of the engine is 87 per cent., and the efficiency of the propeller is 61 per cent. What percentage of the energy in the coal is applied in propelling the ship?

24. The calorific value of a certain coal is 14,000 B.T.U. per pound. The combined efficiency of the engine and boiler is 11·5 per cent. What is the coal consumption per horse power hour?

25. If 12·383 cubic feet of dry air at 32°F. weigh 1 lb., what is the weight of 30 cubic feet at 98°F? Pressure remains constant.

26 An engine develops 2,100 H.P. and uses $14\frac{1}{2}$ lb. of steam per horse power per hour. The exhaust steam enters the condenser at 180°F. and the temperature of the hotwell is 130°F. Find the amount of heat given to the circulating water per hour and the equivalent horse power.

27. Steam is admitted to a cylinder at 205 lb. per square inch gauge. The cut-off is at 0·35 of the stroke and the clearance volume is equal to 5 per cent. of the volume swept out by the piston. Find the pressure when the piston is at 0·9 stroke.

28. A boiler is stated to evaporate 9·6 lb. of water from and at 212°F. per lb. of coal. If the calorific value of the coal is 13,500 B.T.U. per lb., find the efficiency of the boiler.

29. A triple expansion engine has cylinders 15, 25, and 40 inches diameter. The boiler pressure is 216 lb. per square inch. The stroke is 2 feet 3 inches, and the heating surface of the boiler 1,888 square ft. Find the nominal horse power. Use the formula:—

$$\text{N.H.P.} = \frac{(3\,\text{H} + \text{D}^2\ \sqrt[3]{\text{S}})}{700}$$

Where H = square feet of heating surface.
D = diameter of L.P. in inches.
S = length of stroke in inches.
P = pressure per square inch by gauge.

30. At the beginning of a voyage there were 90 tons of fresh water in a ship's boilers. During the voyage 3,800 tons of water were evaporated and the density rose to $3\frac{1}{2}$ times the sea density. What was the average density of the feed water ?

31. A boiler has 6 furnaces, each 5 feet 6 inches long and 44 inches diameter. The steam pressure is 165 pounds gauge. What are the diameters of the safety valves, there being 2 valves fitted ?

Note.—Combined area of valves

$$= \frac{\text{Area of firegrate in square feet}}{2} \times \frac{75}{\text{Gross pressure}}$$

32. An air compressor cylinder is 12 inches diameter and the stroke is 16 inches. The pressure at 8 inches of the stroke is 12 lb. gauge, and at 13 inches of the stroke 39 lb. gauge. Assuming that the compression is according to the law $p\ v$ = constant, find the clearance volume of the cylinder.

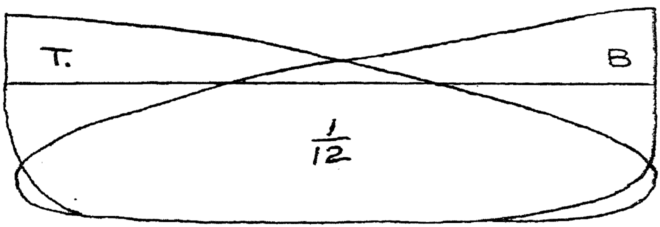
33. A ship's bunker holds 1,200 tons of coal which stows at 42 cubic feet per ton, and whose calorific value is 14,000 B.T.U. This is found to be sufficient for an 18 days' voyage. On another voyage the bunker is filled with a different quality coal, having a calorific value of 12,500 B.T.U. per pound, and stowing at 45 cubic feet per ton. For what length of voyage should this latter coal be sufficient ?

34. The lap of a slide valve is $1\frac{3}{4}$ inches. The maximum port opening to steam is 0·6 of the steam port, which is 2·4 inches in depth. What is the travel of the valve, and why is such a deep steam port necessary in the cylinder face ?

35. The diameter of a cylinder is 12 inches and the clearance volume is equal to 5 per cent. of the volume swept out by the piston. The pressure at the point of cut-off, which is 8 inches from the beginning of the stroke, is 75 lb. gauge. At the end of the stroke it is 35 lb. gauge. Find the stroke of the engine.

36. Steam is admitted to a cylinder at 200 lb. per square inch gauge. The cut-off is at 0·3 of the stroke. The clearance volume is equal to $6\frac{1}{2}$ per cent. of the volume swept out by the piston. What is the terminal pressure ?

37. The indicator cards given are from one cylinder of a triple expansion engine. The diameters of the cylinders are 30 inches, 44 inches and 80 inches, and the stroke is 51 inches. The revolutions are 70 per minute. If each engine indicates the same power, and the coal consumption is 1·6 lb. per I.H.P. hour, find the amount of coal burnt per day.



38. The travel of a valve is $6\frac{3}{4}$ inches. The maximum port opening is $1\frac{3}{4}$ inches and the lead $\frac{1}{4}$ inch. What is the angle of advance ?

39. An air vessel contains 60 pounds of air at a temperature of 63°F., and at a pressure of 500 lb. per square inch gauge. Owing to a fire in the vicinity the pressure rises to 650 lb. per

square inch gauge. Taking the specific heat of air at constant volume to be 0·17, find how many units of heat have been given to the air.

40. The lead of a slide valve is $\frac{1}{8}$ inch and the maximum port opening to steam is 1·677 inches. The eccentric sheave has an angle of advance of $31\frac{1}{2}$ degrees. Calculate the travel of the valve.

41. The area of an indicator card taken from an internal combustion engine is 6 square centimetres and the length of the card 2·74 inches. The spring used in the indicator was $\frac{1}{360}$ (1 inch = 360 lb. per square inch). Find the mean effective pressure in kilograms per square centimetre.

42. A two-stage compressor has cylinders $2\frac{1}{2}$ inches and $9\frac{1}{2}$ inches diameter, and a stroke of 8 inches. It is used to fill a blast bottle, with hemispherical ends, 12 inches diameter and 6 feet long overall. Air is taken into compressor at 15 lb. per square inch absolute, pressure in bottle is to be 900 lb. per square inch by gauge. Assuming revolutions to be constant at 120 per minute, and volumetric efficiency to be 0·9, find the ime it would take to fill the bottle.

43. In a tank 40 feet long by 20 feet wide there are 50 tons of oil at 80° Fahrenheit. The specific gravity of the oil at 60°F. is 0·9. If the volumetric expansion of the oil is 0·00034 per degree Fahrenheit, find the depth of oil in the tank.

44. A steel steam pipe is 40 feet 6 inches long, 10 inches internal diameter and $\frac{3}{8}$ inch thick, when at a temperature of 85°F. Find the length when the pipe is heated to 410°F. The co-efficient of linear expansion is 0·0000121 per degree Centigrade.

45. Steam is admitted to a cylinder at a pressure of 200 lb. per square inch by gauge, and is cut off at 0·45 of the stroke. The clearance volume is $6\frac{1}{2}$ per cent. of the volume swept out by the piston. Assuming expansion according to the law $p\ v =$ const. find the terminal absolute pressure, and the gauge pressure at 0·75 of the stroke.

46. The high pressure cylinder of a triple expansion engine is 18 inches diameter. The stroke is 3 feet and the connecting rod is 7 feet long. The boiler pressure is 180 lb. per square inch gauge and the back pressure on the H.P. piston 70 lb. per square inch gauge. Calculate the thrust in the connecting rod in pounds, and the twisting moment in the shaft in foot pounds due to the H.P. engine, when the crank is 30 degrees from top centre.

47. $6\frac{1}{2}$ litres of water are heated 50 degrees Centigrade in 14 minutes. Find the calories absorbed by the water per second and the equivalent energy in foot pounds per minute.

48. An air compressor has a piston 310 millimetres diameter and a stroke of 420 millimetres. At 210 mm. from the beginning of the stroke the pressure inside the cylinder is 14 lb. per square inch gauge, and at 350 mm. of the stroke the pressure is 45 lb. per square inch gauge. Assuming isothermal compression what is the clearance volume in cubic inches ?

49. 56 kilograms of ice at — 10°C. were placed in a tank containing 250 kilograms of fresh water at 30°C. and 10 kilograms of superheated steam at 650°F. were blown into the mixture. The saturation temperature of the steam was 490°F. Find the final temperature of the water, taking the specific heat of the superheated steam to be 0·6 and the latent heat of fusion for the ice to be 143 B.T.U. per lb.

50. The cylinder diameters of a triple expansion engine are 24 ins., 40 ins., and 64 ins. The mean pressure referred to the L.P. is 33 lb. per square inch. The I.P. engine generates 3 per cent. more power than the L.P. engine and 5 per cent. more than the H.P. engine. Find the mean effective pressure in each cylinder.

51. A boiler contains 40 tons of water. After 8 tons of fresh water are pumped in and then 8 tons of the boiler water blown out, the density is found to be 14·5 ounces per gallon. If it were practicable to blow the 8 tons out of the boiler first and then pump in 8 tons of fresh water, what would be the final density of the water in the boiler ?

52. The mechanical efficiency of a Diesel engine is 74 per cent. The heat loss to the cooling water is 27·5 per cent. and 28 per cent. is lost in the exhaust gases. The calorific value of the fuel used is 19,000 B.T.U. per lb. Find the lb. of fuel used per B.H.P. per hour.

53. The stroke of an air compressor was 2 feet 7 inches and the clearance volume equal to $1\frac{1}{2}$ inches of the stroke. The suction pressure was 1 lb. per square inch below atmospheric pressure and the delivery valves opened when the pressure reached 55 lb. per square inch by gauge. Assume isothermal compression and find how far the piston was from the beginning of its stroke when the delivery valves opened.

54. The travel of a slide valve is 7·26 inches and the lead is 0·275 inch. The depth of the port is 3·214 inches and this is 56

per cent. greater than the maximum port opening to steam. Find the steam lap, and state why the area of the port is greater than the area of the opening to steam.

55. A fuel tank 8 feet 6 inches diameter and 6 feet high when filled with oil would supply fuel for 24 hours. The specific gravity of the oil is 0·92 and the fuel consumption of the engine 0·4 lb. per I.H.P. per hour. Find the I.H.P. of the engine. If the calorific value of the fuel is 18,500 B.T.U. per lb. find also the thermal efficiency of the engine.

56. A shaft 26 inches diameter runs at 95 revs. per minute. The co-efficient of friction between the shaft and the bearings is 0·021 and the load on the bearings is 92 tons. Find the horse power required to turn the shaft at this speed and the heat units generated at the bearings per minute.

57. During a Diesel engine trial for a period of one hour, the following data were taken :—I.H.P. of engine = 3,500 ; cooling water inlet = 60°F., outlet = 127°F., and 47·5 tons of water passed through in one hour ; calorific value of the fuel = 19,300 B.T.U. per lb., and fuel consumption = 1,120 lb. for one hour. Find the thermal efficiency of the engine, and the percentage of the total heat of the fuel carried away by the cooling water.

58. A fuel oil contains 84·43% carbon, 11·15% hydrogen, 2·24% oxygen, 0·59% sulphur, and the remainder impurities. Find the calorific value of this oil, taking the C.V. of hydrogen as 62,000 B.T.U. per lb., carbon 14,000 B.T.U. per lb. and sulphur 4,200 B.T.U. per lb.

59. A mild steel bar of 1 square inch cross section and 6 feet long, is suddenly subjected to an axial load of 10 tons and extends one-twentieth of its length. Find the rise in temperature of the bar if the heat is uniformly distributed, and no heat is lost by radiation.

60. Find the B.T.U. of heat required to convert 9 lb. of ice at — 5° Cent. to dry saturated steam at 195° Cent.

61. An internal combustion engine uses 55·77 lb. of fuel per hour when running on half load. The engine consumes 17% more fuel per B.H.P. per hour on half load than when on full load. Find the weight of fuel used by the engine on full load for one day's continuous running, in kilograms.

62. 25 lb. of ice at 32°F. are dropped into 100 lb. of water at 55°F. contained in a vessel whose water equivalent is 10 lb.

How many lb. of steam at atmospheric pressure having 200° of superheat must be blown into the water to raise its temperature 60°F. above its original temperature? Specific heat of superheated steam = 0·48.

63. In an opposed piston engine, the stroke of each piston varies inversely as the weights of the moving parts which are in the ratio of 6 to 7·5. Find the strokes of the top and bottom pistons if the combined stroke is 108 inches. The cylinder diameter is 27 inches, mean effective pressure 97 lb. per sq. inch, speed 100 revs. per minute, find the brake horse power, taking the mechanical efficiency to be 92%.

64. Air is drawn into a cylinder at an absolute pressure corresponding to 381 millimetres of mercury, and compressed to 8·5 atmospheres Find the clearance volume in the cylinder as a percentage of the distance travelled by the piston, when this pressure is reached, assuming isothermal compression, and taking the barometer reading to be 30 inches of mercury.

65. Owing to a leaking condenser, the hotwell water density is 0·11 of the sea density. Blowing out is resorted to, to keep the boiler water at a certain density. The density of the water blown out of the boiler is 0·8 of the sea density, the feed water temperature is 130°F., and the temperature of the steam is 375°F. Find the percentage increase in consumption due to blowing down.

66. The total horse power generated by a compound engine is 969, and the mean effective pressure in the H.P. cylinder is 67 lb. per sq. inch. If the diameter of the L.P. piston is 2·2 times the diameter of the H.P. piston, and 1·15 times as much power is developed in the L.P. cylinder as in the H.P. cylinder, find the mean effective pressure in the L.P.

67. A rectangular block of copper measures 23 ins. by 5 ins. by 3 ins. Find the increase in volume in cubic inches, and the percentage increase, when it is heated through 300°F., taking the co-efficient of linear expansion for copper as 0·000018 per deg. Cent.

68. During an experimental run of a single cylinder steam engine, the following data were observed. Diameter of cylinder 14 inches, stroke 20 inches, mean effective pressure 45 lb. per sq. inch, average speed 165 revs. per minute. Load on brake 600 lb., effective radius of brake 5 feet. Weight of steam condensed per hour 1,950 lb. Find (*a*) I.H.P., (*b*) B.H.P., (*c*) mechanical efficiency of engine, (*d*) weight of steam used per I.H.P. per hour.

69. An air compressor with pistons 3, $13\frac{1}{2}$ and 15 inches diameter, and stroke 15 inches, runs at 140 revs. per min. and the volumetric efficiency is 88%. Find what volume of air at a gauge pressure of 1,000 lb. per sq. inch will be discharged in 30 minutes, assuming the suction to be at atmospheric pressure.

70. The hole in a crank web is 12·75 inches diameter at 65°F. Find to what temperature it must be heated so that the diameter will be increased to 12·79 inches. Co-efficient of linear expansion = 0·000006 per deg. F.

71. A compressor cylinder has a stroke of 28 centimetres, and the volume when delivery begins is 550 cubic centimetres. The cylinder has a clearance volume of 5% of the stroke volume. The pressure at the beginning of compression is 13·5 lb. per sq. inch absolute, and at the end of compression 68 lb. per sq. inch abs. If the law of the compression curve is $p\ v^{1 \cdot 35}$ = constant, find the cylinder diameter.

72. A closed vessel contains 500 c.c. of gas under pressure. The pressure is measured by a U tube containing mercury, one end of the tube being attached to the vessel and the other end open to the atmosphere. The difference in level of the mercury is 8 inches. More mercury is now poured into the open end until the difference in level is 14 inches. Find the percentage change in volume of the gas, if the temperature does not change. The mercury barometer stands at 30 inches.

73. An air compressor delivers 10 lb. weight of air at 25 atmospheres pressure and 120°F. to an air storage vessel. The suction air to the compressor was at atmospheric pressure and 70°F. What is the volume in cu. feet of the vessel? Note: 1 cu. foot of air at 32°F. and at atmospheric pressure weighs 0·0807 lb.

74. An eight cylinder double acting four stroke cycle heavy oil engine develops 4,182 B.H.P. when running at 110 revs. per minute. The cylinders are $26\frac{3}{4}$ ins. dia. and the stroke is 55 ins. The mechanical efficiency is 0·82, and the ratio of the power developed in the top and bottom of the cylinder is 10 : 7. If the piston rod is 9 ins. diameter, calculate the mean indicated pressure on the top and on the under side of the piston.

75. In a heavy oil engine having a mechanical efficiency of 0·78, the heat carried away by the exhaust gases and cooling water are 28% and 30% respectively of that contained in the fuel. If 60% of the heat in the exhaust gases is recovered in a waste heat boiler, calculate,

(*a*) What proportion of the available heat is represented by the power transmitted to the shaft.

(*b*) How many heat units are lost per lb. of fuel consumed, if the calorific value is 19,500 B.T.U. per lb.

76. A steam engine drives a 100 kilowatt generator, and is supplied with steam from an oil fired boiler. If the overall efficiency of the plant is 7%, what is the fuel consumption per kilowatt hour, if the fuel has a calorific value of 19,500 B.T.U. per lb. ?

77. In a single cylinder engine the maximum torsional stress induced in the crank shaft, which is 3·25 inches diameter, is 7,250 lb. per square inch. The stroke of the engine is 10·5 inches, and the maximum effective pressure acting on the piston is 95 lb. per square inch. Find the piston diameter, neglecting the effect of the angularity of the connecting rod and frictional resistances.

$$\text{For Torsion } \frac{T}{J} = \frac{q}{r}, \text{ and } J = \frac{\pi r^4}{2}$$

78. A solid brass sphere is heated through 167°F., and the increase in diameter is found to be 0·021 inch. If the co-efficient of cubical expansion of brass is 0·0000315 per degree F., find the original diameter of the sphere.

79. The stroke of a gas engine is 18 inches and the clearance volume is equivalent to 3·5 inches of the stroke volume. A plate is attached to the piston so that the clearance volume is reduced to the equivalent of 3 inches of the stroke volume. If the gas and air mixture is taken in at 14 lb. per square inch absolute in both cases, and the index of the law of compression is 1·33, find the final compression pressure before and after the alteration. Show by a sketch the effect of the alteration upon the final compression pressure.

80. A gram calorie is $\frac{1}{100}$th part of the heat required to raise the temperature of 1 gram of water from 0°C. to 100°C. An oil bath contains 13 litres of oil of specific gravity 0·9, and specific heat 0·47. The temperature is raised from 40°C. to 66°C. in 14 minutes. Find (*a*) the heat given in B.T.U., (*b*) the heat given per second in gram calories.

81. Describe how you would obtain the B.H.P. of a small oil engine. The following data were obtained during a test :—
I.H.P. = 120, B.H.P. = 90, Oil consumption per hour = 44 lb. Calorific value of the oil = 19,200 B.T.U. per lb. Calculate (*a*) the mechanical efficiency, (*b*) the indicated thermal efficiency.

82. The swept volume of the cylinder of an engine is 4·63 cubic feet per stroke, and the mean piston speed is 1,100 feet per minute. The stroke is 22 inches, the mechanical efficiency is 0·78, and 300 B.H.P. is developed. Find the number of B.T.U. expended in work done per hour per square inch of piston area.

83. In a four stroke Diesel engine the maximum lift of the exhaust valve is 2 inches. It opens at 32° before the bottom dead centre, and closes at 18° after the top dead centre. The valve is full open from 18° before the bottom dead centre to 6° before the top dead centre. If the engine runs at 120 revolutions per minute, find (*a*) the time the valve is open, (*b*) the time it is full open, (*c*) the mean velocity of opening, (*d*) the mean velocity of closing.

84. An auxiliary engine, running at full load, consumes 640 kilograms of fuel per day. At half load the consumption per horse power is 18% more than at full load. Assuming the increase in consumption per horse power varies directly as the reduction of load, calculate the consumption at three-quarters of full load.

85. A solid sphere, 12 inches diameter, is made of material of specific gravity 2·56, and specific heat 0·212. Find the increase in diameter after 1,036 B.T.U. have been given to the sphere if the co-efficient of cubical expansion is 0·0000384 per degree Fahrenheit.

86. One pound weight of oil gas, of calorific value 18,500 B.T.U. per lb., is mixed with 23 lb. of air at 80°F., and completely burnt under constant volume conditions. If the initial pressure was 20 lb. per sq. inch absolute, what is the pressure when combustion is complete ?

The specific heat at constant volume is 0·169.

87. A steam pipe, 60 feet long and 4 inches external diameter, connects a boiler and an engine. At the boiler end of the pipe the steam pressure is 180 lb. per sq. inch absolute, temperature 373·1°F., and the steam is dry and saturated. At the engine end of the pipe the steam pressure is 165 lb. per sq. inch absolute, temperature 366°F., and the steam is 0·95 dry.

Find (*a*) the heat lost by radiation, etc., per minute, when 54 lb. weight of steam pass through the pipe per minute, (*b*) the loss of heat per square foot of pipe surface per minute.

88. The mean area of the indicator diagrams from a steam reciprocating engine is 5·175 sq. inches, the spring used being 1 inch = 72 lb. The length of the diagram represents the stroke of the engine to a scale of 1 inch = 1 foot. The cylinder is 26 inches diameter and the engine runs at 75 revs. per minute. Find the indicated horse power.

89. The scavenge ports of a two-stroke cycle Diesel engine are just covered when the piston is 800 m.m. from the top of the stroke. The contents of the cylinder at this instant are at 3 lb. per sq. inch by gauge and 105°F. The piston diameter is 700 m.m. and the clearance is equivalent to 70 m.m. Find the weight of air taken in per cycle, if the scavenge efficiency is 0·95.

Note.—1 cu. ft. of air at 32°F. and 14·7 lb. per sq. inch absolute weighs 0·0807 lb.

90. The error in marking off an eccentric keyway was $\frac{1}{2}$ inch in advance of the correct position. The intended angle of advance was 30°. The shaft was 12 inches diameter; valve travel 6 inches; maximum port opening to steam $1\frac{3}{4}$ inches. Find the lead the valve will have, and how much must be added to the lap in order to make the lead the same amount as was originally intended.

91. A steel shaft is 10 inches diameter and has a brass sleeve shrunk upon it. They are at 60°F. Find the temperature to which both must be raised so that the diameter of the sleeve is 0·02 inch more than that of the shaft. Neglect strains due to shrinkage. The coefficient of linear expansion for brass is 0·0000136 per 1°F., and for steel 0·0000067.

92. The feed water of a boiler weighed 1010 ozs. per cubic foot, and the boiler pressure was 215 lb. per sq. inch by gauge. The feed pump takes 20 horse power to drive it and it delivers 110 gallons per minute. Find the work done in pumping one pound weight of water into the boiler, and the mechanical efficiency of the pump.

93. A vessel is 18 inches high and 12 inches diameter. It contains water to a depth of 16 inches at 60°F. Find the depth of water at 200°F. The co-efficient of volumetric expansion of water is 0·00012 per 1°F.

94. A slide valve has 1·43 inches steam lap and 0·21 inch lead. The angle of advance is 29° 40′. The steam port is 26·5 inches broad and 2·75 inches deep. What is the greatest area of opening to steam ?

95. A hand pump, with a stroke of 9 inches, is used to pump air into a vessel where the pressure is 45 lb. per sq. inch abs. The atmospheric pressure is 15 lb. per sq. inch abs., and the index of the law of compression is 1·4. How far will the piston move down the stroke before any air enters the vessel, if the piston is moved quickly ?

96. The diameters of a cylinder and its piston valve liner are 30 ins. and $12\frac{1}{2}$ ins. respectively, and the mean piston speed is 700 feet per minute. If one third of the area through the cylinder

ports of the liner is obstructed by diagonal guide bars, find the depth of these ports to limit the speed of the exhaust steam to 8,500 feet per minute.

97. The water in a boiler has a density of 1,075 ounces per cu. foot. 10 tons of water at a density of 1,007 ounces per cu. foot are pumped in, and 10 tons at a density of 1,055 ounces per cu. foot are blown out. What is the weight of water contained in the boiler ?

98. 2 lb. of ice are converted into superheated steam having the temperature of 400°F. The temperature of saturation is 300°F., and the specific heat of superheat is 0·48. Describe the changes that occur to the ice during its conversion into steam, and give the quantity of heat supplied to effect the change at each stage. The initial temperature of the ice was 22°F.

99. Explain why intercooling is used in multi-stage air compressors. Show by a carefully sketched diagram the effect of the intercooling, and mark on the diagram the pressures and temperatures.

100. State why impulse blading is generally employed, in preference to reaction blading, at the high pressure end of a turbine where the steam may be highly superheated.

101. 300 grams of cast iron, specific heat 0·13 are held in the funnel gases until the temperature is the same as that of the gases. The cast iron is then placed in 0·16 litre of water and the final temperature of the water becomes twice the initial temperature. If no water is evaporated, find the maximum possible temperature of the gases.

SECOND-CLASS EXAMINATION QUESTIONS

NAVAL ARCHITECTURE

1. The engines on one occasion indicated 1,560 horse power, the speed was $14\frac{1}{2}$ knots, and the pressure on the thrust 52 pounds per square inch. Owing to a head wind the speed is reduced to 12 knots and the horse power is increased to 1,680. What is now the pressure on the thrust ?

Note.—The pressure on the thrust block varies directly as the horse power and inversely as the speed of the ship.

2. A ship has a consumption of 30 tons per day at $11\frac{1}{2}$ knots. If the speed is reduced to 9 knots, find the saving in consumption per day and on a voyage of 1,900 nautical miles.

3. A vessel takes cargo in two holds. The centre of gravity of the forward hold is 94 feet forward of the centre of gravity of the ship. The distance between the centres of the two holds is 150 feet. If the weight of cargo taken in is 530 tons, how much should be placed in each hold if the trim of the ship is not to be altered ?

4. The area of the water level of a steamer is 14,020 square feet. The draught is 21 feet forward and 22 feet 3 inches aft. After taking in 430 tons of bunkers the ship is on an even keel. What is her draught ?

5. The draught on arrival in port is 21 feet 6 inches forward and 23 feet 9 inches aft. The area of the water level is 12,525 square feet and 850 tons of coal have been burnt on the voyage. The ship left port on an even keel. What was her draught ?

6. A steamer of 9,000 tons displacement has a water plane area of 12,000 square feet. What is her change in draught in passing from a river whose density is 1,018 ounces per cubic foot into the sea where the density is 1,026 ounces per cubic foot ?

7. The draught of a ship is 25·7 feet forward and 26·8 feet aft. After taking in 324 tons of coal the draught is 26·9 feet forward and 28·4 feet aft. What is the "tons per inch immersion," and what is the area of the water plane ?

8. A rectangular pontoon, 25 feet by 12 feet, has a draught of 20·5 inches in sea water. It is covered on the inside of the bottom with teak 4 inches thick. What is now the draught if the teak weighs 48 lb. per cubic foot ?

9. The mean effective pressure of an engine is 29 lb. per square inch and the revolutions 69 per minute. What will be the revolutions when the mean effective pressure is reduced to 25 pounds per square inch ? If the I.H.P. is 1,200 in the first case, what will it be in the second case ?

Note.—The mean effective pressure varies as the number of revolutions squared.

10. The horse power of a marine engine is 1,400, the revolutions 62 per minute, referred mean pressure 28 pounds per square inch. Owing to a head wind the revolutions are reduced to 58 per minute. If the horse power has been kept constant, what per cent. increase of pressure has been necessary ?

11. A cubic foot of solid coal weighs 80 lb. The same coal broken up stows at 44 cubic feet per ton. A bunker when full contains 175 tons of coal. If the compartment is flooded with sea water, how many tons of water will it hold ?

12. The tons per inch immersion of a steamer is 40. Find (*a*) the volume of displacement of a layer 4 inches deep in the same plane ; (*b*) the area of the surface of the plane.

13. When the speed of a ship was 12 knots the indicated horse power was 12,600, and the pressure on the thrust was 48 lb. per square inch. When the horse power was changed to 12,450 the speed was 11 knots. Find the pressure on the thrust.

Note.—The pressure on the thrust varies directly as the horse power and inversely as the speed.

14. A ship floating in water of a density of 1·015 has its draught increased $1\frac{3}{4}$ inches when 80 tons of coal are put on board. What is the water plane area of the ship ?

15. When the speed of a certain ship is 10 knots, the consumption is 35 tons per day. The speed is increased to 11 knots for 12 hours, and then reduced to 9 knots for the remaining 12 hours. Find the consumption for the day.

16. A bulkhead is 40 feet wide and 20 feet deep. It is flooded with sea water on one side to a depth of 16 feet. Find the total pressure on the bulkhead, and state the position of the point at which the whole pressure may be taken to act.

17. A barge, 60 feet long, 30 feet beam and 11 feet deep, floats at 6 feet draught, in water whose density is 1·024. At what draught will it float in fresh water, and what is the weight of the barge ?

18. The frictional resistance of a ship's hull is 0·3 pound per square foot at a velocity of 600 feet per minute. The wetted area of the ship is 18,000 square feet. What is the speed in knots when the horse power expended in driving the ship is 2,000 ? Take the friction to vary as the velocity squared.

19. A ship steaming at 15 knots burns 100 tons of coal per day. There are 900 nautical miles to go and there are 160 tons of coal on board. The ship arrives with 10 tons of coal left, find the time taken in hours to complete the voyage.

20. The friction of an experimental vessel is found to be 0·35 lb. per square foot of wetted surface when drawn through the water at 600 feet per minute. Find the horse power required to drive a similar ship of 20,000 square feet of wetted surface at 18 knots, taking the frictional resistance to vary as the $(\text{speed})^{2 \cdot 1}$

21. The frictional resistance of an experimental vessel similar to that of a ship's hull was found to be 0·54 lb. per square foot of wetted surface when towed through the water at 800 feet per minute. If the frictional resistance varies as the $(\text{velocity})^{2 \cdot 2}$, find the speed in knots of a ship of similar form with a wetted surface of 21,000 square feet and a thrust horse power of 2,634.

22. The speed of a ship is decreased 11 per cent. Find (*a*) the percentage decrease in daily consumption of fuel, and (*b*) the percentage decrease in the cost of the fuel for a voyage of 3,000 miles.

23. A rectangular box barge 58·5 feet long and 15·75 feet broad floats at a draught of 3·25 feet in sea water which has a density of 1,016 ozs. per cubic foot. Find the weight of the barge in tons, and find what the increase in draught will be if 68·5 tons of coal are put aboard. If the barge was floating in fresh water and loaded with this 68·5 tons of coal, what would be the draught ?

24. A manhole on a tank top has semi-circular ends 9 inches radius, and the overall length is 24 inches. The centres of the studs are $1\frac{1}{8}$ ins. from the edge of the manhole, and the area of each stud at the bottom of the thread is 0·304 sq. inch. There are 16 studs altogether. Find the stress set up in the studs due to water pressure on the door when the tank is pressed up with sea water rising to 24 feet up the sounding pipe above the tank top. Take the area of door exposed to water pressure as that within the pitch perimeter of the studs.

25. A ship of 8,450 tons displacement has a block co-efficient of 0·78. If the beam is 0·14 of the length and 2·12 times the draught, find the beam.

26. A ship steaming at 17 knots burns 170 tons of fuel per day. The speed is reduced and the consumption is now 128·7 tons per day. If the horse power varies as the $(\text{speed})^3$ theoretically, and it is found that the fuel consumption at the reduced speed is 10% greater than the theoretical figure, what is the reduced speed, and what is the saving in fuel for a voyage of 3,000 nautical miles ?

27. A tanker with engines aft has a displacement of 12,000 tons. 150 tons of oil are transferred from a forward tank to a cross bunker aft. The C.G. of the forward tank is 200 feet forward of the ship's original C.G. and the C.G. of the after tank is 150 feet aft of the ship's original C.G. Find the shift of the ship's centre of gravity due to the transference of the oil. If, in the first place, the after tank had 250 tons of oil and 200 tons are burnt after the above 150 tons of oil are transferred, what would be the shift of the ship's centre of gravity ?

28. Before bunkering, the draught of a vessel was 18 feet 9 inches forward and 21 feet 4 inches aft. After loading 650 tons of coal, the draught was 21 feet 7 inches forward and 23 feet 4 inches aft, the density of the water being 1,026 by the hydrometer. If the co-efficient of the water plane area of the vessel is 0·7, and the length to breadth ratio is 7 to 1, what is the length of the vessel ?

29. A box shaped barge, 120 feet long and 20 feet broad, floats on an even keel at 8 feet draught in sea water, the hydrometer reading being 1026. The forward compartment is 10 feet long, and water of hydrometer reading 1010, is pumped in to a depth of 5 feet. Neglecting the effect of change of trim, find the horizontal distance that the centre of gravity of the vessel will move.

30. A cube of wood, having sides 2 feet long, is placed in a cubical tank the sides of which are 3 feet long. Initially, the depth of water in the tank was 2 feet. If the specific gravity of the wood is 0·8, find the pressure on the bottom and sides of the tank when the wood is floating freely in the water.

31. A vessel of 7,000 tons displacement is floating on an even keel. She has two bunkers of rectangular cross section and identical shape, each 35 feet deep. The forward one contains 550 tons of coal and is full, the after one is empty. Find the weight of coal to move from the forward bunker to the after one to lower the centre of gravity of the ship by 6 inches. Assume the vessel remains on an even keel.

SECOND-CLASS EXAMINATION QUESTIONS.

ELECTRICITY.

1. A conducting wire 200 centimetres long and 0·004 sq. centimetre cross sectional area has a resistance of 1·65 ohms. Calculate the specific resistance of the material.

2. A shunt across an ammeter has a resistance $\frac{1}{50}$th of the resistance of the ammeter. The current in the circuit is 165 ampéres. Calculate the current passing through the ammeter and the current passing through the shunt.

3. A copper conductor 500 yards long and $\frac{1}{8}$ inch diameter carries a current of 200 ampéres. Calculate the voltage required to cause this flow of current, given that the resistance of a copper wire 74·1 yards long and 0·048 inch diameter is one ohm.

4. An emergency battery consists of two hundred electric cells. The cells are arranged in two series groups of 100 each, the groups being connected in parallel. The E.M.F. of each cell is 1·9 volts and its internal resistance is 0·1 ohm. The resistance of the external circuit is 4 ohms. What is the maximum horse-power available in an emergency ?

5. The internal resistance of a series wound dynamo is 0·5 ohm, the resistance of the external circuit is 10 ohms and the voltage across the output terminals is 200. Find (*a*) the output of the dynamo in electrical horse power, and (*b*) the horse power to drive it neglecting all sources of loss except those due to internal resistance.

6. The current passing through a copper wire at 30° C. was 2 ampéres and the P.D. on its ends was 220 volts. Find the current passing when the temperature of the resistance is raised to 70°C. and the P.D. is kept constant.

Note. $R_t = R_0 (1 + a\ t)$ where R_t = resistance at $t°$ C., R_0 = resistance at 0° C., a = temperature co-efficient which is 0·00428 per deg. C. for copper, t = temperature in degrees C.

7. A current of 2 ampéres flows through a circuit which has a resistance of 52 ohms. What is the energy dissipated in one hour, expressed in units of heat ? What is the potential difference across the terminals ?

8. An electric cell has an E.M.F. of 1·5 volts and an internal resistance of 3 ohms. It is connected to two resistances which are in parallel, of 10 ohms and 15 ohms respectively. Find (*a*) the total current flowing in the circuit, and (*b*) the P.D. between the terminals.

9. An electric circuit consists of three hundred and fifty 100 volt, 60 watt lamps, and one 12 B.H.P. motor which has an efficiency of 81%. The dynamo supplying current to this circuit has an efficiency of 91% and is driven by a steam engine whose mechanical efficiency is 84%. Find (*a*) the output of the generator in kilowatts, (*b*) the I.H.P. of the engine.

10. What is meant by "volumetric resistivity"? Find the resistivity of a wire 2·4 millimetres diameter, 9 metres long, which has a resistance of 0·34 ohm.

11. A conductor having an active length of 30 centimetres moves at right angles to a magnetic field at a velocity of 25 metres per second. If the flux density of the field is 8,500 lines per sq. centimetre, find the E.M.F. induced in the conductor in volts.

12. Six electric cells are connected together in series, the E.M.F. of each cell is 1·05 volts and its internal resistance is 0·5 ohm. Three wires, A, B and C, whose resistances are 3, 30 and 300 ohms respectively, can be connected between the battery terminals. Determine the current which flows when each wire is inserted separately; and also determine the current which flows when they are all inserted together as a parallel group.

13. An electric heater raises the temperature of 5·8 tons of oil through 100 F°. in 24 hours. The specific heat of the oil is 0·45 and the P.D. across the heater is 220 volts. Find the equivalent foot lb. of energy dissipated by the heater, the equivalent horse power, and the current supplied in ampéres.

14. An accumulator is charged with a steady current of 3 ampéres for 24 hours and then discharged at the rate of 1·8 ampéres for 36 hours. Find its ampére hour efficiency.

15. A group of three resistances in parallel, of 3, 4 and 6 ohms respectively, is connected in series with a resistance of 2 ohms. The voltage across the extreme ends is 100. Find the total current flowing and the current flowing through each resistance.

16. A generator is driven by a six cylinder four stroke single acting oil engine at 240 revs. per min. The output of the generator is 60 kilowatts. The mean effective pressure of the engine is 85 lb. per sq. inch, the stroke is $1\frac{1}{2}$ times the cylinder diameter, and the mechanical efficiency is 80%. Assuming the efficiency of the generator to be 92%, find the diameter of the engine cylinders.

17. A 1·5 kilowatt heater which has an efficiency of 0·9 is used to heat two gallons of fresh water. Find the rise in temperature of the water after half an hour.

18. An armature of a 4 pole dynamo rotates at 600 revs. per min. The area of each pole face is 900 sq. centimetres and the

flux density in the air gap is 9,200 lines per sq. centimetre. Find the average E.M.F. induced in one conductor.

19. An electric motor working off a 400 volt D.C. supply, drives a pump which lifts 180,000 gallons of fresh water per hour to a height of 33 feet. The efficiency of the electric motor is 90% and that of the pump 60%. Determine the brake horse power of the electric motor and the current taken by it.

20. Calculate the quantity of electricity in ampére-hours and the energy consumed in B.O.T. units by a heating element having a resistance of 180 ohms working off a 220 volt supply system for 24 hours.

21. A 110 volt dynamo supplies two hundred and seventy 30 candle power lamps. The efficiency of the dynamo is 0·9 and it is driven by a 15 B.H.P. engine. Calculate the consumption of the lamps in watts per candle power.

22. Find the force required (*a*) in dynes, (*b*) in grams, (*c*) in lb., to move a conductor 45 centimetres long, carrying a current of 109 ampéres, across a magnetic field of 8,000 lines per sq. centimetre.

Note. 981 dynes = 1 gram.

23. The voltage drop across a conductor is 15 when 6 ampéres flow through. Find the energy dissipated in ft. lb. per hour, and the heat units generated.

24. A current of 2·2 ampéres flows through a copper conductor when its temperature is 18° C. Find the current which flows when the temperature rises to 36° C. if the voltage remains constant.

$R_t = R_o \ (1 + a\ t) \qquad a = 0{\cdot}00428$ per deg. C. for copper.

25. When a current of 10 ampéres is passed through a solution of copper sulphate 8 grams of copper are deposited. If the electro-chemical equivalent of copper is 0·00033 gram per coulomb, and the chemical equivalent of copper is 31·8, find how long the current flowed, and how much hydrogen was liberated from the solution.

26. The filament of an electric lamp is 60 cms. long and 0·04 m.m. diameter. The volumetric resistivity of the material of the filament is 0·0000165 ohm per centimetre cube. Calculate the current consumed by the lamp when in a 110 volt circuit, if the resistance of the filament when hot is five times greater than when cold.

SECOND-CLASS EXAMINATION QUESTIONS

SOLUTIONS TO GENERAL ENGINEERING SCIENCE.

1. Let distance $= x$, then total length $= x + 12$.

By moments about fulcrum :—

$3^2 \times 0{\cdot}7854 \times 90 \times x = (12 + x)\ 90$

$\therefore\ 7{\cdot}0686\ x = 12 + x$

$\therefore\ 6{\cdot}0686\ x = 12$

$$x = \frac{12}{6{\cdot}0686} = 1{\cdot}977 \text{ inches. Ans.}$$

$30^2 \times 1$ -	900	By Simpson's Rule.
$34^2 \times 4$ -	4624	
$36^2 \times 2$ -	2592	
$34^2 \times 4$ -	4624	
$30^2 \times 1$ -	900	
12	12)13640	
	1136·6	

Volume $= 0{\cdot}7854 \times 1136{\cdot}6 \times 48$ cubic inches.

$$\text{Volume in gallons} = \frac{0{\cdot}7854 \times 1136{\cdot}6 \times 48 \times 6{\cdot}25}{1{,}728}$$

$= 155$ gallons. Ans.

3. Area supported $\times$ Pressure $=$ Area stay $\times$ Normal stress

$$\therefore\ \text{Normal stress} = \frac{8^2 \times 160}{1{\cdot}625^2 \times 0{\cdot}7854}$$

When adjacent stay is broken the stress is $\frac{1}{3}$ more, i.e., the actual stress is $\frac{4}{3}$ normal stress.

$$\therefore\ \text{Actual stress} = \frac{8^2 \times 160 \times 4}{1{\cdot}625^2 \times 0{\cdot}7854 \times 3}$$

$= 6{,}584$ pounds per sq. inch. Ans.

4. Tank is 51 inches deep.
It is filled at $\frac{51}{45}$ inches per minute.
And emptied at $\frac{51}{75}$ inch per minute.
With both injection open and pump running it is filled :—

$$\frac{75 \times 51 - 45 \times 51}{45 \times 75} \qquad \frac{30 \times 51}{45 \times 75} \text{ inch}$$

per minute.

Depth to be filled $= 51 - 18 = 33$ inches.

$$\therefore \text{ Time} = \frac{33 \times 45 \times 75}{30 \times 51} = 72{\cdot}8 \text{ minutes. Ans.}$$

5. Total length of lever $= 3\frac{1}{4} + 36\frac{3}{4} = 40$ inches.

Pressure $\times$ Area plunger $\times\ 3\frac{1}{4}$ = load on handle $\times$ 40.

$$\therefore \text{ Load on handle} = \frac{430 \times 0{\cdot}7854 \times 2^2 \times 3{\cdot}25}{40}$$

$= 109{\cdot}8$ pounds. Ans.

6. $T - 10$
$\times\ 120 = P$

$$\therefore 180 = \frac{T - 10}{3} \times 120$$

$\therefore$ T $= 14{\cdot}5$ hundredths.

$= 0{\cdot}145$ inch. Ans.

7. Volume in cubic feet $\times$ Weight per cubic foot $\times$ Average height lifted = Work done in 45 minutes $\times$ Efficiency.

$\therefore 124 \times 42 \times 3 \times (19{\cdot}5 + 1{\cdot}5) \times 64 = 18 \times 33{,}000 \times 45 \times$ Eff.

$$\therefore \text{ Efficiency} = \frac{124 \times 42 \times 3 \times 21 \times 64}{18 \times 33{,}000 \times 45}$$

$= 0{\cdot}7857$ or $78{\cdot}57$ per cent.

$\therefore$ Loss $= 100 - 78{\cdot}57 = 21{\cdot}43$ per cent. Ans.

8. Percentage strength of plate

$$\frac{\quad}{P} \times 100 = \frac{\quad}{8\frac{1}{2}} = 85{\cdot}3 \text{ per cent.}$$

Percentage strength of rivets

$$\frac{A \times N \times f_s \times \text{Factor}}{P \times t \times f_t} \quad \frac{1\frac{1}{4}^2 \times 0{\cdot}7854 \times 5 \times 1\frac{3}{4} \times 23}{8\frac{1}{2} \times 1\frac{1}{8} \times 28} \times 100$$

$= 92{\cdot}24$ per cent.

Strength of joint $= 85{\cdot}3$ per cent. Ans.

9. Area of one bolt

$$\frac{8 \times 1\frac{1}{2}(8 \times 1\frac{1}{2} - 1)}{100} \quad \frac{12 \times 11}{100} = 1{\cdot}32 \text{ square ins.}$$

$$36^2 \times 0{\cdot}7854 \times 160 = f \times 25 \times 1{\cdot}32$$

$$f \quad \frac{36^2 \times 0{\cdot}7854 \times 160}{25 \times 1{\cdot}32}$$

$= 4{,}936$ pounds per square inch. Ans.

10. Specific gravity of cast iron is $7{\cdot}21$.

$\therefore$ $\frac{3}{4}$ ton of cast iron in water weighs $\dfrac{7{\cdot}21 - 1}{7{\cdot}21} \times \frac{3}{4}$

$= 0{\cdot}646$ ton.

$\therefore$ Increase in draught $= \dfrac{5 \times 12 \times 0{\cdot}646}{\quad}$

$= 12{\cdot}92$ inches. Ans.

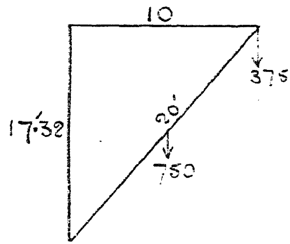
11. B.M. $= 3 \times 12 \times 2{,}800$ inch pounds

$$\text{Stress} = \frac{6\ M}{B\ D^2}$$

$$\frac{\overline{6\ M}}{\quad} \qquad \frac{\overline{\times 3 \times 12 \times 2{,}800}}{6{,}000 \times 2{\cdot}75}$$

$= 6{\cdot}06$ inches. Ans.

12. Since the centre of gravity is at mid-length, the weight of the derrick acting downward through the centre of gravity is equivalent to $\frac{750}{2}$ = 375 lb. acting at the end of the derrick.



$$\text{Force in tie} = \frac{10}{17\cdot32} \times 375$$

$$= 216\cdot5 \text{ pounds.} \quad \text{Ans.}$$

$$\text{Force in jib} = \frac{20}{17\cdot32} \times 375 = 433 \text{ pounds.} \quad \text{Ans.}$$

13. Circumference = 12 × 3·1416

$$40 \text{ per cent. of circumference} = \frac{40 \times 12 \times 3\cdot1416}{100}$$

9 per cent. of this is cut away, i.e., 0·4 × 12 × 3·1416 × 0·09

Amount left = 0·4 × 12 × 3·1416 — 0·036 × 12 × 3·1416

= 0·364 × 12 × 3·1416

∴ Area = 0·364 × 12 × 3·1416 × 13

= 178·4 square inches. Ans.

14.

$$\text{Percentage strength of plate} = \frac{7\cdot5 - 1\cdot625}{7\cdot5} \times 100$$

$$\frac{5\cdot875}{7\cdot5} \times 100$$

$$\frac{2\,t\,S}{D} \qquad \frac{\text{Per cent. strength of plate}}{100}$$

$$\therefore S = \frac{P \times D \times 100}{2\,t \times \text{per cent. strength of plate}}$$

$$\frac{180 \times 12 \times 12 \times 7\cdot5}{2 \times 1\cdot5 \times 5\cdot875}$$

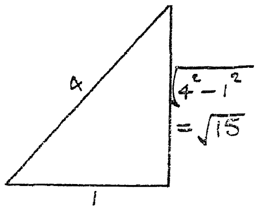
= 11,030 pounds per square inch. Ans.

15. By moments about centre of shaft :—

$$90 \times 18 \times 0{\cdot}79 = W \times \frac{\quad + 1\frac{1}{4}}{2} = 6{\cdot}625\ W$$

$$\frac{90 \times 18 \times 0{\cdot}79}{6{\cdot}625} \text{ pounds. Ans.}$$

16. Load on piston $= 72^2 \times 0{\cdot}7854 \times 20$ pounds.



$\therefore$ Load on crank pin when crank is horizontal

$$= 72^2 \times 0{\cdot}7854 \times 20 \times \frac{4}{\sqrt{15}}$$

$$= 400 \times 14 \times L$$

$$\therefore L = \frac{72^2 \times 0{\cdot}7854 \times 20 \times 4}{400 \times 14 \times \sqrt{15}} = 15{\cdot}02 \text{ inches. Ans.}$$

17. By Simpson's Rule

$42{\cdot}5 \times 1$	$42{\cdot}5$
$41{\cdot}5 \times 4$	166
40×2	80
36×4	144
25×1	25
12	$12)457{\cdot}5$
	$38{\cdot}125$ = mean width

$$\text{Weight of coal} = \frac{38{\cdot}125 \times 48 \times 26}{46}$$

$= 1{,}034$ tons. Ans.

18. Work done $= 30 \times 0{\cdot}8333 \times 33{,}000 \times$ Time in minutes $= 2{,}500 \times 64 \times 22$

$$\therefore \text{Time} = \frac{2{,}500 \times 64 \times 22}{30 \times 0{\cdot}8333 \times 33{,}000}$$

$= 4{\cdot}265$ minutes. Ans.

19. ·1 T 5·1 T

$$= \frac{5{\cdot}1 \times 11 \times 14^2 \times 0{\cdot}7854 \times 32}{(3\tfrac{7}{8})^3}$$

$= 4{,}749$ pounds per square inch. Ans.

Yes. Shaft is strong enough as stress allowed is 7,700 pounds per square inch.

20. Area of edge of jointing + Area of end of roller = Area of end of roll.

2·5 millimetres $= 2{\cdot}5 \times 0{\cdot}03937$ inches.

$\therefore$ Length in feet $\times 12 \times 2{\cdot}5 \times 0{\cdot}03937$

$$= 0{\cdot}7854\,(10^2 - 2^2)$$
$$= 0{\cdot}7854\,(10 + 2)\,(10 - 2)$$

$$\therefore \text{Length} = \frac{0{\cdot}7854 \times 12 \times 8}{12 \times 2{\cdot}5 \times 0{\cdot}03937} = 63{\cdot}83 \text{ feet. Ans.}$$

21 Throw $= \dfrac{3\frac{1}{4}}{2} = 1\frac{5}{8}$ inches.

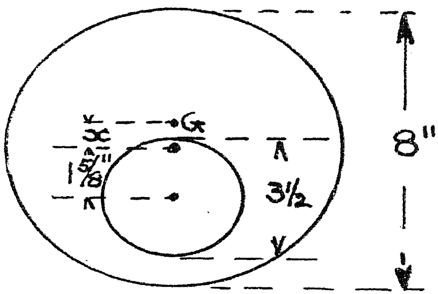
By moments about centre of sheave :—

$$(0{\cdot}7854 \times 8^2 \times 0) - (3\tfrac{1}{2}^2 \times 0{\cdot}7854 \times 1\tfrac{5}{8}) = 0{\cdot}7854\,(8^2 - 3\tfrac{1}{2}^2)\,x$$

$$\therefore x = \frac{-3\frac{1}{2}^2 \times 1\frac{5}{8}}{\quad} = -0{\cdot}385 \text{ inch from centre. Ans.}$$

Note.—The negative sign indicates that the centre of gravity is on the opposite side of the centre of the sheave to that on which moments were taken, i.e., on the opposite side of the centre of the sheave to the shaft hole.

Alternative solution as follows :—



Alternative method :—

By moments about top.

$$(0{\cdot}7854 \times 8^2 \times 4) - (0{\cdot}7854 \times 3{\cdot}5^2 \times 5{\cdot}625) = 0{\cdot}7854\ (8^2 - 3{\cdot}5^2)\ x$$

$$(8^2 \times 4) - (3{\cdot}5^2 \times 5{\cdot}625)$$
$$= 3{\cdot}615 \text{ inches.}$$

$\therefore$ Centre of gravity is $4 - 3{\cdot}615 = 0{\cdot}385$ inch from centre of sheave. Ans.

22 Assuming 12 inches of water over C.C. tops,

Q to be blown out = 264 cubic feet.

Q per minute $= 2{\cdot}5 \times 1{\cdot}25^2 \times \sqrt{180}$ cubic feet.

$$\text{Time} = \frac{264}{2{\cdot}5 \times 1{\cdot}25^2 \times \sqrt{180}} = 5{\cdot}038 \text{ minutes.}$$

= 5 minutes 2·28 seconds. Ans.

Neglecting the weight of the beam since the material is not given :—

Bending moment at centre

$$W\,L \qquad 2 \times 2{,}240 \times 8 \times 12$$
inch pounds.

$$\text{Stress} = \frac{6\ M}{B\ D^2} \quad \frac{6 \times 2 \times 2{,}240 \times 8 \times 12}{4 \times 6 \times 9^2}$$

$= 1{,}327$ pounds per square inch. Ans.

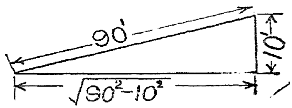
24. $S = \sqrt[3]{\dfrac{D\ W}{11{,}000}}$

$$\therefore S^3 = \frac{D\ W}{11{,}000} \quad \therefore D = \frac{\times}{W}$$

$$\frac{0{\cdot}75^3 \times 11{,}000}{180 \times 4^2 \times 0{\cdot}7854} = 2{\cdot}054 \text{ inches.}$$

$\therefore$ Outside diameter $= 2{\cdot}054 + 0{\cdot}75 = 2{\cdot}804$ inches. Ans.

25. Work done in 30 secs. = Work against gravity + Work against friction.



Work done in 30 secs.

$= 5 \times 2{,}240 \times 10 + \frac{10}{100} \times 5 \times 2{,}240 \times \sqrt{90^2 -}$

$= 212{,}200$ foot lb.

Work done per min. $= 212{,}200 \times \frac{60}{30}$

$$\therefore \text{H.P.} = \frac{212{,}200 \times 60}{33{,}000 \times 30} = 12{\cdot}88 \text{ H.P.} \quad \text{Ans.}$$

26. Length of shaft — Collars and couplings

$= 8 \times 12 - 8 \times 2{\cdot}5 - 2 \times 3{\cdot}5$

$= 96 - 27$

$= 69$ inches.

Volume $= 0{\cdot}7854\ (69 \times 12{\cdot}5^2 + 20^2 \times 8 \times 2{\cdot}5 + 25^2 \times 2 \times 3{\cdot}5)$

$= 0{\cdot}7854\ (10{,}781 + 8{,}000 + 4{,}375)$

$= 0{\cdot}7854 \times 23{,}156$ cubic inches.

$$\therefore \text{Weight} = \frac{0{\cdot}7854 \times 23{,}156 \times 0{\cdot}283}{2{,}240}$$

$= 2{\cdot}298$ tons. Ans.

27.

$$\text{Number of turns} = \frac{\quad}{1\frac{1}{4}} = 7 \text{ turns.}$$

Mean diameter of turn $= 10 - 1\frac{1}{4} = 8\frac{3}{4}$ inches.

Length of each turn $= 3{\cdot}1416 \times 8\frac{3}{4} - \frac{1}{2}$

Total length $= (3{\cdot}1416 \times 8{\cdot}75 - \frac{1}{2}) \times 7$

$= 188{\cdot}44$ inches.

$= 15$ feet $8{\cdot}44$ inches. Ans.

28. Part circumference $= \qquad = \sqrt{12}$

$$\text{Pitch} = \frac{6 \times 2\pi}{\sqrt{12}} \times 2 = 2\pi\sqrt{12} = 21{\cdot}76 \text{ feet.} \quad \text{Ans.}$$

29. Best efficiency is when plate strength = rivet strength

$$\therefore \frac{P - d}{P} = \frac{A\,N\,f_s}{P\,t\,f_t}$$

$$\therefore P = \frac{A\,N\,f_s}{t\,f_t} + d = \frac{0{\cdot}875^2 \times 0{\cdot}7854 \times 2 \times 26}{0{\cdot}625 \times 28} + 0{\cdot}875$$

$= 2{\cdot}661$ inches. Ans.

30.

$$1 \text{ cu. ft.} = 6{\cdot}25 \text{ galls.} \quad \therefore \text{cu. ft. per min.} = \frac{1{,}000}{6{\cdot}25}$$

H.P. $\times$ Efficiency

$$= \frac{\text{Pressure per sq. foot} \times \text{Volume per minute in cubic feet}}{33{,}000}$$

$$\therefore \text{H.P.} = \frac{160 \times 144 \times 1{,}000}{33{,}000 \times 6{\cdot}25 \times 0{\cdot}6}$$

$= 186{\cdot}2$ H.P. Ans.

31. Load on stay = Area of cross section of pin $\times 1\frac{1}{2} \times$ Stress allowed.

$$20 \times 2{,}240 = 0{\cdot}7854\ D^2 \times 1{\cdot}5 \times 5{,}000$$

$$\therefore\ D = \sqrt{\frac{20 \times 2{,}240}{0{\cdot}7854 \times 1{\cdot}5 \times 5{,}000}}$$

$= 2{\cdot}758$ inches. Ans.

From first formula, $135 = \dfrac{99{,}000\ T^2}{6{\cdot}5 \times 42}$

$$\therefore\ T = \sqrt{\frac{135 \times 6{\cdot}5 \times 42}{99{,}000}}$$

$= 0{\cdot}6101$ inch.

From second formula, $135 \quad \dfrac{9{,}900\ T}{42}$

$$\therefore\ T = \frac{42 \times 135}{9{,}900}$$

$= 0{\cdot}5727$ inch.

Thickness $= 0{\cdot}6101$ inch. Ans.

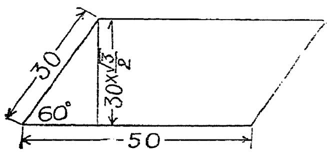
33. Vertical height

$$= 30 \times \frac{\sqrt{3}}{2}$$

$= 25{\cdot}98$ feet.

Area $= 50 \times 25{\cdot}98$

$= 1{,}299$ square feet. Ans.



34. Diagonal $= 3 \times \sqrt{2} = 4{\cdot}242$ feet.

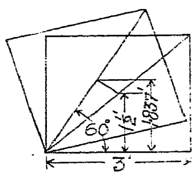
C.G. is at $\frac{1}{2}$ of this length

$= 2{\cdot}121$ feet.

Height of C.G. after tilting

$$= 2{\cdot}121 \times \frac{\sqrt{3}}{2}$$

$= 1{\cdot}837$ feet.



Height C. G. is lifted $= 1{\cdot}837 - \frac{3}{2} = 0{\cdot}337$ foot.

$\therefore$ Work done $= 4{,}455 \times 0{\cdot}337$

$= 1{,}501$ foot lb. Ans.

35. By moments about fulcrum :—

$$P \times \frac{\pi}{4} \times (\tfrac{1}{2})^2 \times 1\tfrac{1}{2} = \frac{\pi \times 6^3}{6 \times 2} \times \frac{62{\cdot}5}{1{,}728} \times 15$$

$$P = \frac{6^3 \times 62{\cdot}5 \times 15 \times 4 \times 4 \times 2}{2 \times 6 \times 1{,}728 \times 3}$$

$= 104{\cdot}2$ lb. square inch. Ans.

36. Pressure on immersed surface
= Depth of C. G. of area × Area × Weight of unit volume of liquid.

Assuming the dock water is sea water :—
Load on end of stone $= \frac{2}{2} \times 2 \times 2 \times 64 = 256$ lb.
Load on each side $= \frac{2}{2} \times 3 \times 2 \times 64 = 384$ lb.
Load on bottom $= 2 \times 3 \times 2 \times 64 = 768$ lb.
256 lb. ; 384 lb. ; 768 lb. Ans.

37. $$\text{Error} = \frac{\text{Weight} \times \text{Difference in length}}{\text{Correct length}}$$

$$= \frac{18 \times 16 \times \frac{1}{16}}{12}$$

$= 1{\cdot}5$ ounces Ans. (app.)

$$\frac{\text{Work done in foot lb. per minute}}{33{,}000} = \text{H.P.}$$

$$\frac{120 \times 3 \times 3{\cdot}1416 \times 180}{33{,}000} = 6{\cdot}168 \text{ H.P.} \quad \text{Ans.}$$

39. H.P. $\propto$ Twisting moment $\times$ Revolutions.

$$\therefore \frac{\text{H.P.}}{\text{T} \times \text{N}} = \text{a constant, where N} = \text{Revs.}$$

Twisting moment $\propto$ Stress $\times$ Diameter^3

$$\therefore \frac{\text{H.P.}}{\text{S} \quad \text{D}^3} = \text{constant.}$$

$$\text{H.P.}_2$$

$$\text{S}_2 \ \text{D}_2^3 \ \text{N}_2$$

$$\text{H.P.}_2 = \quad \text{I.P.}_1 \times \text{S}_2$$

$$\frac{1{,}000 \times 100 \times 8^3 \times 80}{140 \times 10^3 \times 72} = 406{\cdot}4 \text{ H.P.} \quad \text{Ans.}$$

40. Pressure at depth of liquid = depth $\times$ wt. per unit volume.

$$\text{H}_1 \ \text{W}_1 = \text{H}_2 \ \text{W}_2$$

$$\frac{29 \times 849}{12} = \text{H}_2 \times 55$$

$$\frac{29 \times 849}{12 \times 55} = 37{\cdot}29 \text{ feet.} \quad \text{Ans.}$$

41. Strength solid shaft : strength hollow shaft : : $\text{D}^3 : \dfrac{\text{D}^4 \cdot}{\text{D}}$

$$\therefore \text{Strengths} \propto 12^3 : \frac{12^4 - 5^4}{12}$$

Solid shaft is $\dfrac{12^3}{\dfrac{12^4 - 5^4}{12}}$ times as strong as hollow shaft.

$$\frac{12^4}{12^4 - 5^4} = 1{\cdot}031 \text{ times.}$$

$\therefore$ Solid shaft is 3·1 per cent. stronger than hollow shaft. Ans.

42. $$W\,P = \frac{100\ (T + 1)^2}{S - 6}$$

$$\frac{100\ (11 + 1)^2}{9{\cdot}25^2 - 6} \qquad \frac{14{,}400}{79{\cdot}56} = 181 \text{ lb. per sq. inch.}$$ Ans.

43. Strength $\propto$ Twisting moment $\propto$ Diameter^3

$$\therefore \frac{T\,M}{D^3} = \text{constant.}$$

$$\frac{9{,}600}{1^3} = \frac{270 \times}{D^3} \qquad \frac{70 \times 15}{9{,}600} = 0{\cdot}75 \text{ in.}$$ Ans.

44. In 24 feet there are $\frac{24}{3}$ = 8 beams 15 feet long.

$\therefore$ Weight of beams $= 8 \times 15 \times 32 = 3{,}840$ lb.
Weight of plating $= 24 \times 15 \times 10 = 3{,}600$ lb.
Weight of teak $= 24 \times 15 \times \frac{50}{4} = 4{,}500$ lb.

Total weight $=$ $= 11{,}940$ lb.

5·33 tons. Ans.

45. 1 fathom = 6 feet.

$$\therefore \text{Weight of rope} = \frac{80 \times 20}{} = \frac{1600}{6} \text{ lb.}$$

$$\text{Work done on rope} = \frac{1{,}600}{2 \times 6} \times 80 \qquad 10{,}666 \text{ ft. lb.}$$

$$\text{Work done on weight} = 60 \times 80 \qquad 4{,}800 \text{ ft. lb.}$$

Total work $= 15{,}466$ ft. lb.

46. Volume of tank $= 12 \times 4 \times 3 = 144$ cubic feet.

Volume flowing through pipe per minute

$$\frac{2^2 \times 0{\cdot}7854}{144} \times 80 \text{ cubic feet per minute.}$$

$$\text{Time in minutes} = \frac{144 \times 144}{2^2 \times 0{\cdot}7854 \times 80}$$

$= 82{\cdot}47$ minutes. Ans.

47. By moments about fulcrum :—

$$85 \times 13 + 0{\cdot}7854\,D^2 \times 5 \times 3 = 0{\cdot}7854\,D^2 \quad 90 \times 3$$

$$D^2 = \frac{85 \times 13}{0{\cdot}7854 \times 3\,(90 - 5)}$$

$D = 2{\cdot}349$ inches. Ans.

48. Strength of plate section :—

$$P - d \qquad \frac{3\tfrac{7}{8} - 1}{3\tfrac{7}{8}} = \tfrac{23}{31} = 0{\cdot}7419 = 74{\cdot}19 \text{ per cent.}$$

Strength of rivet section :—

$$\frac{A\,N\,f_s}{P\,t\,f_t} \qquad \frac{0{\cdot}7854 \times 1^2 \times 3 \times 23 \times 16}{3{\cdot}875 \times 11 \times 28}$$

$= 0{\cdot}7263 = 72{\cdot}63$ per cent.

$$\text{Working Pressure} = \frac{2\ t\ f_t \times \text{Efficiency}}{D \times \text{Factor of Safety}}$$

$$= \frac{2 \times 11 \times 28 \times 2{,}240 \times 0{\cdot}7263}{16 \times 78 \times 5{\cdot}5}$$

$= 146$ lb. per square inch. Ans.

49. $\text{Stress} = \dfrac{6\,M}{B\,D^2}$, M for cantilever with load at end $= W\,L$.

$$\therefore B = \frac{6\,W\,L}{S\,D^2} \qquad \frac{6 \times 9{,}000 \times 33}{22{,}000 \times 6^2}$$

$= 2{\cdot}25$ inches. Ans.

50. Let x = speed of current in knots.

Distance up stream $= 4\ (10 - x)$

Distance down stream $= 2\frac{1}{2}\ (10 + x)$

$\therefore\ 4\ (10 - x) = 2\frac{1}{2}\ (10 + x)$

$40 - 4\,x = 25 + 2\frac{1}{2}\,x$

$6\frac{1}{2}\,x = 15$

$$x = \frac{15}{6{\cdot}5} = 2{\cdot}307 \text{ knots.} \quad \text{Ans.}$$

51. The 5th second begins at the end of the 4th second, when

Velocity, $V = g\,t$

$= 32{\cdot}2 \times 4$

$= 128{\cdot}8$ feet per second.

This is the initial velocity for the following 3 seconds

$\therefore$ Velocity after 3 more seconds:—

$V = u + g\,t$

$= 128{\cdot}8 + 32{\cdot}2 \times 3$

$= 128{\cdot}8 + 96{\cdot}6$

$= 225{\cdot}4$ feet per second.

$$\text{Average velocity} = \frac{128{\cdot}8 + 225{\cdot}4}{2}$$

$= 177{\cdot}1$ feet per second.

Distance = Average velocity $\times$ time

$= 177{\cdot}1 \times 3$

$= 531{\cdot}3$ feet.

177·1 feet per second. Ans.

531·3 feet. Ans.

52. Assuming ballast is salt water.

Pressure $= 19 \times 64 = 1{,}216$ lb. per square foot.

$$\text{Volume per minute} = \frac{5^2 \times 0{\cdot}7854}{144} \times 230 = 31{\cdot}36 \text{ cubic feet.}$$

$$\text{H.P.} \times \text{Efficiency} = \frac{\text{Pressure} \times \text{Volume per minute}}{33{,}000}$$

$$\therefore \text{H.P.} = \frac{1{,}216 \times 31{\cdot}36}{33{,}000 \times 0{\cdot}62}$$

$= 1{\cdot}864$ H.P. Ans.

53. Volume of body $= 12 \times 12 \times 12 = 1{,}728$ cubic inches.

Volume of hole $= \left(\frac{6\frac{1}{2} + 6}{2}\right. \times \qquad \times 12 = 368{\cdot}3$ cubic inches.

Volume of pins $= (6\frac{1}{2})^2 \times 0{\cdot}7854 \times 7 \times 2 = 464{\cdot}5$ cubic inches.

Total volume $= 1{,}728 - 368{\cdot}3 + 464{\cdot}5 = 1{,}824{\cdot}3$ cubic inches.

Weight $= 1{,}824{\cdot}3 \times 0{\cdot}283$
$= 516{\cdot}3$ lb. Ans.

54. $$\text{Depth of oil} = \frac{\text{Volume of oil}}{\text{Area of tank}}$$

$$= \frac{90 \times 1{,}728}{6{\cdot}25 \times 0{\cdot}7854 \times 25 \times 25}$$

$= 50{\cdot}66$ inches.

Depth used $= 50{\cdot}66 - 39 = 11{\cdot}66$ inches.

$$\text{Oil used per day} = \tfrac{90}{8} \times \frac{11{\cdot}66}{50{\cdot}66}$$

$= 3{\cdot}453$ gallons. Ans.

55. Difference in longitude $= 63° \ 17' - 5° \ 30' = 57° \ 47'$ $= 3{,}467'$

$$\text{Difference in time} = \frac{3{,}467}{360 \times 60} \times 24 \times 60$$

$= 231{\cdot}13$ minutes.

$\therefore$ Clock is put forward 3 hours $51{\cdot}1$ minutes. Ans.

56. Let x = distance from A at which bar will balance.
Then $(8 - x)$ = distance from B at which bar will balance.

By moments about balancing point,

$$1{\cdot}73\,x = 1\,(8 - x)$$

$$2{\cdot}73\,x = 8$$

$$x = \frac{}{2{\cdot}73} = 2{\cdot}93 \text{ feet.}$$

= 2 feet 11·16 inches from A. Ans.

57. Final velocity = 0

Acceleration = — 32·2 feet per second per second (the negative sign being used because the velocity decreases).

$$v = u + g\,t$$

$$0 = 1{,}000 + (-\,32{\cdot}2 \times t)$$

$$1{,}000 = 32{\cdot}2\,t$$

$$t = \frac{1{,}000}{32{\cdot}2} = 31{\cdot}05 \text{ seconds.}$$

$$h = u\,t + \frac{}{2}$$

$$= 1{,}000 \times 31{\cdot}05 + \left(-\,\frac{32{\cdot}2}{2} \times (31{\cdot}05)^2\right.$$

$$= 15{,}530 \text{ feet.}$$

15,530 feet and 31·05 seconds. Ans.

Acceleration is the rate of change of velocity, and is generally stated in feet per second per second units. Ans.

58. Distance = radians × radius × efficiency

$$= 21{\cdot}75 \times 9 \times 0{\cdot}9$$

$$= 176{\cdot}175 \text{ feet.}$$ Ans.

59.

(i.) Bending Moment $= \dfrac{w\,L^2}{2}$ or $\dfrac{W\,L}{2}$

Where w = weight per unit length
W = total weight
L = length

(ii.) $B\,M = \dfrac{W\,L}{4}$ where W = load, L = length.

For proofs of above, *see* Chapter on "Bending moments."

Weight of beam $= 10 \times \frac{1}{4} \times \frac{1}{2} \times 450$ lb.

$$B\,M = \frac{W\,L}{8} = \frac{10 \times 0{\cdot}25 \times 0{\cdot}5 \times 450}{} \text{ inch lb.}$$

$$\text{Stress} = \frac{6\,M}{B\,D^2} \qquad \frac{6 \times 0{\cdot}25 \times 0{\cdot}5 \times 450 \times 10 \times 120}{3 \times 6 \times 6 \times 8}$$

= 469 lb. per square inch. Ans.

60. Velocity ratio = 5 for blocks.

Velocity ratio = 18 for incline.

$$\therefore \text{ Force} = \frac{2 \times 2{,}240}{5 \times 18} = 49{\cdot}78 \text{ lb.} \quad \text{Ans. (i.)}$$

If blocks are reversed, then force exerted on fall of blocks is helping to draw weight up incline.

$$\text{Then } \frac{2 \times 2{,}240}{18} - F = 5\,F.$$

$$= \frac{2{,}240}{6 \times 18} = 41{\cdot}48 \text{ lb.} \quad \text{Ans. (ii.)}$$

61.

Since reactions are to be equal, each must equal $\dfrac{}{2}$

= 6 tons.

Let x = distance of 7 ton weight from nearer end.

Then 5 ton weight is 30 — 8 = 22 feet from the same end.

Taking moments about this end :—

$$6 \times 30 = 5 \times 22 + 7 \times x$$

$$x = \frac{180 - 110}{} = 10 \text{ feet. Ans.}$$

62. From the properties of a circle, when a triangle is drawn on a diameter with its apex on the circumference, the angle at the apex is a right angle. The hypotenuse of the right angled triangle given must therefore be a diameter.

$$\therefore (\text{diameter})^2 = (\text{hypotenuse})^2 = (7{\cdot}9)^2 + (9{\cdot}2)^2$$

$$\therefore \text{Area of circle} = 0{\cdot}7854\ [(7{\cdot}9)^2 + (9{\cdot}2)^2]$$

$$= 115{\cdot}5 \text{ square inches. Ans.}$$

63. The strength of a beam is inversely proportional to its length, and directly proportional to its width, and the square of its depth.

$$\text{From above } \frac{W\ L}{B\ D^2} = \text{constant.}$$

$$\therefore \frac{500 \times 4}{1\frac{1}{4} \times 2\frac{1}{2} \times 2\frac{1}{2}} = \frac{W \times 6\frac{1}{4}}{3 \times 6\frac{1}{2} \times 6\frac{1}{2}}$$

$$W = \frac{500 \times 4 \times 3 \times 6\frac{1}{2} \times 6\frac{1}{2}}{1\frac{1}{4} \times 2\frac{1}{2} \times 2\frac{1}{2} \times 6\frac{1}{4}} = 5{,}192 \text{ lb. Ans.}$$

64. Let S = stroke.

$$\text{Linear velocity of crank pin} = 8{\cdot}5 \times \frac{}{2} \times 60 \text{ feet per minute}$$

$$\text{Velocity of piston} = \text{velocity of crank pin} \times \frac{}{\pi}$$

$$8\cdot5 \times \frac{S}{\quad} \times 60 \times \frac{2}{\quad} = 500$$

$$S = \frac{500 \times \pi}{8\cdot5 \times 60} = 3\cdot08 \text{ feet.} \quad \text{Ans.}$$

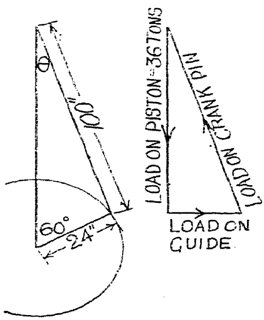
65. For Moment of Resistance *see* Chapter on Bending Moments.

Resisting moment of square beam $= \frac{\quad}{6}$

$$\frac{p \times 1{,}000}{10\cdot2} = \frac{p\,s^3}{6}$$

$$\therefore\ s = 10\sqrt[3]{\frac{6}{10\cdot2}} = 8\cdot38 \text{ inches.}$$

$\therefore$ Section is 8·38 inches square. Ans.



$$\begin{array}{cc} 100 & 24 \\ \text{Sin. } 60^\circ & \text{Sin. } \theta \end{array}$$

$$\therefore \text{ Sin. } \theta = \frac{24 \text{ Sin. } 60^\circ}{100}$$

$$= \frac{24 \times 0\cdot866}{100} = 0\cdot2$$

$$\therefore\ \theta = 12^\circ$$

Load on crank pin

$$\begin{array}{cc} 36 & 36 \\ \text{Cos. } 12^\circ & 0\cdot9781 \end{array} = 36\cdot82 \text{ tons.} \quad \text{Ans. (1)}$$

Load on guide $= 36$ Tan. $\theta = 36 \times 0\cdot2126$

$= 7\cdot654$ tons. Ans. (2)

67. $p\,B\,D^2$

$$D^2 = (3\ B)^2 = 9\ B^2$$

$$\therefore B \times 9\ B^2 = 6 \times 324$$

$$B = \sqrt[3]{\frac{6 \times 324}{9 \times 8}} = 3 \text{ inches.}$$

$$\text{Area} = B \times D = B \times 3\ B = 3\ B^2$$

$$= 3 \times 9 = 27 \text{ square inches.} \quad \text{Ans.}$$

Load on pin = Load on piston.

$$\text{Length} \times \text{Diameter} \times 500 = \tfrac{11}{14} \times 72^2 \times 20$$

$$\therefore 1{\cdot}2\ D \times D \times 500 = \tfrac{11}{14} \times 72^2 \times 20$$

$$D = \sqrt{\frac{\frac{11}{14} \times 72^2 \times 20}{1{\cdot}2 \times 500}} = 11{\cdot}65 \text{ inches.} \quad \text{Ans.}$$

69. Let distance from port left by slower ship $= x$

Then faster ship does $200 - x$ miles.

$$\frac{\text{Distance}}{\text{Speed}} = \text{time.}$$

$$\frac{x}{10} \quad \frac{200 - x}{12} - 3 = \frac{200 - x - 36}{12}$$

$$\frac{x}{10} = \frac{164 - x}{12}$$

$$\therefore 12\ x = 1{,}640 - 10\ x$$

$$\therefore 22\ x = 1{,}640$$

$$\therefore x = \frac{1{,}640}{22} = 74{\cdot}\dot{5}\dot{4} \text{ miles.}$$

Ships pass each other $74{\cdot}\dot{5}\dot{4}$ miles from port left by slower ship. Ans.

70. The number of bolts is not stated.

Area rod × Stress = Area bolts × Stress

If N = No. of bolts

Then $7^2 \times 7{,}000 = N\,D^2 \times 6{,}500$

$$N\,D^2 = \frac{7^2 \times 7{,}000}{6{,}500} = 52{\cdot}77$$

If there are 4 bolts, $D^2 = \dfrac{52{\cdot}77}{4} = 13{\cdot}19$

and $D = 3{\cdot}63$ inches diameter.

If there are 2 bolts, $D^2 = \dfrac{52{\cdot}77}{2} = 26{\cdot}39$

and $D = 5{\cdot}137$ inches diameter.

3·63 inches for 4 bolts. Ans.

5·137 inches for 2 bolts. Ans.

71. Side of large cube = 10 cms. = 3·937 inches.

2 saw cuts $= 2 \times \frac{3}{64} = 0{\cdot}09375$ inch

∴ side of each small cube = $3{\cdot}937 - 0{\cdot}09375$

$= 1{\cdot}281$ inches.

Vol. of each small cube $= (1{\cdot}281)^3 = 2{\cdot}101$ cu. ins.

2·101 cu. ins. Ans.

72. Let L = distance of crosshead from centre of shaft.

When crank is on top centre, $L = 9'\,9'' + 2'\,3''$

$= 117 + 27 = 144''$

When crank is horizontal L =

$= 12\sqrt{90}$

$= 113{\cdot}84$ inches.

Distance of piston from top of stroke = Distance moved by crosshead $= 144 - 113{\cdot}84$

$= 30{\cdot}16$ inches. Ans.

73. $\dfrac{\text{Distance}}{\text{Time}} = \text{Speed}$

Distance ships have gone when first ship is overtaken $= 14 \times 36 = 504$ miles.

$$\frac{504}{36 + 12} = 10{\cdot}5 \text{ knots.}$$

$10{\cdot}5$ knots. Ans.

Area of equilateral triangle $= 0{\cdot}433\ S^2$

Area of hexagon $= 6 \times 0{\cdot}433\ S^2$

But S = Radius of circle $= 7\frac{1}{2}$ inches.

$\therefore$ Area of hexagon $= 6 \times 0{\cdot}433 \times (7\frac{1}{2})^2$

$= 146{\cdot}1$ square inches. Ans.

Normal pressure $= \dfrac{155}{7{\cdot}5}$ lb. per square inch.

Pressure when ship is heeled $= \dfrac{155}{7{\cdot}5} \times \dfrac{\sqrt{3}}{2}$

$= 17{\cdot}9$ lb. per square inch. Ans.

76. Volume of hollow sphere $= \dfrac{\pi}{6}(D^3 - d^3)$

$d^3 = (D - \frac{3}{4})^3$

$= D^3 - \frac{9}{4} D^2 + \frac{27}{16} D - \frac{27}{64}$

Weight = Volume in cubic inches $\times 0{\cdot}26 = 32$

π

$$-\tfrac{3}{16}D+\tfrac{9}{64}) = \frac{32}{0{\cdot}26}$$

$$\frac{32}{0{\cdot}26} \times \frac{6}{\pi} = 235$$

Multiplying through by 4 :—

$9\ D^2 - \frac{27}{4}\ D + \frac{27}{16} = 940$

Dividing through by 9 :—

$D^2 - \frac{3}{4}\ D + \frac{3}{16} = 104{\cdot}\dot{4}$

$\therefore\ D^2 - \frac{3}{4}\ D = 104{\cdot}\dot{4} - \frac{3}{16}$

$= 104{\cdot}2$ (to nearest fourth place).

Completing the square :—

$D^2 - \frac{3}{4}\ D + (\frac{3}{8})^2 = 104{\cdot}2 + \frac{9}{64}$

$= 104{\cdot}34$

$D - \frac{3}{8} = \pm\sqrt{104{\cdot}34} = \pm\ 10{\cdot}23$

$\therefore\ D = 10{\cdot}23 + 0{\cdot}375$

$= 10{\cdot}605$ inches. Ans.

77. Force = force against gravity + force to overcome friction.

$\therefore\ 9{,}300 = W\ \text{Sin.}\ 4^\circ\ 30' + \mu\ W\ \text{Cos.}\ 4^\circ\ 30'$

$= W\ (0{\cdot}0785 + 0{\cdot}11 \times 0{\cdot}9969)$

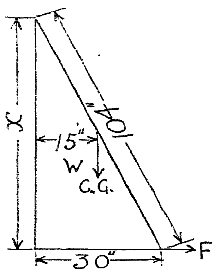
$= W\ (0{\cdot}0785 + 0{\cdot}1097)$

$= W \times 0{\cdot}1882$

$\frac{9{,}300}{0{\cdot}1882}$ pounds.

$$\frac{9{,}300}{0{\cdot}1882 \times 2{,}240} = 22{\cdot}06 \text{ tons.} \quad \text{Ans.}$$

78. $x = \sqrt{104^2 - 30^2} = 99{\cdot}58$ ins.



By moments about top end :—

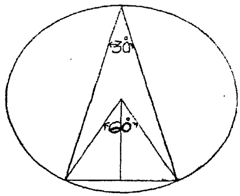
$W \times 15 = F \times 99{\cdot}58$

$$\frac{1\frac{1}{2} \times 2{,}240 \times 15}{99{\cdot}58}$$

$= 506$ pounds. Ans.

79. When a triangle is inscribed in a circle, the angle at the apex is half of the angle formed at the apex of another triangle, on the same base, but having its apex at the centre of the circle.

In the case given, the angle at the apex of the isosceles triangle is 30°. The angle formed at the centre of the circle in the other triangle is therefore 60°, and the triangle must be equilateral. The radius of the circle is therefore equal to the length of the base, i.e., 1 inch.



$\therefore$ Diameter $=$ 2 inches. Ans.

80. Let x = hypotenuse.

Then $30 - 10 - x$ = perpendicular $= 20 - x$

$x^2 = 10^2 + (20 - x)^2$

$x^2 = 100 + 400 - 40\,x + x^2$

$40\,x = 500$

$x = 12{\cdot}5$ = hypotenuse.

Perpendicular $= 30 - 12{\cdot}5 - 10 = 7{\cdot}5$ inches.

12·5 inches and 7·5 inches. Ans.

(or alternatively).

Let y = perpendicular and x = hypotenuse

$x^2 - y^2 = 10^2 \quad \ldots \quad \ldots \quad \ldots \quad$ (i.)

$x + y + 10 = 30$

$\therefore x + y = 20 \ldots \quad \ldots \quad \ldots \quad$ (ii.)

Dividing (i.) by (ii.)

$$\frac{(x + y)\,(x - y)}{x + y} =$$

$\therefore x - y = 5 \quad \ldots \quad$ (iii.)

Adding (ii.) and (iii.)

$x + y = 20$

$x - y = 5$

$2\,x \quad = 25$

$$\therefore\ x = 12{\cdot}5$$

$$12{\cdot}5 + y = 20$$

$$\left.\begin{aligned} y &= 7{\cdot}5 \text{ inches} \\ \text{and } x &= 12{\cdot}5 \text{ inches} \end{aligned}\right\} \text{Ans.}$$

81.

$$\text{Difference in area} = \frac{}{4}\,(\mathrm{D} + d)\,(\mathrm{D} - d) = 28 \quad \text{(i.)}$$

$$\text{Difference in circumference} = \pi\,(\mathrm{D} - d) = 8 \qquad \text{(ii.)}$$

$$\text{and } (\mathrm{D} - d) = \frac{8}{\pi}$$

Dividing (i.) by (ii.)

$$\frac{\frac{\pi}{4}\,(\mathrm{D} + d)\,(\mathrm{D} - d)}{\pi\,(\mathrm{D} - d)} = \tfrac{28}{8}, \qquad \left.\begin{aligned} \therefore\ \mathrm{D} + d &= 14 \\ \mathrm{D} - d &= \frac{8}{\pi} \end{aligned}\right\} \text{Add}$$

$$2\,\mathrm{D} = 14 + \frac{8 \times 7}{22}$$

$$\mathrm{D} = 7 + \tfrac{28}{22} = \frac{77 + 14}{11} = \tfrac{91}{11} = 8\tfrac{3}{11} \text{ feet.} \quad \text{Ans.}$$

Or,

$$\frac{}{4}\,(\mathrm{D}^2 - d^2) = 28$$

$$\pi\,(\mathrm{D} - d) = 8$$

$$\mathrm{D} - d = \frac{8}{\pi} = \frac{8 \times 7}{22} = \tfrac{28}{11}$$

$$d = \mathrm{D} \qquad \tfrac{\ }{11}$$

$$\frac{}{4}\,(\mathrm{D}^2 - d^2) = 28$$

$$\mathrm{D}^2 - d^2 = \frac{28 \times 4 \times 7}{22} = \frac{7 \times 28 \times 2}{11}$$

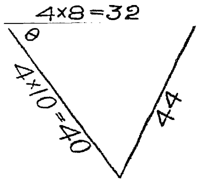
$$\therefore \ D^2 - D^2 + \frac{2 \times 28}{} D - \frac{28^2}{} = \frac{7 \times 28 \times 2}{11}$$

$\therefore \ 2 \ D - \frac{28}{11} = 7 \times 2$

$D - \frac{14}{11} = 7$

$D = 7 + \frac{14}{11} = 8\frac{3}{11}$ feet.

82. From formula given for solution of triangles in Chapter on Trigonometry,



$44^2 = 40^2 + 32^2 - 2 \times 40 \times 32 \text{ Cos. } \theta$

$\therefore \ 11^2 = 10^2 + 8^2 - 2 \times 40 \times 2 \text{ Cos. } \theta$ (dividing by 4^2)

$\therefore \ 121 = 164 - 160 \text{ Cos. } \theta$

$\therefore \ \text{Cos. } \theta = \frac{43}{160} = 0{\cdot}2688$

$\therefore \ \theta = 74° \ 24' =$ angle between courses. Ans.

83. Let the sides be a and b.

Then area $= a \ b = 42$

$\therefore \ 2 \ a \ b = 84$ ... ... (i)

Also $a^2 + b^2 = (12{\cdot}5)^2 = 156{\cdot}25$

Adding (1) and (2) :—

$a^2 + 2 \ a \ b + b^2 = 240{\cdot}25$

Taking the square root of each side

$a + b = 15{\cdot}5$ ... ... (iii)

Subtracting (i) from (ii)

$a^2 - 2 \ a \ b + b^2 = 72{\cdot}25$

Taking the square root of each side

$a - b = 8{\cdot}5$ ... ... (iv)

Adding (iii) and (iv)

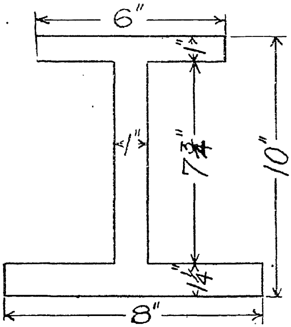
$2\,a = 24$

$\therefore\ a = 12$

$$b = \frac{42}{a} = \frac{42}{12} = 3{\cdot}5$$

$\therefore$ Sides are 3·5 inches and 12 inches. Ans.

84. By moments about bottom of beam,



$$(1\tfrac{1}{4} \times 8) \times \tfrac{5}{8} + (7\tfrac{3}{4} \times 1)\,(3\tfrac{7}{8} + 1\tfrac{1}{4}) + (6 \times 1)] \times 9\tfrac{1}{2}$$
$$= \left[(1\tfrac{1}{4} \times 8) + (7\tfrac{3}{4} \times 1) + (6 \times 1)\right] x$$

$\therefore\ 6{\cdot}25 + 39{\cdot}72 + 57 = (10 + 7{\cdot}75 + 6)\,x$

$$x = \frac{102{\cdot}97}{23{\cdot}75}$$

$= 4{\cdot}336$ inches from bottom. Ans.

85. Weight varies as the volume, and since the thickness is uniform the volume will vary as the area. Area of similar figures vary as the square of their corresponding dimensions.

Ratio of dimensions, $1 : (1 - 0{\cdot}0105) = 1 : 0{\cdot}9895$

Ratio of areas, $1^2 : (0{\cdot}9895)^2 = 1 : 0{\cdot}97911$

$\therefore$ Area is $\dfrac{1 - 0{\cdot}97911}{1} \times 100$ per cent. too small

$= 2{\cdot}089$

$\therefore$ Weight is $2{\cdot}089$ per cent. too small. Ans.

See page 176 for explanation of centrifugal force.
C.F. $= 0{\cdot}00034\ W\ R\ N^2 = 0{\cdot}00034 \times 220 \times \frac{10}{12} \times 240^2$
$= 3{,}590$ lb. Ans.

Strain per inch $= \dfrac{\text{extension}}{\text{original length}} \quad 318$

Stress $= E \times$ Strain $= \dfrac{14{,}000}{16 \times 318}$

$= 2{\cdot}752$ tons per sq. inch.

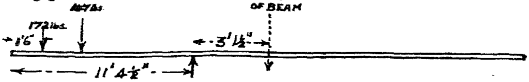
or $2{\cdot}752 \times 2{,}240 = 6{,}163$ lb. per square inch. Ans.

88. Centrifugal force $= 0{\cdot}00034\ W\ R\ N^2$
$= 0{\cdot}00034 \times 5{\cdot}5 \times 3{\cdot}5 \times 70^2 = 32{\cdot}08$ lb.

When the weight is passing over the top the tension in the string will be centrifugal force — weight $= 32{\cdot}08 - 5{\cdot}5$
$= 26{\cdot}58$ lb. Ans.

When the weight is passing round the bottom the tension in the string will be centrifugal force $+$ weight
$= 32{\cdot}08 + 5{\cdot}5 = 37{\cdot}58$ lb. Ans.

89. Weight of beam (W) acts through its centre of gravity which is at mid length.



Taking moments about fulcrum :—

Clockwise moments $=$ anticlockwise moments.

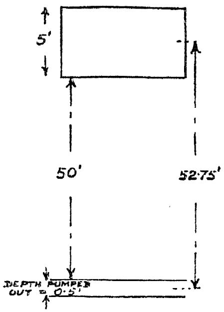
$W \times 3{\cdot}125 =$
$\{172 \times (11'\ 4\tfrac{1}{2}'' - 1'\ 6'')\} + \{147 \times (11'\ 4\tfrac{1}{2}'' -$

$$W \times 3{\cdot}125 = \{172 \times 9{\cdot}875\} + \{147 \times 8{\cdot}125\}$$

$$W = \frac{8 + 1{,}195}{3{\cdot}125} = 926 \text{ lb.}$$

Weight of beam = 926 pounds. Ans.

90.



Volume of present-use tank $= 10 \times 6 \times 5$
$= 300$ cubic feet.

$\therefore$ Depth of water pumped out of double bottom $= \frac{300}{600} = 0{\cdot}5$ foot.

C.G. of water is raised $50 + 2{\cdot}5 + 0{\cdot}25$
$= 52{\cdot}75$ feet.

Work done = weight $\times$ distance C.G. is raised.

$= 300 \times 62{\cdot}5 \times 52{\cdot}75$

$= 989{,}000$ ft. lb. Ans.

91. Change of momentum = Change of velocity $\times$ mass
force $\times$ time

$$\frac{(12 - 10) \times 6{,}080}{3{,}600} \times \frac{5{,}000 \times 2{,}240}{32{\cdot}2}$$

$= \text{force} \times 15 \times 60$

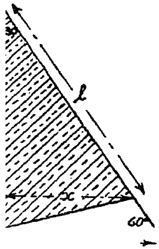
$\therefore$ Force in lb. to cause increase of velocity

$2 \times 6{,}080 \times 5{,}000 \times 2{,}240$

$3{,}600 \times 32{\cdot}2 \times 15 \times 60$

1,308 lb. Ans.

92.



The shaded part is the largest isosceles triangle.

$l = 1\frac{1}{4} \times \tan. 60^\circ = 1\frac{1}{4} \times \sqrt{3}$ inches.

$x = l \times \text{Sin. } 30^\circ = 1\frac{1}{4} \times \sqrt{3} \times 0{\cdot}5$

$$\text{Area} = \frac{x \times l}{2}$$

$$1\tfrac{1}{4} \times \sqrt{3} \times 0{\cdot}5 \times 1\tfrac{1}{4} \times \sqrt{3}$$

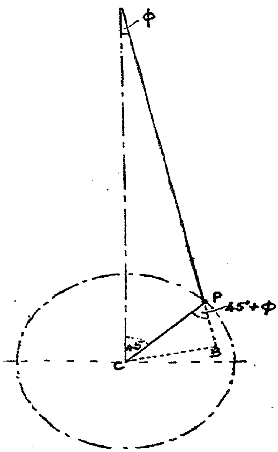
$= (1\frac{1}{4})^2 \times 0{\cdot}25 \times 3$

Volume of prism $= (1\frac{1}{4})^2 \times 0{\cdot}25 \times 3 \times 6$ cubic inches.

At 0·303 lb. per cu. in., weight of prism

$\times\ 0{\cdot}25 \times 3 \times 6 \times 0{\cdot}303$

$= 2{\cdot}13$ lb. Ans.



C P $\quad$ A P

Sin. ϕ $\quad$ Sin. 45°

$$\therefore \text{Sin. } \phi = \frac{520 \times 0{\cdot}7071}{1{,}960}$$

Sin. $\phi = 0{\cdot}1874 \quad \therefore \phi = 10^\circ\ 48'$

C B $= 520 \times \text{Sin. } (45^\circ + 10^\circ\ 48')$
$= 430{\cdot}1$ mm. $= 0{\cdot}4301$ metre.

Twisting moment = load in con. rod × C B.

∴ load in con. rod

$$= \frac{8{,}750}{0{\cdot}4301} \text{ kilograms.}$$

Load on guide = load in con. rod × sin. ϕ

$$\frac{8{,}750}{0{\cdot}4301} \times 0{\cdot}1874 \text{ kilos.}$$

$$\frac{8{,}750}{0{\cdot}4301} \times 0{\cdot}1874 \times 2{\cdot}2 \text{ lb.}$$

$= 8{,}384$ lb. Ans.

94. $$\text{Mean T.M.} = \frac{63{,}000 \times \text{H.P.}}{\text{Revs.}} = \frac{63{,}000 \times 5{,}000}{90} \text{ inch lb.}$$

$$\text{Maximum T.M.} = \frac{1{\cdot}5 \times 63{,}000 \times 5{,}000}{90} \text{ inch lb.}$$

The shaft must be made strong enough to transmit the maximum twisting moment.

$$\text{For a hollow shaft, T.M.} = \frac{\quad}{16} \times \frac{(D^4 - d^4)}{D} \times q$$

$$\therefore \frac{1{\cdot}5 \times 63{,}000 \times 5{,}000}{90} = \frac{\pi}{16} \times \frac{(D^4 - d^4)}{D} \times 8{,}000$$

D D

$$\frac{1{\cdot}5 \times 63{,}000 \times 5{,}000}{90} \quad \frac{\pi \times 15 \times D^3 \times 8{,}000}{16 \times 16}$$

$$\frac{{\cdot}5 \times 63{,}000 \times 5{,}000 \times 16 \times 16}{90 \times \pi \times 15 \times 8{,}000}$$

$= 15{\cdot}28$ inches. Ans.

95. Let L = length of raft in feet.

For equilibrium, Downward force = Upward force

Weight of raft + $2\frac{1}{2}$ tons = Weight of water displaced

$(L \times 9{\cdot}75 \times 0{\cdot}75 \times 62{\cdot}5 \times 0{\cdot}56) + (2{\cdot}5 \times 2{,}240)$

$= L \times 9{\cdot}75 \times \frac{8}{12} \times 62{\cdot}5$

$256\ L + 5{,}600 = 406{\cdot}3\ L$

$$L = \frac{5{,}600}{150{\cdot}3} = 37{\cdot}25 \text{ feet. Ans.}$$

Or alternatively :—

Raft would float at a draught of $0{\cdot}56 \times 9 = 5{\cdot}04$ inches when unloaded, then difference in draught due to a load of $2\frac{1}{2}$ tons $= 8 - 5{\cdot}04 = 2{\cdot}96$ ins.

$\therefore$ Water displaced by draught of 2·96 ins. $= 2\frac{1}{2} \times 2{,}240$ lb.

$$\therefore L \times 9{\cdot}75 \times \frac{\quad}{12} \times 62{\cdot}5 = 2{\cdot}5 \times 2{,}240$$

$$L = \frac{2{\cdot}5 \times 2{,}240 \times 12}{9{\cdot}75 \times 1{\cdot}96 \times 62{\cdot}5}$$

$= 37{\cdot}25$ feet. Ans. (as before).

96. Radius from centre of drum to centre of rope $= \frac{32}{2} + \frac{1}{2}$ $= 16$ inches.

At 72 r.p.m. winch lifts at the rate of $2 \times \frac{22}{7} \times \frac{16}{12} \times 72$ feet per minute.

Work done per minute $= 2 \times \frac{22}{7} \times \frac{16}{12} \times 72 \times 13 \times 112$ ft. lb.

Then horse power of winch $=$

$$\frac{2 \times 22 \times 16 \times 72 \times 13 \times 112}{7 \times 12 \times 33{,}000 \times 0{\cdot}87}$$

$= 30{\cdot}6$ H.P. Ans.

97.

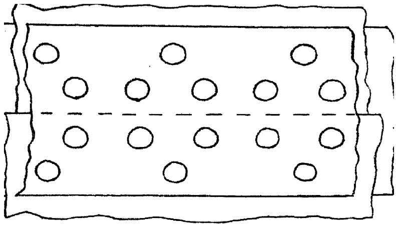
Plate strength at outer row $= \dfrac{p - d}{p} \times 100$

$$= \frac{5{\cdot}875 - 1{\cdot}1875}{5{\cdot}875} \times 100 = 79{\cdot}78 \text{ per cent.}$$

Rivet strength $= \dfrac{a \times n \times f_s}{p \times t \times f_t} \times 1{\cdot}875 \times 100$

$$= \frac{11 \times 19 \times 19 \times 3 \times 8 \times 32 \times 23 \times 15 \times 100}{14 \times 16 \times 16 \times 47 \times 35 \times 28 \times 8}$$

$= 79{\cdot}69$ per cent.



Strength of joint at inner rows

$$\frac{p - 2\,d}{p} \times 100 + \text{Strength of one rivet in shear}$$

$$\frac{5{\cdot}875 - 2{\cdot}375}{5{\cdot}875} \times 100 + \frac{79{\cdot}69}{3}$$

$= 59{\cdot}58 + 26{\cdot}56 = 86{\cdot}14$ per cent.

Strength of seam $= 79{\cdot}69$ per cent.

Safe stress $= \dfrac{28 \times 2{,}240}{4{\cdot}5}$ lb. per sq. inch.

Working pressure $= \dfrac{2\,t\,s}{} \times$ seam strength

$$\frac{2 \times 35 \times 28 \times 2{,}240 \times 0{\cdot}7969}{32 \times 69 \times 4{\cdot}5} = 352{\cdot}2 \text{ lb. per sq. inch. Ans.}$$

98. Efficiency $= \dfrac{\text{Work got out}}{\text{Work put in}}$

135 centimetres $= \dfrac{135}{2{\cdot}54}$ inches.

In turning screw through one revolution :—

Work got out $= 6{\cdot}75 \times 2{,}240 \times 0{\cdot}475$ inch lb.

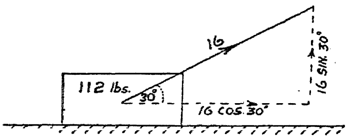
Work put in $= 28 \times 2{\cdot}2 \times {\cdot}2 \times \pi \times \dfrac{135}{2{\cdot}54}$ inch lb.

$$\therefore \text{ Efficiency} = \frac{6 \cdot 75 \times 2{,}240 \times 0 \cdot 475 \times 7 \times 2 \cdot 54}{28 \times 2 \cdot 2 \times 2 \times 22 \times 135}$$

$= 0{\cdot}349$ or $34{\cdot}9$ per cent.

Then, percentage lost in friction $= 100 - 34{\cdot}9 = 65{\cdot}1$ per cent. Ans.

99.



A force of 16 lb. acting at 30° to the horizontal is equivalent to two forces, one of $16 \times$ Cos. 30° lb. acting horizontally and another of $16 \times$ Sin. 30° lb. acting vertically upwards.

Pressure between surfaces $= 112 - 16 \times$ Sin. $30° = 112 - 8 = 104$ lb.

Force to overcome friction $= 16 \times$ Cos. $30° = 16 \times 0{\cdot}866$ lb.

$$\text{Co-efficient of friction} = \frac{\text{force to overcome friction}}{\text{pressure between surfaces}}$$

$$\frac{16 \times 0{\cdot}866}{104} = 0{\cdot}133. \quad \text{Ans.}$$

100. Weight of sea water displaced $= 12 \times \frac{11}{12} \times \frac{4}{12} \times \frac{1022}{16} = 234{\cdot}1$ lb.

Weight of plank $= 234{\cdot}1 - 51 = 183{\cdot}1$ lb. Ans. (*a*)
Specific gravity of wood =

$$\frac{\text{Weight of the wood}}{\text{Weight of an equal volume of fresh water}}$$

$$= \frac{183{\cdot}1}{234{\cdot}1 \times \frac{1000}{1022}} = 0{\cdot}7994. \quad \text{Ans. } (b)$$

101. Acceleration is rate of change of velocity and is generally expressed in feet per second per second.

$$\text{Retardation} = \frac{\text{Change of velocity}}{\text{Time to change}}$$

Change of velocity $= 19 - 13 = 6$ knots.

Then, retardation $= \dfrac{6 \times 6{,}080}{60 \times 60 \times 5 \times 60}$

$= 0{\cdot}03378$ ft. per sec. per sec. Ans. (*a*)

Space passed over = Average velocity × Time

$= \dfrac{19 + 13}{2} \times \dfrac{6{,}080}{60 \times 60} \times 5 \times 60 = 8{,}107$ ft. Ans. (*b*)

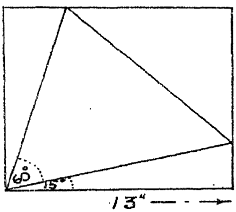
102 Area of square $= 13 \times 13 = 169$ sq. ins.

Let x = area of triangle, and when triangle has been cut out, the area of the remaining portion $= (169 - x)$ sq. ins. Area of triangle is two-fifths of this, therefore

$x = \frac{2}{5}(169 - x)$, from which $x = \dfrac{169 \times 2}{}$ sq. ins.

Area of equilateral triangle $= 0{\cdot}433 \times \text{side}^2$

$\therefore$ Side $= \sqrt{\dfrac{169 \times 2}{7 \times}}$ inches. Ans. (*a*)



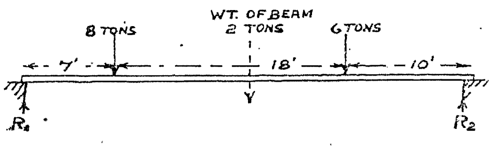
Side of largest equilateral triangle

$\dfrac{13}{\text{Cos. } 15^\circ} \quad \dfrac{13}{0{\cdot}9659}$ ins.

Area $= 0{\cdot}433 \times \left(\dfrac{1}{0{\cdot}9}\right.$

$= 78{\cdot}45$ square inches. Ans. (*b*)

103. When a force acts on a body laterally a stress is set up in the sections of the body parallel to the force. The shear stress is the intensity of this load per unit area.



By moments about R_1 :—

$$(8 \times 7) + (6 \times 25) + (2 \times 17 \cdot 5) = R_2 \times 35$$

$$\frac{56 + 150 + 35}{35} = 6 \cdot 885 \text{ tons.}$$

$R_1 = (8 + 6 + 2) - 6 \cdot 885 = 9 \cdot 115$ tons.

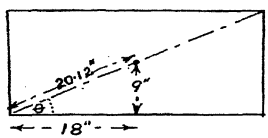
Shearing force at left end $= R_1 = 9 \cdot 115$ tons.

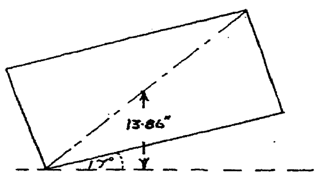
Shearing force at right end $= R_2 = 6 \cdot 885$ tons. Ans.

104. Weight of prism $= (1 \cdot 5)^2 \times 3 \times \frac{450}{2240}$ tons.

Tan. $\theta = \frac{9}{18} = 0 \cdot 5 \therefore \theta = 26° \, 34'$

$$\text{Half diagonal} = \frac{9}{\text{Sin. } 26° \, 34'} = 20 \cdot 12 \text{ ins.}$$





When tilted, diagonal makes an angle of $26° \, 34' + 17° = 43° \, 34'$ to the horizontal, then height of the centre of gravity from horizontal is now $20 \cdot 12 \times \text{Sin. } 43° \, 34' = 13 \cdot 86$ ins.

Distance that C.G. has been raised against gravity

$= 13 \cdot 86 - 9 = 4 \cdot 86$ ins.

$\therefore$ Work done $= (1 \cdot 5)^2 \times 3 \times \frac{450}{2240} \times 4 \cdot 86 = 6 \cdot 59$ inch tons. Ans.

105. Twisting moment transmitted by bolts = twisting moment in shaft.

$$\frac{\pi}{4} \times d^2 \times 6 \times 5{,}000 \times 12 = \frac{63{,}000 \times 4{,}000}{125}$$

$$\therefore d = \sqrt{\frac{63{,}000 \times 4{,}000 \times 4}{125 \times 12 \times 5{,}000 \times 6 \times}} = 2{\cdot}67 \text{ ins.}$$

Diameter of coupling bolts = 2·67 inches. Ans.

106. Length of short side $= \sqrt{\frac{1{,}307}{2{\cdot}5}}$ cms. = distance across flats of hexagon.

Length of side of hexagon $= \sqrt{\frac{1{,}307}{2{\cdot}5}} \times \frac{1}{2} \times \frac{1}{0{\cdot}866}$ cms.

Area of hexagon $= 6 \times 0{\cdot}433 \times \text{side}^2$

$$= 6 \times 0{\cdot}433 \times \quad 1{,}307 \quad \times \tfrac{1}{3} \text{ sq. cms.}$$

$$= 6 \times 0{\cdot}433 \times \frac{1{,}307}{2{\cdot}5} \times \tfrac{1}{3} \times (0{\cdot}3937)^2 \text{ sq. ins.}$$

= 70·2 square inches. Ans.

107. Total force to pull up incline = force to overcome gravity + force to overcome friction.

$\therefore$ F = W Sin. θ + μ W Cos. θ.

In these cases where the angle of inclination is small, the cosine of the angle may be taken as unity.

Then, 0·084 W = W Sin. θ + 0·007 W

$\therefore$ 0·084 = Sin. θ + 0·007

$\therefore$ Sin. θ = 0·084 — 0·007 = 0·077

$\therefore$ θ, the angle of the incline = 4° 25′

or, incline is 1 in $\frac{1}{0{\cdot}077}$ = 1 in 13. Ans.

108. Weight of 1 lb. of cast iron in oil = weight in air — weight of oil displaced.

$\therefore$ weight of oil displaced = 1 — 0·89 = 0·11 lb.

Specific gravity of oil

$$= \frac{\text{Weight of oil}}{\text{Weight of an equal volume of fresh water}}$$

Specific gravity of cast iron = 7·21

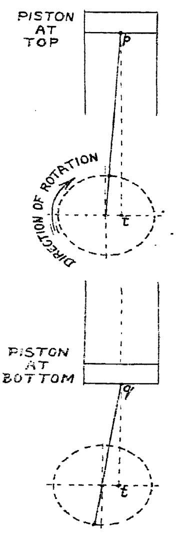
$\therefore$ Specific gravity of oil $= \frac{0\cdot11}{\frac{1}{7\cdot21}} = 0\cdot11 \times 7\cdot21$

$= 0\cdot7931$

$$\text{Degrees Beaumé} = \frac{140}{\text{Sp. G. at } 60^\circ\text{F.}} - 130$$

$$\frac{140}{0\cdot7931} - 130 = 46\cdot5 \text{ degrees.} \quad \text{Ans.}$$

109.



Piston is at the top of its stroke when crank and connecting rod are in line.

Sine of angle between connecting rod and line of stroke $= \frac{4}{96} = 0\cdot04167$

$\therefore$ Angularity = 2° 13′

The piston is therefore at its highest position when the connecting rod is 2° 13′ to the line of stroke, that is, when the crank is 2° 13′ past the vertical.

The crank and connecting rod are again in line with each other when the piston is at the bottom of its stroke as shown.

Sine of angularity $= \frac{4}{48} = 0\cdot08334$

$\therefore$ Angularity = 4° 47′

The piston is therefore at its lowest position when the crank is 4° 47′ past the vertical.

$\therefore$ Angle turned through by the crank while the piston travels from the top of its stroke to the bottom

= 180 + (4° 47′ — 2° 13′)

= 182° 24′. Ans.

$$p\,t = \sqrt{96^2 - 4^2} = \sqrt{100 \times 92} = 95{\cdot}92 \text{ ins.}$$

$$\overline{\times\ 44} = 47{\cdot}84 \text{ ins.}$$

Therefore stroke of piston $= 95{\cdot}92 - 47{\cdot}84$

$= 48{\cdot}08$ inches. Ans.

110.

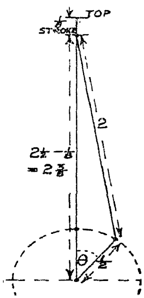
Centrifugal Force $= \qquad = 0{\cdot}00034\ W\ r\ N^2$

$$= 0{\cdot}00034 \times 4 \times \frac{3{\cdot}5}{12} \times (300)^2$$

$= 35{\cdot}7$ lb.

Force in each spring $= 35{\cdot}7$ lb. Ans.

111.



Let length of stroke $= 1$

then crank $= \frac{1}{2}$

and connecting rod $= 2$

When piston is at top of its stroke, its distance from shaft centre is $2 + \frac{1}{2} = 2\frac{1}{2}$

When piston has travelled $\frac{1}{8}$ stroke, its distance from shaft centre is $2\frac{1}{2} - \frac{1}{8} = 2\frac{3}{8}$

By cosine rule :—

$$\text{Cos. } \theta = \frac{(2\frac{3}{8})^2 + (\frac{1}{2})^2 - 2^2}{2 \times 2\frac{3}{8} \times \frac{1}{2}}$$

$= 0{\cdot}7958$

$\therefore\ \theta = 37^\circ\ 16'$

If total angle of opening is 43°, then the valve must open when the crank is $43^\circ - 37^\circ\ 16' = 5^\circ\ 44'$ before top centre. Ans.

112. Compression load on spring

$$= 0{\cdot}714 \times 2{,}240 \times \frac{0{\cdot}8125}{0{\cdot}625} = 2{,}079 \text{ lb.}$$

Total load keeping valve on its seat $= 2{,}079 + 37 = 2{,}116$ lb.
Area of valve $= \frac{11}{14} \times (3\cdot25)^2$ sq. ins.

$$\therefore \text{ Blow off pressure } = \frac{2{,}116 \times 14}{(3\cdot25)^2 \times 11}$$
$$= 255 \text{ lb. per sq. inch.} \quad \text{Ans.}$$

113. One foot head of oil exerts a pressure of $0\cdot89 \times 62\cdot5$ lb. per sq. foot.

$\therefore$ 38·75 feet head of oil exerts a pressure of $0\cdot89 \times 62\cdot5 \times 38\cdot75$ lb. per sq. ft.

$\therefore$ Pressure on outer bottom
$$= 0\cdot89 \times 62\cdot5 \times 38\cdot75$$
$$= 2{,}156 \text{ lb. per sq. foot.} \quad \text{Ans.}$$

Pressure on inner bottom $= 0\cdot89 \times 62\cdot5 \times 35$ lb. per sq. ft.
Area of inner bottom supported by one rivet

$$= 2\cdot5 \times 7 \times \frac{}{8 \times 12} \text{ sq. feet.}$$

Total load supported by one rivet

$$= 0\cdot89 \times 62\cdot5 \times 35 \times 2\cdot5 \times 7 \times \frac{}{8 \times 12} \text{ lb.}$$

Cross section of rivet $= \frac{11}{14} \times (\frac{7}{8})^2$ sq. inches.

$$\therefore \text{ Stress} = \frac{\text{load}}{\text{area}}$$
$$= \frac{0\cdot89 \times 62\cdot5 \times 35 \times 2\cdot5 \times 7 \times 7 \times 14 \times 8 \times 8}{8 \times 12 \times 11 \times 7 \times 7}$$
$$= 4{,}130 \text{ lb. per sq. inch.} \quad \text{Ans.}$$

114. Strength cut away = Strength replaced by ring
$$\{12 + (2 \times 1\tfrac{11}{16})\} \times 1\tfrac{5}{8} \times 29 = (W - 1\tfrac{11}{16}) \times 2 \times 1\tfrac{5}{8} \times 26$$
$$15\tfrac{3}{8} \times 1\tfrac{5}{8} \times 29 = (W - 1\tfrac{11}{16}) \times 2 \times 1\tfrac{5}{8} \times 26$$
$$= \frac{15\tfrac{3}{8} \times 1\tfrac{5}{8} \times 29}{2 \times 1\tfrac{5}{8} \times 26}$$

$$\therefore \; W = \frac{123 \times 29}{8 \times 2 \times 26}$$

$$= 8{\cdot}574 + 1{\cdot}6875 = 10{\cdot}26 \text{ inches width.} \quad \text{Ans.}$$

Shearing strength of rivets in one half = Tensile strength of ring.

$$\tfrac{11}{14} \times (1\tfrac{1}{16})^2 \times 23 \times N = 15\tfrac{3}{8} \times 1\tfrac{5}{8} \times 29$$

$$\therefore \; N = \frac{123 \times 13 \times 29 \times 14 \times 16 \times 16}{8 \times 8 \times 23 \times 11 \times 27 \times 27}$$

$$= 14{\cdot}08, \text{ say } 15 \text{ rivets.}$$

Total number of rivets

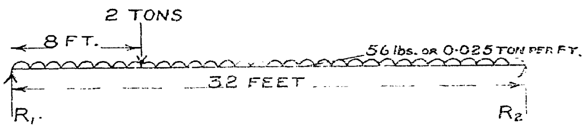
$$= 2 \times 15 + \text{two rivets on the minor axis.}$$
$$= 30 + 2$$
$$= 32 \text{ rivets.} \quad \text{Ans.}$$

115. The piston moves with simple harmonic motion, therefore its velocity is $\omega \, r \sin. \theta$ feet per second.

When $\theta = 45°$,

$$\text{Speed of piston} = 12{\cdot}28 \times 2 \times 0{\cdot}7071$$
$$= 17{\cdot}37 \text{ ft. per sec.}$$
$$\text{or } 17{\cdot}37 \times 60 = 1{,}042{\cdot}2 \text{ ft. per minute.} \quad \text{Ans.}$$

116.



Total distributed load $= 32 \times 0{\cdot}025 = 0{\cdot}8$ ton.

Moments about R_1 :—

$$(2 \times 8) + (0{\cdot}8 \times 16) = R_2 \times 32$$
$$16 + 12{\cdot}8 = R_2 \times 32$$

$$\therefore \; R_2 = \frac{28{\cdot}8}{32} = 0{\cdot}9 \text{ ton.}$$

$$R_1 = 2 + 0{\cdot}8 - 0{\cdot}9 = 1{\cdot}9 \text{ tons.}$$

Shearing force at $R_2 = 0{\cdot}9$ ton.

S.F. at 4 ft. from $R_2 = 0{\cdot}9 - 4 \times 0{\cdot}025 = 0{\cdot}8$ ton.

„ $= 0{\cdot}9 - 8 \times 0{\cdot}025 = 0{\cdot}7$ „

12 $= 0{\cdot}9 - 12 \times 0{\cdot}025 = 0{\cdot}6$ „

16 $= 0{\cdot}9 - 16 \times 0{\cdot}025 = 0{\cdot}5$ „

20 $= 0{\cdot}9 - 20 \times 0{\cdot}025 = 0{\cdot}4$ „

24 $= 0{\cdot}9 - 24 \times 0{\cdot}025 = 0{\cdot}3$ „

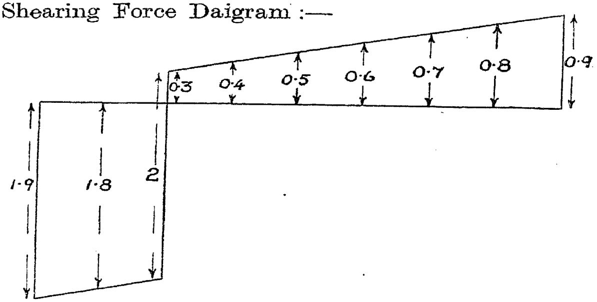
28 $= 0{\cdot}9 - 28 \times 0{\cdot}025 - 2$

$= - 1{\cdot}8$ tons.

32 $= 0{\cdot}9 - 32 \times 0{\cdot}025 - 2$

$= - 1{\cdot}9$ tons.

Shearing Force Daigram :—



For the bending moment diagram, it is convenient to consider the two different types of loading separately :—

Considering concentrated load only,

$R_2 = \frac{8}{32} \times 2 = 0{\cdot}5$ ton, $R_1 = 2 - 0{\cdot}5 = 1{\cdot}5$ tons.

Bending moment at $R_2 = 0$

M at 4 ft. from $R_2 = 0{\cdot}5 \times 4$ 2 ft. tons.

8 $= 0{\cdot}5 \times 8$ 4 „

12 $= 0{\cdot}5 \times 12$ 6 „

16 $= 0{\cdot}5 \times 16$ 8 „

20 $= 0{\cdot}5 \times 20$ 10 „

24 $= 0{\cdot}5 \times 24$ 12 „

28 $= 0{\cdot}5 \times 28$ $2 \times 4 = 6$ ft. tons.

32 $= 0{\cdot}5 \times 32 - 2 \times 8 = 0$

Considering uniformly distributed load only,

$R_1 = R_2 = 0{\cdot}4$ ton.

Bending moment at x ft. from R_2

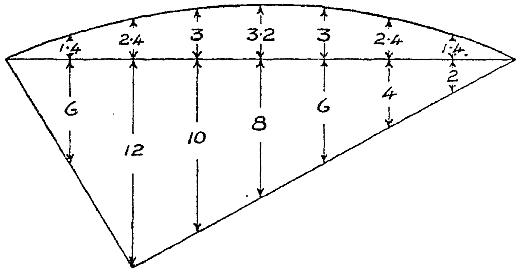
$$= 0{\cdot}4 \times x - 0{\cdot}025 \times x \times$$

$$= 0{\cdot}4\,x - 0{\cdot}0125\,x^2$$

When $x = 0$, $M = 0$

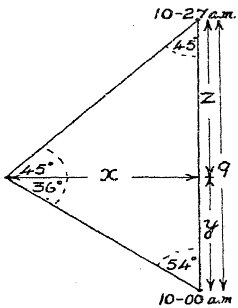
$$x = 4,\ M = 0{\cdot}4 \times 4 - 0{\cdot}0125 \times 4^2 = 1{\cdot}4 \text{ ft.tons}$$
$$x = 8,\ M = 0{\cdot}4 \times 8 - 0{\cdot}0125 \times 8^2 = 2{\cdot}4 \quad ,,$$
$$x = 12,\ M = 0{\cdot}4 \times 12 - 0{\cdot}0125 \times 12^2 = 3{\cdot}0 \quad ,,$$
$$x = 16,\ M = 0{\cdot}4 \times 16 - 0{\cdot}0125 \times 16^2 = 3{\cdot}2 \quad ,,$$
$$x = 20,\ M = 0{\cdot}4 \times 20 - 0{\cdot}0125 \times 20^2 = 3{\cdot}0 \quad ,,$$
$$x = 24,\ M = 0{\cdot}4 \times 24 - 0{\cdot}0125 \times 24^2 = 2{\cdot}4 \quad ,,$$
$$x = 28,\ M = 0{\cdot}4 \times 28 - 0{\cdot}0125 \times 28^2 = 1{\cdot}4 \quad ,,$$
$$x = 32,\ M = 0{\cdot}4 \times 32 - 0{\cdot}0125 \times 32^2 = 0 \quad ,,$$

Bending Moment diagram :—



Note.—When the S.F. and B.M. diagrams to scale are asked for, without particularly asking for the values to be calculated at regular intervals along the beam, the diagram can be drawn from only a few of the calculations shown above.

117.



From 10 a.m. to 10-27 a.m. $= \frac{9}{20}$ hour

Distance travelled $= \frac{9}{20}$ of 20

$= 9$ nautical miles.

$x \times \tan. 45° + x \times \tan. 36° = 9$

$x\ (1 + 0{\cdot}7265) = 9$

$$\therefore\ x = \frac{9}{1{\cdot}7265} = 5{\cdot}213 \text{ nautical miles.}$$

The nearest position of the ship is 5·213 nautical miles from lighthouse. Ans.

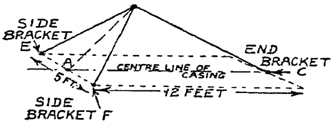
$Z = x, \ \therefore\ y = 9 - 5{\cdot}213$

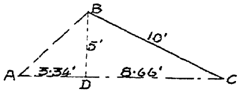
$= 3{\cdot}787$ nautical miles.

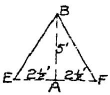
$$\therefore \text{ Time taken } = \frac{3{\cdot}787}{20} \times 60 \text{ minutes} = 11{\cdot}361 \text{ mins.}$$

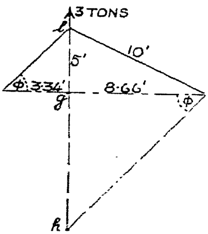
10-00 a.m. + 11·361 minutes = 10 hrs. 11·36 mins. a.m. Ans.

118.









The diagram shows the arrangement of the chains. The C.G. of the casing must be vertically below the ring.

$$\text{Sin. D B C} = \frac{8{\cdot}66}{10} = 0{\cdot}866$$

$\therefore$ D B C = 60°, and D B = 5 ft.

$$\text{A B} = \sqrt{5^2 - (3{\cdot}34)^2} = \sqrt{36{\cdot}16}$$
$$= 6{\cdot}012 \text{ feet.}$$

E B =
$$= 6{\cdot}511 \text{ feet.}$$

The two short chains each measure 6·511 feet. Ans.

$$\text{Tan. } \phi = \frac{5}{3{\cdot}34} = \frac{g\ h}{8{\cdot}66}$$

$$\therefore g\ h = \frac{5 \times 8{\cdot}66}{3{\cdot}34} = 13 \text{ feet.}$$

$l\ h = 5 + 13 = 18$ feet.

$l\ h$ represents 3 tons

$\therefore$ tension in back chain

$$= \frac{10}{18} \times 3 = 1\tfrac{2}{3} \text{ tons. Ans.}$$

119. Let x ft. per sec. = speed of one train,

then $1{\cdot}5\,x$ = speed of other train.

Relative velocity of one train to the other

$$= x + 1{\cdot}5\,x$$
$$= 2{\cdot}5\,x \text{ ft. per sec.}$$

Imagine one train stopped and the other moving at $2{\cdot}5\,x$ ft. per sec., it travels $2 \times 150 = 300$ feet while passing.

$$\text{Speed} = \frac{\text{Distance}}{\text{Time}} \qquad \therefore\ 2{\cdot}5\,x = \frac{300}{}$$

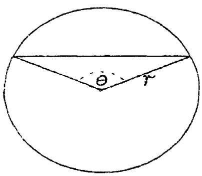
$$\frac{300}{3 \times 2{\cdot}5} = 40 \text{ ft. per sec.}$$

$$40 \text{ ft. per sec.} = \frac{40 \times 60 \times 60}{5{,}280} = 27{\cdot}27 \text{ miles per hour.}$$

$\therefore$ Speed of one train $= 27{\cdot}27$ miles per hour. Ans.

Speed of other $= 1{\cdot}5 \times 27{\cdot}27 = 40{\cdot}9$ miles per hour. Ans.

120.



Area of the segment of a circle $=$

Area of sector $-$ Area of triangle

$$= \pi r^2 \times \frac{\theta \text{ (in radians)}}{2\,\pi} - \frac{}{2} \times \text{Sin.}$$

$\therefore$ Area of segment

$$= \frac{r^2}{} \left\{ \theta - \text{Sin. } \theta \right\}$$

This expression may be used to find the area of the small segment, or of the large segment. If the angle θ of the sector is greater than $180°$, then the sine of that angle has a negative value.

Volume of combustion chambers, furnaces, etc.

$$= 0{\cdot}32 \times \pi \times \times 11{\cdot}5$$
$$= 739{\cdot}82 \text{ cu. ft.}$$

$$\text{Half width of water level} = \frac{146}{11{\cdot}5 \times 2} = 6{\cdot}35 \text{ feet.}$$

Angle in large sector $= 360° - 105° \, 4' = 254° \, 56'$

$$254° \, 56' - \frac{254° \, 56'}{360°} \times 2\pi \text{ radians} = 4{\cdot}447 \text{ radians.}$$

$\text{Sin. } 254° \, 56' = -\text{Sin. } 74° \, 56' = -0{\cdot}9656$

Area of large segment

$$\frac{r^2}{2} \left. \theta - \text{Sin. } \theta \right\}$$

$$= 32 \{ 4{\cdot}447 - (-0{\cdot}9656) \} = 173{\cdot}2 \text{ sq. ft.}$$

Volume of water in boiler $= 173{\cdot}2 \times 11{\cdot}5 - 739{\cdot}82 =$ 1,252 cubic feet.

$$\therefore \text{Weight of water} = 1{,}252 \times \frac{62{\cdot}5}{2{,}240} = 34{\cdot}94 \text{ tons. Ans.}$$

121.

Maximum bending moment $= \dfrac{\text{W L}}{4}$

$$\therefore 53{,}120 = \frac{8{\cdot}25 \times 2{,}240 \times \text{L}}{} \text{ ft. lb.}$$

$$\frac{53{,}120 \times 4}{8{\cdot}25 \times 2{,}240} = 11{\cdot}5 \text{ feet (nearly) length of beam. Ans.}$$

$$\frac{\text{M}}{\text{I}} \quad \frac{p}{y}, \quad \text{I} = \frac{\text{B D}^3}{12} \text{ and } y = \frac{\text{D}}{2}$$

$$\frac{\text{M} \times 12}{\text{B}} \quad \frac{p \times 2}{\text{D}} \quad \frac{6 \text{ M}}{\text{B D}^2} = p$$

$$\therefore \text{D} = \sqrt{\frac{6 \times \text{M}}{\text{B} \times p}} = \sqrt{\frac{6 \times 53{,}120 \times 12}{4 \times 3{\cdot}65 \times 2{,}240}}$$

$= 10{\cdot}82$ inches depth. Ans.

122. $$\text{V.R.} = \frac{2\ \text{R}}{} \text{ or } \frac{2\ \text{D}}{d_2 - d} \text{ (see page 212)}$$

Effective diameters of axle are $7\frac{1}{2} + \frac{1}{2} = 8$ ins. $= d_2$
and $5 + \frac{1}{2} = 5\frac{1}{2}$ ins. $= d_1$

$$\therefore \text{ V.R.} = \frac{2 \times 32}{8 - 5\frac{1}{2}} = \frac{64}{2\cdot5} = 25\cdot6. \quad \text{Ans.}$$

$$\text{M.A.} = \frac{\text{W}}{\text{P}} = \frac{300}{25} = 12. \quad \text{Ans.}$$

$$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}} = \frac{12}{25\cdot6} = 0\cdot4688 \text{ or } 46\cdot88\%. \text{ Ans.}$$

123. Horizontal force that would cause man to slip $= 0\cdot42 \times 175$ lb.

Horizontal force required to cause movement of weight $= 0\cdot36 \times$ W lb.

Therefore, $0\cdot42 \times 175 = 0\cdot36 \times \text{W}$

$$\text{W} = \frac{0\cdot42 \times 175}{0\cdot36} = 204\tfrac{1}{6} \text{ lb.} \quad \text{Ans.}$$

124. The law of Archimedes states that a floating body displaces an amount of water equal to its own weight.

$$\text{Volume of water displaced by wood} = \frac{250}{62\cdot5} \text{ cubic feet.}$$

$$\text{Rise of water level in tank} = \frac{250}{62\cdot5 \times 5 \times 2} = 0\cdot4 \text{ foot.}$$

New depth of water $= 4 + 0\cdot4 = 4\cdot4$ feet.
$\text{P} = \text{H A } w$

Pressure on bottom $= 4{\cdot}4 \times 5 \times 2 \times 62{\cdot}5 = 2{,}750$ lb. Ans.

$$,, \quad \text{sides} = \frac{4{\cdot}4}{2} \times 5 \times 4{\cdot}4 \times 62{\cdot}5 = 3{,}025 \text{ lb. Ans.}$$

$$,, \quad \text{ends} = \frac{4{\cdot}4}{2} \times 2 \times 4{\cdot}4 \times 62{\cdot}5 = 1{,}210 \text{ lb. Ans.}$$

125. Let x nautical miles = distance between ports.

$$\text{Time taken by slow ship} = \frac{x}{14{\cdot}25} \text{ hours.}$$

$$\text{Time taken by fast ship} = \frac{x}{15{\cdot}75} \text{ hours.}$$

The fast ship takes $6 + 15{\cdot}25 = 21{\cdot}25$ hours less than the slow ship,

$= 21{\cdot}25$

14·25 15·75

Multiply throughout by $14{\cdot}25 \times 15{\cdot}75$:—

$$15{\cdot}75\,x - 14{\cdot}25\,x = 21{\cdot}25 \times 14{\cdot}25 \times 15{\cdot}75$$

$$\therefore x = \frac{21{\cdot}25 \times 14{\cdot}25 \times 15{\cdot}75}{1{\cdot}5}$$

$= 3{,}179$ nautical miles. Ans.

126. Volume of water displaced = 1 cubic foot.

$$\text{Rise of water level} = \frac{1}{3 \times 3} = \tfrac{1}{9} \text{ ft.}$$

New depth of water $= 2 + \frac{1}{9} = 2\frac{1}{9}$ ft.

$P = H\,A\,w$

Total pressure on bottom $= 2\frac{1}{9} \times 3 \times 3 \times 62{\cdot}5$

$= 1{,}187{\cdot}5$ lb. Ans.

Total pressure on each side

$$= \frac{2\frac{1}{9}}{2} \times 3 \times 2\frac{1}{9} \times 62 \cdot 5 = 417 \cdot 9 \text{ lb.} \quad \text{Ans.}$$

Weight of water displaced $= 62 \cdot 5$ lb.

$\therefore$ Weight of iron in water, or tension in rope

$= 450 - 62 \cdot 5$

$= 387 \cdot 5$ lb. Ans.

127. Let x = speed of slow ship
and $x + 5$ = speed of fast ship.

Time taken by slow ship $= \dfrac{490}{x}$ hours.

Time taken by fast ship $= \dfrac{490}{x + 5}$ hours.

Fast ship takes 9 hours less to do the 490 miles,

$$\frac{490}{x} \qquad \frac{490}{x + 5}$$

Multiplying throughout by $x \times (x + 5)$:—

$$490\,(x + 5) - 490\,x = 9 \times x \times (x + 5)$$
$$490\,x + 2{,}450 - 490\,x = 9\,x^2 + 45\,x$$
$$\therefore 9\,x^2 + 45\,x = 2{,}450 \text{ or, } x^2 + 5\,x = 272 \cdot 2$$

Completing the square—

$$x^2 + 5\,x + (2 \cdot 5)^2 = 272 \cdot 2 + (2 \cdot 5)^2$$
$$\therefore x + 2 \cdot 5 = \pm \sqrt{278 \cdot 45}$$
$$\therefore x = \pm 16 \cdot 69 - 2 \cdot 5 = 14 \cdot 19$$

$\therefore$ Speed of slow ship $= 14 \cdot 19$ knots. Ans.

128. See page 178 for the investigation of the bursting of flywheels.

Stress (lb. per sq. inch) $= \dfrac{w\ R^2\ N^2}{245}$

= weight per cubic inch.

= radius of wheel in feet.

$= \text{revolution per minute}$

$$N^2 = \frac{245 \times \text{Stress}}{w\ R^2}$$

$$N = \sqrt{\frac{245 \times 1{,}000 \times 1{,}728}{450 \times 4^2}} \qquad 242{\cdot}5 \text{ revs. per min.}$$

Ans.

129. $$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}} \quad \therefore \text{V.R.} = \frac{\text{M.A.}}{\text{Eff.}} \quad \frac{12}{0{\cdot}42}$$

$$\text{V.R.} = \frac{2\ D}{D - d} \qquad \frac{12}{0{\cdot}42}$$

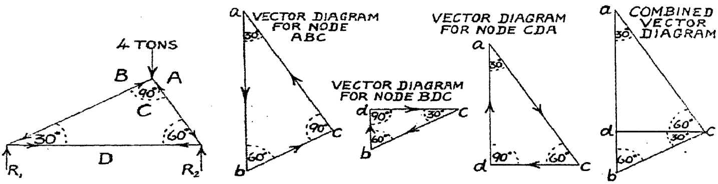
$$\frac{2\ D}{D - 9} \qquad \frac{12}{0{\cdot}42}$$

$$\therefore 2 \times 0{\cdot}42 \times D = 12\ D - 108$$

$$108 = 12\ D - 0{\cdot}84\ D = 11{\cdot}16\ D$$

$$\therefore D = \frac{108}{11{\cdot}16} = 9{\cdot}678 \text{ inches.} \quad \text{Ans.}$$

130.



$$\frac{\quad}{a\ b} = \text{Cos. } 30° \quad \therefore a\ c = a\ b \times \text{Cos. } 30°$$

$\therefore$ Force in A C $= 4 \times 0{\cdot}866 = 3{\cdot}464$ tons (compressive).

Ans.

$$\frac{b\ c}{a\ b} = \text{Sin. } 30^\circ \quad \therefore b\ c = a\ b \times \text{Sin. } 30^\circ$$

$\therefore$ Force in B C $= 4 \times 0{\cdot}5 = 2$ tons (compressive). Ans.

$$\frac{d\ c}{a\ c} = \text{Sin. } 30^\circ \quad \therefore d\ c = a\ c \times \text{Sin. } 30^\circ$$

$\therefore$ Force in D C $= 3{\cdot}464 \times 0{\cdot}5 = 1{\cdot}732$ tons (tensile). Ans.

$$\frac{b\ d}{b\ c} = \text{Sin. } 30^\circ \quad \therefore b\ d = b\ c \times \text{Sin. } 30^\circ$$

$\therefore$ Force B D (which is R_1) $= 2 \times 0{\cdot}5 = 1$ ton. Ans.

$\therefore$ Force A D (which is R_2) $= 4 - 1 = 3$ tons. Ans.

131. By Simpson's rule :—

$$\text{Area} = \frac{h}{3}(a + 4\,b + 2\,c + 4\,d + e$$

Distance between ordinates $= \dfrac{\text{Width}}{4} = \dfrac{L}{4}$

Cross sectional area of steam space

$$= \frac{L}{4 \times 3}(0 + 4 \times 3 + 2 \times 3\tfrac{3}{4} + 4 \times 3 + 0)$$

$$= \frac{L}{12} \times 31\tfrac{1}{2} \text{ sq. feet.}$$

Volume of steam space $=$ Area $\times$ length

$$\therefore 390 = \frac{L \times 31{\cdot}5}{12} \times L$$

$$\therefore L = \sqrt{\frac{390}{3}} \text{ feet.}$$

Length of boiler $= 12{\cdot}19$ feet. Ans.

132. Linear expansion due to heat $= K\ L\ T$ (See page 317).

When cooled down, strain in wire $= \dfrac{K\ L\ T}{L} = K\ T$

Stress $=$ Strain $\times E = K \times T \times E$

$= 0{\cdot}0000067 \times (236 - 60) \times 30 \times 10^6$ lb. per sq. in.

$= 35{,}370$ lb. per sq. inch, or $15{\cdot}79$ tons per sq. inch. Ans.

Equivalent load $= 35{,}370 \times (0{\cdot}1)^2 \times 0{\cdot}7854$ lb.
$= 277{\cdot}8$ lb. Ans.

133. Circumference of driving wheel $= 80 \times 1$ inches $= \frac{80}{12}$ feet.

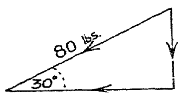
Work done in one revolution $= 250 \times \frac{80}{12}$ ft. lb.

Work done per minute $= 250 \times \frac{80}{12} \times$ revs. per min.

$\therefore 250 \times \frac{80}{12} \times$ revs. per min. $= 25 \times 33{,}000$

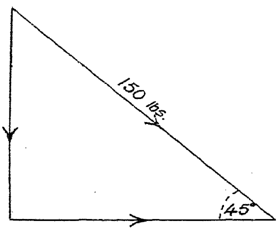
$\therefore$ Revs. per minute $= \dfrac{25 \times 33{,}000 \times 12}{250 \times 80} = 495$. Ans.

134.



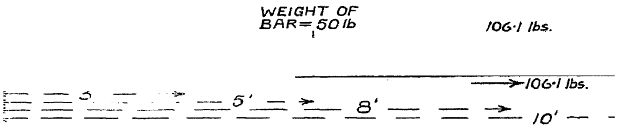
Vertical component of 80 lb. force
$= 80 \times \text{Sin. } 30°$
$= 80 \times 0{\cdot}5 = 40$ lb.

Horizontal component $= 80 \times \text{Cos. } 30°$
$= 80 \times 0{\cdot}866 = 69{\cdot}28$ lb. pushing towards hinge.



Vertical component of 150 lb. force
$= 150 \times \text{Sin. } 45° = 150 \times 0{\cdot}7071$
$= 106{\cdot}1$ lb.

Horizontal component
$= 150 \times \text{Cos. } 45°$
$= 150 \times 0{\cdot}7071 = 106{\cdot}1$ lb.
pulling on hinge.



Moments about A :—

$$(40 \times 3) + (50 \times 5) + (106{\cdot}1 \times 8) = R_B \times 10$$

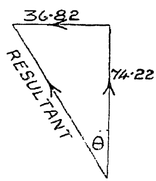
$$120 + 250 + 848{\cdot}8 = R_B \times 10$$

$$1218{\cdot}8 = R_B \times 10$$

$$\therefore R_B = \frac{1218{\cdot}8}{10} = 121{\cdot}88 \text{ lb.} \quad \text{Ans.}$$

Upward force of hinge A $= 40 + 50 + 106{\cdot}1 - 121{\cdot}$
$= 74{\cdot}22$ lb.

Pull of hinge to the left $= 106{\cdot}1 - 69{\cdot}28 = 36{\cdot}82$ lb.



$$\text{Tan. } \theta = \frac{36{\cdot}82}{74{\cdot}22} = 0{\cdot}4961$$

$$\therefore \theta = 26^\circ\, 23'$$

$$\text{Resultant} = \frac{36{\cdot}82}{\text{Sin. } \theta} = \frac{36{\cdot}82}{0{\cdot}4444}$$

$$= 82{\cdot}83 \text{ lb.}$$

Force exerted by hinge $= 82{\cdot}83$ lb. Ans.

Direction of this force $= 26^\circ\, 23'$ to the vertical.
or $90^\circ - 26^\circ\, 23' = 63^\circ\, 37'$ to the bar. Ans.

135. $$\text{V.R. of jack} = \frac{\text{Area of ram}}{\text{Area of plunger}} \qquad \frac{\times\ 3^2}{\times\ (\frac{3}{4})^2} \qquad \frac{3^2 \times 4^2}{3^2}$$
$$= 16.$$

M.A. of jack and lever $=$ Efficiency $\times$ V.R.
$= 0{\cdot}8 \times 16 \times 10 = 128$

Load lifted $=$ M.A. $\times$ effort $= 128 \times 140$
$= 17{,}920$ lb. or 8 tons. Ans.

136. The centrifugal force of the car, when turning, is opposed by the friction force between the tyres and the road. If the former exceeds the latter, then the car will skid. Equating centrifugal force and friction force,

$$\frac{W\,v^2}{r\,g} = \mu\ W, \text{ W will cancel.}$$

$$\frac{}{\times\ 32{\cdot}2} = 0{\cdot}63,\ v^2 = 80 \times 32{\cdot}2 \times 0{\cdot}63 = 40{\cdot}29 \text{ ft. per sec.}$$

$40{\cdot}29$ ft. per sec. $= 40{\cdot}29 \times \frac{60}{88} = 27{\cdot}47$ miles per hour. Ans.

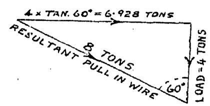
137. The moment of a force is its turning effect about a point. The magnitude of the moment is the product of the force and the perpendicular distance from the line of action of the force to the point.

If two equal and opposite forces do not act in the same straight line they constitute a couple, and the perpendicular distance between their lines of action is the "arm" of the couple. The moment of the couple is one force multiplied by the arm.

The turning moment of the 6 lb. force is $6 \times 3 = 18$ ft. lb.

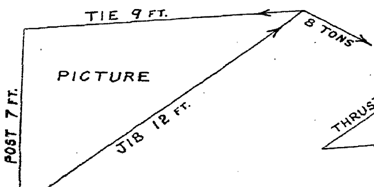
$\times\ 3 = 18,\quad P_1 = 6$ lb. Ans. (*a*)
$\times\ 9 = 18,\quad P_2 = 2$ lb. Ans. (*b*)
$\times\ \frac{5}{12} = 18,\quad P_3 = 43{\cdot}2$ lb. Ans. (*c*)

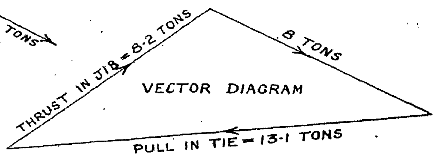
138. In order to pull the load to 60° from the vertical, a horizontal force of 4 tan. 60° $= 4\sqrt{3}$ tons will be required. The resultant pull in the wire from the jib head



$$\overline{4^2} \qquad 48$$

$= \sqrt{64} = 8$ tons.





Force in jib $= 8{\cdot}2$ tons, force in tie $= 13{\cdot}1$ tons. Ans.

139. Velocity of inflow $=$ Co-efficient $\sqrt{2\,g\,h}$,

$\therefore$ Velocity $\propto \sqrt{\text{head}}$, co-efficient and $\sqrt{2g}$ being constant.

But the time to fill the tank $\propto \dfrac{1}{\text{Velocity}}$

$$\therefore \text{Time} \propto \frac{1}{\sqrt{\text{head}}}$$

Time $\times \sqrt{\text{head}} =$ constant.

$1\frac{1}{4} \times \sqrt{16} =$ time for 25 feet head $\times \sqrt{25}$

$\therefore$ Time to fill tank when head is 25 feet

$$= \frac{1\frac{1}{4} \times \sqrt{16}}{\sqrt{25}} = 1 \text{ hour, or } 60 \text{ minutes.}$$

$\therefore$ fraction of tank filled per minute when the head is 25 feet $= \frac{1}{60}$

Also, time to pump tank out $=$ 1 hour 55 minutes $=$ 115 minutes, and the fraction pumped out per minute $= \frac{1}{115}$

$\therefore$ when both are in operation the fraction filled per min.

$$= \frac{1}{60} - \frac{1}{115} = \frac{11}{1{,}380}$$

Fraction of tank to be filled

$$= \frac{3 \text{ ft. } 5 \text{ ins.} - 9 \text{ ins.}}{3 \text{ ft. } 5 \text{ ins.}} = \frac{32}{41}$$

Time to fill tank

$=$ Fraction to be filled $\div$ Fraction filled per min.

$$= \frac{32}{41} \div \frac{11}{1{,}380} = \frac{32}{41} \times \frac{1{,}380}{11} = 97{\cdot}91 \text{ mins.}$$

Ans.

140. When the ships pass, each has travelled the same distance. Let this be x nautical miles.

$$\text{Time A has been under way} = \frac{\quad}{10} \text{ hours.}$$

$$\text{B} = \frac{\quad}{12} \text{ hours,}$$

and the difference between these times is 2 hours.

$$\therefore \frac{\quad}{10} - \frac{\quad}{12} = 2, \quad 6x - 5 \quad = 120, \; x = 120 \text{ miles.}$$

$$\text{Time by B} = \frac{120}{12} = 10 \text{ hours.}$$

$$\text{,, \quad ,, \quad C} = \frac{120}{16} = 7{\cdot}5 \text{ hours}$$

$\therefore$ C must leave $10 - 7{\cdot}5 = 2{\cdot}5$ hours after B. Ans.

141. Area of door exposed to water pressure

$$= \frac{\pi}{4} \times 14{\cdot}5 \times 18{\cdot}5 \times \frac{1}{144} \text{ sq. feet.}$$

Head of water above centre of door $= 20$ feet.

Load on door $= H \times A \times w$

$$= 20 \times \frac{\pi}{4} \times 14{\cdot}5 \times 18{\cdot}5 \times \frac{1}{144} \times 62{\cdot}5$$

$$= 1{,}829 \text{ lb.} \quad \text{Ans.}$$

The door joint fits around the spigot of the door, and since data is not given respecting the door clearance, it is reasonable to assume that the dimensions of the joint are 12 ins. by 16 ins. inside, and 14 ins. by 18 ins. outside.

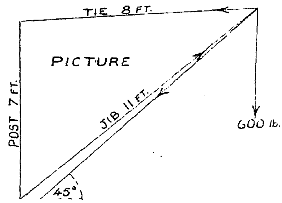
Superficial area of joint

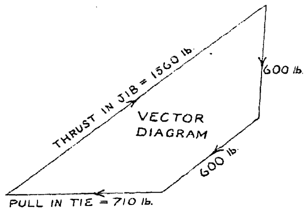
$$= \frac{\pi}{4}\Big[14 \times 18 - 12 \times 16\Big] = \frac{\pi}{4} \times 60 \text{ sq. ins.}$$

Average load per sq. inch on the joint

$$\frac{1{,}829}{\frac{\pi}{4} \times 60} = 38{\cdot}81 \text{ lb. per sq. inch. Ans.}$$

142.





Force in tie = 710 lb.; force in jib = 1,560 lb. Ans.

143. Volume of the block of cast iron

$$\frac{60}{7{\cdot}21 \times 62{\cdot}5} \text{ cu. ft.}$$

Weight of an equal volume of the water

$$\frac{60}{7{\cdot}21 \times 62{\cdot}5} \times 1{\cdot}05 \times 62{\cdot}5 = \frac{60 \times 1{\cdot}05}{7{\cdot}21}$$

When the iron is completely immersed, it appears to lose weight equal to the weight of the water it displaces.

Tension is wire = apparent weight

$$= 60 \quad \frac{60 \times 1{\cdot}05}{7{\cdot}21} = 60\left(1 - \frac{1{\cdot}05}{7{\cdot}21}\right)$$

Tension in Wire

$$= 60\left(\frac{7{\cdot}21 - 1{\cdot}05}{7{\cdot}21}\right) = 60 \times \frac{6{\cdot}16}{7{\cdot}21} = 51{\cdot}27 \text{ lb.}$$

Ans.

The diameter of the wire is not required unless the stress was asked for.

$$\text{Stress in wire} = \frac{51{\cdot}27}{(0{\cdot}0625)^2 \times 0{\cdot}7854} = 16{,}710 \text{ lb. per sq. inch.}$$

144. Weight of vessel

$$= [\pi \times 3{\cdot}5 \times 2{\cdot}5 + \frac{}{4} \times (3{\cdot}5)^2 \times 2] \times 3{\cdot}$$

$$= (8{\cdot}75 + 6{\cdot}125) \times 3{\cdot}8 \times \pi = 177{\cdot}5 \text{ lb.}$$

$$\text{Volume of the vessel} = \frac{}{4} \times (3{\cdot}5)^2 \times 2{\cdot}5 = 24{\cdot}05 \text{ cu. feet.}$$

Weight of water displaced by the vessel

$$= 24{\cdot}05 \times \frac{1024}{16} = 1539{\cdot}2 \text{ lb.}$$

$\therefore$ Weight of oil in vessel $= 1539{\cdot}2 - 177{\cdot}5 = 1361{\cdot}7$ lb.

$$\text{Weight per cu. ft. of oil} = \frac{1361{\cdot}7}{24{\cdot}05}$$

$$\text{Specific gravity of the oil} = \frac{1361{\cdot}7}{24{\cdot}05 \times 62{\cdot}5} = 0{\cdot}9056.$$

145. Stress $= \dfrac{12\,w\,v^2}{g}$, where w is the weight of 1 cu. inch of the shroud.

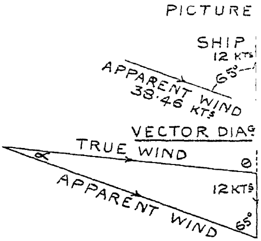
v may be taken as the velocity of the shroud at its mean diameter.

$$v = \frac{50}{12} \times \pi \times \frac{450}{60} = 98{\cdot}14 \text{ ft. per sec.}$$

$$\text{Stress} = \frac{12 \times 0{\cdot}283 \times (98{\cdot}14)^2}{32{\cdot}2} = 1016 \text{ lb. per sq. inch.}$$

Ans.

146.



$$65 \text{ ft. per sec.} = \frac{65}{1{\cdot}69} = 38{\cdot}46 \text{ knots.}$$

The velocity of the wind relative to the ship is given, therefore stop the ship. Give to each a velocity of 12 knots due S, and the apparent wind is the resultant of the true wind and 12 knots due S.

From the vector diagram :—

True wind

$$\overline{- 2 \times 12 \times 38{\cdot}46 \times \text{Cos. } 65^\circ}$$

$$= \sqrt{1233} = 35{\cdot}11 \text{ knots, or } 59{\cdot}33 \text{ ft. per sec.}$$

$$\frac{35{\cdot}11}{\text{Sin } 65^\circ} \quad \frac{12}{\text{Sin. } \alpha} \qquad \frac{12}{35{\cdot}11} \times \sin 65^\circ, \quad \alpha = 18^\circ\ 3'$$

$$\theta = 65^\circ + 18^\circ\ 3' = 83^\circ\ 3'.$$

True wind has a velocity of 35·11 knots, or 59·33 ft. per sec., and its direction is from N 83° 3′ W. Ans.

147. Let V be the volume of the wood in cu. feet.

$$\text{Then V} = \frac{140}{62{\cdot}5 \times \text{Sp. Gr.}}$$

Total downward weight = Weight of oil displaced

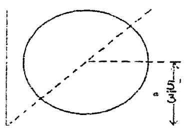
$$\therefore 140 + 60 = \text{V} \times 0{\cdot}91 \times 62{\cdot}5$$

$$\therefore\ 200 = \frac{140}{62{\cdot}5 \times \text{Sp. Gr.}} \times 0{\cdot}91 \times 62{\cdot}5$$

$$200 \times \text{Sp. Gr.} = 140 \times 0{\cdot}91$$

$$\text{Sp. Gr.} = \frac{140 \times 0{\cdot}91}{200} = 0{\cdot}637. \quad \text{Ans.}$$

148.



Area of circle $= \dfrac{5^2}{4}$ sq. inches.

Since the centre of the circle is at $\frac{1}{3}$ of the diagonal from one corner, therefore it is at $\frac{1}{3}$ of the side from each side.

Taking moments about $\overline{o\ o}$

$$\text{C.G. from } \overline{o\ o} = \frac{5^2 \times \frac{5}{2} - \frac{5^2}{4} \times \frac{5}{3}}{\qquad}$$

$$= \frac{\frac{5}{2} - \frac{5}{12}}{1 - \frac{1}{4}} \text{ (cancelling each term by } 5^2\text{).}$$

$$\text{C.G. from } \overline{o\ o} = \frac{\frac{25}{12}}{\frac{3}{4}} = \frac{25}{9} = 2\tfrac{7}{9} \text{ inches from each side.}$$

Ans.

149.

$$\omega = \frac{200}{60} \times 2\,\pi = \frac{20\,\pi}{3} \text{ radians per sec.}$$

$$\text{Centrifugal force} = \frac{W}{g} \times \omega^2\, r$$

$$\frac{2}{32{\cdot}2} \times \qquad\qquad \times 3 = 81{\cdot}74 \text{ lb.}$$

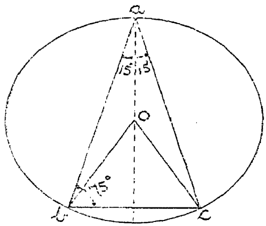
Tension at highest point $= 81{\cdot}74 - 2 = 79{\cdot}74$ lb. Ans.

Tension at lowest point $= 81{\cdot}74 + 2 = 83{\cdot}74$ lb. Ans.
At 45° from the highest point the tension is :—

Centrifugal force — Component of 2 lb. acting down the wire

$= 81{\cdot}74 - 2 \sin. 45^\circ = 81{\cdot}74 - 1{\cdot}414 = 80{\cdot}326$ lb. Ans.

150.



r = radius of the plate.

$\pi r^2 = 804{\cdot}25, \quad r = \sqrt{\dfrac{804{\cdot}25}{\pi}}$

$= 16$ inches.

Apex angle $= 180^\circ - 2 \times 75^\circ = 30^\circ$
Angle $b\,o\,c$ at centre $= 30^\circ \times 2 = 60^\circ$ because the angle at the centre is always twice the angle at the circumference.

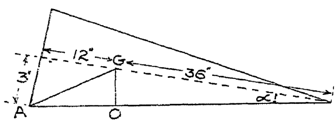
Triangle $b\ o\ c$ is equilateral, and the base of the triangle is 16 inches long.

$= \sin. 15^\circ$, $\quad . \ a\ b =$ $\qquad = 30{\cdot}91$ inches.
$a\ b$ $\qquad\qquad\qquad \sin.\ 15^\circ$

Area $= \frac{1}{2} \times (30{\cdot}91)^2 \times \sin. 30^\circ = 238{\cdot}8$ sq. inches.

Sides are 30·91 ins., 30·91 ins. and 16 ins.
Area is 238·8 sq. ins. } Ans.

151.



Tan $\alpha = \frac{3}{48} = 0{\cdot}0625$
α (semi-vertical angle) $= 3^\circ 34'$
G is at $\frac{3}{4}$ of 48 $= 36''$ from apex.
G O $= 36 \sin. 3^\circ 34' = 2{\cdot}239$ inches.

A G $= \sqrt{3^2 + 12^2} = 12{\cdot}37$ ins.
Rise of G $= 12{\cdot}37 - 2{\cdot}239$
$= 10{\cdot}131$ inches.

Weight of the pyramid $= (\frac{1}{2})^2 \times \frac{1}{3} \times 450 = 150$ lb.

$$\text{Work done} = 150 \times \frac{10\cdot131}{12} = 126\cdot64 \text{ ft. lb.} \quad \text{Ans. } (a)$$

$$\text{Rise of G} = 36 - 2\cdot239 = 33\cdot761 \text{ inches.}$$

$$\text{Work done} = 150 \times \frac{33\cdot761}{12} = 422 \text{ ft. lb.} \quad \text{Ans. } (b).$$

152.

$$\text{Working pressure} = \frac{2\ t}{\text{D}} \times \frac{\text{Tensile strength of material}}{\text{Factor of safety}}$$

The smaller value will be taken for the tensile strength of the material of the shell.

Factor of safety

$$\frac{2 \times \frac{1}{2} \times 23 \times 2240}{150 \times 23} = \frac{224}{15} = 14\cdot93. \quad \text{Ans.}$$

Stress in the weld

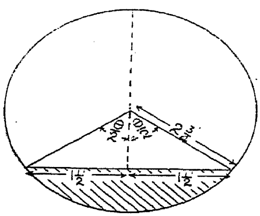
$$\frac{23 \times 2240}{14\cdot93} = 3450 \text{ lb. per sq. inch.} \quad \text{Ans.}$$

Or, imagine a ring of the cylinder 1 inch wide, the bursting force is 150 × 23 lb. The resisting force is $2 \times \frac{1}{2} \times$ stress

$$\therefore\ 2 \times \tfrac{1}{2} \times \text{stress} = 150 \times 23.$$

$$\text{Stress} = 150 \times 23 = 3450 \text{ lb. per sq. inch.}$$

153.



$$\text{in.}\left(\frac{\theta}{2}\right) = \frac{1\frac{1}{2}}{2\frac{3}{4}}$$

$$\left(\frac{\theta}{2}\right) = 33^\circ\ 3'$$

Angle of the sector $= \theta = 66^\circ\ 6'$

Area of the segment (this is the cross sectional area of the water)

$$\left[\theta\ (\text{radians}) - \sin.\ \theta\right]$$

Area of segment

$$= \frac{121}{32}\left[\frac{66\cdot1 \times 2\pi}{360} \qquad \sin. 66^\circ 6'\right.$$

$$= \frac{121}{32}\left[1\cdot154 - 0\cdot9142\right] = 0\cdot907 \text{ sq. feet.}$$

Weight of water $= 0\cdot907 \times 16 \times 62\cdot5 = 907$ lb. Ans.

154. Head of oil over bottom of tank $= 35 + 3\frac{3}{4} = 38\frac{3}{4}$ feet.

Load $= H \times A \times w$

Load per sq ft. on bottom $= 38\frac{3}{4} \times 1 \times 0\cdot92 \times 62\cdot5$ $= 2{,}228$ lb. Ans.

Average head of oil over sides and ends $= 35 + 1\frac{7}{8}$ $= 36\frac{7}{8}$ feet.

Load per sq. foot on sides and ends $= 36\frac{7}{8} \times 0\cdot92 \times 62\cdot5$ $= 2{,}120$ lb. Ans.

This is the average load per sq. foot on the sides and ends. It is greater towards the bottom and less towards the top of the tank.

Pitch of the rivets $= 7 \times \frac{7}{8} = 6\frac{1}{8}$ inches.

Number of rivets along the sides $= \dfrac{60}{6\frac{1}{8}} = 10$

Number of rivets along the ends $= \dfrac{30}{6\frac{1}{8}} = 5$

Total rivets $= 2\ (10 + 5) = 30$.

Load on top of tank $= 35 \times 0\cdot92 \times 62\cdot5 \times 2\frac{1}{2} \times 5$ lb. $= 25{,}160$ lb.

Average shear stress on rivets

$$= \frac{25160}{(\frac{7}{8})^2 \times 0\cdot7854 \times 30} = 1395 \text{ lb. per}$$

Weight of oil in tank and pipe

$$= [30 \times 60 \times 45 + [2^2 \times 0\cdot7854 \times 35 \times 12] \times \frac{0\cdot92 \times 62\cdot5}{1728}$$

$$= [81000 + 1320] \times \frac{0\cdot92 \times 62\cdot5}{1728}$$

$= 2{,}740$ lb.

Load on trestles $= 2{,}740$ lb. Ans.

155. Let A be the faster ship, and x knots be its speed.

Let B be the slower ship, and y knots be its speed.

Then A would do the whole distance in $\frac{420}{x}$ hours,

and B in $\frac{420}{y}$ hours.

Since both ships left port at the same time, then $\frac{420}{x} - 9$ hours is the time A steams before they meet, and B has taken $\frac{420}{y} - 16$ hours.

These must be equal, therefore :—

$$\frac{420}{x} - 9 = \frac{420}{y} - 16$$

$$\frac{420 - 9x}{x} \qquad \frac{420 - 16y}{y}$$, and by cross multiplication,

$$y\ (420 - 9x) = x\ (420 - 16y)$$

$$420y - 9xy = 420x -$$

$$7xy = 420x - 420y$$

$$xy = 60x - 60y \ \dots \qquad \dots \qquad \dots \quad (1)$$

The distance travelled by A in 9 hours $= 9x$ miles,

and the distance by B in 16 hours $= 16y$ miles.

The sum of these two distances must be 420 miles.

$$\therefore\ 9x + 16y = 420$$

$16y = 420 - 9x,\ y = \frac{420 - 9x}{16}$ Substitute in (1).

$$\frac{x\,(420 - 9x)}{16} = 60x - \frac{60\,(420 - 9x)}{16},$$

multiply each term by 16.

$$420x - 9x^2 = 960x - 25200 + 540x$$
$$25200 = 1080x + 9x^2, \text{ cancel by } 9$$
$$2800 = 120x + x^2$$

or $x^2 + 120x = 2800$, a quadratic equation.

$$x^2 + 120x + \left(\tfrac{120}{2}\right)^2 = 2800 + 3600$$
$$= 6400$$

$$x + 60 = \pm\ 80$$

$\therefore\ x = 80 - 60 = 20$ knots, taking the positive value.

$$y \quad \frac{420 - 9x}{16} \quad \frac{420 - 180}{16} \quad \frac{240}{16} = 15 \text{ knots.}$$

The speeds are 20 and 15 knots. Ans.

156. The velocity of the aeroplane relative to the carrier is $150 - 20 = 130$ knots, when flying directly away from the carrier. It is $150 + 20 = 170$ knots when flying directly towards the carrier.

If the carrier may be assumed to be stationary, the plane moves away from it at 130 knots, and towards it at 170 knots.

Let the plane fly away for x hours, and return in $2 - x$ hour.

The distance flown in each direction must be the same, considering the carrier stationary,

$$\therefore\ 130x = 170\,(2 - x)$$
$$= 340 - 170x, \qquad 300x = 340$$

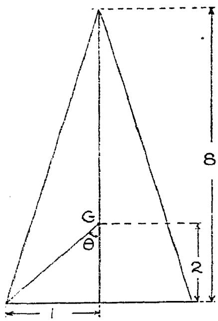
$$x = \frac{340}{300} \text{ hours} = \frac{340 \times 60}{300} = 68 \text{ minutes}$$

$$= 1 \text{ hour } 8 \text{ minutes.}$$

$\therefore$ the plane may fly directly away from the carrier for 1 hour 8 minutes, and it must turn back at 7 hours 8 min. a.m. Ans.

157. $= 0{\cdot}3 = \tan \qquad = 16^\circ\ 42'$.

The angle of the incline upon which the cone would just slide is $16^\circ\ 42'$.



Let the radius of the base of cone be 1. Then its diameter is 2, and the vertical height is $4 \times 2 = 8$.

The C.G. of a cone is at $\dfrac{\text{height}}{4}$ above the base, and $\frac{8}{4} = 2$

Tan $\theta = \frac{1}{2}$, $\theta = 26^\circ\ 34'$.

If the cone is tilted until G is vertically above A, then the cone would be in a state of unstable equilibrium. The cone may be tilted through $26^\circ\ 34'$ before it becomes unstable. When on the point of sliding down the incline, it will have been tilted through $16^\circ 42'$, and it is therefore in a state of stable equilibrium. Ans.

If the height was 8 times the diameter of the base, then if the radius is 1, the height is 16, and G is $\frac{16}{4} = 4$ above the base.

$$\text{Tan } \theta = \tfrac{1}{4},\ \theta = 14^\circ\ 2'.$$

This cone could only be tilted through $14^\circ\ 2'$ before it became unstable. When the inclination of the plane is slightly greater than $14^\circ\ 2'$, the cone would overturn, but sliding would not occur until the inclination was $16^\circ\ 42'$.

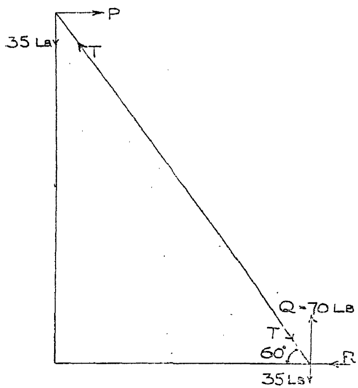
$\therefore$ this cone would overturn before it would slide. Ans.

158. The ladder is in equilibrium under the action of a system of forces, and therefore :—

The algebraic summation of the vertical forces must be zero.

The algebraic summation of the horizontal forces must be zero.

The algebraic summation of first moments about any point must be zero. (See page 199).



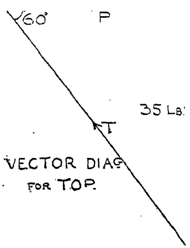
The bulkhead is smooth and therefore there is no friction force acting at the top end of the ladder.

The weight of the ladder acts vertically down, therefore the vertical reaction of the deck must be 70 lb., the weight of the ladder.

The reaction of the bulkhead (P) is a horizontal force, and there must be an equal and opposite force R acting at the foot of the ladder.

Since the ladder is uniform, half of its weight (35 lb.) can be supposed to act at each end.

Draw a vector diagram for the forces acting at the top end of the ladder.



$$\frac{35}{P} = \tan 60°, \quad P = \frac{35}{\tan 60°} = 20 \cdot 2 \text{ lb.}$$

The horizontal friction force at the foot of the ladder is R, and R = 20·2 lb.

Suppose the vertical reaction of the deck was not known.

Let this be Q and the length of the ladder be l.

Taking moments about the top end of the ladder:—

Q provides a counter-clockwise moment, R and 35 lb. clockwise moments.

$$Q \times l \cos 60° = 35 \times l \cos 60° + 20 \cdot 2 \times l \sin 60°.$$

Cancel l, and divide each term by cos 60°.

$$Q = 35 + \frac{20 \cdot 2 \times \sin 60°}{\cos 60°}$$

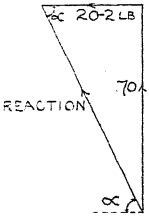
$$\text{but, } 20 \cdot 2 = \frac{35}{\tan 60°} \quad \text{and} \quad \frac{\sin 60°}{\cos 60°} = \tan 60°.$$

$$\therefore\ Q = 35 + \frac{35 \tan 60^\circ}{\tan 60^\circ} = 70 \text{ lb.}$$

Vertical reaction of the deck = 70 lb. ; reaction of bulkhead = 20·2 lb. Ans.

If the reaction of the deck, in magnitude and direction was asked for :—

The reaction is the resultant of 70 lb. vertically upwards and 20·2 lb. horizontally.



$$\text{Reaction} = \sqrt{70^2 + (20{\cdot}2)^2} = 72{\cdot}86 \text{ lb.}$$

$$\text{Tan } \alpha = \frac{70}{20{\cdot}2} = 3{\cdot}464, \quad \alpha = 73^\circ\ 54'$$

The reaction acts at 73° 54′ to the deck.

159. The weight of any volume of a substance, compared to the weight of an equal volume of fresh water, is the specific gravity of that substance.

10·89 metres = 10890 millimetres.

From the data given it is evident that a column of oil 10890 m.m. high gives the same pressure as a column of mercury 747 m.m. high.

The pressure is proportional to the product of the height and the weight per unit volume. 1 cu. ft. of mercury weighs 849 lb.

$\therefore$ 10890 × Wt. of 1 cu. ft. of oil = 747 × 849

$$\therefore \text{ Wt. of 1 cu. ft. of oil} = \frac{747 \times 849}{10890} = 58{\cdot}23 \text{ lb. Ans.}$$

160. $$\text{30 inches of mercury} = \frac{30 \times 849}{1728} = 14{\cdot}74 \text{ lb. per sq. inch.}$$

Absolute pressure on one side
= 7 + 14·74 = 21·74 lb. per sq. inch abs.

Absolute pressure on other side
= 30— 25 = 5 ins. mercury = 2·457 lb. per sq. in.

With big pressure on top side of piston :—
Effective load

$$= 21{\cdot}74 \times \quad {}^2 - (6\tfrac{1}{2})^2 \Big] - 2{\cdot}457 \times \frac{\pi}{4} \quad 10^2 \Big] \text{lb.}$$

Effective load

$$= \frac{\pi}{4} \times \frac{1}{2240}\Big[21{\cdot}74 \times 8982{\cdot}75 - 2{\cdot}457 \times 8925\Big] \text{tons}$$

Effective load $= 60{\cdot}782$ tons. Ans.

With big pressure on under side of piston :—
Effective load

$$= 21{\cdot}74 \times \frac{\pi}{4}\Big[95^2 - 10^2\Big] - 2{\cdot}457 \times \frac{\pi}{4}\Big[95^2 - (6\tfrac{1}{2})^2\Big] \text{lb.}$$

Effective load

$$= \frac{\quad}{4} \times \quad 21{\cdot}74 \times 8925 - 2{\cdot}457 \times 8982{\cdot}75 \Big] \text{tons.}$$

Effective load $= 60{\cdot}236$ tons. Ans.

161. Component of force in tow rope parallel to the surface of the water $= 4 \cos 20° = 3{\cdot}7588$ tons.

Component of $3{\cdot}7588$ in the direction of motion of the ship $= 3{\cdot}7588 \cos 15° = 3{\cdot}6306$ tons.

$\therefore$ total force urging ship forward $= 2 \times 3{\cdot}6306$
$= 7{\cdot}2612$ tons. Ans.

$$\text{H.P.} = \frac{7{\cdot}2612 \times 2240 \times 3 \times 6080}{33000 \times 60} = 149{\cdot}8. \quad \text{Ans.}$$

162. Special riveting is not employed, and there are 3 rivets in the pitch.

Strength of plate left between rivet holes
$= (6 - d)\ 1\tfrac{1}{8} \times 28 = 189 - 31{\cdot}5\ d$ tons.

Strength of rivets in the pitch

$$= d^2 \times \frac{\pi}{4} \times 23 \times 3 \times \qquad 101{\cdot}61d^2 \text{ tons.}$$

These are to be equal,

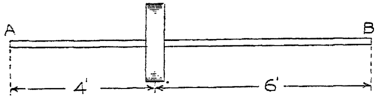
$\therefore\ 101{\cdot}61d^2 = 189 - 31{\cdot}5d$. Divide all by $101{\cdot}61$.

$d^2 = 1{\cdot}861 - 0{\cdot}31d$

$d^2 + 0{\cdot}31d = 1{\cdot}861$, a quadratic equation from which
$d = 1{\cdot}218$ inches.

Diameter of rivets $= 1{\cdot}218$ inches. Ans.

163.



Taking moments about B

$$A \times 10 = (0{\cdot}75 \times 6) + (1{\cdot}5 \times 5) = 12.$$

$$\therefore A = \frac{12}{10} = 1{\cdot}2 \text{ tons.}$$

The static load on A is 1·2 tons.

Let the force due to centrifugal force action of the unbalanced wheel be P tons. $\frac{6}{10}$ of P affects the load on bearing A.

$$\therefore \text{ max. load on A} = 1{\cdot}2 + 0{\cdot}6\ P = 1{\cdot}7 \text{ tons}$$

$$\text{minimum load on A} = 1{\cdot}2 - 0{\cdot}6\ P = 0{\cdot}7 \text{ ton}$$

Subtracting $\qquad 1{\cdot}2\ P = 1$

$$\text{or } P = \frac{\quad}{1{\cdot}2} \text{ ton.}$$

$$\text{Centrifugal force} = 0{\cdot}00034\ W\ r\ N^2, \text{ or } \frac{W\ r\ N^2}{2935}$$

where N = revs. per minute.

$$\text{or, Centrifugal force} = \frac{W}{g} \times \omega^2 r, \text{ where } \omega = \text{radians per second.}$$

$$100 \text{ r.p.m.} = \frac{100}{60} \text{ revs. per sec.} = \frac{100}{60} \times 2\pi$$

$$= \frac{10\pi}{3} \text{ radians per sec.}$$

$$P = \frac{W}{g} \times \omega^2 r \qquad r = \frac{P \times g}{W \times \omega^2} \text{ feet.}$$

$$r = \frac{1 \times 32{\cdot}2 \times 9}{1{\cdot}2 \times 0{\cdot}75 \times 100 \times \pi^2} \times 12 \text{ inches}$$

$$= 3{\cdot}916 \text{ inches. Ans.}$$

164. Full area through valve $= d^2 \times \dfrac{\pi}{4} \times \dfrac{1}{144}$ sq. feet.

Volume of water flowing per minute

$$= d^2 \times \frac{\pi}{4} \times \frac{1}{144} \times v \times 60 \text{ cu. ft.}$$

Weight flowing per minute

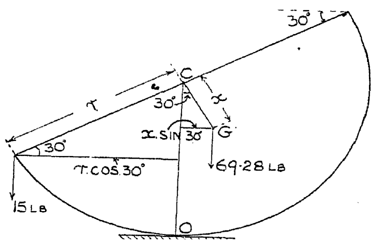
$$= d^2 \times \frac{\pi}{4} \times \frac{1}{144} \times v \times 60 \times 62{\cdot}5 \text{ lb.}$$

$$= 20{\cdot}45\, d^2 v \text{ lb.} \quad \text{Ans.}$$

When $d = 6\frac{1}{2}$ ins. and $v = 8{\cdot}5$ ft. per sec.

Weight $= 20{\cdot}45 \times (6\frac{1}{2})^2 \times 8{\cdot}5 = 7345$ lb., or $3{\cdot}28$ tons. Ans.

165.



The horizontal table is a tangent to the hemisphere, and therefore the vertical from the point of contact O must pass through the centre C of the flat surface.

Let r be the radius of the hemisphere.

Let x be the distance from G to C.

Taking moments about the vertical O C.

$$15 \times r \cos 30^\circ = 69{\cdot}28 \times x \sin 30^\circ$$

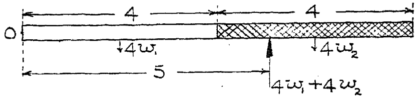
$$\therefore \frac{x}{r} = \frac{15 \cos 30^\circ}{69{\cdot}28 \sin. 30^\circ} = \frac{15}{69{\cdot}28 \tan 30^\circ} \quad \frac{15\sqrt{3}}{69{\cdot}28}$$

$$= 0{\cdot}375$$

$$x = 0{\cdot}375\, r$$

or the centre of gravity is at $0{\cdot}375$ of radius from centre C. Ans.

166.



Let each rod have a sectional area of 1 and a length of 4 units.

Let the density of the lighter material be w_1 per unit volume, and of the other w_2.

The weight of the lighter rod is $4w_1$, and of the heavier $4\ w_2$.

Take moments about O.

$(4\,w_1 \times 2) + (4\,w_2 \times 6) = (4\,w_1 + 4\,w_2)\,5$

$8\,w_1 + 24\,w_2 = 20\,w_1 + 20\,w_2$

$\therefore\ 4\,w_2 = 12\,w_1$, and $\dfrac{w_2}{w_1} = \dfrac{12}{4} = 3$

The heavier material has a density 3 times that of the lighter. Ans.

167. Area of surfaces in contact $= 6 \times \pi \times 5$ sq. inches.

Total normal pressure between seat and chest
$= 6 \times \pi \times 5 \times 400$ lb.

Friction force to be overcome to start seat
$= \mu \times$ normal pressure.

Friction force $= 0{\cdot}4 \times 6 \times \pi \times 5 \times 400$ lb.

The friction force is zero at the instant the seat is withdrawn.

$\therefore$ Average force $= \dfrac{0{\cdot}4 \times 6 \times \pi \times 5 \times 400}{2}$ lb.

Work done $= \dfrac{0{\cdot}4 \times 6 \times \pi \times 5 \times 400}{2} \times \dfrac{}{12}$

$= 3141{\cdot}6$ ft. lb. Ans.

168. When load-extension curve is carefully plotted, it begins to deviate from the straight line at a load of 140 lb. This marks the limit of proportionality, or the elastic limit.

Stress $= \dfrac{\text{Load}}{\text{Sectional area}} \qquad \dfrac{140}{(0{\cdot}06)^2 \times 0{\cdot}7854}$

$= 49500$ lb. per sq. inch, or $22{\cdot}1$ tons per sq. inch. Ans.

$$\text{Strain at elastic limit} = \frac{0{\cdot}091}{5 \times 12} = 0{\cdot}001516$$

$$\text{Modulus of elasticity} = \frac{49500}{0{\cdot}001516}$$

$= 32{,}650{,}000$ lb. per sq. inch, or $14{,}570$ tons per sq. inch. Ans.

169. $$\text{T (inch lb.)} = \frac{d^3}{5{\cdot}1} \times \text{stress, and T} = \frac{63000 \times \text{H.P.}}{\text{revs.}}$$

$$\frac{d^3 \times \text{stress}}{5{\cdot}1} \quad \frac{63000 \times \text{H.P.}}{\text{revs.}}$$

$$\frac{d^3 \times \text{stress} \times \text{revs.}}{\text{H.P.}} = \text{constant, and when HP is constant}$$

$d^3 \times \text{stress} \times \text{revs.} = \text{constant.}$

$162 \times (6\tfrac{1}{2})^3 \times 7300 = 388 \times (4\tfrac{1}{2})^3 \times \text{stress}$

$$\therefore \text{ stress in 2nd shaft} = \frac{162 \times 7300}{388} \times \left(\frac{6\frac{1}{2}}{4\frac{1}{2}}\right.$$

$= 9186$ lb. per sq. inch. Ans.

170. With this form of riveting there are 4 rivets in the pitch. (See page 292).

Strength of plate equal to solid pitch$=4\tfrac{1}{2} \times 1 \times 28 = 126$ tons.

Strength of plate left between rivet holes, in outer rows $= (4\tfrac{1}{2} - d) \times 1 \times 28 = 126 - 28d$ tons.

Shearing strength of rivets in pitch

$= d^2 \times 0{\cdot}7854 \times 24 \times 4 = 75{\cdot}4d^2$ tons.

From the data of the question :—

$$126 - 28d = \frac{110}{100} \times 75{\cdot}4d^2$$

$126 = 82{\cdot}94d^2 + 28d$. Divide by $82{\cdot}94$,

$d^2 + 0{\cdot}3376d = 1{\cdot}519$, a quadratic equation.

$d = 1{\cdot}075$ inches. Ans.

$126 - 28d = 126 - 28 \times 1{\cdot}075 = 95{\cdot}9$ tons.

$75{\cdot}4d^2 = 75{\cdot}4 \times (1{\cdot}075)^2 = 87{\cdot}18$ tons.

This is the least strength.

Efficiency of joint $= \dfrac{87{\cdot}18}{} = 0{\cdot}692$, or $69{\cdot}2\%$. Ans.

SOLUTIONS TO SECOND-CLASS EXAMINATION QUESTIONS.

HEAT AND HEAT ENGINES

1. Tons evaporated   Rise in boiler density

Tons in boiler   Average feed density

$$\therefore \text{Rise in density} = \frac{\frac{125}{24} \times 9 \times 1}{39} = 1{\cdot}202 \text{ thirty seconds,}$$

$$\therefore \text{Boiler density} = 1 + 1{\cdot}202 = 2{\cdot}202 \text{ thirty seconds.}$$

Ans.

Heat gained by ice = Heat lost by water.

Weight $(143 + 50 - 32) = 200\ (80 - 50)$

$161\ W = 6{,}000$

$W = \frac{6000}{161} = 37{\cdot}28$ pounds. Ans.

3. Total heat given to steam = Sensible + Latent

$= 8{\cdot}2\ \{367 - 126 + 966 - 0{\cdot}7\ (367 - 212)\}$

$= 8{\cdot}2 \times 1{,}098{\cdot}5$ B.T.U.

$$\text{Equivalent evaporation} = \frac{8{\cdot}2 \times 1{,}098{\cdot}5}{966}$$

$= 9{\cdot}324$ pounds. Ans.

4. Rise in temperature $= 731 - 576 = 155$ F. degrees.

$60 = (1 + 0{\cdot}000455 \times 155) \times$ first consumption.

$$\therefore \text{First consumption} = \frac{60}{1 + 0{\cdot}000455 \times 155}$$

$= 56{\cdot}05$ tons per day. Ans.

5. Exhaust opening when crank is on centre :-

$= \text{Lap} + \text{Lead} - \text{exhaust lap.}$

$\therefore \text{Lap} = 2\frac{1}{4} - \frac{1}{16} + (-\frac{3}{32}).$

$= 2\frac{3}{32}$ inches. Ans.

6. $$\frac{\text{Water evaporated}}{\text{Water in boiler}} = \frac{\text{Rise in density}}{\text{Average feed density}}$$

$$\therefore \text{Water evaporated} = \frac{95 \times 3{\cdot}5 \times 5}{0{\cdot}4}$$

$= 4{,}156$ tons. Ans.

Let T = final temperature.

Heat lost by steam = Heat gained by water

$\therefore 18\{377{\cdot}7 - T + 966 - 0{\cdot}7\ (377{\cdot}7 - 212)\} =$
$1{,}900\ (T - 95)$

$18\ (1{,}227{\cdot}7 - T) = 1{,}900\ (T - 95)$

$\therefore T = 105{\cdot}6°F.$ Ans.

8. Pounds of steam per hour

$$= \frac{0{\cdot}5^2 \times 0{\cdot}7854 \times 15 \times 16 \times 2 \times 62{\cdot}5 \times 0{\cdot}75 \times 60}{12}$$

$\therefore$ Pounds per I.H.P. hour

$$\frac{0{\cdot}5^2 \times 0{\cdot}7854 \times 15 \times 16 \times 2 \times 62{\cdot}5 \times 0{\cdot}75 \times 60}{1{,}250 \times 12}$$

17·67 pounds. Ans.

9. Heat lost by steam = Heat gained by feed.

$W\{275 - 215 + 966 - 0{\cdot}7\ (275 - 212)\} = 215 - 120$

$W\{60 + 966 - 44{\cdot}1\} = 95$

$$\frac{95}{981{\cdot}9}$$

$= 0{\cdot}0967$ pound. Ans.

10. Exhaust opening at bottom,

i.e., when engine is passing top centre

$= \text{top (lap + lead)} - \text{bottom exhaust lap}$

$= 2\frac{3}{4} + \frac{1}{8} - \frac{1}{8} = 2\frac{3}{4}$ inches. Ans. (*a*)

Exhaust opening at top, i.e., when engine is passing bottom centre $= 2\frac{5}{8} + \frac{1}{4} - \frac{1}{4} = 2\frac{5}{8}$ inches. Ans. (*b*)

11. Scale deposited $= 1{\cdot}406 + 0{\cdot}033 = 1{\cdot}439$ grams for every 2·2 lb. of water.

2·2 lb. = 1,000 grams.

$$\therefore \text{Scale per ton of water} = \frac{2{,}240 \times 1{\cdot}439}{1{,}000}$$

$= 3{\cdot}221$ pounds. Ans.

12. I.H.P.

$$\frac{\{(26^2 \times 67) + (69^2 \times 14{\cdot}75)\} \times 0{\cdot}7854 \times 4 \times 2 \times 60}{33{,}000}$$

$= 1{,}322$ I.H.P. Ans.

13. $(29 \times 18) - (4 \times 18) + (4 \times 0) = 29\,x$

$\therefore 25 \times 18 = 29\,x$

$$\therefore x = \frac{25 \times 18}{29} = 15{\cdot}52 \text{ ounces per gallon. Ans.}$$

14. Heat lost by steam = Heat given to cooling water.

H.P. $\times 14{\cdot}5 \times 965 = (86 - 67) \times 1{,}300 \times 2{,}240$

$$\text{H.P.} = \frac{19 \times 1{,}300 \times 2{,}240}{14{\cdot}5 \times 965} = 3{,}954 \text{ I.H.P.} \quad \text{Ans.}$$

15.

$$\text{Percentage increase in consumption} = \frac{37{\cdot}5 - 34}{34} \times 100$$

$$= \frac{3{\cdot}5 \times 100}{34}$$

Rise in temperature $= 827 - 600 = 227$ F degrees.

$$\therefore \text{ Percentage increase per degree} = \frac{3{\cdot}5 \times 100}{34 \times 227}$$

$= 0{\cdot}04535$ per cent. Ans.

The increase in funnel temperature will be due to deposits of soot inside the tubes, and to the formation of scale on the heating surfaces of the boiler.

16. Water evaporated Rise in density

Water in boilers Average feed density

$$\therefore \text{ Feed density} = \frac{119 \times 3{\cdot}5 \times 5}{4{,}500} = 0{\cdot}4627 \text{ oz. per gallon.} \quad \text{Ans.}$$

17.

$$\frac{\text{I.H.P.}}{\text{N.H.P.}} = \frac{P\,A{\cdot}L\,N}{33{,}000 \times \text{N.H.P.}}$$

$$= \frac{96^2 \times 0{\cdot}7854 \times 42 \times 4{\cdot}5 \times 2 \times 90{\cdot}6}{33{,}000 \times 960}$$

$= 7{\cdot}823$ times. Ans.

18. $\frac{1}{2}$ travel $=$ lap $+$ M.P.O.

$= 2\frac{1}{4} + 2 = 4\frac{1}{4}$ inches.

$\therefore$ Travel $= 8\frac{1}{2}$ inches. Ans. (*a*)

Distance from mid-position $=$ lap lead $= 2\frac{1}{4} + \frac{3}{32}$ $= 2\frac{1}{32}$ inches. Ans. (*b*)

19. Heat given to cooling water per hour $= 4{,}000 \times 16 \times 1{,}000$

$$\text{Equivalent H.P.} = \frac{4{,}000 \times 16 \times 1{,}000}{2{,}545}$$

$= 25{,}150$ Horse Power. Ans.

Heat carried away per pound of cooling water

$= 82 - 51 = 31$ B.T.U.

$$\therefore \text{Cooling water per day} = \frac{4{,}000 \times 16 \times 1{,}000 \times 24}{31 \times 2{,}240}$$

$= 22{,}120$ tons per day. Ans.

20. $f \times (6\tfrac{1}{2})^2 \times 0{\cdot}7854 = 25^2 \times 0{\cdot}7854 \times (160 - 49 - 3)$

$$f = \frac{25^2 \times 108}{(6{\cdot}5)^2}$$

$= 1{,}597$ pounds per square inch when running. Ans.

When starting, $f = 1{,}597 \times \frac{160}{108} = 2{,}367$ pounds per square inch. Ans.

21.
$$\text{Mechanical Efficiency} = \frac{120}{120 + 22{\cdot}8} = \frac{120}{142{\cdot}8} \quad 0{\cdot}8403$$

$= 84{\cdot}03$ per cent. Ans. (a)

$$\text{H.P.} = 142{\cdot}8 = \frac{28 \times 20^2 \times 0{\cdot}7854 \times S}{33{,}000}$$

$$\therefore S = \frac{142{\cdot}8 \times 33{,}000}{28 \times 20^2 \times 0{\cdot}7854}$$

$= 535{\cdot}6$ feet per minute. Ans. (b)

22. Water evaporated per minute

$$\frac{120 \times 16 \times 8\tfrac{1}{2}}{60 \times 62{\cdot}5} \text{ cubic feet.}$$

Volume of water to be evaporated before water is out of sight $= 280 \times \frac{3}{4}$ cubic feet.

$$\text{Time} = \frac{\text{Total volume}}{\text{Volume evaporated per minute}}$$

$$\frac{280 \times 0{\cdot}75 \times 60 \times 62{\cdot}5}{120 \times 16 \times 8{\cdot}5} \text{ min.}$$

$= 48{\cdot}28$ minutes. Ans.

23. Combined efficiency $= 0{\cdot}68 \times 0{\cdot}175 \times 0{\cdot}87 \times 0{\cdot}61$ $= 0{\cdot}06314$ or $6{\cdot}314\%$. Ans.

24. Heat equivalent of 1 horse power hour $= 2{,}545$ B.T.U.

$$\frac{\text{Heat converted into work}}{\text{Heat supplied}} = \text{Efficiency.}$$

$$\frac{2{,}545}{14{,}000 \times W} = 0{\cdot}115$$

$$\therefore W = \frac{2{,}545}{14{,}000 \times 0{\cdot}115} = 1{\cdot}58 \text{ pounds. Ans.}$$

25. Volume $\propto$ absolute temperature.

$$\therefore \frac{V}{T} = \text{constant.}$$

$32°$ F. $= 32 + 460 = 492°$ F. absolute.

$98°$ F. $= 98 + 460 = 558°$ F. absolute.

$$\therefore \frac{30}{558} = \frac{V}{492}$$

$\therefore V = \frac{492}{558} \times 30 =$ volume of 30 cubic feet at $558°$ F. when cooled to $492°$ F.

$$\therefore \text{Pounds of air} = \frac{492 \times 30}{558 \times 12{\cdot}383} = 2{\cdot}136 \text{ pounds. Ans.}$$

26. Heat given up per lb. of steam

$= 180 - 130 + 966 - 0{\cdot}7\ (180 - 212)$

$= 1{,}038{\cdot}4$ B.T.U.

Total heat given up per hour

$= 2{,}100 \times 14{\cdot}5 \times 1{,}038{\cdot}4$

$= 31{,}619{,}280$ B.T.U. per hour.

1 H.P. hour = 2,545 B.T.U. per hour

$$\therefore \text{Equivalent H.P.} = \frac{31{,}619{,}280}{2{,}545}$$

$= 12{,}424$ H.P.

31,619,280 B.T.U. Ans.

12,424 H.P. Ans.

Pressure × Volume = Constant.

$\therefore p_1 v_1 = p_2 v_2$

$p_1 = 205 + 15 = 220$ lb. per square inch absolute.

$v_1 = 0{\cdot}35 + 0{\cdot}05 = 0{\cdot}4$ of the stroke.

$v_2 = 0{\cdot}9 + 0{\cdot}05 = 0{\cdot}95$ of the stroke.

$220 \times 0{\cdot}4 = p_2 \times 0{\cdot}95$

$$\frac{220 \times 0{\cdot}4}{0{\cdot}95} = 92{\cdot}63 \text{ lb. square inch absolute.}$$

Ans.

$= 77{\cdot}63$ lb. square inch gauge. Ans.

$$\text{Efficiency} = \frac{\text{Heat given out in steam}}{\text{Heat supplied}}$$

$$= \frac{9{\cdot}6 \times 966}{13{,}500} = 0{\cdot}687$$

$= 68{\cdot}7$ per cent. Ans.

29. $$\text{N.H.P.} = \frac{(3 \times 1{,}888 + 40^2) \sqrt[3]{216}}{700}$$

$$\frac{(5{,}664 + 1{,}600 \times 3)\ 6}{700} = 89{\cdot}7 \text{ N.H.P.} \quad \text{Ans.}$$

30. $$\frac{\text{Rise in density}}{\text{Average feed density}} = \frac{\text{Water evaporated}}{\text{Water in boilers}}$$

$$\frac{3{\cdot}5 \times 5}{\text{F.D.}}$$

$$\therefore \text{F.D.} = \frac{3{\cdot}5 \times 5 \times 90}{3{,}800}$$

$= 0{\cdot}4145$ ounce per gallon. Ans.

31. $$0{\cdot}7854 \text{ D}^2 \times 2 = \frac{6 \times 5{\cdot}5 \times 3{\cdot}667 \times 75}{2 \times 180}$$

32. Let x = clearance in inches of stroke.

At 8 inches of the stroke the volume of air is proportional to $16 - 8 + x = 8 + x$ inches.

At 13 inches of the stroke the volume of air is proportional to $16 - 13 + x = 3 + x$ inches.

$p_1 v_1 = p_2 v_2$

$\therefore (12 + 15)(8 + x) = (39 + 15)(3 + x)$

$27(8 + x) = 54(3 + x)$

$\therefore 8 + x = 2(3 + x)$

$8 + x = 6 + 2x$

$x = 2$ inches.

Clearance volume $= 0{\cdot}7854 \times 12^2 \times 2$

$= 226{\cdot}2$ cubic inches. Ans.

33. Length of voyage $\propto$ coal carried

$$\text{,,} \quad \text{,,} \quad \propto \frac{1}{\text{Cubic feet per ton}}$$

$$\text{,,} \quad \text{,,} \quad \propto \text{Calorific value.}$$

$$\frac{\text{Voyage} \times \text{Cubic feet per ton}}{\text{Calorific value}} = \text{constant}$$

$$\frac{18 \times 42}{14{,}000} = \frac{\text{Voyage} \times 45}{12{,}500}$$

$$\text{Voyage} = \frac{18 \times 42 \times 12{,}500}{14{,}000 \times 45} \qquad \text{Ans.}$$

34. Maximum port opening $= 0{\cdot}6 \times 2{\cdot}4 = 1{\cdot}44$ inches.

$\frac{1}{2}$ travel $=$ lap $+$ M.P.O.

$= 1{\cdot}44 + 1{\cdot}75 = 3{\cdot}19$ inches.

$\therefore$ Travel $= 3{\cdot}19 \times 2 = 6{\cdot}38$ inches. Ans.

The steam port deals with both the steam going into the cylinder, and the exhaust. After expansion the steam has a greater volume, and the steam port which opens fully to exhaust is made deeper than the max. port opening to steam, to accommodate the increased volume. Ans.

35. The volume at any portion of the stroke is proportional to the distance moved by the piston, plus the clearance as a proportion of the stroke.

Let stroke $= x$ inches.

Volume at cut-off is proportional to 8 + 5 per cent of x

$= 8 +$

20

Volume at end of stroke is proportional to

$x + 5$ per cent. of x

$$\frac{x}{20} \qquad \frac{21}{20}$$

$p_1 \; v_1 =$

$$\therefore (75 + 15)\left(8 + \frac{x}{20}\right) = (35 + 15)\ \tfrac{21}{20}\ x$$

$$\therefore 90\left(8 + \frac{x}{20}\right) = 50 \times \tfrac{21}{20}\ x$$

$$\therefore 8 + \frac{x}{20} = \frac{50 \times 21}{90 \times 20}\ x = \tfrac{105}{180}\ x$$

$$\tfrac{105}{180} \qquad \frac{x}{20} = 8 \qquad \frac{105\ x - 9\ x}{180}$$

$$\therefore 96\ x = 8 \times 180$$

$$\therefore x = \frac{8 \times 180}{96} = 15 \text{ inches.} \quad \text{Ans.}$$

36.

$= 200 + 15 = 215$

$v_1 = 0{\cdot}3 + 0{\cdot}065 = 0{\cdot}365$

$v_2 = 1 + 0{\cdot}065 = 1{\cdot}065$

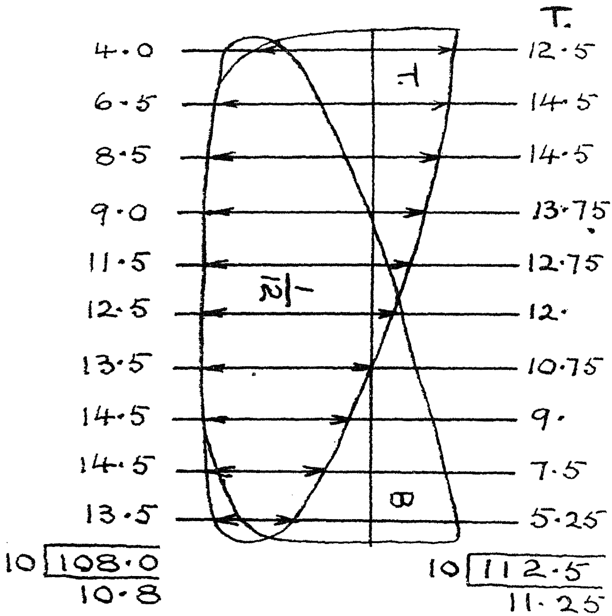
$$\frac{215 \times 0{\cdot}365}{1{\cdot}065}$$

$= 73{\cdot}69$ lb. square inch absolute.

$= 73{\cdot}69 - 15 = 58{\cdot}69$ lb. square inch gauge.

$58{\cdot}69$ lb. per sq. inch gauge. Ans.

37. M.E.P. of L.P. engine = 11 lb. per square inch.



$$\text{H.P.} = \frac{\text{P A L N}}{33{,}000}$$

$$= \frac{11 \times 80^2 \times 0{\cdot}7854 \times 51 \times 2 \times 70}{33{,}000 \times 12}$$

$= 997$ H.P.

H.P. of whole engine $= 997 \times 3 = 2{,}991$ H.P.

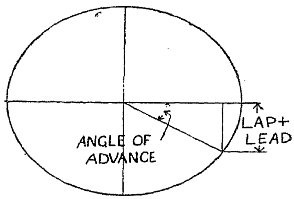
$$\text{Consumption per day} = \frac{2{,}991 \times 1{\cdot}6 \times 24}{2{,}240}$$

$= 51{\cdot}27$ tons. Ans.

38. $\frac{1}{2}$ Travel $=$ Maximum port opening $+$ Lap

$\therefore$ Lap $= 3\frac{3}{8} - 1\frac{3}{4} = 1\frac{5}{8}$ inches.

Lap $+$ Lead $= 1\frac{5}{8} + \frac{1}{4} = 1\frac{7}{8}$ inches.



$$\frac{\text{Lap} + \text{Lead}}{\frac{1}{2}\text{ Travel}} = \text{Sine of angle of advance}$$

$$\frac{1\frac{7}{8}}{3\frac{3}{8}} = 0{\cdot}5556$$

$\therefore$ Angle of advance $= 33° \ 45'$. Ans.

39. Volume remains constant, then absolute pressure varies as the absolute temperature.

$$\frac{p_1}{T_1} = \frac{p_2}{T_2} \quad \therefore \ T_2 = \frac{(460 + 63) \times (650 + 15)}{(500 + 15)}$$

$= 675{\cdot}3°$ F abs.

$t_2 = 675{\cdot}3 - 460 = 215{\cdot}3°$ F.

Heat given to air $=$ weight $\times$ specific heat $\times$ rise in temperature

$= 60 \times 0{\cdot}17 \times (215{\cdot}3 - 63)$

$= 1{,}553$ B.T.U. Ans.

40. Sine of angle of advance $= \dfrac{\text{lap} + \text{lead}}{\text{half travel}}$

Let $x =$ half travel. Sine of $31{\cdot}5° = 0{\cdot}5225$

Then lap $= 0{\cdot}5225\,x - 0{\cdot}125$

Also, Lap $+$ M.P.O. $=$ half travel

$\therefore$ $0{\cdot}5225\,x - 0{\cdot}125 + 1{\cdot}677 = x$

$0{\cdot}4775\,x = 1{\cdot}552 \quad \therefore \ x = 3{\cdot}25$ inches.

Travel of valve $= 2 \times 3{\cdot}25 = 6{\cdot}5$ inches. Ans.

41. $2\cdot54$ centimetres $= 1$ inch.

Area of card $= \dfrac{\quad}{(2\cdot54)^2}$ square inches.

Mean height $= \dfrac{6}{(2\cdot54)^2 \times 2\cdot74}$ inches.

$\therefore$ M.E.P. $= \dfrac{6 \times 360}{(2\cdot54)^2 \times 2\cdot74}$ lb. per sq. inch.

$$\frac{6 \times 360}{(2\cdot54)^2 \times 2\cdot74 \times 2\cdot2 \times (2\cdot54)^2} \text{ kilogs. per sq. cm.} = 8\cdot607 \text{ Ans.}$$

42. Volume of bottle $= \left\{\frac{11}{14} \times 1^2 \times 5\right\} + \left\{\frac{\quad}{6} \times 1^3\right\}$

$= 4\cdot4518$ cu. ft.

If temperature of air is the same after being compressed and cooled, as the atmosphere, then $p\,v =$ constant, and $4\cdot4518$ cu. ft. of air at 915 lb. per square inch abs. is equal to

$\dfrac{4\cdot4518 \times 915}{15}$ cu. ft. at 15 lb. per square inch abs.

$= 271\cdot6$ cu. feet.

Volume of free air to be pumped $= 271\cdot6 - 4\cdot4518 = 267\cdot1428$ cu. ft., because we assume bottle to contain $4\cdot4518$ cu. ft. of atmospheric air before pumping begins.

Volume of free air taken into L.P. compressor per minute

$= \frac{11}{14}\left[(9\tfrac{1}{2})^2 - (2\tfrac{1}{2})^2\right] \times \frac{1}{144} \times \frac{7}{12} \times 0\cdot9 \times 120$

$$\frac{11 \times 12 \times 7 \times 8 \times 0\cdot9 \times 120}{14 \times 144 \times 12} = 33 \text{ cu. feet.}$$

Then time to pump up bottle $= \dfrac{267\cdot1428}{33}$

$= 8\cdot095$ minutes. Ans.

43. Volume of oil in tank at 60° F. $= \dfrac{50 \times 2{,}240}{62{\cdot}5 \times 0{\cdot}9}$ cu. ft.

Increase in volume due to 20° increase in temperature

$$\frac{50 \times 2{,}240}{62{\cdot}5 \times 0{\cdot}9} \times 20 \times 0{\cdot}00034 \text{ cu. ft.}$$

Volume of oil at 80° F.

$$= \frac{50 \times 2{,}240}{62{\cdot}5 \times 0{\cdot}9} + \frac{50 \times 2{,}240}{62{\cdot}5 \times 0{\cdot}9} \times 20 \times 0{\cdot}00034$$

$$\frac{50 \times 2{,}240}{62{\cdot}5 \times 0{\cdot}9}(1 + 20 \times 0{\cdot}00034) \text{ cu. ft.}$$

$$\therefore \text{Depth of oil} = \frac{50 \times 2{,}240 \times 1{\cdot}0068}{62{\cdot}5 \times 0{\cdot}9 \times 40 \times 20}$$

$= 2{\cdot}506$ feet. Ans.

44. Increase in temperature $= 410 - 85 = 325$ Fah. degrees $= 325 \times \frac{5}{9}$ Cent. degrees.

Increase in length = Original length × increase in temp. × co-efficient of linear expansion

$= 486 \times (325 \times \frac{5}{9}) \times 0{\cdot}0000121$

$= 1{\cdot}062$ inches.

Length of pipe when heated = 40 ft. 6 ins. + 1·062 ins. = 40 feet 7·062 inches. Ans.

45. Let p_2 = terminal pressure and p_3 = pressure at 0·75 of the stroke.

$p_1 \times v_1 = p_2 \times v_2$
$(200 + 15) \times (0{\cdot}45 + 0{\cdot}065) = p_2 \times (1 + 0{\cdot}065)$

$$\frac{215 \times 0{\cdot}515}{1{\cdot}065} = 104 \text{ lb. per sq. in. abs.}$$

Terminal pressure = 104 lb. per sq. inch absolute. Ans. (*a*)

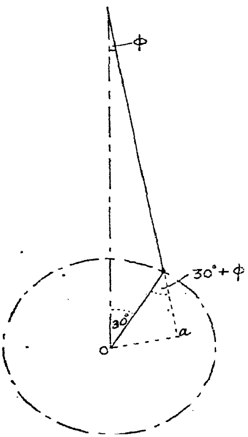
$p_1 \times v_1 = p_3 \times v_3$

$215 \times 0{\cdot}515 = p_3 \times (0{\cdot}75 + 0{\cdot}065)$

$$\therefore p_3 = \frac{215 \times 0{\cdot}515}{0{\cdot}815} = 135{\cdot}8 \text{ lb. per sq. in. abs.}$$

Pressure at 0·75 stroke = 135·8 lb. per sq. inch absolute.
= 135·8 — 15 = 120·8 lb. per sq. inch gauge. Ans. (*b*)

46.



Load on piston $= \frac{11}{14} \times 18^2 \times (180 - 70)$ lb.

1·5 7

Sin. ϕ Sin. 30°

$\therefore$ Sin. ϕ = 1·5 × 0·5 = 0·1071

$\therefore \phi = 6° \ 9'$, and $30° + \phi = 36° \ 9'$

Thrust in connecting rod =

load on piston

Cos. 6° 9′

$$\frac{\frac{11}{14} \times 18^2 \times 110}{0{\cdot}9942} = 28{,}160 \text{ lb.}$$

Twisting moment = Load on connecting rod × **O** a
= 28,160 × 1·5 Sin. 36° 9′
= 28,160 × 1·5 × 0·5899
= 24,900 ft. lb. Ans.

47. 1 litre = 1,000 cu. cms. which weighs 1,000 grams.

Heat units given to water = 6·5 × 1,000 × 50 gram calories in 14 mins.

$$\therefore \text{Heat units per second} = \frac{6{\cdot}5 \times 1{,}000 \times 50}{14 \times 60}$$

= 386·9 gram calories. Ans. (*a*)

252 gram calories $=$ 1 British thermal unit,

$$\therefore \ 386\cdot9 \text{ gram calories per sec.} = \frac{386\cdot9}{252} \times 60 \times 778 \text{ ft. lb. per min.}$$

$= 71{,}680$ ft. lb. Ans. (*b*)

48. For isothermal compression, $p_1 \, v_1 = p_2 \, v_2$

$\therefore \ (45 + 15) \times (420 - 350 + x) =$

$(14 + 15) \times (420 - 210 + x)$

$\therefore \ 60 \times (70 + x) = 29 \times (210 + x)$

$4{,}200 + 60\,x = 6{,}090 + 29\,x$

$\therefore \ x = \frac{1890}{31} = 60\cdot97$ mms.

$$\text{Clearance volume} = \frac{60\cdot97}{25\cdot4} \times \frac{11 \times 310^2}{14 \times (25\cdot4)^2}$$

$= 281$ cubic inches. Ans.

49. $$30^\circ \text{ C.} = \frac{30 \times 9}{5} + 32 = 86^\circ \text{ F.}$$

$$10^\circ \text{ C.} = \frac{-10 \times 9}{5} + 32 = 14^\circ \text{ F.}$$

Let T $=$ final temperature

Heat lost by steam $=$

Heat gained by water $+$ Heat gained by ice

$10 \times 2\cdot2\ [(650 - 490)\ 0\cdot6 + \{966 - 0\cdot7\ (490 - 212)\} + (490 - \mathrm{T})] =$
$250 \times 2\cdot2\ (\mathrm{T} - 86) + 56 \times 2\cdot2\ [(32 - 14)\ 0\cdot5 + 143 + (\mathrm{T} - 32)]$
$[96 + 771\cdot4 + 490 - \mathrm{T}) = 25\ [\mathrm{T} - 86] + 5\cdot6\ [9 + 143 + \mathrm{T} - 32]$
$\therefore \ 96 + 771\cdot4 + 490 - \mathrm{T} = 25\,\mathrm{T} - 2{,}150 + 672 + 5\cdot6\,\mathrm{T}$
$2{,}835\cdot4 = 31\cdot6\,\mathrm{T}$

$$\therefore \ \mathrm{T} \text{ (final temperature)} = \frac{2{,}835\cdot4}{31\cdot6} = 89\cdot73^\circ \text{ F.} \quad \text{Ans.}$$

50. I.P. generates 1·03 times L.P. power. I.P. generates 1·05 times H.P. power.

$$\therefore \text{ H.P. generates } \frac{1\cdot03}{1\cdot05} = 0\cdot981 \text{ times L.P. power.}$$

Ratio of powers, as L.P. : I.P. : H.P. : : 1 : 1·03 : 0·981
$1 + 1\cdot03 + 0\cdot981 = 3\cdot011$

$$\therefore \text{ L.P. generates } \frac{}{3\cdot011} \text{ of the total power,}$$

$$\text{I.P. generates } \frac{1\cdot03}{3\cdot011} \text{ of the total power,}$$

$$\text{H.P. generates } \frac{0\cdot981}{3\cdot011} \text{ of the total power.}$$

$$\text{Mean effective pressure in L.P.} = \frac{1}{3\cdot011} \times 33$$

$= 10\cdot96$ lb. per sq. inch. Ans.

$$\text{Mean effective pressure in I.P.} = 10\cdot96 \times \frac{64^2}{40^2} \times 1\cdot03$$

$= 28\cdot9$ lb. per sq. inch. Ans.

$$\text{Mean effective pressure in H.P.} = 10\cdot96 \times \frac{64^2}{24^2} \times 0\cdot981$$

$= 76\cdot47$ lb. per sq. inch. Ans.

51. Let x be the boiler density at first, then :—

$(40 \times x) + (8 \times 0) - (8 \times 14\cdot5) = 40 \times 14\cdot5$
$\therefore 40\,x = 48 \times 14\cdot5$
$x = 17\cdot4$ ozs. per gall.

Now let y be the final density of the boiler water in the second case :—

$(40 \times 17\cdot4) - (8 \times 17\cdot4) + (8 \times 0) = 40 \times y$
$32 \times 17\cdot4 = 40\,y$
$13\cdot92 = y$

$\therefore$ Final density would be 13·92 ozs. per gall. Ans.

52. Power developed in cylinder $= 100 \quad 27{\cdot}5 - 28 = 44{\cdot}5$ per cent. of the heat supplied.

$\therefore$ thermal efficiency $= 44{\cdot}5$ per cent. or $0{\cdot}445$.

Overall efficiency = mechanical efficiency $\times$ thermal efficiency.

$= 0{\cdot}74 \times 0{\cdot}445$

$= 0{\cdot}3293$

$$\text{Efficiency} = \frac{\text{heat got out}}{\text{heat supplied}}$$

$$. \; 0{\cdot}3293 = \frac{\dfrac{33{,}000 \times 60}{778}}{W \times 19{,}000} \quad \frac{2{,}545}{W \times 19{,}000}$$

W, the weight of fuel used per B.H.P. per hour

$$\frac{2{,}545}{0{\cdot}3293 \times 19{,}000}$$

$= 0{\cdot}4067$ lb. Ans.

$v_1 = 2' \, 7'' \quad 1{\cdot}5'' = 32{\cdot}5''$

Let x = distance from end of stroke when delivery valves open, then $v_2 = x + 1{\cdot}5$ ins.

Pressure at beginning of compression $= 15 - 1 = 14$ lb. per sq. in. abs.

Pressure at end of compression $= 55 + 15 = 70$ lb. per sq. in. abs.

$p_1 \, v_1 = p_2 \, v_2$

$\therefore 14 \times 32{\cdot}5 = 70 \times (x + 1{\cdot}5)$

$$x = \frac{14 \times 32{\cdot}5}{70} - 1{\cdot}5 = 5 \text{ inches.}$$

Delivery valves open 5 inches from the end of the stroke, which is $2' \, 7'' - 5'' = 2$ ft. 2 ins. from beginning of stroke. Ans.

54. Maximum port opening $= \frac{100}{156} \times 3{\cdot}214 = 2{\cdot}06$ inches.

Steam lap = half travel — M.P.O.

$= 3{\cdot}63 - 2{\cdot}06 = 1{\cdot}57$ inches. Ans.

The area of the steam port is greater than the area of steam opening because it has to allow the larger volume of expanded steam to pass through during the exhaust period.

55. Weight of fuel used in 24 hours =

$\frac{11}{14} \times (8{\cdot}5)^2 \times 6 \times 62{\cdot}5 \times 0{\cdot}92 = \text{I.H.P.} \times 0{\cdot}4 \times 24$ lb.

$$\text{I.H.P.} = \frac{11 \times (8{\cdot}5)^2 \times 6 \times 62{\cdot}5 \times 0{\cdot}92}{14 \times 0{\cdot}4 \times 24} = 2{,}040.$$ Ans. (*a*)

$$\text{Thermal efficiency} = \frac{\dfrac{33{,}000 \times 60}{778}}{0{\cdot}4 \times 18{,}500} = \frac{2{,}545}{0{\cdot}4 \times 18{,}500}$$

$= 0{\cdot}344$ or $34{\cdot}4\%$. Ans. (*b*)

56. Force applied at circumference to turn shaft
$= 0{\cdot}021 \times 92 \times 2{,}240$ lb.

Work done in turning shaft one revolution
$= 0{\cdot}021 \times 92 \times 2{,}240 \times \pi \times \frac{26}{12}$ ft. lb.

Work done per minute

$= 0{\cdot}021 \times 92 \times 2{,}240 \times \pi \times \frac{26}{12} \times 95$ ft. lb.

$$\therefore \text{Horse power} = \frac{0{\cdot}021 \times 92 \times 2{,}240 \times \pi \times 26 \times 95}{33{,}000 \times 12}$$

$= 84{\cdot}82$ H.P. Ans. (*a*)

Heat units generated per minute

$$\frac{84{\cdot}82 \times 33{,}000}{778} = 3{,}597 \text{ B.T.U. per min.}$$ Ans. (*b*)

57. Heat equivalent of I.H.P. of engine

$= 3{,}500 \times 2{,}545$ B.T.U. per hour.

Heat in fuel consumed $= 1{,}120 \times 19{,}300$ B.T.U. per hour.

$$\therefore \text{ Thermal efficiency } = \frac{3{,}500 \times 2{,}545}{1{,}120 \times 19{,}300} = 0{\cdot}4121, \text{ or } 41{\cdot}21\%. \quad \text{Ans.}$$

Heat carried away in the cooling water

$= (127 - 60) \times 47{\cdot}5 \times 2{,}240$ B.T.U. per hour.

$= 67 \times 47{\cdot}5 \times 2{,}240$ B.T.U.

$\therefore$ Per cent. of total heat carried away

$$\frac{67 \times 47{\cdot}5 \times 2{,}240 \times 100}{1{,}120 \times 19{,}300} \quad 32{\cdot}97\%. \quad \text{Ans.}$$

58. Hydrogen available for combustion

$$= \left(H - \frac{O}{8}\right) = \left(11{\cdot}15 - \frac{2{\cdot}24}{8}\right) = 10{\cdot}87\%$$

Heat from combustion of

$$\begin{aligned} C &= 0{\cdot}8443 \times 14{,}000 = 11{,}820 \text{ B.T.U.} \\ H &= 0{\cdot}1087 \times 62{,}000 = 6{,}739 \quad ,, \\ S &= 0{\cdot}0059 \times 4{,}200 = 24{\cdot}8 \quad ,, \\ & \qquad\qquad\qquad\quad \overline{18{,}583{\cdot}8} \quad ,, \end{aligned}$$

Calorific value $= 18{,}583{\cdot}8$ B.T.U. per lb. Ans.

59. Weight of the bar $= 1 \times 72 \times 0{\cdot}283 = 20{\cdot}376$ lb.

Extension of the bar $= \dfrac{72}{20}$ inches $= \dfrac{3}{10}$ foot.

Since the load is suddenly applied, then work done on the bar

$$= \text{Load} \times \text{Extension} = \frac{10 \times 2{,}240 \times 3}{10} = 6{,}720 \text{ ft. lb.}$$

$$\text{Heat equivalent of work done} = \frac{6{,}720}{778} \text{ B.T.U.}$$

Let T be the rise in temp. of the bar.
Sp. heat of steel $= 0{\cdot}116$.

$$\text{Then, } 20{\cdot}376 \times 0{\cdot}116 \times T = \frac{6{,}720}{778}$$

$$T = \frac{6{,}720}{778 \times 20{\cdot}376 \times 0{\cdot}116} = 3{\cdot}654 \text{ F}^\circ. \quad \text{Ans.}$$

60. $-5^\circ \text{ C.} = -5 \times \frac{9}{5} + 32 = 23^\circ \text{ F.}$
$195^\circ \text{ C.} = 195 \times \frac{9}{5} + 32 = 383^\circ \text{ F.}$

Heat to convert 1 lb. of ice into steam
$= (32-23)\ 0{\cdot}5 + 143 + (383-32) + 966 - 0{\cdot}7\ (383-212)$
$= 1344{\cdot}8$ B.T.U.

Heat for 9 lb. of ice $= 1{,}344{\cdot}8 \times 9 = 12{,}103{\cdot}2$ B.T.U. Ans.

61. Let x = B.H.P. at half load, and $2x$ = B.H.P. at full load.

$$\text{Fuel per B.H.P. hour at half load} = \frac{55{\cdot}77}{x} \text{ lb.}$$

$$\text{,, ,, full ,,} = \frac{55{\cdot}77}{x} \times \frac{100}{117} \text{ lb.}$$

$$\text{hour at full load} = \frac{55{\cdot}77 \times 100 \times 2x}{x \times 117} \text{ lb.}$$

$$\text{day ,, ,, ,,} = \frac{55{\cdot}77 \times 100 \times 2 \times 24}{117} \text{ lb.}$$

$$\text{,, ,, ,, ,,} = \frac{55{\cdot}77 \times 100 \times 2 \times 24}{117 \times 2{\cdot}2} \text{ kilograms}$$

$= 1{,}040$ kilograms. Ans.

62. Final temperature $= 55 + 60 = 115° F.$

Since the water equivalent of the vessel is 10 lb., then there is, in effect,

$100 + 10 = 110$ lb. of water.

Let x lb. of steam be required.

Heat lost by Steam

= Heat gained by (Water + Vessel + Ice)

$$x\{(200 \times 0{\cdot}48) + 966 + (212 - 115)\}$$
$$= 110\,(115 - 55) + 25\,(143 + 115 - 32)$$

$$1{,}159\,x = 6{,}600 + 5{,}650$$

$$x = \frac{12{,}250}{1{,}159} = 10{\cdot}57 \text{ lb.} \quad \text{Ans.}$$

63. $6 + 7{\cdot}5 = 13{\cdot}5$

$\dfrac{6}{13{\cdot}5}$ of $108 = 48$ inches. Also $\dfrac{7{\cdot}5}{13{\cdot}5}$ of $108 = 60$ inches.

The weight of the moving parts in connection with the top piston will be greater than for the bottom piston.

∴ The top piston has 48 inches travel.
,, bottom ,, 60 ,, ,,

The engine is a two cycle engine.

$$\text{B.H.P.} = \frac{97 \times 27^2 \times 0{\cdot}7854 \times 108 \times 100 \times 0{\cdot}92}{33{,}000 \times 12}$$

$$= 1{,}394. \quad \text{Ans.}$$

64. Compression pressure $= 8{\cdot}5 \times 30 \times 25{\cdot}4$ mm. of mercury.

Let the stroke volume be 100, this also represents the stroke.

Let the clearance volume be x

$$p_1\,v_1 + p_2\,v_2$$
$$381\,(100 + x) = 8{\cdot}5 \times 30 \times 25{\cdot}4\,x$$

$$38{,}100 + 381\,x = 6{,}477$$

$$x = \frac{38{,}100}{6{,}096} = 6{\cdot}25$$

Clearance $= 6{\cdot}25\%$ of stroke Ans.

65. Fraction of the feed blown out.

$$\frac{\text{Feed density}\quad 0{\cdot}11}{\text{Boiler density}\quad 0{\cdot}8} \qquad \frac{11}{80}$$

This means that if 80 lb. are fed in, 11 lb. will be blown out, and 80 — 11 = 69 lb. will be left to form steam. Or, in order to form 1 lb. of steam when blowing down, $\frac{80}{69}$ lb. of feed must be taken, $\frac{11}{69}$ lb. will be blown out, and the remainder will form steam.

The heat to form 1 lb. of steam before blowing was resorted to

$$= (375 - 130) + 966 - 0{\cdot}7\,(375 - 212)$$
$$= 245 + 851{\cdot}9 = 1096{\cdot}9 \text{ B.T.U.}$$

When blowing down, the heat to form 1 lb. of steam will be

$$\tfrac{80}{69} \times 245 + 851{\cdot}9 \text{ B.T.U.}$$

The extra heat required will be the sensible heat to $\frac{11}{69}$ lb. of water which is blown out.

$$\tfrac{11}{69} \times 245 = 39{\cdot}06 \text{ B.T.U.}$$

The daily consumption will increase in direct proportion to the extra heat required to form 1 lb. of steam.

$\therefore$ % increase in consumption

$$= \frac{39{\cdot}06}{1096{\cdot}9} \times 100 = 3{\cdot}56\%. \quad \text{Ans.}$$

66. Since the piston speed of each engine is the same, then Horse power $\propto$ Mean pressure $\times$ Area of piston.

$$\frac{\text{Horse power}}{\text{Mean press.} \times \text{Area}} = \text{constant.}$$

Let the horse power of H.P. be 1, then that of L.P. is 1·15
Let the area of H.P. be 1, then that of L.P. is $(2{\cdot}2)^2 = 4{\cdot}84$

$$\begin{array}{cc} 1 & 1{\cdot}15 \\ 67 \times 1 & \text{M.E.P.} \times 4{\cdot}84 \end{array}$$

$$\therefore \text{ M.E.P. of L.P.} = \frac{67 \times 1{\cdot}15}{4{\cdot}84} = 15{\cdot}92 \text{ lb. per sq. inch.}$$
Ans.

67. Co-efficient of linear expansion per Fah. degree
$= 0{\cdot}000018 \times \frac{5}{9} = 0{\cdot}00001$

Coefficient of cubical expansion $= 3 \times 0{\cdot}00001$ (see page 317)
Initial volume $= 23 \times 5 \times 3 = 345$ cu. ins.
Increase in volume $= 345 \times 3 \times 0{\cdot}00001 \times 300 = 3{\cdot}105$ cu. ins. Ans.

$$\text{Increase per cent.} = \frac{3{\cdot}105}{345} \times 100 = 0{\cdot}9. \quad \text{Ans.}$$

Alternatively, the dimensions, when the block is heated, become :—

$23\ (1 + 0{\cdot}00001 \times 300)$; $5\ (1 + 0{\cdot}00001 \times 300)$; $3\ (1 + 0{\cdot}00001 \times 300)$

$$\begin{aligned} \text{Final volume} &= 23 \times 5 \times 3\ (1 + 0{\cdot}003)^3 \\ &= 345\ (1 + 0{\cdot}009027027) \end{aligned}$$

$\therefore$ Increase in volume =
$345 \times 0{\cdot}009027027 = 3{\cdot}114324315$ cu. ins.

$$\text{Increase per cent.} = \frac{3{\cdot}114324315}{345} \times 100 = 0{\cdot}9027.$$

The difference between this answer and the previous one is negligible.

$$\text{B.H.P.} = \frac{600 \times 2\pi \times 5 \times 165}{33{,}000} \quad 94{\cdot}25 \quad \text{Ans. } (b)$$

$$\text{I.H.P.} = \frac{45 \times 14^2 \times 0{\cdot}7854 \times 20 \times 330}{33{,}000 \times 12} = 115{\cdot}5$$
Ans. (*a*)

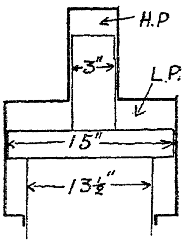
$$\text{Mechanical effy.} = \frac{\text{B.H.P.}}{\text{I.H.P.}} \quad \frac{94{\cdot}25}{115{\cdot}5} = 0{\cdot}816,$$

or 81·6%. Ans.

Steam per I.H.P. per hour

$$= \frac{1950}{115{\cdot}5} = 16{\cdot}88 \text{ lb.} \quad \text{Ans. } (d).$$

69.



Assuming that the volumetric efficiency of 0·88 is that of each stage, that the compressor is single acting and is arranged as shown by the diagram. Also, that the air is delivered at the same temperature as it entered the L.P. stage, and that atmospheric pressure is 15 lb. per sq. inch absolute, then :—

Volume of atmospheric air taken in in 30 minutes

$$\frac{(15^2 - 3^2)\ 0{\cdot}7854 \times 15 \times 0{\cdot}88}{1{,}728} \times 140 \times 30 \text{ cu. ft.}$$

$$\text{Volume} \propto \frac{}{\text{Pressure}}, \text{ when temperature is constant.}$$

∴ Volume at 1,015 lb. per sq. inch abs.

$$\frac{(15^2 - 3^2) \times 0{\cdot}7854 \times 15 \times 0{\cdot}88}{1{,}728} \times 140 \times 30 \times \frac{15}{1{,}015}$$

$= 80{\cdot}43$ cu. feet. Ans.

70. Let T be the increase in temperature.

Then final diam. $= 12{\cdot}75\ (1 + 0{\cdot}000006 \times \text{T}) = 12{\cdot}79$

$12{\cdot}75 \times 0{\cdot}000006 \times \text{T} = 12{\cdot}79 - 12{\cdot}75$

$$\frac{0{\cdot}04}{12{\cdot}75 \times 0{\cdot}000006} \quad 523\ \text{F}^\circ$$

Final temp. $= 65 + 523 = 588^\circ$ F. Ans.

71. Let v_1 be the initial volume.

$$13{\cdot}5 \times v_1^{1\cdot35} = 68 \times (550)^{1\cdot35}$$

By logs.

$$\log. 13{\cdot}5 + 1{\cdot}35 \log. v_1 = \log. 68 + 1{\cdot}35 \log. 550$$

$$\therefore \log. v_1 = \frac{\log. 68 + 1{\cdot}35 \log. 550 - \log. 13{\cdot}5}{1{\cdot}35}$$

Putting in the values, and working out

$$\log. v_1 = 3{\cdot}2605$$

$$\therefore v_1 = 1{,}822 \text{ c.c.}$$

Now this includes the clearance volume, or it is
$1{\cdot}05 \times$ Stroke volume.

$$\therefore \text{Stroke vol.} = \frac{1{,}822}{1{\cdot}05} = 1{,}736 \text{ c.c.}$$

$$d^2 \times 0{\cdot}7854 \times 28 = 1{,}736$$

$$\therefore \text{Diameter} = \sqrt{\frac{1{,}736}{0{\cdot}7854 \times 28}} = 8{\cdot}88 \text{ cms. Ans.}$$

72. $p_1 = 30 + 8 = 38$ inches of mercury.
$p_2 = 30 + 14 = 44$,, ,,
$p \times v =$ constant when the temperature does not change.

$$\therefore 38 \times 500 = 44 \times v_2$$

$$\frac{38 \times 500}{44} = 431{\cdot}8 \text{ c.c.}$$

$$\text{Change of volume} = 500 - 431{\cdot}8 = 68{\cdot}2 \text{ c.c.}$$

$$\text{Change per cent.} = \frac{68{\cdot}2}{500} \times 100 = 13{\cdot}64\%$$

Alternatively :—

$$v_2 = \frac{38}{44} \text{ of } 500 \text{ c.c.}$$

$$\text{Change of volume} = 500 - \frac{38}{44} \times 500 = \frac{6}{44} \times 500 \text{ c.c.}$$

$$\therefore \text{Change per cent.} = \frac{6}{44} \times 500 \times \frac{100}{500} = 13{\cdot}64\% \quad \text{Ans.}$$

73. 1 lb. of air at 32° F. and atmospheric pressure occupies

$$\frac{1}{0{\cdot}0807} = 12{\cdot}39 \text{ cu. ft.}$$

1 lb. of air at 70° F. and atmospheric pressure occupies

$$12{\cdot}39 \times \frac{(460 + 70)}{(460 + 32)} = 13{\cdot}34 \text{ cu. ft.}$$

$$\frac{p\ v}{\mathrm{T}} = \text{constant for any gas.}$$

Let v cu. feet be the volume of 1 lb. of air at 25 atmos. and 120° F.

$$\frac{1 \times 12{\cdot}39}{492} = \frac{25 \times v}{(460 + 120)}, \quad v = 0{\cdot}5843 \text{ cu. ft. per lb.}$$

Volume of 10 lb. weight $= 10 \times 0{\cdot}5843 = 5{\cdot}843$ cu. feet.

Probably the answer expected is that the volume of the vessel is 5·843 cu. feet, whereas this is simply the volume of the air delivered by the compressor. The vessel might have any volume greater than this, and the compressor could deliver the air to it. Possibly it is to be assumed that the vessel contains air at atmospheric pressure, and 70°F., before the 10 lb. weight is delivered. The incoming air would compress this to 25 atmospheres pressure.

Let the volume of the vessel be $5{\cdot}843 + x$ cu. feet. This volume, at atmos. pressure, is compressed to 25 atmos. pressure.

The weight does not change,

$$\frac{5{\cdot}843 + x}{13{\cdot}34} \qquad \frac{x}{0{\cdot}5843}$$

$$5{\cdot}843 + x = 22{\cdot}84\,x, \quad x = 0{\cdot}2558 \text{ cu. foot.}$$

Volume of vessel $= (5{\cdot}843 + 0{\cdot}2558) = 6{\cdot}0988$ cu. feet, but even this is not an entirely satisfactory solution.

74. $$\text{I.H.P.} = \frac{4{,}182}{0 \cdot 82} = 5{,}100$$

Power developed on top side of pistons

$$= \frac{10}{(10 + \cdot 7)} \text{ of } 5{,}100 = 3{,}000 \text{ I.H.P.}$$

Power developed on under side of pistons

$$= 5{,}100 - 3{,}000 = 2{,}100 \text{ I.H.P.}$$

For top side :—

$$\frac{3{,}000}{8} = \frac{p \times (26 \cdot 75)^2 \times 0 \cdot 7854 \times 55 \times \frac{110}{2}}{33{,}000 \times 12}$$

from which mean indicated pressure
= 87·35 lb. per sq. inch. Ans.

For under side :—

$$\frac{2{,}100}{8} = \frac{p \times \left[(26 \cdot 75)^2 - 9^2)\right] \times 0 \cdot 7854 \times 55 \times \frac{110}{2}}{33{,}000 \times 12}$$

from which mean indicated pressure
= 68·93 lb. per sq. inch. Ans.

Alternatively for second part :—

H.P. $\propto$ mean pressure $\times$ effective area

$$\frac{\text{H.P.}}{\times a} = \text{constant}$$

$$\frac{3{,}000}{87 \cdot 35 \times (26 \cdot 75)^2} = \frac{2{,}100}{p \times \left[(26 \cdot 75)^2 - 9^2\right]}$$

from which p = 68·93 lb. per sq. inch, as before.

75. Loss due to exhaust gases = 28%
,, ,, cooling water = 30%
total = 58%

If there are no other losses then I.H.P. represents $100 - 58 = 42\%$ of the available heat.

$$\text{B.H.P. represents } 0{\cdot}78 \text{ of } 42\% = 32{\cdot}76\% \qquad \text{Ans. } (a)$$

$42 - 32{\cdot}76 = 9{\cdot}24\%$, which may be regarded as friction loss. If 60% of the heat in the exhaust gases is recovered then 40% is lost.

$$40\% \text{ of } 28\% = 11{\cdot}2\%$$

Loss due to exhaust gases	=	11·2%
,, ,, cooling water	=	30%
,, ,, friction	=	9·24%
total	=	50·44%

$$50{\cdot}44\% \text{ of } 19{,}500 = 9{,}836 \text{ B.T.U.} \qquad \text{Ans. } (b)$$

Heat equivalent of 1 kilowatt hour = 3,410 B.T.U.

Let x lb. of oil be required,

$$\text{Then } 0{\cdot}07 = \frac{3{,}410}{x \times 19{,}500}$$

$$x = \frac{3{,}410}{0{\cdot}07 \times 19{,}500} = 2{\cdot}498 \text{ lb.} \qquad \text{Ans.}$$

77. Let d = diameter of piston in inches.

$$\text{Piston load} = d^2 \times \frac{\cdot\cdot}{} \times 95 \text{ lb.}$$

$$\text{Crank length} = \frac{10{\cdot}5}{} = 5{\cdot}25 \text{ inches.}$$

Maximum twisting moment, neglecting the effect of the angularity of the connecting rod

$$= d^2 \times \frac{\pi}{4} \times 95 \times 5{\cdot}25 \text{ inch lb.}$$

Radius of shaft $= \dfrac{3{\cdot}25}{\ } = 1{\cdot}625$ inches.

$$\frac{T}{\ } = \frac{q}{r}, \qquad T = \frac{J}{\ } \times q$$

$$d^2 \times \frac{\pi}{4} \times 95 \times 5{\cdot}25 = \frac{\pi \times (1{\cdot}625)^4 \times 7{,}250}{2 \times 1{\cdot}625}$$

$$d = \sqrt{\frac{\pi \times (1{\cdot}625)^4 \times 7{,}250 \times 4}{2 \times 1{\cdot}625 \times \pi \times 95 \times 5{\cdot}25}} = 11{\cdot}17 \text{ ins.} \quad \text{Ans.}$$

78. Coefficient of cubical expansion = 3 × coeff. of linear expansion.

∴ Coefficient of linear expansion

$$\frac{0{\cdot}0000315}{3} \qquad 0{\cdot}0000105$$

Increase in diameter =
Original dia. × Coefficient × Change of temperature.

$$\therefore \text{ Original dia.} = \frac{0{\cdot}021}{0{\cdot}0000105 \times 167} = 11{\cdot}98 \text{ inches.}$$

Ans.

79. The law of compression is $p\,v^{1\cdot33}$ = constant.

Before alteration, $14 \times (18 + 3{\cdot}5)^{1\cdot33} = p_2 \times (3{\cdot}5)^{1\cdot33}$

By logs, Log 14 + 1·33 log. 21·5 = Log p_2 + 1·33 log. 3·5

∴ Log p_2 = log. 14 + 1·33 (log. 21·5 — log. 3·5)

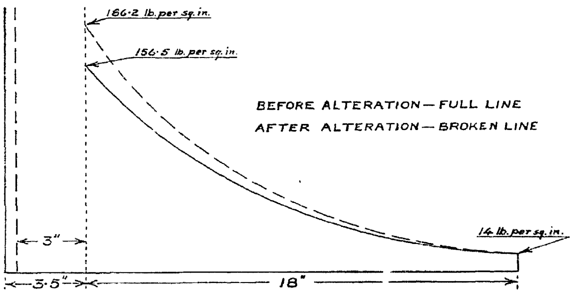
= 1·1461 + 1·33 × 0·7883 = 2·1945

p_2 = 156·5 lb. per square inch. Ans.

After alteration, $14 \times (18 + 3)^{1\cdot33} = p_2 \times (3)^{1\cdot33}$

By logs. Log. 14 + 1·33 log. 21 = log. p_2 + 1·33 log. 3

$$\text{Log. } p_2 = \log. 14 + 1{\cdot}33 (\log. 21 - \log. 3)$$
$$= 1{\cdot}1461 + 1{\cdot}33 \times 0{\cdot}8451 = 2{\cdot}2700$$
$$p_2 = 186{\cdot}2 \text{ lb. per square inch.} \quad \text{Ans.}$$



80. 13 litres = 13 × 1,000 cubic centimetres.

Weight of the oil = 13 × 1,000 × 0·9 grams.

Rise of temperature = 66 — 40 = 26 C degrees.

Heat given to the oil

$$= \text{Weight} \times \text{Rise of temp.} \times \text{Specific heat}$$
$$= 13 \times 1{,}000 \times 0{\cdot}9 \times 26 \times 0{\cdot}47 \text{ gram calories}$$
$$= \frac{13 \times 1{,}000 \times 0{\cdot}9 \times 26 \times 0{\cdot}47}{252} = 567{\cdot}3 \text{ B.T.U.} \quad \text{Ans. } (a)$$

Heat given per second

$$\frac{567{\cdot}3 \times 252}{14 \times 60} = 170{\cdot}2 \text{ gram calories.} \quad \text{Ans. } (b)$$

81. The brake horse power could be determined by fitting a rope brake upon the rim of the flywheel of the engine. A suitable weight should be hung from one end of the brake, and a spring balance at the other end. If W lb. = the weight, and S lb. = the reading of the balance, then (W — S) lb. is the effective load on the flywheel rim. If the flywheel is *d* feet diameter, and the engine runs at N revolutions per minute, then

$$\text{B.H.P.} = \frac{(W - S) \times \pi d \times N}{33{,}000}$$

Mechanical Efficiency

$$= \frac{\text{B.H.P.}}{\text{I.H.P.}} = \frac{90}{120} = 0{\cdot}75, \text{ or } 75\%. \quad \text{Ans. } (a)$$

Oil used per I.H.P. per hour $= \dfrac{44}{120}$ lb.

Indicated thermal efficiency

$$\frac{2{,}545}{\dfrac{44}{120} \times 19{,}200} = 0{\cdot}3615, \text{ or } 36{\cdot}15\%. \quad \text{Ans. } (b)$$

82. Indicated horse power $\dfrac{300}{0{\cdot}78}$

Heat equivalent of the indicated horse power

$$\frac{300}{0{\cdot}78} \times 2{,}545 \text{ B.T.U. per hour.}$$

$$\text{Piston area} = \frac{\text{Swept volume}}{\text{Stroke}} = \frac{4{\cdot}63 \times 1{,}728}{22} \text{ sq. ins.}$$

$\therefore$ heat equivalent of the work done per square inch of piston area per hour

$$\frac{\text{Heat equivalent of I.H.P.}}{\text{Piston Area}}$$

$$\frac{300 \times 2{,}545}{0{\cdot}78} \times \frac{22}{4{\cdot}63 \times 1{,}728} = 2{,}692 \text{ B.T.U.}$$

Ans.

83. The exhaust valve is open for $32° + 180° + 18° = 230°$ of the crankshaft movement.

Time for one revolution of crank $= \dfrac{60}{120} = \frac{1}{2}$ second.

$\therefore$ Valve is open for $\dfrac{230}{360}$ of $\frac{1}{2} = 0{\cdot}3194$ second. Ans. (*a*)

The valve is full open for $18° + 180° - 6°$
$= 192°$ of the crank movement.

$\therefore$ Valve is full open for $\dfrac{192}{360}$ of $\frac{1}{2} = 0{\cdot}2667$ second.
Ans. (*b*)

The valve opens during $32° - 18° = 14°$ of the crank movement.

$\therefore$ Time of opening $= \dfrac{14}{360}$ of $\frac{1}{2}$ second, and during this

time the valve lifts 2 inches, or $\frac{1}{6}$ foot.

Time of opening $\times$ Average velocity of opening $=$ Valve lift.

$\therefore$ Average velocity of opening

$$= \frac{1}{6} \div \left(\frac{14}{360} \times \frac{1}{2}\right) = 8\tfrac{4}{7} \text{ ft. per sec.} \quad \text{Ans. } (c)$$

The valve closes during $6° + 18° = 24°$ of the crank movement.

$\therefore$ Average velocity of closing

$$= \frac{1}{6} \div \left(\frac{24}{360} \times \frac{1}{2}\right) = 5 \text{ ft. per sec.} \quad \text{Ans. } (d)$$

84. Increase in consumption per H.P. $\propto$ Reduction in load.

Increase in consumption per H.P. from
Full to Half load $= 18\%$
Full to $\frac{3}{4}$ load $= 9\%$

Consumption per day at full load $= 640$ kilograms.

$\therefore$ Consumption per day at $\frac{3}{4}$ load $= 640 \times \frac{3}{4} \times \dfrac{109}{100}$

$= 523{\cdot}2$ kilograms. Ans.

85.

$$\text{Weight of sphere} = \frac{}{6} \times 1^3 \times 2{\cdot}56 \times 62{\cdot}5 \text{ lb.}$$

Heat given = Weight of sphere × Change of temp. × Sp. heat.

∴ Change of temp.

$$\frac{1{,}036}{\frac{\pi}{6} \times 1^3 \times 2{\cdot}56 \times 62{\cdot}5 \times 0{\cdot}212} = 58{\cdot}34 \text{ F}^\circ.$$

Coefficient of cubical expansion

= 3 × coeff. of linear expansion.

∴ Coeff. of linear expansion

$$\frac{0{\cdot}0000384}{3} = 0{\cdot}0000128$$

Increase in diameter

$= 12 \times 0{\cdot}0000128 \times 58{\cdot}34 \quad 0{\cdot}008982$ inch. Ans.

86. 18,500 B.T.U. are given to 1 + 23 = 24 lb. of gases, under constant volume conditions.

Heat given = Weight × Change of temp. × Specific heat.

∴ Change of temperature

$$\frac{18{,}500}{24 \times 0{\cdot}169} = 4{,}561 \text{ F degrees.}$$

Initial temperature = 80 + 460 = 540° F. absolute.

Final temperature = 540 + 4,561 = 5,101° F. absolute.

Now $\dfrac{\text{Absolute pressure}}{\text{Absolute temp.}}$ is constant when the volume is constant.

$$\therefore \frac{20}{540} = \frac{\text{Final pressure}}{5{,}101}$$

$$\text{Final pressure} = \frac{20 \times 5{,}101}{540} = 188{\cdot}9 \text{ lb. per sq. inch absolute. Ans.}$$

87. Latent heat of dry steam at 373·1° F.
$= 966 - 0{\cdot}7\ (373{\cdot}1 - 212) = 853{\cdot}23$ B.T.U. per lb.

Sensible heat above 32 °F. $= 373{\cdot}1 - 32$
$= 341{\cdot}1$ B.T.U. per lb.

$\therefore$ Total heat above 32° F.
$= 341{\cdot}1 + 853{\cdot}23 = 1194{\cdot}33$ B.T.U. per lb.

Latent heat of dry steam at 366° F.
$= 966 - 0{\cdot}7\ (366 - 212) = 858{\cdot}2$ B.T.U. per lb.

Latent heat of steam of dryness 0·95
$= 858{\cdot}2 \times 0{\cdot}95 = 815{\cdot}29$ B.T.U. per lb.

Sensible heat above 32°F. $= 366 - 32 = 334$ B.T.U. per lb.

$\therefore$ total heat above 32°F. $= 334 + 815{\cdot}29$
$= 1149{\cdot}29$ B.T.U. per lb.

Heat lost per lb. during passage along pipe
$= 1194{\cdot}33 - 1149{\cdot}29$
$= 45{\cdot}04$ B.T.U.

Heat lost by 54 lb. $= 54 \times 45{\cdot}04 = 2432{\cdot}16$ B.T.U.
Ans. (*a*)

External surface of the pipe

$$= \frac{4 \times 3{\cdot}1416 \times 60}{12} = 62{\cdot}832 \text{ sq. feet.}$$

Heat lost per minute per sq. foot

$$= \frac{2432{\cdot}16}{62{\cdot}832} = 38{\cdot}71 \text{ B.T.U.} \quad \text{Ans. } (b)$$

88. Let the stroke of the engine be l feet, then the length of the diagram is l inches.

Average height of diagram

$$= \frac{\text{Mean area}}{\text{length}} = \frac{5{\cdot}175}{l} \text{ inches.}$$

Mean effective pressure $= \dfrac{5{\cdot}175}{l} \times 72$ lb. per sq. in.

$$\text{I.H.P.} = \frac{p\,A\,l\,N}{33{,}000}$$

$$\frac{5{\cdot}175 \times 72 \times 26^2 \times 0{\cdot}7854 \times l \times 2 \times 75}{l \times 33{,}000} = 899.$$

Ans.

Volume enclosed when scavenge ports are closed

$$= (700)^2 \times 0{\cdot}7854 \times (800 + 70) \text{ cu. m.m.}$$

$$\frac{(700)^2 \times 0{\cdot}7854 \times 870}{(25{\cdot}4)^3 \times 1728} = 11{\cdot}83 \text{ cu. feet.}$$

$\therefore$ Volume of air taken in $= 11{\cdot}83 \times 0{\cdot}95 = 11{\cdot}24$ cu. ft.

Now 0·0807 lb. of air at 14·7 lb. per sq. in. and 492°F.abs. occupies 1 cu. ft.

$\therefore$,, ,, ,, 17·7 ,, ,, 492°F.abs. ,, $\dfrac{14{\cdot}7}{17{\cdot}7}$,,

and ,, ,, ,, 17·7 565°F.abs. ,, $\dfrac{14{\cdot}7 \times 565}{17{\cdot}7 \times 492}$

$= 0{\cdot}9535$ cu. ft.

$\therefore$ 0·0807 lb. of air under the cylinder conditions occupies 0·9535 cu. foot.

$\therefore$ Weight of air $= 0{\cdot}0807 \times \dfrac{11{\cdot}24}{0{\cdot}9535} = 0{\cdot}951$ lb.

Ans.

90. Radius of shaft = 6 inches.

$$\frac{\frac{1}{2}}{6} = \frac{1}{12}\text{ radian} = \frac{360}{12 \times 2\pi} = 4{\cdot}775 \text{ degrees} = 4^\circ\ 47'$$

$\therefore$ Actual angle of advance of the eccentric

$= 30^\circ + 4^\circ\ 47' = 34^\circ\ 47'$

Steam lap = Half travel — max. p.o. to steam

$= 3 - 1\frac{3}{4} = 1\frac{1}{4}$ inches.

Lap + lead = Half travel $\times$ sine of angle of advance

$= 3 \times \sin. 34^\circ\ 47' = 1{\cdot}711$ inches.

$\therefore$ lead the valve has $= 1{\cdot}711 - 1{\cdot}25 = 0{\cdot}461$ inch. Ans.

The intended angle of advance was 30°

$\therefore$ Lap + intended lead $= 3 \times \sin. 30^\circ = 1{\cdot}5$ inches.

Intended lead $= 1{\cdot}5 - 1{\cdot}25 = 0{\cdot}25$ inch.

$\therefore$ Amount to add to lap $= 0{\cdot}461 - 0{\cdot}25 = 0{\cdot}211$ inch.

Ans.

91. Let t° be the increase in temperature in Fah. degrees.
Increase in diameter of sleeve $= 10 \times t \times 0{\cdot}0000136$.
Increase in diameter of shaft $= 10 \times t \times 0{\cdot}0000067$.
$\therefore (10 \times t \times 0{\cdot}0000136) - (10 \times t \times 0{\cdot}0000067) = 0{\cdot}02$

$$10 \times t \times 0{\cdot}0000069 = 0{\cdot}02.$$

$$t = \frac{0{\cdot}02}{0{\cdot}000069} = 290 \text{ F}^\circ.$$

Final temp. $= 60 + 290 = 350^\circ$ F. Ans.

92.

Volume of 1 pound of feed water $= \dfrac{16}{1010}$ cu. ft.

Work done in pumping one pound
= Press. per sq. foot $\times$ Vol. in cu. feet

$$= 215 \times 144 \times \frac{16}{1010} = 490{\cdot}5 \text{ ft. lb.} \quad \text{Ans.}$$

$$110 \text{ gallons} = \frac{110}{6{\cdot}25} \text{ cu. feet}$$

$$\frac{110}{6{\cdot}25} \times \frac{1010}{16} \text{ lb. of water pumped per min.}$$

$\therefore$ H.P. output of pump

$$= \frac{110}{6{\cdot}25} \times \frac{1010}{16} \times \frac{490{\cdot}5}{33000} = 16{\cdot}51.$$

Mechanical efficiency of pump

$$= \frac{16{\cdot}51}{20} = 0{\cdot}8255, \text{ or } 82{\cdot}55\% \quad \text{Ans.}$$

93. Vol. of water at 60°F. $= 12^2 \times 0{\cdot}7854 \times 16$ cu. ins.
Vol. of water at 200°F. =
$12^2 \times 0{\cdot}7854 \times 16 + 12^2 \times 0{\cdot}7854 \times 16 \times 0{\cdot}00012 \times (200 - 60)$

$$= 12^2 \times 0{\cdot}7854 \times 16\,[1 + 0{\cdot}0168]$$
$$= 12^2 \times 0{\cdot}7854 \times 16 \times 1{\cdot}0168 \text{ cu. ins.}$$

$$\therefore \text{ new depth } = \frac{12^2 \times 0{\cdot}7854 \times 16 \times 1{\cdot}0168}{12^2 \times 0{\cdot}7854}$$
$$= 16{\cdot}2688 \text{ inches. Ans.}$$

94. $$\frac{\text{Lap} + }{\text{Half travel}} = \sin 29^\circ\ 40'$$

$\therefore$ half travel $= (1{\cdot}43 + 0{\cdot}21) \div \sin.\ 29^\circ\ 40' = 3{\cdot}313$ ins.
Max. port opening $= 3{\cdot}313 - 1{\cdot}43 = 1{\cdot}883$ inches.
$\therefore$ Greatest area of opening $= 1{\cdot}883 \times 26{\cdot}5 = 49{\cdot}9$ sq. ins
Ans.

95. The law of compression is $p\ v^{1\cdot4}$ = constant
$\therefore\ 15 \times 9^{1\cdot4} = 45 \times v^{1\cdot4}$
$9^{1\cdot4} = 3 \times v^{1\cdot4}$
By logs. $1{\cdot}4 \log 9 = \log 3 + 1{\cdot}4 \log v$. Divide each term by 1·4.

$$\text{Log } 9 = \frac{\log\ 3}{1{\cdot}4} + \log v.$$

$$\therefore \text{ Log } v = \log 9 - \frac{\log\ 3}{1{\cdot}4} = 0{\cdot}9542 - 0{\cdot}3408 = 0{\cdot}6134$$

$v = 4{\cdot}106$.
$\therefore$ piston has moved $9 - 4{\cdot}106 = 4{\cdot}894$ inches. Ans.

96. Let d inches be the depth of the ports in the liner.
The free passage through the ports is :—
$\frac{2}{3} \times$ circumference of the liner $\times$ depth of the ports.
The ratio of maximum piston speed to mean piston speed is $\pi : 2$.

$$\therefore \text{ maximum piston speed } = \frac{\pi}{2} \times 700 \text{ feet per minute}$$

Area of cylinder $\times$ max. piston speed $=$ Area through ports $\times$ steam speed

$$\frac{\pi}{4} \times 30^2 \times \frac{\pi}{2} \times 700 = \tfrac{2}{3} \times \pi \times 12\tfrac{1}{2} \times d \times 8500$$

$$d = \frac{\pi \times 30^2 \times \pi \times 700 \times 3}{4 \times 2 \times 2 \times \pi \times 12\frac{1}{2} \times 8500}$$
$$= 3{\cdot}49 \text{ inches. Ans.}$$

97. When 10 tons of water, at 7 ounces of solid matter per cu. ft. have been pumped in, the boiler has attained its final density. This is 55 ounces of solid matter per cu. ft.

Let the boiler hold W tons of water.

$$(W \times 75) + (10 \times 7) - (10 \times 55) = (W \times 55)$$
$$W(75 - 55) = 10(55 - 7)$$
$$20\,W = 10 \times 48$$
$$W = 24 \text{ tons. Ans.}$$

98. The first heat given to the ice raises its temperature from 22°F. to 32°F. This is Sensible Heat, and the specific heat of ice is 0·5.

Therefore, Sensible Heat given to the ice
$$= 2(32 - 22) \times 0{\cdot}5 = 10 \text{ B.T.U.}$$

When the ice has reached the temperature of 32°F., Latent Heat must be given to change its physical state from solid into liquid. Whilst any ice remains unmelted the temperature of the resulting water remains at 32°F. The Latent Heat of water is 143 B.T.U. per lb.

$\therefore$ 2 × 143 = 286 B.T.U. must be supplied to melt the ice.

The volume occupied by the water is slightly less than the volume of the ice.

When all the ice has melted, further application of heat causes the temperature of the water to rise. At first the volume of the water decreases a little until 39°F. is reached, afterwards the volume increases.

The heat supplied to raise the temperature to 300°F. is Sensible Heat, and the quantity is 2 (300 — 32) = 536 B.T.U.

The water has now reached the temperature at which it is changed into steam. Latent heat is required to effect the change, and the volume of the steam formed is much greater than that of the water. At the pressure due to 300°F., i.e., about 68 lb. per sq. inch abs., it is about 400 times as great.

The Latent Heat of formation of steam is approximately 2 [966 — 0·7 (300 — 212)] = 1808·8 B.T.U., and the steam is now dry and saturated.

When all the water has been evaporated, the temperature of the steam may be raised above 300°F. The steam becomes superheated, and the heat to superheat is Sensible Heat.

The amount required is $2\,(400 - 300) \times 0{\cdot}48 = 96$ B.T.U Steam, in practice, is always superheated at constant pressure.

The volume increases, and it is approximately $\dfrac{400 + 460}{300 + 460}$

$= 1{\cdot}13$ times the volume it occupied as saturated steam.

The total heat supplied

$$= 10 + 286 + 536 + 1808{\cdot}8 + 96$$
$$= 2736{\cdot}8 \text{ B.T.U.}$$

99. Inter-cooling is employed :—

(*a*) To prevent dangerously high temperatures of the air being reached, otherwise there will be risk of internal explosions in the compressor and difficulty with internal lubrication.

(*b*) Less work is done in compressing the air.

The ideal form of compression would be isothermal, but this is not possible in practice. By compressing the air in stages, and cooling between the stages, isothermal conditions are approximated to.

The following calculations would not be expected in answer to the question, but they are given here to assist the student.

Let a 3 stage air compressor compress air from 15 lb. per sq. inch abs. to 960 lb. per sq. inch abs. Let the initial volume be 100 units, and the initial temperature be 80°F. (540°F. abs.)

Total number of compressions $= \dfrac{960}{15} = 64$

If the ratio of compression is the same in each stage, then this will be $\sqrt[3]{64} = 4$ in each stage.

In the 1st stage the compression is from
15 lb. to $4 \times 15 = 60$ lb. per sq. inch.

In the 2nd stage the compression is from
60 to $4 \times 60 = 240$ lb. per sq. inch.

In the 3rd stage the compression is from
240 to $4 \times 240 = 960$ lb. per sq. inch.

Suppose the air was compressed adiabatically in one stage to 960 lb.

Let the law be $p \times v^{1\cdot4} =$ constant, then :—

$15 \times 100^{1\cdot4} = 960 \times v^{1\cdot4}$, from which the final volume is 5·13 units.

Now $\dfrac{p \times v}{} =$ constant

$$\therefore \frac{15 \times 144 \times 100}{540} = \frac{960 \times 144 \times 5\cdot13}{T}, \text{ from which } T = 1780^\circ\text{F. abs.,}$$

or 1320°F., a very high temperature.

If the air was compressed isothermally in one stage, the law is $p \times v =$ constant. $\therefore 15 \times 100 = 960 \times v$.

The final volume would be 1·5625 units, and the final temperature 80°F.

Now imagine the air is compressed adiabatically from 15 lb. per sq. inch to 60 lb. per sq. inch.

$15 \times 100^{1\cdot4} = 60 \times v^{1\cdot4}$, from which $v = 37\cdot2$ units.

Also, $\dfrac{p \times v}{T} =$ constant.

$$\frac{15 \times 144 \times 100}{540} \quad \frac{60 \times 144 \times 37\cdot2}{} \quad \text{From which } T = 805^\circ\text{F. abs.,}$$

or 345°F.

If the air is now cooled at constant pressure (60 lb. per sq. inch) to 80°F., it will occupy the same volume it would have had if compressed isothermally.

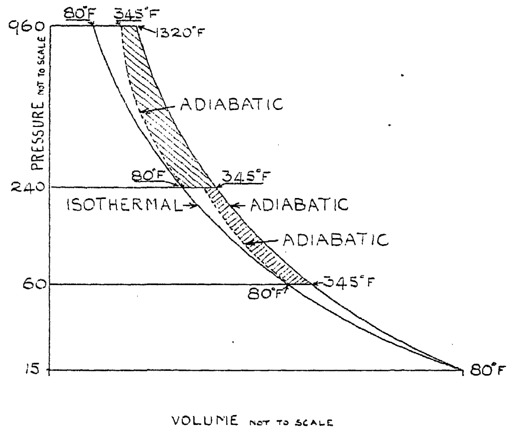
$$15 \times 100 = 60 \times v, \quad v = 25 \text{ units.}$$

Now imagine these 25 units of volume at 60 lb. per sq. inch and 80°F. are compressed adiabatically to 240 lb. per sq. inch. Calculating as before, the volume would be 9·26 units, and the temperature 345°F. If the air is cooled at constant pressure to 80°F., its volume will become the same as if the compression had been isothermal.

$$60 \quad 25 = 240 \times \qquad\qquad 6{\cdot}25 \text{ units.}$$

These 6·25 units of volume at 240 lb. per sq. inch and 80°F. can now be compressed adiabatically to 960 lb. per sq. inch, when the volume would be 2·32 units and the temperature 345°F. Cooling this air to 80°F. would result in its volume becoming 1·5625 units.

All these results could be plotted to scale, but would not give a satisfactory diagram. The values are shown on the diagram below, but they are not drawn to scale, some of the values being considerably exaggerated.



SHADED AS SHOW WORK SAVED BY INTERCOOLING

100. In the first stage nozzles of the impulse turbine it would be possible to expand the steam to such a pressure that all superheat is expended in generating velocity, and superheated steam would not come in contact with the

moving blades. Also, since there is no expansion of the steam as it passes through the moving blades, tip clearance of the blades need not be kept to a minimum.

If reaction blading was employed, the first rings of fixed and moving blades would be exposed to high temperature steam. Also, since there is fall in pressure between entrance and exit of both fixed and moving blades, tip clearance must be kept as small as possible in order to limit loss due to leakage of steam.

101. Since the weights are given in the metric system, it is possible that the temperatures are intended to be taken on the centigrade scale.

Let T°C. be the temperature of the gases, and the temperature of the iron before being placed in the water.

Since no water is evaporated its highest temperature will be 100°C. and its initial temperature $\frac{100}{2} = 50$°C.

Heat lost by iron = Heat gained by water.

$$300\,(T - 100) \times 0{\cdot}13 = 0{\cdot}16 \times 1000\,(100 - 50)$$

$$T - 100 = \frac{0{\cdot}16 \times 1000 \times 50}{300 \times 0{\cdot}13} \qquad 205$$

$$= 205 + 100 = 305°\text{C.} \quad \text{Ans.}$$

If, however, the temperatures are taken on the Fahrenheit scale, the highest possible temperature will be 212°F., and the initial temperature of the water $\frac{212}{2} = 106$°F. Then :—

$$300\,(T - 212) \times 0{\cdot}13 = 0{\cdot}16 \times 1000\,(212 - 106)$$

$$T - 212 = \frac{0{\cdot}16 \times 1000 \times 106}{300 \times 0{\cdot}13} = 435$$

$$\therefore T = 435 + 212 = 647°\text{F.} \quad \text{Ans.}$$

647°F is a higher temperature than 305°C.

SOLUTIONS TO SECOND-CLASS EXAMINATION QUESTIONS.

NAVAL ARCHITECTURE

1. Pressure on thrust $\propto$ H.P.

Pressure on thrust $\propto \dfrac{1}{\text{Speed}}$

$$\frac{P \times S}{\text{H.P.}} = \text{constant.}$$

$52 \times 14{\cdot}5 \qquad P \times 12$

$1{,}560 \qquad 1{,}680$

$$\therefore\ P = \frac{52 \times 14{\cdot}5 \times 1{,}680}{12 \times 1{,}560} = 67{\cdot}67 \text{ pounds per sq. in.}$$

Ans.

Consumption $\propto$ speed^3

$$\therefore\ \frac{\text{Consumption}}{\text{Speed}^3} = \text{constant.}$$

$30 \qquad \text{New consumption}$

$(11{\cdot}5)^3 \qquad 9^3$

$$\therefore\ \text{New consumption} = \frac{30 \times 9^3}{(11{\cdot}5)^3}$$

$$= 14{\cdot}38 \text{ tons per day.}$$

$\therefore$ Saving per day $= 30 - 14{\cdot}38 = 15{\cdot}62$ tons.

Consumption for 1,900 miles at 11·5 knots $= \dfrac{1{,}900 \times 30}{11{\cdot}5 \times 24}$

$= 206{\cdot}5$ tons.

Consumption for 1,900 miles at 9 knots $= \dfrac{1{,}900 \times 14{\cdot}38}{9 \times 24}$

$= 126{\cdot}5$ tons.

$\therefore$ Saving on voyage $= 206{\cdot}5 - 126{\cdot}5 = 80$ tons.

15·62 tons per day. Ans.

80 tons on voyage. Ans.

3. Let x tons be placed in forward hold.

Then $530 - x$ tons are placed in after hold.

Taking moments about C.G. of ship :—

$94\,x = (150 - 94)\ (530 - x)$

$94\,x = 56 \times 530 - 56\,x$

$150\,x = 56 \times 530$

$$x = \frac{56 \times 530}{150}$$

$= 197{\cdot}9$ tons in forward hold. Ans.

332·1 tons in after hold. Ans.

Increased draught $= \dfrac{430 \times 35}{14{,}020} = 1{\cdot}073$ ft. $= 1$ ft. $0{\cdot}876$ in.

Orig. mean draught $= \dfrac{21 + 22{\cdot}25}{2} = 21{\cdot}625$ ft.
$= 21$ ft. $7{\cdot}5$ ins.

New draught $= 21$ feet $7{\cdot}5$ inches $+ 1$ foot $0{\cdot}876$ inch $= 22$ feet $8\frac{3}{8}$ inches. Ans.

Mean draught on arrival $= \dfrac{21{\cdot}5 + 23{\cdot}75}{} = 22{\cdot}625$ feet.

$$\frac{\text{Difference in draught} \times 12{,}525}{35} = 850$$

$$\therefore \text{Difference} = \frac{850 \times 35}{21{,}525} = 2{\cdot}376 \text{ feet.}$$

$$\therefore \text{Draught} = 22{\cdot}625 + 2{\cdot}376 \text{ feet.}$$

$\doteq$ 25 feet. Ans.

6.
$$\text{Draught} \times 12{,}000 \times \frac{1{,}018}{16 \times 2{,}240} = 9{,}000$$

$$\therefore \text{Draught} = \frac{9{,}000 \times 16 \times 2{,}240}{12{,}000 \times 1{,}018}$$

$$\text{Similarly draught in second case} = \frac{9{,}000 \times 16 \times 2{,}240}{12{,}000 \times 1{,}026}$$

Difference in draught

$$= \frac{9{,}000 \times 16 \times 2{,}240}{12{,}000}\left(\frac{1}{1{,}018} - \frac{1}{1{,}026}\right)$$

$$= \frac{9{,}000 \times 16 \times 2{,}240}{12{,}000}\left(\frac{1{,}026 - 1{,}018}{1{,}018 \times 1{,}026}\right)$$

$$= \frac{9{,}000 \times 16 \times 2{,}240 \times 8}{12{,}000 \times 1{,}018 \times 1{,}026}$$

$= 0{\cdot}206$ foot $= 2{\cdot}472$ inches. Ans.

7.
$$\text{Mean draught before coaling} = \frac{25{\cdot}7 + 26{\cdot}8}{2} = 26{\cdot}25 \text{ feet.}$$

$$\text{Mean draught after coaling} = \frac{26{\cdot}9 + 28{\cdot}4}{2} = 27{\cdot}65 \text{ feet.}$$

Difference in draught $= 27{\cdot}65 - 26{\cdot}25 = 1{\cdot}4$ feet $= 16{\cdot}8$ inches.

$$\therefore \text{Tons per inch immersion} = \frac{324}{16{\cdot}8} = 19{\cdot}28 \text{ tons. Ans. } (a)$$

Area of water plane $\times$ change in draught $= 324 \times 35$

$$\therefore \text{Area} = \frac{324 \times 35}{1{\cdot}4} = 8{,}100 \text{ square feet. Ans. } (b)$$

8. Increase of draught $= 4 \times \frac{48}{64} = 3$ inches.

$\therefore$ Draught $= 20{\cdot}5 + 3 = 23{\cdot}5$ inches. Ans.

$$P \qquad = \text{constant.}$$

$$\therefore \frac{29}{69^2} \qquad 25$$

$$\therefore N = \frac{\times 69^2}{29} = 64{\cdot}06 \text{ revolutions per minute.}$$

Ans. (

$$\text{I.H.P.} \propto P \times N \qquad \frac{\text{I.H.P.}}{P \times N} = \text{constant.}$$

$$\frac{1{,}200}{69 \times 29} \qquad \frac{\text{I.H.P.}}{64{\cdot}06 \times 25}$$

$$\therefore \text{I.H.P.} = \frac{64{\cdot}06 \times 25 \times 1{,}200}{\times 29}$$

$= 960$ I.H.P. Ans. (*b*)

10. If H.P. is constant, then revolutions $\times$ pressure $=$ constant.

$$\therefore 62 \times 28 = 58 \times P$$

$$\therefore P = \frac{62 \times 28}{58} = 29{\cdot}93 \text{ pounds per square inch.}$$

Increased pressure $= 29{\cdot}93 - 28 = 1{\cdot}93$ pounds per square inch.

$$\frac{1{\cdot}93}{28} \times 100 = 6{\cdot}892 \text{ per cent. Ans.}$$

11. Vol. of bunker $= 175 \times 44$ cubic ft. $= 7{,}700$ cubic feet.

Actual vol. of coal $= \dfrac{175 \times 2{,}240}{80}$ cubic ft. $= 4{,}900$ cubic feet.

$\therefore$ Vol. to be occupied by water $= 7{,}700 - 4{,}900 = 2{,}800$ cubic feet.

$\therefore$ Wt. of sea water $= \frac{2800}{35} = 80$ tons. Ans.

12. Tons displaced $= 40 \times 4$

$\therefore$ Volume $= 40 \times 35 \times 4 = 5{,}600$ cubic feet.

Area $\times \frac{1}{12} = 40 \times 35$

$\therefore$ Area $= 40 \times 35 \times 12$

$= 16{,}800$ square feet.

(*a*) 5,600 cubic feet. (*b*) 16,800 square feet. Ans.

13. $P \propto$ H.P. and $\propto \dfrac{1}{S}$

$$\therefore \frac{P \times S}{\text{H.P.}} = \text{constant.}$$

$$\frac{48 \times 12}{12{,}600} \qquad \frac{P \times 11}{12{,}450} \qquad P = \frac{12{,}450 \times 48 \times 12}{12{,}600 \times 11}$$

$= 51{\cdot}73$ lb. per square inch. Ans.

14. Density $= 1{\cdot}015 = 1{,}015$ ozs. per cubic foot.

$$\text{Area} \times \frac{1{\cdot}75}{12} \times \frac{1{,}015}{16} = 80 \times 2{,}240$$

$$\therefore \text{Area} = \frac{80 \times 2{,}240 \times 12 \times 16}{1{\cdot}75 \times 1{,}015}$$

$= 19{,}370$ square feet. Ans.

15. Consumption for given time $\propto$ (speed)^3

$$\therefore \frac{C}{S^3} = \text{constant.}$$

$$\therefore \frac{C_1}{S_1^3} = \frac{C_2}{S_2^3} \qquad \therefore C_2 = \frac{C_1\ S_2^3}{S_1^3}$$

Daily consumption at 11 knots $= 35 \times \dfrac{}{10^3}$

$= 46{\cdot}585$ tons.

At 9 knots $= 35 \times \dfrac{9^3}{10^3}$

$= 25{\cdot}515$ tons.

Under conditions given, daily consumption

$$\frac{46{\cdot}585}{2} \qquad \frac{25{\cdot}515}{2}$$

$= 36{\cdot}05$ tons. Ans.

16. Total pressure $= H\ A\ w$

$$16 \qquad \frac{64 \times 40 \times 16}{2{,}240} = 146{\cdot}285 \text{ tons.} \quad \text{Ans.}$$

The centre of pressure is at one-third of the depth of bulkhead from the bottom, i.e., at 5 feet 4 inches from the bottom. Ans.

17. Draught is inversely proportional to the density of water.

$\therefore$ Draught $\times$ Density $=$ Constant.

$$\therefore \text{Draught in fresh water} = \frac{6 \times 1{\cdot}024}{1} = 6{\cdot}144 \text{ feet.}$$

$= 6$ feet $1{\cdot}7$ inches. Ans. (*a*)

Weight of barge $=$ Weight of water displaced.

$$\frac{60 \times 30 \times 6{\cdot}144 \times 62{\cdot}5}{2{,}240}$$

18. Let V = Velocity of ship in feet per minute.
Frictional resistance varies as V^2

$$\therefore \text{ Resistance per square foot } = \frac{0\cdot3 \times V^2}{600^2} \text{ pounds.}$$

$$\text{Total resistance of hull } = \frac{18{,}000 \times 0\cdot3 \times V^2}{600^2} \text{ pounds.}$$

$$\frac{\text{Resistance in pounds} \times \text{Velocity in feet per minute}}{33{,}000} = \text{H.P.}$$

$$\frac{18{,}000 \times 0\cdot3 \times V^2 \times V}{600^2 \times 33{,}000} = 2{,}000$$

$$\therefore V^3 = \frac{2{,}000 \times 600^2 \times 33{,}000}{18{,}000 \times 0\cdot3}$$

$$\therefore V = \sqrt[3]{\frac{2{,}000 \quad {,}000}{18{,}000 \times 0\cdot3}} \text{ feet per minute.}$$

$$\therefore \text{ Speed in knots } = \tfrac{60}{6080} \sqrt[3]{\frac{2{,}000 \times 600^2 \times 33{,}000}{18{,}000 \times 0\cdot3}}$$

$= 16\cdot18$ knots. Ans.

19. At 15 knots, ship would take $\frac{900}{15} = 60$ hours,
then coal consumption would be $100 \times \frac{60}{24} = 250$ tons.

Consumption varies as the $(\text{speed})^2$ per voyage,

$$\frac{}{S_1^2} \qquad \frac{}{S_2^2} \qquad \frac{250}{15^2} \qquad \frac{150}{S_2^2}$$

$$\therefore \text{ Reduced speed, } S_2, = \sqrt{\frac{15^2 \times 150}{}}$$

$= 15 \times \sqrt{\frac{15}{25}}$ knots.

$$\text{Time to do 900 miles } = \frac{900}{15 \times} = 77\cdot47 \text{ hours.}$$

Ans.

20. $$18 \text{ knots} = \frac{18 \times 6{,}080}{60} = 1{,}824 \text{ feet per minute.}$$

Force to drive ship of 20,000 sq. ft. of wetted surface at 1,824 ft. per min.

$$\frac{0{\cdot}35 \times 20{,}000 \times (1{,}824)^{2\cdot1}}{(600)^{2\cdot1}} \text{ lb.}$$

$$\text{Work done} = \frac{0{\cdot}35 \times 20{,}000 \times (1{,}824)^{2\cdot1}}{(600)^{2\cdot1}} \times 1{,}824 \text{ ft. lb. per min.}$$

$$= \frac{0{\cdot}35 \times 20{,}000 \times (1{,}824)^{3\cdot1}}{(600)^{2\cdot1}} \text{ ft. lb. per min.}$$

$$\therefore \text{Horse power} = \frac{0{\cdot}35 \times 20{,}000 \times (1{,}824)^{3\cdot1}}{33{,}000 \times (600)^{2\cdot1}} = 3{,}995. \text{ Ans.}$$

21. 21,000 sq. ft. of wetted surface drawn through the water at V ft. per min. requires a force of

$$0{\cdot}54 \times 21{,}000 \times \frac{}{(800)^{2\cdot2}} \text{ lb.}$$

Work done = force × distance

$$\therefore 2{,}634 \times 33{,}000 = 0{\cdot}54 \times 21{,}000 \times \frac{}{(800)^{2\cdot2}} \times V \text{ ft. lb. per min.}$$

$$V^{2\cdot2} \times V = V^{3\cdot2}$$

$$\therefore V = \sqrt[3\cdot2]{\frac{2{,}634 \times 33{,}000 \times (800)^{2\cdot2}}{0{\cdot}54 \times 21{,}000}}$$

$$= \sqrt[3\cdot2]{\frac{2{,}634 \times 33{,}000 \times (800)^{2\cdot2}}{0{\cdot}54 \times 21{,}000}} \times \tfrac{60}{6080} \text{ knots.}$$

$= 16$ knots. Ans.

22. Speed is reduced to 89 per cent. of the previous speed.
$\therefore$ Consumption is reduced to $(0{\cdot}89)^3 = 0{\cdot}705 = 70{\cdot}5$ per cent. of the previous consumption per day.
Then decrease in daily consumption $= 100 - 70{\cdot}5 = 29{\cdot}5$ per cent. Ans. (*a*)

Consumption is reduced to $(0{\cdot}89)^2 = 0{\cdot}792 = 79{\cdot}2$ per cent. of the previous consumption per voyage.

Then decrease in consumption for the voyage $= 100 - 79{\cdot}2$ $= 20{\cdot}8$ per cent. Ans. (*b*)

23. Weight of barge = Weight of sea water displaced.

$$\frac{58{\cdot}5 \times 15{\cdot}75 \times 3{\cdot}25 \times 1{,}016}{2{,}240 \times 16} \quad 84{\cdot}9 \text{ tons. Ans. } (a)$$

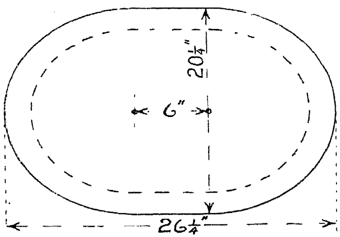
When loaded with coal, draught $= 3{\cdot}25 \times \dfrac{(84{\cdot}9 + 68{\cdot}5)}{84{\cdot}9}$

$= 5{\cdot}873$ ft.

$\therefore$ Increase in draught $= 5{\cdot}873 - 3{\cdot}25 = 2{\cdot}623$ feet. Ans. (*b*)

Draught in fresh water would be $5{\cdot}873 \times \frac{1016}{1000} = 5{\cdot}965$ feet. Ans. (*c*)

24.



Area of door exposed to water pressure

$= 0{\cdot}7854 \times (20\tfrac{1}{4})^2 + 6 \times 20\tfrac{1}{4}$

$\doteq 443{\cdot}7$ sq. inches.

$= \dfrac{443{\cdot}7}{144}$ sq. feet.

Load on door $= H\ A\ w$

Load $= \dfrac{24 \times 443{\cdot}7 \times 64}{144}$

Effective area of studs $= 16 \times 0{\cdot}304$ sq. inches.

$$\text{Stress in studs} = \frac{24 \times 443{\cdot}7 \times 64}{144 \times 16 \times 0{\cdot}304}$$

$= 973$ lb. per sq. inch. Ans.

25. Displacement $= 8{,}450 \times 35$ cu. feet.

$$= l \times b \times d \times 0{\cdot}78$$

$$b = 0{\cdot}14\ l \quad \therefore\ l = \frac{b}{0{\cdot}14}$$

$$= 2{\cdot}12\ d \quad \therefore\ d = \frac{\quad}{2{\cdot}12}$$

$$8{,}450 \times 35 \quad \frac{\quad}{0{\cdot}14} \times \frac{\quad}{2{\cdot}12} \times b \times 0{\cdot}78$$

$$b = \qquad\qquad \overline{\times\ 2{\cdot}12}$$

Let the reduced speed be x knots.

Theoretical consumption per day $= 170 \times \left(\frac{x}{17}\right)^3$ tons.

Actual ,, ,, ,, $= 170 \times \left(\frac{x}{17}\right)^3 \times 1{\cdot}1$

and this is 128·7

$$\frac{128{\cdot}7 \times 17^3}{170 \times 1{\cdot}1}$$

$$\sqrt[3]{\overline{128{\cdot}7 \times 17^2}}$$

Reduced speed is 15·01 knots. Ans.

Consumption for voyage at 17 knots

$$\frac{3{,}000}{17 \times 24} \times 170 \text{ tons.}$$

Consumption for voyage at 15·01 knots

$$\frac{3{,}000}{15{\cdot}01 \times 24} \times 128{\cdot}7 \text{ tons.}$$

$$\text{Saving} = \left\{\frac{3{,}000}{17 \times 24} \times 170\right\} \quad \left\{\frac{3{,}000}{15{\cdot}01 \times 24} \times 128{\cdot}7\right\}$$

$$\frac{3{,}000}{24}\left\{\frac{170}{17} - \frac{128{\cdot}7}{15{\cdot}01}\right\}$$

$$\frac{3{,}000}{24}\left\{10 - 8{\cdot}575\right\} = \frac{3{,}000}{24} \times 1{\cdot}425$$

178·1 tons. Ans.

27. 150 Tons of oil are moved through $200 + 150 = 350$ feet.

Moment to cause shift of C.G. $= 150 \times 350$ ft. tons.

$\therefore$ Displacement of ship $\times$ shift of C.G. $= 150 \times 350$

$$\text{Shift of C.G.} = \frac{150 \times 350}{12{,}000} = 4\tfrac{3}{8} \text{ feet, or 4 ft. 4·5 inches.}$$

C.G. moves aft through $4\frac{3}{8}$ feet. Ans.

C.G. is now $150 - 4\frac{3}{8} = 145\frac{5}{8}$ feet from C.G. of after tank.

When 200 tons have been burnt the ship's displacement is $12{,}000 - 200 = 11{,}800$ tons.

$\therefore 11{,}800 \times \text{Shift of C.G.} = 200 \times 145\frac{5}{8}$

$$\text{Shift of C.G.} = \frac{200 \times 1{,}165}{11{,}800 \times 8} = 2{\cdot}468 \text{ feet.}$$

C.G. moves forward through 2·468 feet. Ans.

Final position of C.G. is $4{\cdot}375 - 2{\cdot}468 = 1{\cdot}907$ feet aft of its original position.

28.

18 feet 9 inches
21 feet 4 inches

40 feet 1 inch

$$\frac{40 \text{ ft. } 1 \text{ inch}}{2} = 20 \text{ ft. } \tfrac{1}{2} \text{ in.}$$ (mean draught before bunkering).

21 feet 7 inches
23 feet 4 inches

44 feet 11 inches

$$\frac{44 \text{ ft. } 11 \text{ inches}}{2} = 22 \text{ ft. } 5\tfrac{1}{2} \text{ ins.}$$ (mean draught after bunkering).

22 feet $5\frac{1}{2}$ inches
20 feet $\frac{1}{2}$ inch

2 feet 5 inches = change of draught.

Increase in volume displaced

$$\frac{650 \times 2{,}240 \times 16}{1{,}026} \text{ cubic feet.}$$

Let L = length of the ship in feet, then $\frac{L}{7}$ = beam.

Water plane area

$$= L \times \frac{L}{7} \times 0{\cdot}7 = \frac{L^2}{10} \text{ sq. feet.}$$

Water plane area × Change of Draught = Increase in vol. displaced

$$\frac{L^2}{10} \times 2\tfrac{5}{12} = \frac{650 \times 2{,}240 \times 16}{1{,}026}$$

$$\sqrt{\frac{650 \times 2{,}240 \times 16 \times 10 \times 12}{1{,}026 \times 29}} \quad 6{\cdot}5 \text{ ft.}$$ Ans.

29. Weight of water pumped in

$$\frac{10 \times 20 \times 5 \times 1{,}010}{2{,}240 \times 16} = 28{\cdot}18 \text{ tons.}$$

Displacement of the barge

$$= \frac{120 \times 20 \times 8 \times 1{,}026}{2{,}240 \times 16} = 549{\cdot}6 \text{ tons.}$$

Centre of gravity of water pumped in is at $\frac{120}{2} - \frac{10}{2}$ $= 55$ feet forward of mid-length of barge.

Final displacement $= 549{\cdot}6 + 28{\cdot}18 = 577{\cdot}78$ tons.

$577{\cdot}78 \times$ Shift of C.G. $= 28{\cdot}18 \times 55$

$$\text{Shift of C.G.} \quad \frac{28{\cdot}18 \times 55}{577{\cdot}78} = 2{\cdot}682 \text{ feet. Ans.}$$

30. Volume of water in tank $= 3^2 \times 2 = 18$ cubic feet.

Volume of wood immersed $= 2^2 \times 2 \times 0{\cdot}8 = 6{\cdot}4$ cubic feet.

Total volume of water and immersed wood $= 18 + 6{\cdot}4$ $= 24{\cdot}4$ cubic feet.

$$\text{New depth of water} = \frac{24{\cdot}4}{3 \times 3} = 2{\cdot}711 \text{ feet.}$$

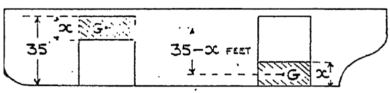
Assuming fresh water in tank :—

Load on bottom $= 2{\cdot}711 \times 9 \times 62{\cdot}5 = 1{,}525$ lb. Ans.

$$\text{Load on each side} = \frac{2{\cdot}711}{2} \times 3 \times 2{\cdot}711 \times 62{\cdot}5 = 689{\cdot}3 \text{ lb.}$$

Load on 4 sides $= 689{\cdot}3 \times 4 = 2{,}757$ lb. Ans.

31.



$$\text{Tons per foot depth of bunker} = \frac{550}{35} = \frac{110}{7} \text{ tons.}$$

Let x feet depth of coal be shifted from forward bunker to after bunker.

$$\text{Weight of coal shifted} = \frac{110}{7} \times x \text{ tons.}$$

The centre of gravity of the coal shifted is lowered by $35 - x$ feet.

Weight of coal $\times$ distance its C.G. is lowered
$= $ Displ. of ship $\times$ shift of C.G. of ship.

$$\therefore \frac{110}{7} x \times (35 - x) = 7{,}000 \times \tfrac{1}{2}$$

$$x \times (35 - x) = \frac{7{,}000 \times 1 \times 7}{2 \times 110} = \frac{2450}{11}$$

$$35\,x - x^2 = \frac{2450}{11}$$

or $x^2 - 35\,x = -\dfrac{2450}{11}$. Solving the quadratic,

$$x = 8{\cdot}36 \text{ feet, or } 26{\cdot}64 \text{ feet.}$$

$$\left.\begin{array}{rl}\text{Weight of coal shifted} & = 8{\cdot}36 \times \tfrac{110}{7} = 131{\cdot}4 \text{ tons} \\ \text{or} & = 26{\cdot}64 \times \tfrac{110}{7} = 418{\cdot}6 \text{ tons}\end{array}\right\}\text{Ans.}$$

Both these answers are possible, but less work would be done in shifting the smaller weight of coal.

SOLUTIONS TO SECOND-CLASS EXAMINATION QUESTIONS.

ELECTRICITY.

1. Specific resistance $= \dfrac{R\,a}{l} = \dfrac{1{\cdot}65 \times 0{\cdot}004}{200}$

$= 0{\cdot}000033$ ohm per cm. cube. Ans.

2. The current divides itself in inverse proportion to the resistances, therefore the current passing through the shunt will be 99 times that passing through the ammeter.

Current through shunt $=$ 99 per cent. of total current.
$= 0{\cdot}99 \times 165 = 163{\cdot}35$ amps. Ans.

Current through ammeter $=$ 1 per cent. of total current
$= 0{\cdot}01 \times 165 = 1{\cdot}65$ amps. Ans.

Specific resistance $= \dfrac{[illegible]}{l}$, $\therefore \dfrac{[illegible]}{l_1} =$

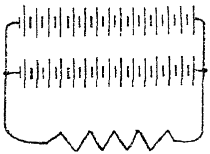
$\therefore R_1$ (the resistance of the conductor)

$$= \frac{1 \times 0{\cdot}048^2 \times \frac{11}{14} \times 500 \times 36}{0{\cdot}125^2 \times \frac{11}{14} \times 74{\cdot}1 \times 36}$$

$$\frac{0{\cdot}048^2 \times 500}{0{\cdot}125^2 \times 74{\cdot}1} \text{ ohms.}$$

$$E = I\,R = \frac{200 \times 0{\cdot}048^2 \times 500}{0{\cdot}125^2 \times 74{\cdot}1} = 199 \text{ volts.} \quad \text{Ans.}$$

4.



$r = 4$ ohms

Total E.M.F. $= 100 \times 1{\cdot}9 = 190$ volts.
Resistance of each group of cells
$= 100 \times 0{\cdot}1 = 10$ ohms.

Equivalent resistance of the two groups in parallel :—

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{10} = \frac{1}{5} \quad \therefore \quad = 5 \text{ ohms.}$$

Total resistance of the circuit $= 5 + 4 = 9$ ohms.

$$\text{Total current flowing} = \frac{E}{R} = \frac{190}{9} \text{ ampéres.}$$

Power output $=$ watts in external circuit $= I^2\,R$

$$= \frac{190^2}{} \times 4$$

$\therefore$ Horse power output

$$\frac{\text{Watts}}{746} \qquad \frac{190^2 \times 4}{9^2 \times 746} = 2{\cdot}39. \quad \text{Ans.}$$

5.

$$\text{Current} = \frac{E}{R} = \frac{200}{10} = 20 \text{ ampéres.}$$

Output in watts $= I^2\,R = 20^2 \times 10 = 4{,}000$ watts.

$$\therefore \text{H.P. output} = \frac{4{,}000}{746} = 5{\cdot}36 \text{ H.P.} \quad \text{Ans. } (a)$$

Watts lost internally $= I^2\,R = 20^2 \times 0{\cdot}5 = 200$ watts.

$\therefore$ Total power $= 200 + 4{,}000 = 4{,}200$ watts.

Input (horse power to drive dynamo)

$$\frac{4{,}200}{746} = 5{\cdot}63 \text{ H.P.} \quad \text{Ans. } (b)$$

6. $$\frac{R_0\,[1 + a\,t_1]}{R_0\,[1 + a\,t_2]} \qquad R_{\cdot} \qquad \frac{0{\cdot}00428 \times 30}{0{\cdot}00428 \times 70}$$

$$= \frac{1{\cdot}1284 \times R_2}{1{\cdot}2996}$$

$E = I\,R$, and assuming E to be kept constant, then $I_1\,R_1 = I_2\,R_2$

$$\therefore\ 2 \times \frac{1{\cdot}1284 \times R_2}{1{\cdot}2996}$$

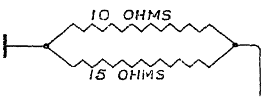
$$\therefore\ I_2 = \frac{2 \times 1{\cdot}1284}{1{\cdot}2996} = 1{\cdot}736$$

Current passing $= 1{\cdot}736$ ampéres. Ans.

Energy lost in t seconds $= I^2\,R\,t$ watt-seconds (or Joules) and 1,056 watt-seconds $=$ 1 B.T.U.

$$\therefore\ \text{Heat generated} = \frac{2^2 \times 52 \times 3{,}600}{1{,}056} = 709 \text{ B.T.U.}$$ Ans.

P.D. across terminals $= E = I\,R = 2 \times 52 = 104$ volts. Ans.



Equivalent resistance of the two in parallel :—

$$\frac{1}{R} = \frac{1}{10} + \frac{1}{15} = \frac{3 + 2}{30} = \frac{5}{30}$$

$\therefore\ R = \frac{30}{5} = 6$ ohms.

Total resistance of circuit $= 3 + 6 = 9$ ohms.

$$I = \frac{E}{R} \quad \therefore\ \text{Current flowing} = \frac{1{\cdot}5}{9}$$

$= 0{\cdot}1\dot{6}$ ampére. Ans. (*a*)

$$\text{P.D. across output terminals} = \frac{1{\cdot}5}{9} \times 6$$

$= 1$ volt. Ans. (*b*)

Note that the voltage lost due to internal resistance of the cell is $\frac{1{\cdot}5}{9} \times 3 = 0{\cdot}5$ volt, therefore output voltage is $1{\cdot}5 - 0{\cdot}5 = 1$ volt. (as before).

9. Power supplied to lamps $= 350 \times 60 = 21{,}000$ watts.

,, ,, motor $= 12 \times 746 \times \frac{100}{81} = 11{,}050$ watts.

Total power $= 21{,}000 + 11{,}050 = 32{,}050$ watts.

$$= \frac{32{,}050}{1{,}000} = 32{\cdot}05 \text{ kilowatts.} \quad \text{Ans. } (a)$$

Equivalent H.P. output of generator $= \frac{32{,}050}{746}$

H.P. input $= \frac{32{,}050}{746 \times 0{\cdot}91} =$ B.H.P. of engine.

$\therefore$ I.H.P. of engine $= \frac{32{,}050}{746 \times 0{\cdot}91 \times 0{\cdot}84} = 56{\cdot}2$ Ans. (b)

10. Volumetric Resistivity is another term for Specific Resistance ; it is the resistance offered to the flow of an electric current between the two opposite faces of a cube of unit dimensions.

Resistance $= s\,l \qquad = \frac{R\,a}{l}$

$\therefore$ Specific resistance, $s \qquad \frac{0{\cdot}34 \times 10^6 \times \qquad \times 0{\cdot}24^2}{9 \times 100}$

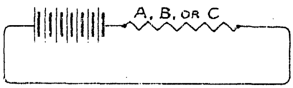
$= 17{\cdot}1$ microhms per cm. cube. Ans.

11. E.M.F. $= B\,l\,v \times 10^{-8}$ volts.

$= 8{,}500 \times 30 \times 25 \times 100 \times 10^{-8}$

$= 6{\cdot}375$ volts. Ans.

12.



Total E.M.F.

$= 6 \times 1{\cdot}05 = 6{\cdot}3$ volts.

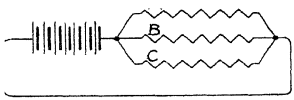
Total resistance of battery

$= 6 \times 0{\cdot}5 = 3$ ohms.

When A is inserted. $I = \dfrac{E}{R} = \dfrac{6{\cdot}3}{3 + 3} = 1{\cdot}05$ amps. Ans.

„ B $\quad I = \dfrac{6{\cdot}3}{3 + 30} = 0{\cdot}1909$ amp. Ans.

$I = \dfrac{6{\cdot}3}{3 + 300} = 0{\cdot}0208$ amp. Ans.



Resistance of parallel group :—

$$\frac{1}{R} = \frac{1}{3} + \frac{1}{30} + \frac{1}{300} = \frac{111}{300}$$

$$\therefore R = \frac{300}{111}$$

Total resistance of circuit =
resistance of battery + resistance of external circuit

$$3 + \frac{300}{111} = \frac{633}{111} \text{ ohms.}$$

$\therefore$ Current flowing $= I = \dfrac{E}{R}$

$$\frac{6{\cdot}3 \times 111}{633} = 1{\cdot}105 \text{ amps. Ans.}$$

13. Heat supplied $= 5{\cdot}8 \times 2{,}240 \times 0{\cdot}45 \times 100$ B.T.U.

$= 5{\cdot}8 \times 2{,}240 \times 0{\cdot}45 \times 100 \times 778$ ft. lb.

$= 454{\cdot}7 \times 10^6$ ft. lb. Ans.

$$\text{Equivalent horse power} = \frac{454\cdot7 \times 10^6}{24 \times 60 \times 33{,}000}$$

$= 9\cdot57$ H.P. Ans.

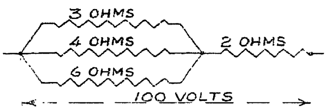
$$\text{Current supplied} = \frac{9\cdot57 \times 746}{220} = 32\cdot45 \text{ ampéres. Ans.}$$

14. $$\text{Ampére hour efficiency} = \frac{\text{Ampére hour discharge}}{\text{Ampére hour charge}}$$

$$\frac{1\cdot8 \times 36}{3 \times 24}$$

$= 0\cdot9$, or 90%. Ans.

15. Equivalent resistance of parallel group :—



1

$$\frac{4 + 3 + 2}{12} \qquad \frac{9}{12}$$

$\therefore R = \frac{4}{3} = 1\frac{1}{3}$ ohms.

Total resistance $= 1\frac{1}{3} + 2 = 3\frac{1}{3}$ ohms.

$$\text{Current flowing} = I = \frac{E}{R} = \frac{100}{3\frac{1}{3}} = 30 \text{ ampéres. Ans.}$$

Volt drop across parallel group ($E = I\,R$) $= 30 \times 1\frac{1}{3}$
$= 40$ volts.

Current flowing thro' 3 ohms res. $= \frac{40}{3} = 13\frac{1}{3}$ amps. Ans.

,, ,, 4 ,, $= \frac{40}{4} = 10$ amps. Ans.

,, ,, 6 ,, $= \frac{40}{6} = 6\frac{2}{3}$ amps. Ans.

,, ,, 2 ,, = total current

= 30 amps. Ans.

16.

$$\text{I.H.P. of engine} = \frac{60 \times 1{,}000}{746 \times 0{\cdot}92 \times 0{\cdot}8} = 109{\cdot}3$$

$$\text{I.H.P.} \quad \frac{P\ A\ L\ N}{33{,}000} \times 6$$

$$\therefore\ 109{\cdot}3 = \frac{85 \times 0{\cdot}7854 \times D^2 \times 1{\cdot}5 \times D \times 240 \times 6}{33{,}000 \times 12 \times 2}$$

$$\therefore\ D = \sqrt[3]{\frac{109{\cdot}3 \times 33{,}000 \times 12 \times 2}{85 \times 0{\cdot}7854 \times 1{\cdot}5 \times 240 \times 6}}$$

$= 8{\cdot}435$ inches. Ans.

17. 1,056 watt-seconds = 1 B.T.U.

B.T.U. of heat given out by heater
= B.T.U. received by water.

$$\frac{1{\cdot}5 \times 1{,}000 \times 30 \times 60 \times 0{\cdot}9}{1{,}056} = 2 \times 10 \times \text{Rise in deg. F}$$

$$\therefore\ \text{Rise in temperature} = \frac{1{\cdot}5 \times 1{,}000 \times 30 \times 60 \times 0{\cdot}9}{1{,}056 \times 20}$$

$= 115$ F°. Ans.

18. Total flux cut in one revolution $= 4 \times 900 \times 9{,}200$ lines.

Time for one revolution $= \frac{60}{600}$ second.

Absolute units of E.M.F. generated = Flux cut per second

$$\text{E.M.F. per conductor} = \frac{4 \times 900 \times 9{,}200 \times 600}{10^8 \times 60} \text{ volts}$$

$= 3{\cdot}312$ volts. Ans.

19.

$$\text{Work done per minute} = \frac{180{,}000 \times 10 \times 33}{60} \text{ ft. lb.}$$

$$\therefore\ \text{B.H.P. of electric motor} = \frac{180{,}000 \times 10 \times 33}{60 \times 33{,}000 \times 0{\cdot}6}$$

$= 50$ B.H.P. Ans.

$$\text{Power supplied to motor} = \frac{50 \times 746}{0\cdot9} \text{ watts.}$$

$$\text{Current taken} = \frac{50 \times 746}{0\cdot9 \times 400} = 103\cdot6 \text{ amps.}$$

20.

$$\text{Current taken} = I = \frac{E}{R} = \frac{220}{180} \text{ amps.}$$

$$\text{Quantity of electricity} = \frac{220}{180} \times 24 \text{ amp. hours.}$$

$$= 29\tfrac{1}{3} \text{ amp. hours. Ans.}$$

$$\text{Power absorbed} = 220 \times \frac{220}{180} \text{ watts.}$$

$$\text{Energy consumed} = \frac{220 \times 220 \times 24}{1{,}000 \times 180} \text{ kilowatt-hours.}$$

$$= 6\cdot453 \text{ kilowatt hours or B.O.T. units. Ans.}$$

21.

$$\text{Watts supplied per lamp} = \frac{15 \times 746 \times 0\cdot9}{270}$$

$$\text{Watts per candle power} = \frac{15 \times 746 \times 0\cdot9}{270 \times 30}$$

$$= 1\cdot243 \text{ watts per c.p. Ans.}$$

22.

$$\text{Force in dynes} = \frac{B\ I\ l}{10}$$

$$\frac{8{,}000 \times 109 \times 45}{10} = 3{,}924{,}000 \text{ dynes. Ans. } (a)$$

$$\frac{3{,}924{,}000}{981} = 4{,}000 \text{ grams.} \quad \text{Ans. } (b)$$

$$\frac{4{,}000 \times 2{\cdot}2}{1{,}000} \quad 8{\cdot}8 \text{ lb.} \quad \text{Ans. } (c)$$

23. Power absorbed = volts $\times$ amps.
$= 15 \times 6 = 90$ watts.

$$\text{Equivalent horse power} = \frac{90}{746}$$

$$\text{Foot lb. per hour} = \frac{90 \times 33{,}000 \times 60}{746}$$

238,900 ft. lb. per hour. Ans.

$$\frac{238{,}900}{778} = 307 \text{ B.T.U.} \quad \text{Ans.}$$

24. Let R_1 = resistance at $t_1°$ C.
and R_2 = resistance at $t_2°$ C.
Then $R_1 = R_0 (1 + a\, t_1)$ and $R_2 = R_0 (1 + a\, t_2)$

$$\frac{R_1}{R_2} \quad \frac{R_0 (1 + a\, t_1)}{R_0 (1 + a\, t_2)} \quad R_0 \text{ cancels.}$$

$$R = \frac{E}{} \quad \frac{E_1 \times I_2}{I_1 \times E_2} \quad \frac{a\, t_1}{a\, t_2}, \quad E_1 = E_2 \text{ and cancels.}$$

$$\frac{1}{1 + a\, t_2} \quad \frac{a\, t_1}{} \qquad I_2 = \quad \times \frac{(1 + a\, t_1)}{(1 + a\, t_2)}$$

$$\therefore I_2 = \frac{2{\cdot}2 \times (1 + 0{\cdot}00428 \times 18)}{(1 + 0{\cdot}00428 \times 36)}$$

$$\frac{2{\cdot}2 \times 1{\cdot}077}{1{\cdot}154} = 2{\cdot}053 \text{ amps.} \quad \text{Ans.}$$

25. Weight of deposit $= \mathrm{I} \times t \times$ E.C.E.

$$8 = 10 \times t \times 0{\cdot}00033$$

$\therefore$ time in minutes $=$

$$10 \times 0{\cdot}00033 \times 60$$

$= 40{\cdot}4$ minutes. Ans.

$$\frac{\text{Wt. of copper deposited}}{\text{Wt. of hydrogen liberated}} \quad \frac{\text{Chemical equivalent of copper}}{\text{Chemical equivalent of hydrogen}}$$

$\therefore$ Wt. of hydrogen liberated

$$\frac{8 \times 1}{31{\cdot}8}$$

$= 0{\cdot}2516$ gram. Ans.

26. Diameter of filament $= 0{\cdot}04$ mm. $= 0{\cdot}004$ cm.

$$\mathrm{R} = \frac{s\,l}{\mathrm{A}}$$

Resistance of filament when hot

$$\frac{0{\cdot}0000165 \times 60}{0{\cdot}7854 \times (0{\cdot}004)^2} \times 5 \text{ ohms.}$$

$$\text{(Ohm's law) } \mathrm{I} = \frac{\mathrm{E}}{\mathrm{R}} = \frac{110 \times 0{\cdot}7854 \times (0{\cdot}004)^2}{0{\cdot}0000165 \times 60 \times 5}$$

$= 0{\cdot}2793$ ampére. Ans.

FIRST-CLASS EXAMINATION QUESTIONS.

ENGINEERING SCIENCE.

1. Two round bars are secured together in a straight line by a cylindrical screwed coupling which is 4·25 inches outside diameter. Allowing a depth of thread of 0·18 inch, find the diameter of the bars so that bars and coupling will have equal stresses in tension.

2. A cylindrical vessel is constructed of plates $1\frac{1}{64}$ inches thick, and all rivets are $1\frac{3}{16}$ inches diameter. The longitudinal seam is a double riveted butt joint with double straps, the pitch of the rivets in the outer rows is $6\frac{1}{8}$ inches and half of this in the inner rows. The circumferential seams are double riveted lap joints with pitch of rivets $3\frac{1}{32}$ inches. The vessel is 4 ft. 9 ins. diameter. Allowing the tensile strength of the plates to be 28 tons per sq. inch, the shearing strength of the rivet material to be 23 tons per sq. inch, and the factor of safety 4·5, calculate the working pressure.

3. A cylindrical vessel with hemispherical ends is 15 inches diameter and 30 inches long overall. It weighs 55 lb. What weight of cast iron must be placed inside to entirely submerge the vessel in sea water ? What weight of cast iron, suspended below the vessel, will entirely submerge it ? One cubic foot of cast iron weighs 450 lb.

4. The balance weight on a crank weighs 1,392 lb., and its effective radius is 2·3 feet. It is held in place by 2 bolts and the stress in these is not to exceed 8,000 lb. per square inch. Find the diameter of the bolts if the maximum speed is 100 revolutions per minute.

5. The lever of a lever safety valve is $\frac{1}{2}$ inch broad and $1\frac{1}{2}$ inches deep. The distance from the centre line of the valve to the point at which the weight is attached is 18 inches. The valve spindle is attached to the lever by a $\frac{3}{4}$ inch diameter pin which passes through the lever. What weight at the end of the lever will cause the stress in the lever, due to bending, to be 5,000 lb. per square inch ?

6. The mean radius of the rubbing surface of the thrust collars is 7 inches. The co-efficient of friction is 0·09. At 102 revolutions per minute the horse power lost in friction is 16. Find the load on the thrust in pounds.

7. A turbine rotor drum is 3·5 feet in diameter, and it runs at 700 revolutions per minute. The stress due to centrifugal force is 2,300 lb. per square inch. Another rotor, of similar material, is 8 feet in diameter and runs at 320 revs. per minute. Find the stress.

Note.—Centrifugal force varies directly as the product of the weight and the square of the velocity.

8. A sheet copper tank, 4 feet by 15 inches by 8 inches, floats in sea water. The copper weighs 18 ozs. per square foot. What weight of iron could be suspended from the tank ? 1 cubic foot of iron weighs 480 lb. The tank is entirely enclosed.

9. The percentage strengths of the drilled plate section and of the rivet section of the longitudinal seam of a boiler are 79·79 and 79·29 respectively. The circumferential seam is a double riveted lap joint which, for its purpose, is 40% stronger than the longitudinal seam. The rivets are $1\frac{3}{16}$ inches diameter, and their shearing strength is 23 tons per sq. inch. The plates are $1\frac{3}{32}$ inches thick, and have a tensile strength of 28 tons per sq. inch. Find the pitch of the rivets in the circumferential seam.

10. The following data were taken from a set of rope pulley blocks having a velocity ratio of 6.

Load in lb.	50	100	150	200	250	300
Effort in lb.	15·6	26·3	37	47·7	58·3	69·3

Plot the graphs of effort and efficiency on a base line of load lifted, to the scales :—1 inch = 50 lb. load. 1 inch = 10 lb. effort. 1 inch = 10% efficiency.

Determine the law connecting load and effort and calculate the effort required to lift 267·2 lb. Read from the graph the efficiency of the lifting machine when the effort applied is 30 lb.

11. A stay is 2·5 inches diameter. The end is swelled to 3·25 inches to allow for a cotter. The thickness of the cotter is 0·75 inch. Find the depth of the cotter to have a shearing strength equal to the tensile strength of the stay.

f_t = 28 tons per square inch.

f_s = 23 tons per square inch.

12. A combustion chamber is 2 feet 3 inches deep from tube plate to back plate. The girders consist of two plates 0·625 inch thick each, and are pitched 9·5 inches apart. The boiler

pressure is 180 lb. per square inch. There is one stay for each girder. Find the depth of the girder, if the stress is not to exceed 10,000 lb. per square inch. The tube plate and the back plate may be assumed to take half the load, and the stay to take the other half.

13. Find the load that can be lifted by a double geared hand winch. The handle is 24 inches radius and the force exerted is 200 lb. The driving pinions have 12 and 15 teeth respectively and the driven wheels 60 and 75 teeth. The barrel is 12 inches diameter and the rope 1 inch diameter. The loss due to friction is 35 per cent.

14. The load at the thrust is 40,000 lb., and the speed is 14·5 knots. The revolutions are 75 per minute. Find the twisting moment on the shaft in inch tons.

15. A deadweight safety valve, 4·5 inches diameter, is loaded with cast iron weights of 650 lb. At what pressure will the valve lift ? Water accumulates in the chest and in the waste steam pipe until its level is 2 feet 4 inches above the valve, and the weights are entirely submerged. At what pressure per square inch will the valve now lift ?

16. When weighed in a balance with unequal arms, a body appears to weigh 6·5 lb., when in one pan, and 5·5 lb. when in the other. What is its true weight ?

True weight = $\sqrt{\text{Product}}$ incorrect weights. Show how this expression is arrived at.

17. A steamer runs the measured mile in 4 minutes 31 seconds with the current and in 7 minutes 19 seconds against the current. What is her true speed in knots and also in statute miles ?

18. The brass for a "single" top end connecting rod is semi-cylindrical. The length fore and aft is 12 inches, and between the flanges 10 inches. It is 11·5 inches wide and 13·5 inches over the flanges. The measurements vertically downwards are :—

Flange, 1 inch. Brass, 1·25 inches. Pin, 10 inches. Brass, 1·25 inches. Flange, 1 inch.

Find the weight. 1 cubic inch brass weighs 0·3 lb.

19. A hollow cast iron column is tapered uniformly for the whole length and is 12 feet overall. The outside diameter at the top is 10 inches, the outside diameter at the bottom 13 inches and it is 2 inches thick. The top flange is 15 inches diameter

7. A turbine rotor drum is 3·5 feet in diameter, and it runs at 700 revolutions per minute. The stress due to centrifugal force is 2,300 lb. per square inch. Another rotor, of similar material, is 8 feet in diameter and runs at 320 revs. per minute. Find the stress.

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True weight $= \sqrt{\text{Product of incorrect weights}}$. Show how this expression is arrived at.

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Flange, 1 inch. Brass, 1·25 inches. Pin, 10 inches. Brass, 1·25 inches. Flange, 1 inch.

Find the weight. 1 cubic inch brass weighs 0·3 lb.

19. A hollow cast iron column is tapered uniformly for the whole length and is 12 feet overall. The outside diameter at the top is 10 inches, the outside diameter at the bottom 13 inches and it is 2 inches thick. The top flange is 15 inches diameter

and 2·25 inches thick. The bottom flange is rectangular, 19 inches by 15 inches, and 2·5 inches thick. Find the weight. 1 cubic foot of cast iron weighs 450 lb.

20. A planing machine table weighs 21 cwts. The stroke is 7 feet, and 4·8 double strokes are made per minute. The coefficient of friction between the table and the bed is 0·11. The speed of the return stroke is four times the speed of the forward stroke. Find the horse power of the engine to drive the machine.

21. A beam 12 inches long, 1 inch broad and 1 inch deep, and supported at the ends, is broken by a weight of 480 lb. at mid-length. Find the breadth of a cantilever of similar material, 9 feet long, that will carry a concentrated load of 3 cwts. at the free end. The depth is to be three times the breadth and the factor of safety is 5.

22. A cylindrical pontoon with hemispherical ends, 8 feet diameter and 50 feet long overall, floats in water having a density of 1,024, with its horizontal axis on the water level. Find the displacement in tons.

23. A beam 10·5 inches deep, and 10 feet long, is supported at the ends. It carries an evenly distributed load, which with the weight of the beam is equal to two tons per foot run, and a concentrated load of 4 tons at mid-length. Find the breadth so that the stress shall not exceed a maximum of 4 tons per sq. inch.

24. A safety valve is 3·5 inches diameter. The mean diameter of the spring is 3 inches, it has 11 coils and the steel for the spring is $\frac{13}{16}$ inch square. The compression to give the necessary load is 0·7 inch. The spring is found to be unsuitable and another one 3·25 inches mean diameter, of the same size steel, and having 13 coils, is substituted. What will be the compression ?

Use the following formula :—

$$\text{Compression (inches)} = \frac{S \times D^3 \times N}{C \times d^4}$$

S = load on spring (lb.)

D = mean diameter of spring (inches).

N = number of coils.

C = 22 for round steel and 30 for square steel.

d = diameter in sixteenths of an inch, or side of square in sixteenths.

25. A piece of brass weighs 23 grams in air and 20·26 grams in fresh water. A piece of cork weighs 1·6 grams in air, and the cork and brass connected together and submerged in the water weighs 15 grams. Find the specific gravities of the brass and cork.

26. A truck requires a force of 22 cwts. to pull it up an incline of 4° 25′. The force required to pull it down is 58 lb. Find the weight of the truck.

27. Two steel pump rods are fastened together by means of a screwed steel hexagonal coupling, the side of which is 1·7 inches. Allowing 0·2 inch on the diameter for the thread, find the diameter of the rods so that the strengths of the rods and coupling are equal in tension.

28. A barge is towed along a canal, the tow-rope being inclined at an angle of 30° to the centre line of the barge. The pull on the rope is 480 lb. Find the force causing forward motion in the barge and the horse power exerted, if the speed of the barge is 1 foot per second.

29. A hollow shaft has an outside diameter of 14 inches. The diameter of a solid shaft is 12·125 inches. The shafts are made of a similar material and each weighs 395 lb. per foot of length. A certain twisting moment induces a stress of 8,000 lb. per square inch in the hollow shaft. What would be the stress induced in the solid shaft by an equal torque ?

30. The area of the valve of a lever safety valve is 8 square inches. The distance from the centre line of valve to fulcrum is 3·25 inches. The valve and spindle weigh 10 lb. The lever weighs 9 lb. and its centre of gravity is 10 inches from the fulcrum. Through what distance must the movable weight of 60 lb. be moved, to allow for a variation of 10 lb. per square inch in the pressure at which the valve will lift ?

31. The depth of the section of a rolled joist is 12 inches. The flanges are of equal breadth, and the flanges and web are each 1 inch thick. The joist is made of mild steel, weighing 490 lb. per cubic foot. It is 12 feet long, and is fixed as a cantilever having a load of 3·5 tons at the free end. Find the breadth of the flanges if the stress is not to exceed 5 tons per square inch, (*a*) Neglecting the weight of the joist, (*b*) Taking weight of joist into account.

Note.—The resisting moment of the section is approximately equal to the area of one flange multiplied by the depth of the section and multiplied by the stress.

32. The load on the piston is 40 tons. The connecting rod is 100 inches long and the crank is 24 inches long. Find the maximum torsional stress in the shaft, which is 13 inches diameter.

33. The valve of a spring loaded safety valve is 3·5 inches diameter. The spring was compressed 1 inch to give the working pressure of 180 lb. per square inch. Find the foot pounds of work done in lifting the valve 0·5 inch against the compression of the spring.

34. A safety valve spring is made of square steel of 0·875 inch side, its mean diameter is 3·25 ins., its length 17 ins., and there are 15 clear coils. Find the length of steel rod required to make this spring neglecting the end parts which are required for bearing surfaces.

35. A chain, weighing 6 lb. per foot run, hangs over a wheel 40 feet on one side and 10 feet on the other. Find the foot pounds of work done in pulling down the short end until the difference in the level of the ends of the chain is 4 feet.

36. A rule for the working pressure of a furnace built up of a series of flanged drums is :—

$$W.P. = \frac{9{,}900 \times t}{3 \times D}\left(5 - \frac{C + 12}{60 \times}\right.$$

t = thickness (inches).
D = diameter (inches).
C = length of drums (inches).

Find the thickness of a furnace 3 feet 3 inches diameter, having drums 26 inches over the flanges, to stand a working pressure of 150 lb. per square inch.

37. A turbine weighs 33 tons. The 'thwartship pedestals are 7 feet apart. When the turbine is at rest an equal load comes upon each foot, but when the turbine is run at 100 revolutions per minute the load on the port feet is twice that on the starboard feet. Find the horse power being developed.

38. A propeller is 14 feet diameter and is driven at 60 revolutions per minute. The speed of the ship is 11 knots. Find the velocity of the tips of the blades through the water in feet per second.

39. Find the diameter of the ram of a hydraulic jack. The load lifted is 4·2 tons. The force pump plunger is 0·625 inch diameter and the leverage of the handle is 17. The effort applied is 38 lb. and the efficiency of the jack is 63 per cent.

40. Calculate the size of the coupling bolts necessary for a hollow shaft, assuming the stress in the bolts is the same as the stress in the shaft. The shaft is 16 inches outer diameter and 8 inches inner diameter. There are 7 bolts in each coupling at a radius of 12 inches.

41. A force of 850 lb., acting parallel to the plane, is required to move a truck weighing 3·5 tons up an incline of 1 in 20. What force would be required to move it down the incline ?

42. Two ships, whose speeds are 10 and 11 knots respectively, are making for the same port at which they will arrive simultaneously. Their courses are at 90° to each other, and at a certain time they are 80 miles apart. How far will they be apart two hours later ?

43. A cast iron beam is 10 feet long, 3·25 inches broad and 5·5 inches deep. It is to be lifted by a crane. When the sling was first put on, it was found to require a man weighing 140 lb. sitting on one end to keep it horizontal. How far must the sling be shifted so that the beam will be lifted horizontally ?

44. Two ships leave a port for another which is 240 miles away. The first ship leaves the port 5 hours before the second one, which is 4 knots faster, and arrives at the distant port 5 hours after the faster one. What are the speeds of the ships ?

45. A screw jack has an efficiency of 40 per cent. The pitch of the screw is 0·25 inch and the weight lifted is 5 tons. Two bars of lengths 30 inches and 40 inches are used. If a force of 40 lb. is applied at the end of the long bar, find what force is necessary at the end of the short bar so that the resistance due to the weight is just overcome. State the direction in which this second force should act.

46. Two pieces of plate, 6 inches broad and 0·5 inch thick, are to be joined together by a lap joint using rivets 0·75 inch diameter. Design the joint for the best number of rivets. Take the tensile strength of the plate at 27 tons per square inch and the shearing strength of the rivets at 23 tons per square inch.

47. A beam, which is supported at the ends on a span of 12 feet, carries a uniformly distributed load of 1 ton per foot run and a concentrated load at mid-span of 5 tons. The beam is 4 inches broad. Find its depth in order that the stress shall not exceed 5 tons per square inch.

48. A solid drawn pipe is 10 feet long and weighs 244 lb. The thickness is 0·276 inch, and the specific gravity is 8·9. Find the internal diameter of the pipe.

49. A ship steaming due West at 16 knots runs into a 3 knot current running South West. Find the resultant direction of the ship and the distance she will travel in 1 hour 35 minutes.

50. A ship, starting from rest, is uniformly accelerated for 5 minutes, during which time she travels 0·384 of a statute mile. What is then her speed in knots ; also what was her speed when she had travelled 0·25 of a nautical mile ?

51. The shell of a vertical boiler is made up of three drums, the thicknesses being 0·625 inch ; 0·5 inch, and 0·75 inch respectively. The longitudinal seams are double riveted lap joints, the rivets are 0·875 inch diameter and the pitch 2·75 inches. The plates have a tensile strength of 28 tons per square inch, and the rivets 23 tons per square inch in shear. The shell is 7 feet in diameter. Find the working pressure, using a factor of safety of 5.

52. A body is projected vertically upwards from the ground with a velocity of 100 feet per second. Two seconds afterwards another body is projected upwards with a velocity of 200 feet per second. To what height will they attain ? At what height will they meet, and how long after leaving the ground ?

53. A cast iron cylinder, open at one end, has an external diameter of 2 feet and an overall length of 2 feet. The base is 1 inch thick. If it floats in sea water at a draught of 2 feet, what is the thickness of the cylinder ? If it is suspended from a wire and totally submerged, what will be the tension in the wire ?

54. The diameter of a boiler is 16 feet, and its length 11 feet 6 inches. The area of the water level is 157·4 square feet. The furnaces, combustion chambers and tubes occupy 32 per cent. of the total volume of the boiler. Find the weight of fresh water in the boiler.

55. The difference between the areas of two equilateral triangles is 24·5 square inches, and the difference between their perimeters is 6 inches. Find the lengths of the sides of the triangles.

56. A force of 1 ton acting parallel to the plane is required to pull a load of 9·25 tons up an incline. A force of 94 lb. will pull the load down. Find the rise of the incline.

57. The valve of a lever safety valve has an area of 8 square inches. The lever weighs 6 lb. and its C.G. is at 7·5 inches from the centre line of the valve. The length of the lever from the centre line of the valve to the end where the weight is hung is 21 inches. It is found that if no weight is hung on the lever, the valve will blow at 3 lb. per square inch, and that when

62 lb. are hung on the end of the lever, the valve will lift at 65 lb. per square inch. Find the weight of the valve, also the distance from the valve to the fulcrum.

58. The diameter of a bolt is 4 inches and its overall length is 27 inches. The head is 6 inches in diameter and 4 inches deep. A ring of the same dimensions as the bolt head is made to slide on the body of the bolt. When this ring is at a certain position on the bolt, the C.G. of the whole is at 15 inches from the point end. Find the position of the ring.

59. The stroke of an engine is 39 inches. When the crank has passed through 30° from the top centre, the load on the piston is 28·8 tons and the load on the crank pin is 29 tons. Find the load on the guide in tons, the length of the connecting rod in inches, and the distance the piston is from the bottom of the stroke.

60. If D is the diameter of a feed check valve in inches.

L is the lift of the valve in inches.

V is the velocity of the water passing through the valve in feet per second, construct a formula to give the pounds of water passing through the valve per minute.

A valve is 3 inches diameter and has a lift of 0·0625 inch. The velocity of the water is 12 feet per second. Find the pounds of water passing through the valve per minute.

61. The load on the collars of the thrust of an engine is 41,800 lb. There are 5 collars and the pressure per square inch on the liners is 70 lb. The area of each liner is two-thirds of the area of the collar. The cube of the external diameter of the collar is four times the cube of the diameter of the shaft. Find the external diameter of the collar.

62. A tank is 15 feet in diameter and 12 feet long. It is supported on columns 15 feet high with its axis vertical. It is to be filled with fresh water through a 3 inch diameter pipe which enters the tank at the top. The pump is placed upon the ground and the water pressure at the pump is 60 lb. per square inch. Find the approximate time to fill the tank.

63. The combustion chamber tube plate of a boiler is 0·75 inch thick, the tubes are 3 inches outside diameter and 0·16 inch thick, spaced 4·25 inches apart horizontally. The width of the combustion chamber is 2 ft. 9 inches. Find the working pressure of the boiler from the formula :—

$$\text{W.P.} = 875 \times \frac{(\text{D} - d)\,t}{\text{W} \times \text{D}}$$

t = thickness of tube plate in thirty-seconds of an inch.
D = horizontal pitch of the tubes, centre to centre, in inches.
d = internal diameter of plain tubes in inches.
W = width of combustion chamber in inches.

Determine, also, the compressive stress set up in the tube plate.

64. A wagon weighing 10 tons requires a force of 11 cwts. acting parallel to the incline to pull it up the incline, and it requires a force of 9 cwts. to prevent it running down. Find the rise of the incline and the friction upon the level in pounds per ton.

65. A cast iron girder has the top flange 6 inches broad and the bottom flange 7 inches broad. The overall depth is 10 inches. The flanges and web are all of the same thickness, and the girder weighs 80 lb. per foot of length. Find the position of the centroid of its section.

66. The area of a circle is equal to twice the area of the inscribed equilateral triangle plus a certain number of square inches. The perimeter of the circle is greater than that of the triangle by the same number of inches. Find the length of the side of the triangle in centimetres.

67. The length of the connecting rod of an engine is 1·95 times the length of the stroke. When the crank has passed through 55° from the top centre, the load on the guide is 5,230 lb. Find the loads on the piston and on the crank pin. What fraction of the stroke has been completed ?

68. A mild steel girder is supported at the ends on a span of 20 feet. It is 9 inches deep overall and the flanges are 6 inches wide, the thickness of the flanges and the web being 1 inch. A load of 4 tons is hung at 4 feet from one end. At what distance from the other end will the stress in the girder be 1 ton per square inch, (*a*) if the weight of the girder is neglected, (*b*) if the weight is taken into account.

69. A circular plate 24 inches diameter, has a round piece 7 ins. diameter cut out, the centre being 6 inches from the centre of the plate. Another round piece 6 ins. diameter is cut out, its centre being 5 inches from the centre of the plate. The centres of the holes and of the plate form a right angled triangle. Calculate the position of the centre of gravity of the plate remaining.

70. The total work done in lifting a weight through a vertical distance of 4 feet by means of a spring is 43 foot lb. and the useful work done is 88·88 per cent. of the total work. Find the weight lifted, also the weight to stretch the spring 1 inch.

71. A treble riveted lap joint has four rivets in a pitch, the pitch in the outer rows being 4·5 inches. The tensile strength of the plate is 28 tons per square inch, and the shearing strength of the rivet material is 23 tons per square inch. The plate is 1 inch thick. Find the diameter of the rivets to give the greatest possible joint strength, and calculate the efficiency of the joint.

72. A body is allowed to fall from a height of 525 feet, and at the same instant another body is projected vertically upwards from the ground at 9,000 feet per minute. How long will they take to meet; at what height above the ground, and what will be the velocity of the impact?

73. A flat bar, weighing 20 lb. per foot of its length, rests upon two supports placed 5 feet apart. The bar is 7·5 feet long and one end overhangs its support by 2 feet. A weight of 140 lb. is placed on this end. What weight placed above the farther support will be necessary to preserve equilibrium?

74. A lever safety valve is 2·5 inches diameter. The valve and spindle weigh 2·5 lb. and the distance from the centre line of the valve to the fulcrum is 2·5 inches. The lever is uniform in section and it weighs 5 lb. per foot of its length. When there is no weight on the end of the lever the valve blows off at 5 lb. per square inch. What weight must be hung at the end for a boiler pressure of 60 lb. per square inch?

75. A connecting rod is 100 inches long and it weighs 2 tons. Its centre of gravity is at 46 inches from the lower end. It hangs vertically. Find the least force, applied at the lower end, that will pull this end 3 feet from the vertical. What would be the value of the horizontal force?

76. The rim of a cast iron flywheel is 7 feet internal diameter. The tensile strength of the cast iron is 9 tons per square inch. Find the revolutions per second at which this rim will burst.

77. The stroke of an engine is 4 feet and the connecting rod is 8 feet 4 inches long. When the crank has moved through 65° from the top centre, the effective load on the piston is 22 tons. The co-efficient of friction between the shoe and the guide is 0·072. Find the frictional force opposing motion of the shoe.

78. A solid rectangular beam is 20 feet long, 5 ins. broad and 12 ins. deep, and is simply supported at each end. It carries a concentrated load of 10 tons at 5 ft. from the left hand end, and another of 6 tons at 4 ft. from the right hand end. Calculate the bending moment at the centre of the beam, and under each load ; find also the maximum stress set up in the beam. Neglect the weight of the beam.

79. A double bottom tank holds 159 tons of oil of specific gravity 0·9. The depth of the tank is 4·5 feet, and its length is 50 feet. The breadth at the top is 29·7 feet ; at three-quarters depth it is 30·1 feet ; at half depth 30·6 feet and at quarter depth 31·2 feet. What is the breadth at the bottom ?

80. An equilateral triangle has an area of 20·84 square centimetres. Find the area of the inscribed circle in square inches.

81. Two ships are steaming at 15 knots towards the same port, and one will arrive one hour before the other. Their courses converge at an angle of 120° and they are 216 miles apart. How far are they from the port ?

82. The diameter of a spring loaded safety valve is 3·75 inches. The compression of the spring for a boiler pressure of 185 lb. per square inch, is 64 per cent. more than would be required for a boiler pressure of 115 lb. per square inch. Find the weight of the valve spindle and spring.

83. The engine room bulkheads are 25 feet apart. A wire rope 28 feet long is attached to each, and upon this wire is placed a snatch block which comes to rest at a horizontal distance of 10 feet from one bulkhead. Find the difference in the heights of the points of attachment of the wire. Find the tension in the wire when a cylinder cover weighing 2·25 tons is hung from the block.

84. The length of the chord of the arc of a circle is 12 feet, and the length of the chord of half the arc is 10 feet. Find the length of the arc of the circle in feet and inches.

85. Construct expressions to find the speed of a vessel (*a*) in statute miles per hour, (*b*) in kilometres per hour, if the vessel travels K nautical miles in N minutes. Make use of the expressions derived to find the speed in statute miles per hour, and also in kilometres per hour if the vessel travels 1 nautical mile in 4 minutes 48 seconds.

86. The larger pulley of the compound sheave of a differential pulley block is $\frac{3}{4}$ inch greater in diameter than the smaller pulley. The mechanical advantage at a certain load is 11 and the efficiency is 39 per cent. Find the diameters of the pulleys.

87. A solid shaft 9 inches diameter transmits 1,000 H.P. at 85 revolutions per minute. A hollow shaft whose internal diameter is one-half of the external diameter transmits 1,500 H.P. at 93 revolutions per minute. If the torsional stress is the same in each shaft, find the dimensions of the hollow shaft.

88. A ship attains her maximum speed of 17 knots when she has travelled 8 kilometres from rest. Find the acceleration, the time to attain the maximum speed and the distance travelled in the first 7 minutes.

89. A truck requires a force of 1,000 lb. acting parallel to the incline to cause it to move up the incline, and a force of 800 lb. to prevent it from running down. The tractional resistance upon the level is 0·56 per cent. of the weight of the truck. Find the weight of the truck, and the rise of the incline.

90. A triangular piece of plate has angles of 45°, 60° and 75°. The shortest side is 2·75 inches long. Find the distance of the C.G. of the plate from each side.

91. A cast iron beam of rectangular section is 4 inches broad and is supported at the ends on a span of 10 feet. It carries a concentrated load of 1,500 lb. at mid-length. Take the weight of the beam into account and find its depth in order that the stress shall not exceed 1,000 lb. per square inch.

92. The perimeter of a rectangle measures 49·5 feet and its diagonal measures 20·02 feet. Find the length of its sides.

93. A cast iron cylinder, which is closed at the ends, has an overall length equal to its external diameter. It weighs 32 kilograms and will float just submerged in oil having a specific gravity of 0·83. Find its diameter in feet, and the volume of the enclosed portion of the cylinder in cubic feet.

94. A weight of 260 kilograms is pulled up an incline, which rises 2·29 metres in 8·275 metres, by a force acting parallel to the plane. The force to overcome friction upon the level is 14 lb. per ton. Find the force required on the incline.

95. Two ships leave a port together on courses which diverge at an angle of 40° 20′. One ship's speed is 6 knots more than the other. After 6 hours the ships are 78 miles apart. Find their speeds.

96. A wire rope, 34 feet long, is attached at its ends to bulkheads which are 28 feet apart, the points of attachment are at the same horizontal level. A snatch block runs freely upon the wire, and a cylinder cover weighing 2 tons is hung from the block. A horizontal force acts on the snatch block and maintains it at a horizontal distance of 10·25 feet from one bulkhead. Find the horizontal force, and the tension in the wire.

97. An open ended steel cylinder is 12 feet long and it weighs 3,450 lb. If its external diameter had been 5 per cent. less and its internal diameter 5 per cent. more, its weight would be reduced by one-half. Find the diameters in feet.

98. A sheet of metal is 30 inches square. Two small squares, one 4 inches by 4 inches and the other 5 inches by 5 inches are cut from adjacent corners. The remaining corners of the plate are folded over on lines 15 inches in each direction from these corners. Find the position of the C.G. of the plate remaining.

99. An angle bar 2 inches by 3 inches outside dimensions, and 12 feet long, is used as a tie bar and carries a load of 15 tons. The bar is found to stretch 0·087 inch. Find its thickness. E = 13,500 tons per square inch.

100. Two ports, M and N, are 425 miles apart. A steamer leaves M for N, and 4 hours afterwards another steamer, whose speed is 3 knots less, leaves N for M. They pass at a distance of 150 miles from N. Find the speeds of the steamers.

101. The section of a bar of metal is an equilateral triangle. Its length is 19 feet and when it is subjected to a tensile force of 23 tons the strain is 0·0019. The modulus of elasticity is 7,500 tons per square inch. Find the size of the section, the stress and the total elongation.

102. The jib of a crane is 27 feet long and the tie is $9\frac{1}{2}$ feet long. The jib and tie make an angle of 55° with each other. Find the forces in the jib and tie when 11 tons is suspended from the end of the jib.

103. A train weighs 310 tons. The tractional resistance on the level is 8 lb. per ton. By what percentage will the speed be reduced when the train is ascending an incline of 1 in 75 if the horse power remains the same ?

104. The speed of a steamer is 17 knots. She runs into a current which increases her speed by 2·8 knots. If the current had been flowing in the opposite direction her speed would have been reduced by 2·5 knots. Find the speed of the current and its direction relative to the steamer's normal course.

105. Two bulkheads are 30 feet apart. A wire rope $27\frac{1}{2}$ feet long is attached to the after bulkhead, and another 10 feet long to the forward one. These two ropes are shackled together and from the shackle a weight of 1·75 tons is hung. If the tension in the wires is equal, find the vertical difference between the points of attachment of the wires to the bulkheads, and the horizontal distance from the after bulkhead to the shackle.

106. A conical shaped buoy weighs 15·7 tons and floats with its apex downwards at a draught of $8\frac{1}{4}$ feet in sea water. The vertical height of the buoy is 8 feet 8 inches. Find the diameter of its base.

107. The working pressure of a furnace made up of Adamson rings may be calculated from :—

$$\text{W.P.} = \frac{C\,(t - 1)^2}{(L + 24)\,D}, \text{ and from :—}$$

$$\text{W.P.} = \frac{C_1}{D}\,[10\,(t - 1) - L].$$

If t = thickness in $\frac{1}{32}$nds ; $C = 1{,}450$; $C_1 = 50$; $L = 30$ inches, and the calculated working pressure is the same in each case, find the thickness in inches.

108. A piece of coal measures 19 inches × 13 inches × 9 inches. It weighs 79·6 lb. per cubic foot. It is suspended from a wire and submerged in sea water having a hydrometer reading of 1,024. If it absorbs 5 per cent. of its own weight of sea water, find the tension in the wire.

109. An isosceles triangle has an area of 144 square inches. The angles at the base are each 40°. Calculate the diameter of the largest circle that can be cut from the triangle, and the area of the parts remaining.

110. Two ships, one 14 knots and the other 10·5 knots, are steaming towards the same port where the slower one will arrive 3 hours before the faster one. If the angle between their courses is 75°, how far were the ships apart when the faster one was 126 miles from the port ?

111. A cylindrical tank with a hemispherical end is 3 feet diameter and 6 feet 6 inches long overall. Its weight is 375 lb. What depth of oil of specific gravity 0·9 must be run into this tank to cause it to float, with its axis vertical, at a draught of 5 feet in fresh water ?

112. A cast iron flywheel is 14·5 feet diameter. When running at 584 revolutions per minute the wheel bursts. What is the tensile strength of the cast iron ?

113. A pipe has a lap welded seam, the efficiency of which is 0·56 of the solid plate. If the tensile strength of the plate is 21·5 tons per square inch and the factor of safety is 9, find thickness in inches. Use the formula :—

$$\text{W.P.} = \frac{(t - 12)\,90}{\text{D}},$$ where t is in hundredths of an inch.

114. A mixture of fuel oil and sea water in equal volumes has a weight of 46·54 grams; the specific gravity of the sea water being 1·028. A further quantity of oil is now added to the mixture until the weights of the oil and water are equal. The mixture now weighs 49 grams. What is the specific gravity of the oil?

115. A stone is in the form of an equilateral triangular prism. The sides of the triangle are 4 feet and the length is 4 feet. Its specific gravity is 2·32. The stone rests on one of its rectangular faces. Find the work done in raising this stone about one of the edges of its length until the base is inclined at 30° to the horizontal. Express the answer in inch tons.

116. The working pressure allowed on a flat surface in a boiler may be calculated from :—

$$\text{W.P.} = \frac{100}{\text{H}^2 + \text{V}^2}(t - 1)^2 + 0{\cdot}084\,t.$$

H is the horizontal pitch of the stays in inches; V the vertical pitch of the stays in inches, and t the thickness of the plate in thirty-seconds of an inch. Find the plate thickness in inches when the pressure is 175 lb. per square inch, and the stays are pitched 18 inches horizontally and 16 inches vertically.

117. The twisting moment on the shaft of a vessel travelling at 19·5 knots is 600 inch tons; the load at the thrust being 32,000 lb. Find the speed of the shaft in revolutions per minute.

118. A cubical tank, open at the top, has sides 4·5 feet long. The plates weigh 7·5 lb. per square foot of surface. This tank floats in sea water at a draught of 4 feet when containing oil of specific gravity 0·89. What is the depth of oil in the tank?

119. The top flange of a rolled joist is 2 inches less in breadth than the bottom flange. The overall depth of the section is 14 inches and the flanges and web are all $1\frac{1}{2}$ inches thick. If the centroid of the section is at 6·6 inches from the bottom flange, what is the breadth of the top flange?

120. A piece of cast iron which weighs 1 lb. in air is found to weigh 0·882 lb. when entirely immersed in an oil. Express the density of the oil in degrees Beaumé.

121. A square is inscribed within a circle, the diagonal of the square being equal to the diameter of the circle. If the difference between the area of the circle and the area of the square, expressed in square inches, is the same as the difference in their perimeters in inches, find the diameter of the circle.

122. Explain the meaning of the term acceleration. A body is projected vertically upwards with sufficient force so that at 200 feet height it still has a velocity of 18 feet per sec. At what velocity does it reach the ground and what has been the total distance passed through ?

123. A cylinder cover lifting beam for an engine room is made of mild steel. The flanges are 8 inches wide and $\frac{1}{2}$ inch thick. The web is $\frac{5}{8}$ inch thick and the depth of the beam is 9 inches. The beam is 18 feet long and it carries loads of 2 tons, 3 tons and 8 tons at distances of 4 feet, 8 feet and 15 feet from one end. A shoe at each end supports the beam and each is riveted to the bulkhead by 7, $\frac{5}{8}$ inch diam. rivets. Calculate the maximum shear stress in the rivets.

124. A body has been projected vertically upwards, and it is observed to pass two points 350 feet apart in 4·1 seconds. How far does the body travel upwards beyond the higher of these points ?

125. Explain what you understand by the "Triangle of Forces." Two ropes are attached to hooks in a horizontal beam. The ropes are connected together and a weight of 640 lb. is hung from the point of connection. If the ropes make angles of 45° and 60° respectively with the beam, what is the tension in each one ?

126. A spring loaded safety valve is $3\frac{1}{2}$ inches diam., and the valve, spindle and spring together weigh 37 lb. What is the percentage increase in the load on the spring when altering the blowing off pressure from 110 lb. per sq. inch to 180 lb. per sq. inch ?

127. The diameter of the base of a cone is 3 feet and its vertical height is 5 feet. It is partly filled with water so that when resting upon its base the depth of the water is $2\frac{1}{2}$ feet. If the cone is inverted, and balanced on its apex, what will then be the depth of the water ?

128. The internal diameter of a pressure cylinder was 7 feet 6 inches. It was made of 4 plates each $1\frac{1}{2}$ inches thick, these being cut obliquely and butt welded, the joints making an angle of 45° with the longitudinal axis of the cylinder. If the working pressure was 350 lb. per sq. inch, find the stress on the longitudinal section and on the circumferential section. Find also the stress normal to the oblique seams.

129. The surface area of the flange of a cast iron cylinder cover is 10 times the section of the studs securing the cover to the cylinder. If the joint starts to blow when the temperature has been raised 300 F°., what was the initial stress in the studs ? Assume the following values :—

Co-efficient of linear expansion of steel is 0·0000067 per degree F.

Co-efficient of linear expansion of cast iron is 0·000006 per degree F.

E for steel = 12,000 tons per sq. inch. E for cast iron = 6,000 tons per sq. inch.

130. The chamber of a fresh water pump has a sectional area of 21 sq. inches, and its stroke is 12 inches. The section of the pipe to the tank is 7 sq. inches and the tail valve of the pump is 14 feet above the water level in the tank. Assuming zero clearance between pump bucket and tail valve, and neglecting the weight of the valve, find the height the water would rise in the pipe line for the first stroke of the pump. The atmospheric pressure would support a column of fresh water 34 feet high.

131. A piece of mild steel 0·75 inch diam. and 3 inches long is subject to an axial tensile load of 5 tons. What is the stress normal to a plane at 45° to the axis, and what is the shear stress on this plane ?

132. A pressure cylinder is $7\frac{1}{2}$ feet diameter. It is made of 3 plates cut obliquely and butt welded, the seams running at 45° to the axis of the cylinder. The tensile strength of the plates is 28 tons per sq. inch, and the thickness $1\frac{1}{2}$ inches. Find the stress on the oblique seams. Factor of safety is 5.

133. A circular copper bar is strengthened by a steel tube of the same length rigidly fixed on the outside. This compound bar is subjected to an axial tensile load of 10 tons. The sectional area of the copper bar is 1·53 sq. inches and of the steel tube 0·885 sq. inch. Find the intensity of the stress in the bar and in the tube. E for steel = 13,400 tons per sq. inch. E for copper = 6,700 tons per sq. inch.

134. An octagonal steel bar is 5·5 feet long. It has a hole 6 inches diameter bored through it from end to end. The weight of the bar is then 900 lb. Find the width of the side of the bar if it is a regular polygon. One cu. inch of steel weighs 0·283 lb.

135. A hydraulic steering gear has 4 rams each 12 inches diam. The rudder stock is 18 inches diam. and its centre is 30 inches from the centre line of the rams. When the rudder had been turned 35° from its mid position it was struck by a heavy sea. If the stress induced in the rudder stock was 10,000 lb. per sq. inch when the spring loaded bye-pass valves lifted, at what pressure were they set to lift ?

136. Two cylindrical tanks of equal capacity are each 7 feet long. One contains an alkali and the other an acid in liquid form. When the liquids mix they form a foam whose volume is 8 times that of the liquids and the foam is discharged through a pipe 2 inches diameter. The engine room is 50 feet long and 75 feet wide and the foam is required to cover this to a depth of 6 inches. Find the diameter of the tanks if 5 per cent. of their volume is allowed for expansion. The hydraulic pressure is equivalent to a head of 36 feet. If the actual time to empty the tanks is 15 minutes find the ratio of the actual time taken to the theoretical time required neglecting friction, etc.

137. Water flows through a smooth bore horizontal pipe. The pipe has a tapered section, the diameter of the large part is 2 feet and that of the smaller part is 1 foot, and the difference in pressure between these two ends is equal to 8 inches of water. Find the quantity of water passing through the pipe in gallons per minute.

138. A horizontal beam 12 feet 9 inches long, 3·75 inches diameter, is made of material which weighs 490 lb. per cu. foot. It is securely fastened at one end and a load W lb. hung from the free end. The maximum stress set up is 9·2 tons per sq. inch. Find the load W.

139. A pair of rope pulley blocks has three sheaves in each block and the end of the rope is made fast to the top block. An effort of 520 lb. just raises a load of 2,000 lb. hanging on the lifting hook. The weight of each block is 60 lb. Neglecting the weight of the rope, find (*a*) the overall efficiency, (*b*) the efficiency of each sheave, and (*c*) the total load on the top hook.

140. A light flexible cord passes over a frictionless pulley. A weight of 25 lb. is hung from one end of the cord, and 56 lb. from the other end. Neglecting the weight of the pulley, find how far the weights will move from rest in 5 seconds.

141. A weight of 2 tons is suspended by 3 chains which hang from a horizontal plane. The chains are joined together at their lower ends by a ring from which the load is suspended. The chains are 5 ft., 5 ft., and 4 ft. long respectively. The top attachments form an isosceles triangle of sides 5 ft., 5 ft., and 6 ft. The points of attachment of the 5 ft. long chains are 6 ft. apart. Find the tension in each chain.

142. Fresh water issues from a hole 0·5 inch diameter in a tank and there is a constant head of 6 feet above the hole. If the co-efficient of velocity is 0·97, and the co-efficient of contraction of area is 0·64, find the weight of water passing through the hole per minute.

143. Prove that the deflection of a closely coiled helical spring is $\dfrac{64\ W\ R^3\ N}{C}$

where W = axial load. R = mean radius of coils.

N = number of coils. C = modulus of rigidity.

d = diameter of wire.

A closely coiled helical spring has 10 coils of steel wire 0·5 inch diameter, the mean diameter of the coils is 3 inches. Find what axial load will cause a deflection of one inch. Take the modulus of rigidity as 12×10^6 lb. per sq. inch.

144. A U-tube of uniform bore, one square inch area, has vertical branches 33 inches high, and contains mercury to a height of 6·8 inches in both sides. Water is now poured into one branch until this branch is full, find the weight of water poured in.

145. Find the strain energy, or resilience, of a steel bar 1·25 inches diameter and 8 feet long which carries an axial load of 10 tons. Draw the diagram of the work done on the bar by this load. E = 13,000 tons per sq. in.

146. A simple unloaded governor has its speed increased from 100 revs. per min. to 120. Find the vertical height through which the weights rise.

147. A tank 30 feet wide has a rectangular vertical division bulkhead. On one side there is fresh water to a depth of 20 feet, and on the other side there is oil of specific gravity 0·77 to a depth of 15 feet. Find the resultant pressure on the bulkhead in tons and the position of the centre of pressure.

148. A hollow cast iron sphere of uniform thickness weighs 32 kilograms and floats just awash in oil of specific gravity 0·83. Find the external and internal diameters.

149. A hollow shaft 15 ins. external diam. and 7 ins. internal diam. is subjected to a twisting moment of 1,600 inch tons. Find the maximum shear stress set up in the shaft in tons per sq. inch, and the angle of twist in degrees over a length equal to twenty times the outside diameter. C = 5,730 tons per sq. in.

150. Water flows through a smooth horizontal pipe of variable diameter. At one point the pressure of the water was 25 lb. per sq. inch and its velocity was 4 feet per second. At another position the velocity is 45 feet per second, what is its pressure here ?

151. A screw jack has a pitch of thread of 0·375 inch and the length of the lever is 19 inches. Find the velocity ratio. If a force of 10 lb. on the end of the lever just lifts a load of 500 lb., and a force of 30 lb. just lifts a load of 2,300 lb., determine the relation between effort and load and estimate the effort required to lift a load of 3,000 lb. and the efficiency at this load.

152. Prove that the height of a Porter governor is

$$w + W\big) \, \frac{g}{\ } \text{ feet when the links are all of equal length.}$$

where w = weight of one ball.

W = central weight.

g = 32·2 ft. per sec. per sec.

ω = angular velocity in radians per sec.

Find the height of a Porter governor revolving at 250 revs. per min., the weight of each ball is 3 lb. and the central weight is 40 lb.

153. Define impulse. A bullet weighing 1 ounce is fired from a gun at a velocity of 1,500 feet per second, and penetrates 8 inches into a block of wood. Find the average force exerted by the wood in pounds.

154. A treble riveted double strap butt joint has every alternate rivet omitted from the outer rows. The maximum pitch of the rivets is 9·375 ins., diameter $1\frac{7}{16}$ ins., and the plate thickness is $1\frac{7}{16}$ ins. Allowing a constant of 1·875 for double shear, and taking the shearing strength of the rivet material to be 23·5 tons per sq. inch, and tensile strength of plates 28 tons per sq. inch, find the strength of the joint at the outer, middle, and inner rows, and state what is the efficiency of the joint compared with the solid plate.

155. In a four-ram electric-hydraulic steering gear, the diameter of the rams is 12 inches, distance between centre-line of rams and centre of rudder stock is 31 inches, maximum angle

of rudder is 35°, and the bye-pass valves lift due to a pressure of 1,100 lb. per sq. inch. If the stress in the tiller arms at $24\frac{1}{2}$ inches from the centre of the rudder stock must not exceed 14,250 lb. per sq. inch, find the diameter of the tiller arms at this section.

156. A tank containing fresh water rests upon the ground and a rivet is blown out of it from a position 3 feet vertically above the base. The jet of water strikes the ground at a horizontal distance of 15 feet from the base of the tank. Neglecting all frictional losses, find the pressure in lb. per square foot on the tank bottom.

157. A weight travels in a circular path in a horizontal plane, suspended by a weightless cord. Prove that the vertical height from the plane to the point of suspension varies inversely as the $(\text{revolutions})^2$. Find the height of a conical pendulum when rotating at 75 revs. per min., and the change in height when the speed falls to 70 revs. per min.

158. A hydrometer stem is divided into 12 equal parts. The top mark represents a specific gravity of 0·5, and the bottom mark is for fresh water. What will be the specific gravity of the liquid in which the hydrometer floats at the 4th mark up from fresh water mark ?

159. Two bars of copper and one bar of steel, all of the same sectional area, are rigidly secured at their ends, the steel bar being in the middle. The temperature of the bars is raised 85F.° Determine the resulting intensity, and nature, of the stress in the bars, and the strain. The co-efficient of linear expansion of steel is 0·0000067 per degree F., and 0·0000095 per degree F. for copper. The modulus of elasticity is 30 million lb. per sq. inch for steel, and 15 million lb. per sq. inch for copper.

160. A jet of fresh water 0·75 inch diam. impinges on a bulkhead and exerts a pressure of 30 lb. on the plate. Assuming there is no splash back, find the initial velocity of the water, and the ft. lb. of energy expended per minute.

161. A grindstone, whose moment of inertia is 600 lb. ft.^{2} units, was started up from rest and attained a speed of 250 revs. per min. in 15 seconds. Find the couple which was applied.

162. An aeroplane, flying horizontally at 240 miles per hour, at a height of 2,000 feet, approaches a railway at right angles to its course. A train was travelling along this line at 60 miles per hour. A bomb was released from the plane and hits the

train. Neglecting air resistances find (*a*) what time elapses from the moment the bomb is released to the moment it strikes, and (*b*) the actual distance from the plane to the train when the bomb is released.

163. A beam of rectangular section is 14 feet long, 5 inches broad, and 9 inches deep, and it is simply supported at its ends. It carries a concentrated load of 8,400 lb. at 5 feet from the right hand end. Neglect the weight of the beam and find the position in the beam where the maximum stress is 2,000 lb. per sq. inch.

164. After having been hardened up, the bottom end connecting rod bolts are subjected to further tightening, resulting in an advance of 0·625 inch measured on the nut at the circumference of the thread. To what additional stress are the bolts subjected if half of the resulting deformation of the material takes place in the bolts. The bolts are mild steel, $2\frac{1}{2}$ inches diameter and 24 inches long. They have six threads per inch. $E = 30 \times 10^6$ lb. per square inch.

165. A cotter has a taper of 1 in 8 divided equally between the two edges. The co-efficient of friction is 0·2. If a force of 600 lb. is required to drive the cotter in, find (*a*) the force holding the parts together, and (*b*) the force necessary to drive the cotter out.

166. A derrick 11 feet long is secured by a tie 8 feet long to a vertical post, the points of attachment to the post are 6 feet apart. A load of 4 tons is suspended by means of a wire rope from the end of the derrick; the wire passes over a pulley at the derrick head and is led down at an angle of 60° to the vertical, to a snatch block on the deck and then to the winch drum. Draw to scale the vector diagram of the forces acting at the crane head, and measure off the forces in the derrick and tie. Find also the horse power indicated by the winch when the load of 4 tons is lifted steadily at the rate of 2 feet per second, taking the efficiency of the lifting gear to be 68% and the mechanical efficiency of the winch as 86%.

167. Two vessels leave one port for another 1,065 nautical miles away. One ship is 3 knots faster than the other. The slow ship leaves 14 hours before the fast one and arrives at her destination 2 hours sooner. If they both took the same course, what were the speeds of the ships?

168. A train weighing 150 tons attains a speed of 30 miles per hour, from rest, in $1\frac{1}{2}$ minutes; the tractive resistance is 16 lb. per ton, find the average force exerted on the train during

this time. Find also the horse power exerted when the train is running at a constant speed of 30 miles per hour.

169. A main steam pipe is firmly fixed to a stop valve on the boiler, and at the other end to a tee-piece on the steam range. The main steam pipe is 10 feet long, 6 ins. outside diameter and 0·375 inch thick. If the steam range expands laterally and causes 0·3 inch deflection in the main steam pipe, find the maximum stress set up in the pipe.

Note.—The main steam pipe may be treated as a cantilever.

$$\text{Deflection} = \frac{W\,l^3}{3\,E\,I}, \quad E = 29 \times 10^6 \text{ lb. per sq. in.}$$

$$\frac{\quad}{64}\,(D^4 - \qquad \text{inch}^4 \text{ units.}$$

170. A short steel tube is rigidly fitted over a copper bar. The lengths of the tube and the bar are exactly equal, and their cross sectional areas are each 1·25 sq. inches. This combined bar carries an axial load of 20 tons. Determine the loads carried by the steel and the copper respectively. E for steel is 13,500 tons per sq. inch, E for copper is 6,000 tons per sq. inch.

171. A solid steel cone, 3 ft. 6 ins. diam. at the base and 6 ft. perpendicular height, rests on its base on horizontal ground. Find how many ft. lb. of work are required to tilt it over through an angle of 40°.

172. A boiler weighing 50 tons is being lowered into a ship by a crane. When lowered, the length of the wire from crane head to boiler is 82 feet, and the boiler is found to be 5 feet horizontally out of position. To rectify this, a pair of quadruple rope pulley blocks are attached to the boiler, arranged to exert a horizontal pull to bring it into position. Find the force required at the fall end of the blocks if 30 per cent. is lost in friction.

173. Prove that the periodic time of a simple pendulum is $2\,\pi\sqrt{\frac{l}{g}}$. A pendulum makes 30 complete vibrations per minute, find its length.

174. Two ropes 3 inches circumference hang from hooks and are connected together at their lower ends; a weight of 4 tons hangs from this connection. One rope hangs at 30° to the horizontal and the other at 45°. If the breaking load in lb. is $6{,}700 \times G^2$ where G is the girth in inches, find the factor of safety.

175. A round steel table 5 feet diam. and 1 inch uniform thickness weighs 801 lb. It has four steel legs 4 ft. long of a uniform cross section, weighing 32 lb. each, welded on to the table at a concentric pitch circle diameter of 3·75 ft. Find the force, which, if applied to the edge of the table, will just tilt it, if the force is applied (*a*) in an upward direction, (*b*) in a downward direction. Find also for both cases (*a*) and (*b*), the ft. lb. of work done in tilting the table until it is just about to fall over.

176. A beam 24 feet long is simply supported at each end. Four loads of 2, 4, 6 and 3 tons respectively, are hung from points at 5, 9, 14 and 20 feet respectively from the left hand support. Neglect the weight of the beam and draw to scale the shearing force and bending moment diagrams using the following scales : 1 inch = 4 feet length, 1″ = 4 tons shearing force, 1″ = 20 ft. tons. of moment. Read off your diagrams, and write down the shearing force and bending moment at (*a*) 3 ft., (*b*) 12 ft., from the left hand support.

177. The lengths of the sides of the base of a rectangular pyramid are 4 inches by 4 inches, and the slant height measured on the edge is $6\sqrt{2}$ inches. Find the volume and the position of the centre of gravity, by Simpson's rule.

178. Define acceleration. A body falls from rest and passes two points 1,542 feet apart in 7·6 seconds. Find the height above the higher point from which the body was released. If the body continued to fall, find its velocity after 12 seconds from point of release.

179. A horizontal smooth bore venturi water meter is 12 ins. diam, at the entrance and 4 ins. at the throat, and the pressure difference between the entrance and throat is equal to 30 inches of fresh water. Calculate the gallons of water passing per minute.

180. A wheel and differential axle has a velocity ratio of 30. It is found that an effort of 25 lb. just lifts a load of 240 lb. and that an effort of 7 lb. just lifts a load of 24 lb. Find what effort would be required to lift a load of 720 lb. and the efficiency at this load.

181. The weight of a motor car in tons, multiplied by the miles per gallon, equals 33. The tractional resistance is 162 lb. per ton. Find the consumption of petrol in pints per B.H.P. per hour.

182. A hollow cylindrical cast iron beam 20 ins. outside diam. and 14 ins. inside diam. is 10 feet long and rests upon two supports, one at each end. Find what load, if placed at mid-length, will induce a maximum stress of 700 lb. per sq. in. in the beam.

183. A coal bunker contains 1,077 tons of coal when stowed at 43 cu. ft. per ton. The length of the bunker is 56 ft. The depth of the bunker is three-sevenths of the width of the bunker across the top. The width across the bottom is half of the width across the top. At a quarter depth the width is 45 ft., at half depth the width is 40 ft., and at three-quarter depth the width is 34 ft. Find the width of the bunker across the top.

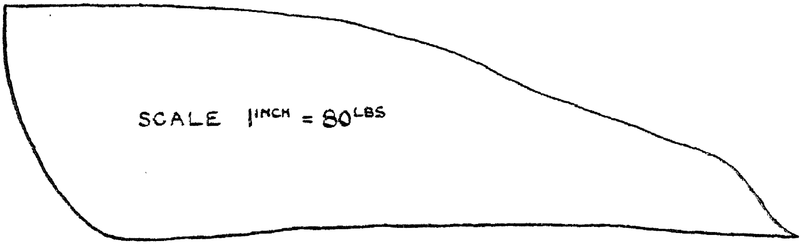
184. The twisting moment on a crank shaft is 547 inch tons. The speed of the ship is 14·3 knots and its total resistance through the water at this speed is 32,000 lb. Find the revolutions turned per minute by the shaft, and the I.H.P. of the engine, if the mechanical efficiency of the engine is 0·83, and the propeller efficiency is 0·7.

185. The weight of a flywheel is 6 tons and its radius of gyration is 4·5 feet. The flywheel shaft is running at 120 revs. per min. in bearings 8 ins. diameter and the co-efficient of friction is 0·01 at all speeds. Find (*a*) the ft. lb. of energy stored in the flywheel, (*b*) the ft. lb. of energy lost to friction per revolution, (*c*) the number of revolutions the flywheel will make in coming to rest when the driving motor is cut out.

186. A solid shaft is to be replaced by a hollow shaft of the same outside diameter, the material of the latter being 30 per cent. stronger than that of the former. Find the ratio of the internal diameter of the hollow shaft to its external diameter, and the percentage saving in weight.

FIRST-CLASS EXAMINATION QUESTIONS.

HEAT AND HEAT ENGINES.



1. The above diagram was taken from the H.P. cylinder of a triple expansion engine. The cylinder was 26 inches diameter, the stroke was 4 feet, and the revolutions were 65 per minute. Calculate the weight of steam used by the engine per hour from this diagram. Assume that the bottom diagram is similar to the one given.

2. An engine develops 1,500 I.H.P., and the steam consumption is 15·5 lb. per hour per I.H.P. The injection water is 57° F., and the discharge is 115° F., whilst the hotwell temperature is 125° F. The exhaust steam from the L.P. is 162° F., and it may be assumed to be dry. Find the tons of circulating water used per day.

3. The cylinder of an engine is 26 inches diameter and cut off takes place at 26 inches. The stroke is 4 feet and the revolutions are 65 per minute. The boiler pressure is 180 lb. per square inch and the back pressure is 55 lb. per square inch, both by gauge. The clearance is 10 per cent. of the volume swept out by the piston. Find the mean effective pressure and the I.H.P. Assume that the steam expands according to the law $p\ v =$ constant.

4. Steam is admitted to a cylinder at 150 lb. per square inch by gauge and the back pressure is 42 lb. per square inch gauge. The clearance is 8 per cent. of the volume swept out by the piston. The slide valve closes to exhaust at 0·86 of the return stroke. Find the pressure in the cylinder at the point of admission, which takes place at the end of the stroke.

5. Owing to a leaky condenser the feed water density is 3·5 ozs. per gallon. The boilers are kept at 18 ozs. per gallon by blowing down. If the steam temperature is 380° F. and the feed temperature is 160° F., and the coal consumption was 20 tons per day before the leakage started, what is the consumption now ?

6. An engine has a stroke of 40 inches. The valve opens to exhaust at 0·9 of the stroke. The pressure by gauge at cut off and at release are 145 lb. and 102 lb. respectively. The clearance is equal to 10 per cent. of the volume swept out by the piston. Find the point in the stroke at which cut off occurs.

7. The sum of the areas of a pair of indicator diagrams is 3·5 square inches. The length of each card is 4·5 inches, and the scale is $\frac{1}{8}$.. Find the I.H.P. of the engine, the cylinder being 20 inches diameter, the stroke 2 feet and the revolutions 80 per minute.

8. The steam pressure is 124 lb. per square inch by gauge. The clearance is equivalent to 1·25 inches of the stroke. At the point of cut off, if all the steam in the cylinder was condensed, the depth of water on the piston would be 0·1875 inch. Find the cut off in inches of the stroke.

$$\text{Relative volume of steam} = 16 \times \frac{1{,}600}{P + 1} \text{ nearly.}$$

P = absolute pressure.

9. The mercury barometer stands at 30 inches when the atmospheric pressure is 14·7 lb. per square inch. What pressure is lost due to air in the condenser when the barometer stands at 29·8 inches and the vacuum gauge reads 23 inches ? The temperature of the condenser is 104° F., which corresponds to an absolute pressure of 1·04 lb. per square inch.

10. A steam engine cylinder is 24 inches diameter. The stroke is 4 feet and the connecting rod is 9 feet long. The boiler pressure is 180 lb. per square inch. Cut off takes place at 0·6 of the stroke. What must be the area of the guide shoe in order that the maximum pressure shall not exceed 100 lb. per square inch ?

11. A mild steel steam pipe, 12 feet long, is rigidly fixed, with no provision for contraction, when at a temperature of 360° F. The temperature falls to 100° F. What will be the stress per square inch ? The co-efficient of linear expansion of mild steel is 0·0000067 per 1° F. and the modulus of elasticity is 13,500 tons per square inch.

12. A triple expansion engine has cylinders 26″, 42″ and 71″. The mean pressures are 73, 27 and 10·5 lb. per square inch respectively. The revolutions are 72 per minute. Find the new revolutions if the vacuum drops back 4 inches.

Note.—Mean pressure varies directly as revolutions squared.

13. The L.P. piston is 67 inches diameter. The piston rod is 7 inches diameter and 5 inches at the bottom of the thread on cone end. The tail rod is 5 inches in diameter. At one point in the up stroke the driving pressure is 12 lb. per square inch by gauge, and the back pressure 5 lb. per square inch absolute. What is then the maximum stress in the rod ?

14. A compound engine uses 33 tons of coal per day. It is converted to a triple expansion engine, and develops 21·67 per cent. more power per ton of coal. If the converted engine is run at an increase in horse power of 18 per cent., what is the consumption ?

15. A steam engine uses 2,700 cubic feet of steam per hour, the steam being carried for the full length of the stroke. The steam pressure is 55 lb. per square inch and the back pressure 5 lb. per square inch, both by gauge. What is the horse power of the engine ?

16. Find the effective horse power of the feed pumps for a marine engine of 2,500 I.H.P., which uses 15 lb. of steam per I.H.P. per hour. The boiler pressure is 180 lb. per square inch.

17. An engine indicates 1,500 I.H.P. The steam consumption is 15 lb. per I.H.P. per hour. The condenser is leaking and the hotwell density is 0·08 of the sea density. The boilers are kept at 3·5 times the sea density by blowing. How many tons are blown out per day ; how many tons leak into the condenser per day, and how many tons flow into the bilges ? Assume that 10 tons of " make-up " feed are required per day, this being supplied from the leakage that is taking place in the condenser.

18. Steam from the L.P. receiver is used to heat the feed water. It raises the temperature from 124°F. to 168°F. The I.H.P. of the engine is 2,500 and the steam consumption is 14·5 lb. per I.H.P. per hour. What is the equivalent horse power of the heat put into the feed water ?

19. Owing to the condenser leaking, the feed water density is 0·3 of the sea density. The boiler is maintained at a density of 3·4 times the sea density. The steam temperature is 370°F. and the feed 120°F. Previous to the condenser leaking the consumption was 35 tons of coal per day. What is it now ?

20. One gallon of sea water has in solution 1,870 grains of sodium chloride ; 4 grains of calcium carbonate ; 93 grains of calcium sulphate. A boiler holds 40 tons of water at the working water level, and was filled with fresh water. At the end of the voyage the density was three times the sea density. How many pounds of scale forming matter have been deposited in the boiler ?

21. A certain percentage of the steam generated by the boiler at 382° F. is passed through a surface heater. The remainder passes through the engines and is delivered to the hot well as water at 125° F. where it mixes with the drain from the heater. All is returned through the heater and is delivered to the boiler at 190°F. What percentage of the steam is used by the heater ?

22. The leads of a slide valve are 0·12 inch at top end and 0·25 inch at the bottom end. The exhaust laps are, Top, — 0·1 inch ; Bottom, + 0·25 inch. The opening to exhaust when the engine is on the top centre is 2 inches. Find the top steam lap.

23. A boiler generates steam of temperature 381° F. and dryness fraction 0·94 from feed water at 160° F. What percentage more fuel will be burnt in order to superheat the steam 50 F.° at constant pressure ? Specific heat of steam = 0·48.

24. A slide valve has a travel of 3·2 inches. The steam lap is 0·7 inch and the port opening to steam when the engine has passed through 90° from the centre is also 0·7 inch. Calculate the lead of the valve.

25. The feed pumps of an engine are driven by means of levers from the engine crosshead. Their mechanical efficiency is 42 per cent. and they require 0·008 of the indicated horse power to drive them. The boiler pressure is 210 lb. per square inch. Calculate the pounds of steam used per I.H.P. per hour.

26. The lead of a slide valve is 0·25 inch. The steam lap is 1·25 inch and the port opening to steam after the crank has turned through 90° from the centre is 2·25 inches. Calculate the valve travel.

27. A slide valve has a travel of 6 inches. The angle of advance of the eccentric is 30°. How far is the valve from its mid-travel when the crank has turned through 35° from the centre ?

28. 20 lb. of water at 120° F. are mixed with a second quantity of water which has the same heat contents, measured from 32° F. The resulting temperature is 150° F. Find the weight and the temperature of the second quantity of water.

29. The horse power of a pump is 15. It carries steam for the full stroke and uses 2,700 cubic feet per hour. If the initial pressure of the steam is 80 lb. per square inch by gauge, what is the back pressure ?

30. If it requires 9 per cent. more heat units to completely dry a sample of wet steam having a temperature of 394° F. and which has been formed from water at 200° F., what was the dryness fraction of the sample ?

31. A piece of copper weighs 2 lb. and has a temperature of 350° F. It is dropped into a quantity of water which has four times the number of heat units that the copper has, both being measured from 32° F. If the resulting temperature is 84° F., find the weight and the temperature of the water.

32. The surface condenser of an engine is leaking, and blowing down from the boilers is resorted to, to keep the density at four times the sea density. The steam temperature is 380° F. and the feed temperature is 140° F. If the increase in the coal consumption is 2 per cent., find the density of the feed water.

33. Air is drawn into the cylinder of a gas engine at 14·7 lb. per square inch absolute, and it is compressed to 114 lb. per square inch absolute, this pressure being attained at the end of the stroke which is 18 inches. If the law for the compression is $p\ v^{\frac{4}{3}} = c$, what is the clearance volume in inches of the stroke ?

34. The water in an evaporator has a density of 1,070 ozs. per cubic foot after being worked for $2\frac{1}{2}$ hours without blowing, or brining. The weight of fresh water made has been 1·25 tons. If the sea water density is 1027 ozs. per cubic foot, find the weight of sea water the evaporator holds.

35. A cubic foot of air at 15 lb. per square inch absolute is compressed to 300 lb. per square inch gauge. Find the final volume of the air, if the compression is isothermal, also if adiabatic.

36. An oil fuel contains 85 per cent. carbon, 13 per cent. hydrogen and 2 per cent. oxygen. The atomic weights are carbon, 12 ; hydrogen, 1 ; oxygen, 16. The atmosphere contains 23 per cent. by weight of oxygen. Find the theoretical weight of air to burn 1 lb. of the oil.

37. 4 kilograms of ice at — 25°C. are placed in a vessel containing 10 kilos. of water at 45° C. 6 kilos of water at 65°C. are added. Find the resulting temperature in F°. when the ice has melted, and assuming that no heat has been lost or gained due to radiation.

38. The travel of a slide valve is 7·75 inches. The angle of advance of the eccentric is 30°, and the port opening to steam when the crank is 100° past the top centre is 1·25 inches. Find the lead of the valve.

39. A triple expansion engine has the H.P. cylinder 26 inches diameter. The stroke is 4 feet ; the connecting rod length is 100 inches and the shaft 13 inches diameter. The boiler pressure is 180 lb. per square inch by gauge, and cut-off takes place in the H.P. cylinder at 0·6 of the stroke. Find the number of times the steam is expanded. Diameter of the shaft for a triple expansion engine with three cranks at 120° is given by :—

$$\sqrt[3]{\frac{C \times P \times D^2}{f\left(2 + \frac{D^2}{d^2}\right)}}$$

Where C $=$ length of crank in inches.

P $=$ boiler pressure (absolute).

D $=$ diameter of L.P. cylinder.

d $=$ diameter of H.P. cylinder

f $=$ 1,110.

40. When the crank has passed through 45° from the bottom centre the effective load on the piston was 47 tons. The length of the connecting rod is 112 inches and the stroke is 4 feet 4 inches. If the shaft is 13 inches diameter, find the torsional stress.

41. Air is drawn into the cylinder of a Diesel engine at atmospheric pressure and it is compressed up to 500 lb. per square inch gauge. If the clearance volume is 7·5 per cent. of the volume swept out by the piston, what is the law connecting pressure and volume ?

42. Water is heated in a sealed vessel to 350°F. An escape valve is now opened and the temperature falls to 212°F. Some of the water evaporates and is allowed to escape. If the experiment is repeated without adding more water to the vessel, to what temperature must the water be heated so that the same weight of water evaporates when the temperature again falls to 212°F.

43. The travel of a slide valve is 6 inches. The steam lap is 1·125 inches and the lead is 0·125 inch. What is the port opening to steam when the crank is 90° past the dead centre ?

44. Find the indicated horse power and the pounds of fuel per I.H.P. per hour, of a 4-stroke Diesel engine which uses 3·6 tons of fuel per day. It has 8 cylinders 508 m.m. bore and 974 m.m. stroke. The revolutions are 85 per minute and the mean effective pressure from the cards is 6·35 kilos per square centimetre.

45. An engine burns 79 tons of coal per day and each pound of coal evaporates 8·5 pounds of water. The condenser is leaking and 40 tons of water are blown out of the boilers per day to maintain a density of $\frac{4}{32}$. What is the density of the feed ?

46. Owing to a leaky condenser scumming is resorted to, and the density of the boiler is kept at 3·7 times the sea density. The feed density is 0·55 of the sea density. The coal consumption is found to have increased 4·3 per cent. since the leaking began. If the feed temperature is 133°F., what is the steam temperature?

47. The stroke of a gas engine is 22 inches and the clearance is 4·3 inches of the stroke. Air and gas are drawn in at atmospheric pressure and compressed to 175 lb. per square inch gauge. If the compression follows the law, $p\ v^n$ = constant, find the value of "n."

48. The data obtained when determining the calorific value of a sample of fuel oil were :—Weight of sample of oil 0·81 gram. Weight of water in calorimeter 2,270 grams. Water equivalent of the calorimeter 412 grams. Rise of temperature of the water due to the combustion of the fuel 3·11 C°. Determine the calorific value of the fuel in gram calories per gram, and in B.T.U. per lb.

49. A boiler has a fire grate area of 95 sq. feet and the ratio of total heating surface to grate area is 36 to 1. Two safety valves are fitted and their diameter is $\frac{3}{4}$ inch greater than if 3 valves had been fitted. Find the boiler pressure given that :—

Total area of safety valves in square inches

$$\frac{1{\cdot}5 \times \text{Heating surface in sq. feet}}{\text{Absolute boiler pressure}}$$

63. Define Latent Heat, and state how it is affected by pressure. In a certain boiler, feed water is supplied at 230°F. and wet steam is produced at 328°F., find the dryness fraction of this steam if it would take an increased fuel consumption of 5% to produce dry steam.

64. A two-stroke engine of 3,600 I.H.P. uses 0·37 lb. of fuel per I.H.P. per hour, and was fitted with an exhaust gas boiler. The data obtained from the boiler were :—Exhaust gas inlet 620°F. Exhaust gas outlet 420° F. Steam temperature 390° F., and dryness fraction 0·96. Feed temperature 120° F. Thermal efficiency of boiler 0·8. The engine used 18 lb. of air per lb. of fuel burnt in the cylinders. Calculate the weight of steam formed by the boiler per hour. Assume the mean specific heat of the gases to be 0·203.

65. The output of a turbo-generator is 7,500 kilowatts. The electrical efficiency is 95 per cent. neglecting all other losses. 35,000 cubic feet of air at 70° F. per minute are blown through the windings at a pressure of 6 inches of water. The windings are kept at 55° C. What is the difference in temperature between the windings and the final temperature of the air ? The weight of 1 cubic foot of air at 32° F. and at a pressure equal to a head of 34 feet of water is 0·0807 lb. The specific heat of air at constant pressure is 0·24.

66. A six cylinder two-stroke Diesel engine developing 3,600 I.H.P. at 85 revolutions per minute, burns 0·35 lb. of fuel per I.H.P. per hour. At each stroke 10 cubic feet of air at 2 lb. per square inch gauge and at 85° F. enter the cylinder. An analysis of the fuel gave 84 per cent. carbon, 15 per cent. hydrogen and 1 per cent. incombustible matter. The atomic weight of carbon is 12, oxygen 16, and hydrogen 1, and the air contains 23 per cent. by weight of oxygen. One cubic foot of air at atmospheric pressure and at 32° F. weighs 0·0807 lb. Find :—(*a*) The theoretical weight of air required per cycle. (*b*) The actual weight of air supplied per cycle. (*c*) The weight of excess air per cycle.

67. A three stage air compressor has cylinders 3 inches, $13\frac{1}{2}$ inches and 15 inches diameter. It is connected to 3 air storage bottles of equal size which have internal diameters of 12 inches and are 8 feet long overall with hemispherical ends. How long will it take to lift the relief valves on the bottles, which are set to lift at 1,200 lb. per sq. inch gauge, starting from atmospheric

pressure ? The stroke of the compressor is 12 inches, the revolutions 150 per minute, and the volumetric efficiency is 0·91. The volume of the pipes, etc., connecting the bottles to the compressor is equal to 10 per cent. of the volume of the bottles.

68. A twin screw Diesel driven ship has 4 air reservoirs having a total capacity of 1,000 cubic feet. The compressor is motor driven and has cylinders 3 inches, $6\frac{1}{2}$ inches and 15 inches diameter, a stroke of 15 inches, and the volumetric efficiency is 0·9. The compressor runs at 180 revolutions per minute. The compressor supplies air to blast bottles at 900 lb. per sq. inch absolute and a leak off from this line supplies the air reservoirs at 350 lb. per sq. inch absolute. While manœuvring, the pressure in the air reservoirs falls to 250 lb. per sq. inch absolute. The compressor is working continuously, and the engine takes 50 per cent. of the air delivered. How long will it take to bring the pressure in the air reservoirs back to 350 lb. per sq. inch absolute ?

69. A slide valve has a travel of 7 inches. The bottom lead is $\frac{1}{4}$ inch and the angle of advance of the eccentric sheave is 37° 22′. Assuming the valve to have outside steam admission, find the angle turned through by the crank from its bottom centre when the steam port is open one inch.

70. A steam turbine was originally supplied with steam at 235 lb. per sq. inch absolute (temperature 395° F.) and having a dryness fraction of 0·94. The steam was expanded down to 2·4 lb. per sq. inch abs. (temperature 133° F.) when its dryness fraction was 0·755. A vacuum augmentor is now fitted and the terminal pressure is reduced to 1 lb. per sq. inch abs. (temperature 101·7° F.) and also the initial steam is superheated 200 F°. Taking the specific heat of superheat as 0·56 and the final dryness fraction of the steam as 0·828, find the percentage increase in power due to these improvements, if the horse power is proportional to the heat drop of the steam during its passage through the turbine.

71. 91·5 kilograms of water at 54° C. and 97·5 kilograms of water at 0° C. are mixed with 31·5 kilograms of ice at — 25° C. If the temperature of the mixture becomes uniform, find the resultant temperature in degrees Centigrade.

72. 3 cubic feet of air at atmospheric pressure are compressed to a gauge pressure of 450 lb. per sq. inch. Find the final volume in cubic inches (*a*) if compressed isothermally, (*b*) if compressed adiabatically, following the law $p\ v^{1\cdot 4}$ = constant.

73. Define boiler efficiency, and equivalent evaporation from and at 212° F. A test was carried out on a boiler with a low grade fuel and the following results were obtained :—

Weight of coal burnt per hour = 160 lb.

Calorific value of coal = 10,110 B.T.U. per lb.

Weight of feed water entering boiler per hour = 816 lb.

Temperature of feed water = 145° F.

Saturation temperature of steam = 380° F.

Superheat temperature of steam = 720° F.

Taking the specific heat of superheated steam as 0·54, find the boiler efficiency and the equivalent evaporation from and at 212° F.

74. A sample of coal weighing 1·34 grams is burnt in a bomb calorimeter and the water, which weighs 1,634 grams, is raised in temperature from 14·3° C. to 20·4° C. The water equivalent of the calorimeter was 223 grams. Find the calorific value of the coal (*a*) in gram calories per gram, (*b*) in B.T.U. per lb. Make a sketch of the calorimeter.

75. The total load on a single collar thrust block is 12 tons, the effective radius of the pads is 7·5 inches, and the speed of the shaft is 77 revs. per min. Find the quantity of oil required to pass through the block, in gallons per hour, to limit the rise in temperature of the oil to 10 F°. Take the co-efficient of friction as 0·05, specific heat of oil 0·48, specific gravity of oil 0·92.

76. Calculate suitable cylinder diameters for a compound steam engine of 1,200 horse power, taking the cut off in the H.P. cylinder at 0·5 stroke, cylinder area ratio 1 : 3·5, diagram factor 0·68, initial steam pressure 140 lb. per sq. in. absolute, back press. 3 lb. per sq. in. abs., piston speed 600 feet per minute.

77. 50 kilograms of dry steam at 100 lb. per sq. in. gauge pressure and at a temperature of 170° C. are blown into 4,000 litres of water at 45° C. Find the resultant temperature.

78. An engine of 7,800 I.H.P. uses 15·3 lb. of steam per I.H.P. per hour. The temperature of the exhaust is 162° F. and its dryness is 0·81. The hotwell is 120° F. If the circulating water inlet is 55° F. and outlet 84° F., and the work done in pumping the circulating water through the condenser is equivalent to pumping it to a height of 30 feet, find the horse power developed by the circulating pump.

79. The stroke of a steam engine is 3 feet 6 inches and cut off takes place when the crank has passed through 130° from its top centre. In this position the load on the piston is 28·9 tons and the load on the crank pin is 29·37 tons. Find (*a*) the total pressure on the guide, (*b*) the length of the connecting rod, and (*c*) the distance that the piston is from the bottom of its stroke.

80. An engine is coupled to a rope brake, and when running at 300 revs. per min. the load on the brake is 370 lb. on one end and 10 lb. on the other. The effective diameter of the brake is 5 feet. 85% of the heat generated by friction at the brake is carried away by a stream of cooling water which has a rise in temperature of 10 C°. Find the gallons of water supplied per hour.

81. The cylinder of a vertical steam engine is 12 ins. diam., the stroke is 18 ins., and the length of the connecting rod is 45 ins. The weight of the moving parts is 275 lb. When the crank is horizontal the effective steam pressure on the piston is 53·2 lb. per sq. inch and the de-acceleration of the piston is 161 feet per sec. per sec. Find the effective crank effort in this position, and also the turning moment.

82. Steam at 390° F. is blown into water whose initial temperature is 60° F. and causes the absolute temperature and weight to be increased by 10% and 5% respectively. Find the dryness fraction of the steam.

83. Two copper pipes of a telemotor system are each 0·5 inch internal diameter and 250 feet long when filled with oil at a temperature of 70° F. If the temperature of the oil rises to 95° F., how much oil will be released into the replenishing tank ? Neglect the amount of oil in the cylinders.

Volumetric expansion of oil = 0·00043 per degree Fahr.

Linear expansion of copper = 0·00001 per degree Fahr.

84. In the manufacture of lead pipes, the hot ingot is forced through a die by a ram having a force of 1,400 kilograms per square centimetre. Find the rise in temperature of the lead in degrees Centigrade, due to it being forced through the die.

85. An oil engine uses 0·38 lb. of fuel per B.H.P. per hour. The mechanical efficiency is 80%. Find the thermal efficiency based on the indicated horse power if the fuel is composed of 86% carbon, 12·1% hydrogen, 1·5% oxygen, and the remainder impurities.

86. State what is meant by the terms "Lower Calorific Value" and "Higher Calorific Value." 0·015 lb. of coal is burnt in a bomb calorimeter whose water equivalent was 3 lb. The weight of water in the calorimeter was 12 lb. and its rise in temperature was 6·8 C°. Find the calorific value of this fuel and state whether your answer is the higher or lower value.

87. Find suitable cylinder diameters for a triple expansion engine to develop 2,500 I.H.P. with a piston speed of 600 feet per min. The initial pressure of the steam is 200 lb. per sq. inch absolute, and the back pressure is 3 lb. per sq. inch absolute. Take the diagram factor as 0·65, number of expansions 12, and ratio of cylinder volumes 1 : 2·7 : 7·2.

88. A throttling calorimeter was fitted on to a steam pipe wherein the steam pressure was 120 lb. per sq. inch (saturation temperature 341° F.) The pressure of steam in the calorimeter was 15·5 lb. per sq. inch (saturation temperature 214·7° F.) and the temperature of the steam was 259° F. Taking the specific heat of superheated steam as 0·5, find the dryness fraction of the steam in the main steam pipe.

89. Steam from a nozzle impinges on the blades of an impulse turbine at a velocity of 3,200 feet per second, the angle of the nozzle to the direction of movement of the blades is 18°, and the linear velocity of the blades is 750 feet per second. Calculate the entrance angle of the blades and, if the exit angle is the same as that at entrance, find, neglecting friction, the absolute velocity of the exit steam.

90. A cold storage room is 15 feet long, 12 feet wide, and 7 feet high, and every wall is covered with 6 inches thickness of insulating material. The co-efficient of thermal conductivity of the insulation is 0·0003 gram-calorie per centimetre cube per second. Find the quantity of heat to be taken away from the room every minute, in B.T.U., to maintain the temperature inside at — 5° C., when the outside temperature is 25° C.

91. The consumption of oil in a boiler is 1,400 lb. per hour. The constituents of the oil are 85% carbon, 13% hydrogen, and 2% oxygen. Find (*a*) the theoretical amount of air required per lb. of fuel for perfect combustion, and (*b*) if the actual amount of air supplied is 70% in excess of the theoretical quantity, find the weight of flue gases passing up the chimney every hour.

92. 8,025 lb. of coal having a calorific value of 12,950 B.T.U. per lb., are burnt in the boilers of a steam engine plant every hour, and 80,800 lb. of dry steam are generated at 210 lb. per

sq. inch by gauge. The temperature of the steam is 395° F. and that of the hotwell water 169° F. Find the efficiency of the boilers, and, if the overall efficiency of the plant is 11%, find the thermal efficiency based on the I.H.P. if the mechanical efficiency of the engine is 80%.

93. Steam at 195 lb. per sq. inch gauge is supplied to the H.P. cylinder of an engine, and is cut off at 0·42 of the stroke. The clearance volume is equal to 8% of the piston swept volume, the back pressure is 58 lb. per sq. inch gauge and the compression pressure at the end of the exhaust stroke is 190 lb. per sq. inch gauge. Find (*a*) the fraction of the exhaust stroke where the valves closes to exhaust, (*b*) mean gross pressure, and (*c*) mean effective pressure. Assume $p\ v$ = constant for both curves.

94. Steam leaves the nozzles of a De Laval turbine at a velocity of 3,600 feet per sec. The steam jet is at an angle of 20° to the direction of motion of the blades, and the linear velocity of the blades is 600 feet per second. If the turbine uses 1,800 lb. of steam per hour, find the force exerted on the blades, (*a*) neglecting friction, (*b*) if the friction loss is 12%, and (*c*) the H.P. developed when the friction loss is 12%. Assume the blades are symmetrical, that is the entrance and exit angles are equal.

95. A refrigerating machine makes $\frac{1}{2}$ ton of ice per hour, at 30° F., from water at 48° F. The brine in the evaporator coils is at 16° F., and the latent heat of CO_2, at this temperature, is 105·5 B.T.U. per lb. On entering the evaporator coils the CO_2 has a dryness fraction of 0·34 and on leaving its dryness fraction is 0·92. Estimate the pounds of CO_2 that pass through the coils per hour.

96. In a vapour compression refrigerating machine the ammonia employed is liquefied at 70° F. After passing through the regulating valve its temperature is 14° F. Find the dryness fraction of the ammonia immediately after passing the valve, upon the assumption that it was entirely liquefied at the higher temperature.

Latent heat of ammonia = 566 — 0·8 t B.T.U. per lb. where t is the temperature in F°.

Specific heat of ammonia = 1·1.

97. The following values were taken from the expansion curve of an indicator diagram. Where the pressure was 100 lb. per sq. inch abs. the volume was 3 cu. feet and where the pressure was 67·85 lb. per sq. inch abs., the volume was 4 cu. ft. If the law of expansion is $p\ v^n$ = constant, find the value of n and the constant.

98. The values of the nett steam force acting on the piston of a horizontal steam engine for one stroke, obtained by measurement from the indicator diagram, and the accelerating forces found by calculation, are given in the table for various crank angles. Plot the piston effort on a base of crank angle turned through and from the curve find the crank angle when the piston effort is zero. If the stroke is 12 ins. and the connecting rod is 30 ins., find the twisting moment when the connecting rod and crank are at 90°.

Crank Angle in Degrees	0	30°	60°	90°	120°	150°	180°
Nett Steam Load in lb.	13100	15400	14200	9800	5600	1300	—5000
Accelerating Force in lb.	4600	3330	1450	—860	—2200	—2300	—3000

99. The fuel valve of a Diesel engine is open for 45° total movement of the crank, and it opens at 7° before the crank reaches the top centre. If the piston moves through a total linear distance of 134 millimetres while the fuel valve is open, find the stroke of the engine in millimetres and in inches.

100. The mean speed of a flywheel of a reciprocating engine is 750 revolutions per minute. If the speed varies from 0·5% above this mean speed to 0·5% below, find the angular acceleration of the flywheel.

Note. At top and bottom of the stroke the acceleration is nil. Assume the acceleration and the retardation to be equal and opposite.

101. 9 kilograms of ice at — 7° C. are mixed with 11·5 kilograms of water at 79° C. Find the resulting temperature in Fahrenheit degrees.

Sp. heat of ice = 0·5. Latent heat of water = 143 B.T.U. per pound.

102. A ship's engines indicate 9,000 H.P. when the speed is 12 knots, and the load on the thrust is 51 tons. The efficiency of the propeller is 90% of the mechanical efficiency of the engine. Find the propeller efficiency and the mechanical efficiency.

103. A turbine is supplied with steam at a pressure of 235 lb. per sq. inch, corresponding temperature 395·6° F., and superheated 200 F°. The steam is exhausted into a condenser at 2·5 lb. per sq. inch absolute, corresponding temperature 134·4°F., and a dryness fraction of 0·94.

If the shaft horse power developed is proportional to the heat lost by the steam whilst passing through the turbine, calculate the per cent. increase of power obtained when the pressure in the condenser is reduced to 1 lb. per sq. inch absolute, corresponding temperature 102·7° F. by fitting a vacuum augmentor. The final dryness fraction is 0·92.

Note.—The Specific heat of superheated steam should be taken as 0·48.

104. Find the difference in the level of the water in the gauge glass and in the boiler, when the temperature of the boiler water is 325° F., and the water in the gauge glass is 250° F. The water level in the glass is 4 feet above the bottom cock on the boiler.

Use the formula $V = 1 + \dfrac{(T - 39{\cdot}2)^2}{711\ (697 + T)}$

V (the volume of water) is unity when the temperature is 39·2° F.

T is the temperature in Fahrenheit degrees.

105. A ship has 3 boilers, each 17 feet diameter and 13 feet long. Before lagging the boilers, the temperature of the boiler plates was 350° F., and the stokehole temperature was 100° F. After lagging the shells and end plates, the temperature of the cleading was 150° F. and the stokehole 90° F. If the calorific value of the coal used is 12,500 B.T.U. per lb., find the tons of coal saved per day by lagging.

$= K\ T^4$, where Q = quantity of heat in B.T.U. radiated per sq. foot of surface per hour. T is the absolute temperature in Fah. degrees. K is a constant of value 16×10^{-10}

106. Steam leaves the guide blades of a reaction turbine at a velocity of 440 feet per second, the exit angle being 20°. The linear velocity of the moving blades is 284 feet per second. Assuming the channel section of fixed and moving blades to be identical, and assuming ideal conditions find (*a*) the entrance angle of the moving blades (*b*) the work done per pound of steam per second in this stage.

107. Wet steam, with 2% moisture, at 200 lb. per sq. inch by gauge (saturation temperature 388° F.), is passed through a non-conducting reducing valve and reduced in pressure to 100 lb. per sq. inch by gauge (saturation temperature 338° F.) Find the temperature of the reduced pressure steam. The specific heat of superheated steam is 0·48.

108. A solid metal sphere at 60° F. was placed into 24 gallons of water at 200° F. When the temperature of the sphere and the water had equalised it was found that the diameter of the sphere had increased by 0·15%. The specific gravity of the metal of the sphere was 2·56; its specific heat was 0·22, and the co-efficient of linear expansion was 0·0000128 per degree Fahrenheit. Find the original diameter of the sphere.

109. 1·2 tons of fuel are burnt every hour in the boiler furnaces of a ship. The analysis of the fuel is 85% carbon, 12% hydrogen, 1·5% oxygen and 1·5% impurities. Find the theoretical weight of air to burn one pound of the fuel. Actually, 50% excess air is supplied. If the stokehole temperature is 76° F., the funnel temperature 550° F., and the specific heat of the gases of combustion is 0·24, find the quantity of heat passing up the funnel every hour.

110. The exhaust steam from a turbine passes into the condenser at 116° F., and it is 13% wet. The hotwell temperature is 104° F. The circulating water enters the condenser at 55° F. and leaves at 90° F., its weight per hour being 520 tons. The tubes of the condenser are $\frac{3}{4}$ inch outside diameter and 12 feet long. If 14 lb. weight of steam are condensed per square foot of tube surface per hour, find the number of tubes in the condenser.

111. A Diesel engine uses 0·47 lb. of fuel per B.H.P. per hour, of calorific value 19,500 B.T.U. per pound. The mechanical efficiency of the engine is 80·5%. Find the per cent. loss of heat in the exhaust gases and cooling water respectively, if the exhaust gases carry away 10% more heat than the cooling water.

112. A copper calorimeter, of weight 12 grams, contains 70 grams of a liquid at 10° C. When 100 grams of steel, at 150° C. are placed in the liquid the temperature rises to 44·5° C. If there is no heat lost, determine the specific heat of the liquid.

113. Two cubic feet of air are compressed in a compression cylinder from atmospheric pressure (15 lb. per sq. inch abs.) to 330 lb. per sq. inch by gauge. The clearance volume of the cylinder is 100 cubic inches. Find the volume of air delivered per stroke if it is compressed (*a*) isothermally, (*b*) adiabatically. State which is the more economical method of compression, and give reasons.

114. When the crank of a high pressure engine has turned through 110° past top dead centre, the load on the crank is 52,700 lb. The connecting rod is 2·15 times the length of the stroke. Find the load on the guide, in tons, at this instant, and the fraction of the stroke at cut off if cut off occurs when the engine is in this position. The diameter of the cylinder is 27 inches, and the steam pressure in the middle pressure receiver is 70 lb. per sq. inch. Find the steam pressure on the high pressure piston at cut off.

115. A rope brake on the flywheel of an engine is kept at a uniform temperature by a stream of water amounting to 530 gallons per hour, the water entering at 20° C. and leaving at 45° C. The flywheel rim is 5 feet diameter, and the engine runs at 70 revs. per minute. Assuming that the water carries away 85 per cent. of the heat generated, find the effective load on the brake, and the brake horse power.

116. The fuel valve of a 2 stroke Diesel engine opens at 9° before the top dead centre, and it closes when the piston has completed 9 per cent. of its downward stroke. If the piston moved through 4 inches whilst the valve is open, find (*a*) the length of the stroke, (*b*) for how many degrees of the crank circle the fuel valve is open.

117. Steam at a pressure of 200 lb. per sq. inch absolute, dryness fraction 0·95 and corresponding temperature 382° F., is supplied to the coils of an evaporator and condensed to water at 260° F. The maximum pressure allowed in the evaporator is 30 lb. per sq. inch absolute, corresponding temperature 250° F. If, during a test, the feed and vapour valves are closed, and 2,000 lb. weight of steam are supplied to the coils per hour, find the minimum diameter of the safety valve if the lift is $\frac{1}{24}$ of the diameter.

Note.—The volume of steam at 30 lb. per sq. inch absolute is 13·72 cu. ft. per one pound, and the velocity of steam through the safety valve should be taken as 750 feet per second.

118. The output of a dynamo is 60 kilowatts at 240 revs. per minute. The dynamo is driven by a 6 cylinder 4-stroke oil engine, the mechanical efficiency of which is 0·8. Find the diameters of the cylinders if the mean effective pressure is 8·5 kilograms per sq. centimetre, and the stroke is 1·5 times the cylinder diameter. Assume the efficiency of the dynamo is 0·92. Give the answer in millimetres.

119. A six-cylinder 4-stroke single-acting internal combustion engine has a mean effective pressure of 85 lb. per sq. in. in each cylinder. The cylinders are 650 mm. diameter, the stroke is twice the cylinder diameter, and the B.H.P. developed is 1,000. If the mechanical efficiency expressed as a percentage is equal to the square root of the product of the revolutions per minute and the mean effective pressure, find the revolutions per minute and the mechanical efficiency.

120. A single screw ship has a quadruple expansion steam engine which uses 1·38 lb. of fuel per I.H.P. per hour. The calorific value of the fuel is 13,650 B.T.U. per lb., and the boiler efficiency is known to be 73%. Find (*a*) the thermal efficiency of the engine, (*b*) the I.H.P. when the consumption is 63 tons per day, (*c*) the total loss of heat per lb. of fuel burnt.

121. Steam leaves the nozzles of an impulse turbine at a velocity of 3,826 ft. per sec., the nozzles being inclined at 18° to the direction of movement of the blades. The blade speed is 347 ft. per sec. and the exit angle of the blades is 24°. The steam loses 20% of its velocity due to friction in passing through the blades. Calculate the entrance angle of the blades, and the absolute velocity and direction of the steam at exit.

122. The average dimensions of an engine room are 36 feet by 56 feet and its permeability is 75%. A carbon dioxide system of fire extinguishing is fitted, which must be capable of producing a saturation of 25% of CO_2 gas in the engine room up to a height of 20 feet. One cubic foot of air at 14·7 lb. per sq. inch absolute and at 32°F. weighs 0·0807 lb., the relative density of CO_2 gas to air is 1·518 to 1. Taking the engine room temperature as 60°F., find the weight of CO_2 gas required. If 2 lb. of liquid CO_2 have the same volume as 3 lb. of water, find the volume of the bottles in which the CO_2 liquid is stored.

123. An evaporator is worked at a constant density of $2\frac{1}{2}$ thirty-seconds. The water supplied to the evaporator is at 70°F., and the vapour temperature is 222°F. The steam to the coils is at 384°F., and the water from the drain 234°F. If 1 lb. of coal burnt in the furnaces of the boiler generates 7·5 lb. of steam, what weight of coal is required to make 24 tons of make-up feed if the evaporator is worked continuously ?

124. The following data were taken from a 4-stroke compression ignition engine during a test. I.H.P. = 308 ; B.H.P. = 195; Consumption of fuel per hour = 109·5 lb., calorific value of this fuel = 19200 B.T.U. per lb. ; weight of circulating water used per minute = 171 lb., inlet temperature 63°F., outlet temperature 118°F. Calculate the indicated and brake thermal efficiencies ; find the necessary data for, and draw up, a heat balance.

125. A turning engine is capable of producing a torque of 1600 inch lb. in its crank shaft, and the gear ratio is 1,000 to 1. The efficiency of the gear is 12%. A bar has been left across the guide and is held in position by two bolts which are each $\frac{7}{8}$ inch diameter. The engine is turned and the guide shoe comes on to the bar when the piston is exactly half way down the cylinder, find the stress set up in the bolts, the stroke of the piston being 42 inches and the connecting rod 7 feet long.

126. An air motor has a stroke of 12 inches and the clearance is equal to 0·5 inch of the stroke. Air is admitted at a pressure of 90 lb. per sq. inch gauge for three-eighths' of the stroke, and is expanded to 15 lb. per sq. inch gauge at the end of the stroke. Find the law connecting pressure and volume and the fraction of the stroke where the pressure is 70 lb. per sq. inch gauge.

127. State what you understand by the term "coefficient of performance" of a refrigerating machine. The capacity of a refrigerating machine is measured by the weight of ice that can be made at 32°F. from water at 32°F. in 24 hours ; find the weight of ice at 27°F. that can be made from water at 67°F. in 24 hours by a 4-ton machine. If the condenser temperature is 70°F., and the coefficient of performance 5·8, what is the temperature of the evaporator ?

128. A compression ignition engine has a compression ratio of 13 to 1, and 35 lb. of air are supplied per lb. of fuel burnt. Air is drawn into the cylinder at 14 lb. per sq. inch absolute and at 100°F. The law of compression is $p\,v^{1 \cdot 4}$ = constant, calorific value of the fuel 18500 B.T.U. per lb., specific heat of air at constant pressure 0·2375. Assume ideal conditions and find the maximum pressure and temperature in the cylinder during the cycle.

129. In a multi-collar thrust block there are 6 shoes and 7 collars, the inside and outside diameters of the rubbing surfaces are 12 and 20 inches respectively, and the pressure on them is 45 lb. per sq. inch. Assume propeller and engine efficiencies combined to be 69% and calculate the I.H.P. of the engine when the ship is travelling at 12 knots. Make a sketch of the shoe showing the bearing surface.

130. A shaft $17\frac{3}{4}$ inches diameter was found to have a flaw in it. It was put in a lathe and the flaw turned out and the diameter is now $16\frac{1}{2}$ inches. It is decided to increase the speed by 10%, what boiler pressure should now be carried if the original boiler pressure was 200 lb. per sq. inch by gauge ?

131. The difference in pressure between the two sides of the piston in a cylinder 20 inches diameter of a horizontal engine, is 110 lb. per sq. inch at a certain part of the stroke, and the acceleration of the piston is then 280 feet per sec. per sec. If the weight of the moving parts is 1400 lb., find the load on the crosshead.

132. A heavy oil engine has a crank pin circle radius of 21 inches, the connecting rod is 6 feet long and the centre line of the cylinder is off-set 2 inches. When the crank has travelled through 30° past the top vertical centre the load on the piston was 20 tons, find the pressure in lb. per sq. inch on the guide if the area of the guide shoe is 173 sq. inches. Find also the distance in inches the piston has moved down its stroke for this position of the crank.

133. An ordinary slide valve has 7·5 inches travel. The lead is 0·24 inch and the positive exhaust lap is 0·17 inch. When the crank is on the top centre the port is 2 inches open to exhaust. Find the steam lap, the angle of advance of the eccentric and the angular position of the crank when compression begins.

134. A marine engine is fitted with a surface heater that receives steam from the main engine supply line. The engine takes 93·6% of the steam from the boilers, the residue going to the heater. The engine steam eventually reaches the air pump as water at 126°F., thence it is lifted to the hotwell where it mixes with the drainage water from the heater coils. All then passes through the heater and is delivered to the boilers at 196°F. Assuming no loss of heat, find the initial temperature of the steam.

FIRST-CLASS EXAMINATION QUESTIONS

NAVAL ARCHITECTURE.

1. The ballast tank of a steamer is pumped up until the level of the water in the sounding pipe is 12 feet above the top of the tank. The vertical transverse floors are 2·2 feet apart and the tank is 28 feet broad. Find the upward load on one space in tons.

2. A cylindrical pontoon is 36 feet long and 5 feet diameter. It floats in sea water having a density of 1,017 ozs. per cu. foot at a draught of 2 ft. 10 inches. Find its displacement in tons.

3. A cubic foot of solid coal weighs 80 lb. and will absorb 8 per cent. of its weight of moisture. The coal is broken up and is stored in a compartment at 44 cubic feet per ton. The compartment holds 175 tons when full. It is flooded with sea water. How many tons enter the compartment ?

4. The areas of the vertical transverse sections of a steamer up to the water level are :—25, 140, 254, 327, 402, 353, 261, 185, 30 square feet. The distance between the sections is 20 feet and the vessel is 200 feet long. The displacement of the foremost portion is 5 tons, and the aftermost, 6 tons. Find the total displacement. The observations were taken when the vessel was floating in sea water.

5. The displacement of a ship is 5,600 tons. When passing from sea into a river the draught changes 3 inches. The sea water weighs 1,026 ozs. per cubic foot and the river water 1,009. Find the water plane area of the ship.

6. A steamer at sea displaces 7,000 tons and has a water plane area of 12,500 square feet. She enters a river where the hydrometer reads 1008, and burns 80 tons of coal in steaming up the river. The hydrometer registered 1027 at sea. Find the change in the draught of the steamer.

7. A forepeak bulkhead is triangular in shape and is 26 feet wide at the top and 15 feet deep. What is the load upon the bulkhead when the tank is full of fresh water ? State the position of the centre of pressure.

8. A ship has a displacement of 3,500 tons and the draught is 16 feet. The centre of gravity of the ship is 15 feet above the keel. 480 tons of cargo are discharged, the centre of gravity of this being 10 feet above the keel; also 200 tons are discharged, the centre of gravity being 17 feet above the keel. 350 tons are now loaded at 14 feet above the keel. Find the position of the centre of gravity of the ship.

9. A vertical sided flat-bottomed barge is 60 feet long overall, 12 feet beam, and floats at a draught of 5 feet in water whose specific gravity is 1·023. The deck plan shows the forward section meeting at an angle of 80°. The after end is semi-circular, and the middle section is rectangular. Find the displacement of the barge in tons, and the co-efficient of the water plane area.

10. A ship is 350 feet long and 40 feet beam. The difference between the tons per inch immersion at one draught and the tons per inch at another draught, is 1 ton. Find the difference between the co-efficients of the water plane areas at these draughts.

11. A ship has a displacement of 3,500 tons. 500 tons of cargo on board the ship are moved vertically downwards through 10 feet. 500 tons are then put aboard at 10 feet above the original centre of gravity. Find the alteration in the position of the C.G. of the ship.

12. A ship is 670 feet in length and has a co-efficient of displacement of 0·7. The original draught was 25 feet. Cargo has been taken on board and the increase in the wetted surface is 2,600 square feet. Find the alteration in draught. Use the expression :—

$$\text{Wetted surface} = 1{\cdot}7 \text{ L D} + \frac{\text{V}}{\text{D}}$$

13. When the speed of a steamer is reduced by 2 knots, the consumption per day is reduced by 52 tons and the consumption on a voyage of 4,500 miles is reduced by 22 per cent. Find the speed of the steamer and the consumption per day.

14. The bulkhead in a steamer is 45 feet wide and 28 feet high. The ship has an accident and water rises to a height of 18 feet on one side of the bulkhead and to 5 feet on the other. Find the nett load on the bulkhead and the position of the resulting centre of pressure.

15. A ship loads in a river where the hydrometer reads 1008. She proceeds to sea and burns 500 tons of coal. The draught is then 1 foot 9 inches less than when in the river. If the sea density is 1,028 and the waterplane area may be taken as 13,000 square feet in each case, find the original displacement of the ship.

16. A ship's speed is 19 knots and the consumption is 21 tons of oil per day. The speed was reduced to 16 knots and at that time there were 31 tons of oil on board. When the ship arrived in port 4 tons remained. How far was the ship from port when the speed was reduced and what was the consumption per day at the reduced speed ?

17. A ship has bunkers just sufficient to last between 2 ports when steaming at full speed. If the speed was reduced by 3 knots the consumption per day would be reduced by 40 per cent., but she would be $4\frac{1}{2}$ hours longer on the voyage. The bunkers left at the end of the voyage when steaming at reduced speed would be $12\frac{1}{2}$ tons. Find :—(*a*) The distance between the ports ; (*b*) The consumption per day at full speed ; (*c*) The full speed.

If now the consumption for the voyage was reduced 40 per cent. when the speed was reduced 3 knots, all other data remaining the same, what would be the distance between the ports, consumption per day and the full speed ?

18. A ship's length is 480·5 feet ; the beam is 48·5 feet and the draught is 24 feet. The co-efficient of displacement is 0·72. Calculate the speed, if the H.P. is 9,375 from the formula :—

$$\text{I.H.P.} = \frac{D^{\frac{2}{3}}\ V^3}{275}$$

D = displacement in tons, V = speed in knots.

19. A ship steams at 17 knots on a consumption of 130 tons of coal per day. When 1,300 miles from port there are 500 tons of coal on board. The speed is then reduced so that the ship takes $3\frac{1}{2}$ days longer to reach port than she otherwise would have taken. What is the consumption per day at the reduced speed and how much coal remains upon arrival ?

20. The I.H.P. of an engine is 2,000 when running at 75·3 revolutions per minute. The pitch of the propeller is 16 feet and its apparent slip is 10 per cent. If the mechanical efficiency of the engine is 0·87 and the efficiency of the propeller is 0·7, what is the load on the thrust in pounds ?

21. The frictional resistance of a plate similar to that of a ship's hull is 0·3 lb. per sq. foot at 600 feet per minute, when towed through fresh water. This frictional resistance is proportional to the square of the speed and also to the density of the water. A steamer has a wetted surface of 20,000 square feet and the draught is 24 feet. The indicated horse power is 1,500 and 70 per cent. of this is expended in overcoming skin friction when in sea water of density 1,026 ozs. per cubic foot. Find the speed of the steamer.

22. The engines of a triple screw steamer develop 19,500 H.P. and this gives a speed of 17·5 knots. The centre engine breaks down. What power should be developed by each of the other engines to give a speed of 12 knots ?

23. A ship has the following dimensions :—length 520 feet, beam 55 feet, draught 26 feet, co-efficient of fineness 0·78. Find the power required to propel this ship at 14 knots, and also the power necessary for a margin of 10 per cent. above this speed.

Use the formula :—

$$\text{I.H.P.} = \frac{D^{\frac{2}{3}}\ V^3}{C}; \quad \begin{array}{l} D = \text{displacement in tons.} \\ V = \text{speed in knots.} \\ C = 250. \end{array}$$

24. The wetted surface of a ship was 33,000 sq. feet and the H.P. expended in overcoming friction was 5,320 at 17·5 knots. The skin friction varies as $(\text{speed})^{1 \cdot 876}$. Find the resistance per sq. foot of an experimental ship which was travelling at 10 feet per second.

25. The wetted surface of a ship is twice that of a smaller ship of similar form and lines. The displacement of the larger vessel is 1,500 tons more than that of the smaller one. Find the displacement of the latter.

Note.—The wetted surface varies directly as $(\text{length})^2$.

26. A ship steams at 20 knots on a consumption of 260 tons of fuel per day. The ship is 2,040 miles from port and at this time it is decided to increase the speed so as to arrive in 96 hours. If the consumption varies as the 3·2 power of the speed, find the amount of fuel on board allowing a margin of one day for bad weather.

27. A rectangular bulkhead is 30 feet high and 47 ft. 9 ins. wide. Due to the ship being damaged, sea water rises to a height of 3 ft. 6 ins. on one side and 20 feet on the other. Find the nett load on the bulkhead and the position of the resultant centre of pressure.

28. Define Statical Stability, Transverse Metacentre, and Transverse Metacentric Height. A weight of 20 tons is shifted 30 feet across the deck of a ship of 6,000 tons displacement, causing the vessel to heel through an angle of 3 degrees. Find the transverse metacentric height.

29. Define Permeability and state what is meant by Reserve Buoyancy. A rectangular box barge is 200 feet long, 40 feet wide, and draws 10 feet of water. Two transverse bulkheads

divide the vessel to form a central compartment 30 feet long. If this compartment has a permeability of 60%, find the draught of the barge when the compartment is bilged and laid open to the sea.

30. A vessel floats at a draught of 21 feet in fresh water and the load water plane area is 14,000 square feet. The water plane areas below this, measured at regular intervals of 5 ft. 3 ins. are, 12,800, 11,300, 8,700, and 0 sq. feet respectively. Find the tons displacement. If this vessel now moves to sea where the hydrometer reading is 1028, find the change in draught neglecting any small difference that there may be in the water plane areas.

31. A vessel of 6,500 tons displacement floats at 22·5 feet draught. The centre of buoyancy is 13 feet above the keel and the transverse metacentre is 4 feet above the centre of gravity. Find the position of the centre of gravity above the keel if the moment of inertia of the water plane area is 1,500,000 feet^4 units.

32. A fore-peak bulkhead is in the form of an isosceles triangle, 18 feet vertical height, and 20 feet wide at the top. If the tank is filled to a height of 14 feet with fresh water, find the total load on the bulkhead in tons. At what position would you place a shore to support this bulkhead ?

33. A forepeak bulkhead is 12 feet deep. Transverse measurements across the bulkhead at regular intervals of 3 feet are, 16, 15, 13, 8·5, and 0, feet respectively. If the tank is filled with sea water, find the total pressure on the bulkhead in tons, and the position of the centre of pressure.

34. A ship is 350 ft. long, 45 ft. beam, and 23 ft. draught. The co-efficient of immersed midship section area is 0·83 and the block co-efficient is 0·72. Find (*a*) the displacement in tons at 35 cu. feet per ton, (*b*) area of immersed midship section, and (*c*) prismatic co-efficient.

35. A watertight door is 6 feet high by 3 feet wide. The door frame is held by 36 bolts on a pitch line at 3 inches from the edge of the door, and the area of each bolt at the bottom of the thread is 0·442 sq. inch. If sea water rises to a height of 30 feet above the sill of the door, find the average stress in the bolts.

Note.—The area upon which the water acts should be taken as that bounded by the bolt pitch lines.

36. A vessel has a displacement of 8,000 tons. The moment to change trim one inch is 800 ft. tons and the tons per inch immersion is 40 when floating on an even keel at 23 ft. 7 ins. draught. Find the new draught fore and aft when 100 tons of cargo are put on board at 80 feet aft of the centre of flotation. Take the centre of flotation to remain unchanged at mid-length.

37. A deep tank bulkhead is 20 feet high and the vertical stiffeners on the bulkhead are 3 feet apart. Find the shearing force at the top and bottom of the stiffeners and the position where the shearing force is zero, when the tank is full with sea water.

38. The following data were taken from the displacement tables of a ship. At the load water plane, corresponding to a draught of 18 feet, the tons per inch immersion was 30·2. The tons per inch at water planes parallel to, and below, the load water plane, measured at regular intervals of 3 feet, were 29·6 ; 28·2 ; 25·5 ; 21·2 ; 13·6 and 0. Calculate the displacement and the distance that the centre of buoyancy is below the load water level.

39. A twin screw vessel travels at $14\frac{1}{2}$ knots with the two engines developing a total I.H.P. of 8,300. If one of the propellers was lost and the ship continued her voyage with one engine running, developing 4,350 I.H.P., what would be her speed ?

40. A vessel has a displacement of 8,000 tons, and floats on an even keel at a draught of 23 ft. 7 ins., the tons per inch immersion being 40. The vessel is 388 feet long, and the longitudinal metacentric height is 410 feet. 100 tons of cargo are put on board at 80 feet aft of the centre of flotation. If the centre of flotation is 12 feet aft of the mid-length of the vessel, find the final draughts.

41. Define "Wake." A ship's speed is 15·5 knots and the speed of the propeller is 114 revs. per min., its pitch being 16 feet. If the speed of the wake is 10% of the ship's speed, find the apparent slip, and the real slip.

42. For the first $8\frac{1}{2}$ hours the speed of a steamer is $22\frac{1}{2}$% above normal. It is then reduced for the remainder of the day so that the consumption for the day is the normal amount. Find the per cent. difference in the distance gone for the day.

43. If the speed of a vessel varies as the $\sqrt[3]{\text{consumption}}$ per unit of time, and as the $\sqrt{\text{consumption}}$ per distance, find the original speed and consumption per day of a certain ship, if, when the speed is reduced by 2 knots the consumption is reduced by 41 tons of fuel per day, and the saving on a voyage of 5,000 miles is 24%.

44. A vertical bulkhead is 24 feet high and is supported by stiffeners 3 feet 6 inches apart. The stiffeners are secured to the tank top by 12 rivets, each 0·875 square inch in area. Find the shearing stress on the rivets when the compartment on one side is filled with sea water. Each stiffener may be regarded as a beam simply supported at its ends.

FIRST-CLASS EXAMINATION QUESTIONS.

ELECTRICITY.

1. A wire 0·139 inch diameter has a resistance of 3·6 ohms. Find the diameter of a wire 4 times as long, and made of the same material, to have a resistance of 6·08 ohms.

2. The efficiency of a dynamo is 0·93. It generates at 110 volts and is driven by a 5 B.H.P. engine. It supplies current to lamps of 25 C.P., each requiring 0·29 ampére. Find the number of lamps.

3. There are 150, 40-watt lamps in an installation, the voltage being 220. What is the horse power required to drive the dynamo if its efficiency is 92 per cent ?

4. A dynamo generates at 110 volts and supplies current for 230 thirty-two candle power lamps. Its efficiency is 92 per cent., and the engine driving the dynamo develops 14 horse power. What is the consumption per candle power ?

5. An electric motor, using current at 100 volts, drives a pump which discharges 360 gallons of sea water per minute against a head of 88 feet. If the combined efficiency of the motor and pump is 0·52, find the current used by the motor.

6. The output of a dynamo is 110 kilowatts at 110 volts. The resistance of the armature is 0·0023 ohm, and of the field winding 0·00055 ohm. The resistance of the brush gear and the brushes causes a drop of potential of 2 volts at this load. Find the voltage generated in the armature and the electrical efficiency.

7. The conductivity of a wire at 0° C. is unity and at t° C. it is $1 - 0{\cdot}003807\, t + 0{\cdot}000009009\, t^2$. The resistance of a wire of a certain material is 5·6 ohms per 1,000 yards at 15° C. What would be the resistance of 3,600 yards of similar wire at 122° F., and also of 215 feet ?

8. The potential difference between the ends of a circuit through which a current of 2 amps. is flowing is 220 volts when the temperature is 15° C. What would be the current when the temperature has risen to 30° C. ? Assume the voltage to remain constant.

$$R_t = R_o\,(1 + a\,t). \quad a = 0{\cdot}00428 \text{ per degree C.}$$

9. An electric heater is capable of raising the temperature of 5·8 tons of oil 100 F°. in 24 hours. The specific heat of the oil is 0·45, and the voltage of the supply 220 volts. Find the current in ampéres, the consumption in kilowatts and the equivalent horse power.

10. The efficiency of a 110 volt lamp is 0·886 watt per candle power. The length of the filament is 65·5 cms., and the diameter is 0·04 millimetre. The specific resistance of the filament is 16·5 microhms per centimetre cube when cold, and five times as much when hot. What is the candle power of the lamp ?

11. A series wound motor on a 220 volt supply has an output of 45 H.P. The resistance of the armature and field winding is 0·15 ohm. Find the electrical efficiency of the motor.

12. The temperature of 600 grams of water is raised 5 C°. per minute by a heating element having a resistance of 2·5 ohms. Find the current supplied to the heating element.

13. A conductor 25 centimetres long and carrying a current of 100 ampéres, is placed at right angles to a magnetic field of 15,000 lines per square centimetre. Find the force exerted on the conductor in pounds.

14. What is meant by "Frequency" ? An alternator has 6 poles and runs at 1,000 revs. per minute, find its frequency. If this alternator drives a synchronous motor having 40 poles, what will be the speed of the motor ?

15. A certain battery can discharge at the rate of 20 ampéres for 8 hours, or at 40 ampéres for 2·6 hours. If $I^n \times t = K$ where I = current in ampéres, t = time, n and K are constants, find for how long the battery would discharge at 30 ampéres.

16. An ammeter is tested by a silver voltameter. The weight of the cathode before deposit was 23·8 grams, and after a steady current was passed through for ½ hour its weight was 39·25 grams. The reading of the ammeter was 7·3 ampéres. Find the error of the ammeter taking the electro-chemical equivalent of silver as 0·001118 gram per coulomb.

17. A two-pole magneto dynamo has a useful flux of 100,000 lines. The armature has a single winding of 480 turns. Find the average E.M.F. generated when running at 1,440 revolutions per minute.

18. An electric motor having an internal resistance of 0·25 ohm runs on a 100 volt supply. At no load the current taken was 1·5 ampéres and at full load at the same speed the current was 35 ampéres. Find (*a*) back E.M.F. at no load, (*b*) back E.M.F. at full load, (*c*) electrical efficiency at full load.

19. Define Electro-Motive-Force, and state Ohm's law. The total generated voltage of a dynamo is 105 and it supplies a current of 75 ampéres. The internal resistance of the dynamo is 0·015 ohm. If the potential difference at the ends of the mains is 100 volts, find the resistance of the mains.

20. The potential difference across a conductor is 110 volts, and the current passing through it is 3 ampéres, when its temperature is 20° C. Due to an increase of temperature, the current falls to 2·8 ampéres. If the P.D. is kept the same, find the increase in temperature.

Temperature co-efficient = 0·00428 per deg. C.

21. A current of 50 ampéres passes through a resistance of 0·25 ohm before going to a motor. The supply voltage to the resistance is 150. Find the volt drop across the resistance. If the supply voltage was increased to 240 and the current increased accordingly, compare the heat generated at the resistance in the two cases.

22. A 4-pole generator has a useful pole flux of $2{\cdot}5 \times 10^6$ lines. The armature has 420 conductors and is lap wound. What E.M.F. will be generated when running at 600 revs. per minute ?

23. One conducting wire is 50 metres long and weighs 100 grams. Another conducting wire of the same material is 90 feet long and weighs 2 ounces. What is the ratio of their resistances ?

24. An electric motor lifts a load of 2 tons by means of a derrick. If 60% of the power of the motor is available for useful work, find the horse power exerted by it when the load is rising steadily at the rate of 4 inches per second. Find also the current taken, the voltage being 220 and the motor efficiency 84%.

25. An electric circuit consists of 350, 110-volt, 60-watt, 40 candle power lamps, and one 15 B.H.P. electric motor whose efficiency is 80%. The dynamo supplying current to this circuit

has an efficiency of 89% and is driven by a steam engine whose mechanical efficiency is 85%. Find (*a*) the consumption of the lamps in watts per candle power, (*b*) the current taken by each lamp, (*c*) the I.H.P. of the engine allowing for an overload of 10%, and (*d*) the diameter of the main conducting wires allowing 1,000 ampéres per sq. inch of cross section.

26. A bipolar dynamo with two conductors has a useful flux of 1,440,000 lines per pole. Calculate the average E.M.F. induced in each conductor when the armature is driven at 1,400 revs. per minute.

27. A circuit has a resistance of 12 ohms, and an inductance of 0·04 henry. Calculate its impedance if the frequency is 50 cycles per second.

28. A ship's saloon is illuminated by forty 60-watt lamps which require a voltage of 110. The mains are of wire which has a resistance of 1·2 ohms per 1,000 yards and the distance from the switchboard to the saloon is 165 feet. What should be the reading of the voltmeter on the switchboard ?

29. A six pole lap wound generator has a useful flux of $5{\cdot}4 \times 10^6$ lines per pole. Find the necessary number of conductors to generate 150 kilowatts at 220 volts when running at 400 revs. per minute.

30. When a direct current is passed through a coil, with a pressure of 60 volts, the power was 300 watts. When an alternating current is passed through this coil, with a pressure of 130 volts, the power was 1,200 watts. Find the reactance of the coil.

31. State what is meant by the R.M.S. value of an alternating current. The mean instantaneous values of an alternating current for a half cycle are as follows :—

Time in Seconds	0 to 0·05	0·05 to 0·1	0·1 to 0·15	0·15 to 0·2	0·2 to 0·25	0·25 to 0·3
Mean Value of Current, in amps.	1	6	11	11	6	1

Plot to scale the alternating current curve on a base of time, and find the R.M.S. value. Mark off the current at 0·1 second.

32. A lead fuse is designed to blow at 7 ampéres. Its melting temperature is 327° C., and at that temperature its specific resistance is 45 microhms per centimetre cube. It loses heat from its surface at the rate of $\frac{1}{4000}$ calorie per square centimetre of surface area per second for each degree Centigrade rise above the atmospheric temperature. If the atmospheric temperature is 17° C., find the diameter of the fuse wire.

33. A circuit has a resistance of 3 ohms and a reactance of 4 ohms, and alternating current passes through it. When the voltage is 100, find (*a*) the current flowing, (*b*) the active E.M.F., (*c*) the E.M.F. of self induction.

34. A wire of 16 ohms resistance is immersed in a tank containing 1 litre of fresh water at 15° C. Find the current flowing if the water is to reach boiling temperature in 10 minutes.

35. A series wound dynamo has an internal resistance of 0·015 ohm. It generates 106 volts and sends 75 ampéres to the external circuit. The E.M.F. at the extreme ends of the mains is 100 volts. Find the resistance of the mains, and the equivalent horse power lost in the mains.

36. A two pole electric motor has a flux density of 3,300 lines per square centimetre. The armature is 16 inches long and it carries 246 conductors. At full load each conductor carries 20 amps. Find the force in pounds on the conductors at full load.

37. A two wire ring mains system is 2,000 yards long. At the supply point A the pressure is 220 volts, and at a point B, 400 yards from A, the load is 110 amps. If the resistance of the mains is 0·032 ohm per 1,000 yards of single wire, find the current in each section of the system, and the voltage at the load.

38. A shunt wound generator gives out 75 ampéres at 150 volts. The armature resistance is 0·06 ohm, and the resistance of the shunt is 30 ohms. Find the electrical efficiency of the generator.

39. An electric motor, on a 110 volt supply, is attached to a rope brake which has a cooling water system. The current taken by the motor is 500 ampéres, and the heat generated at the brake causes 748 gallons of water to be raised through 10 C°. each hour. Assuming 85% of the heat generated at the brake to be carried away by the cooling water, find (*a*) the output of the motor, expressed in kilowatts, (*b*) the efficiency of the motor.

40. An electric current was passed through a copper voltameter for $\frac{1}{2}$ hour, and the ammeter reading was 0·8 ampére. The electro-chemical equivalent of hydrogen is 0·00001044 gram per coulomb, and the chemical equivalent of copper is 31·8. If the total weight of deposit on the cathode was 0·49 gram, find the error of the ammeter, and express it as a per cent.

41. An electric motor, running at 400 revolutions per minute, transmits a torque of 144 ft. lb. If the efficiency is 90%, find the current taken when the voltage of supply is 500.

42. An alternating current, flowing through a circuit, has a maximum voltage of 100, and a maximum current of 60 ampéres. The current lags by 60° behind the voltage, and current and voltage curves are sinusoidal. Plot the current and voltage curves for one cycle, if the frequency is 50 cycles per second.

43. Find the time taken to heat 1 pint of fresh water from 15° C. to boiling temperature by a heating element of 150 ohms resistance, if the efficiency is 80% and the supply voltage is 240.

44. A dynamo is supplied with 13·5 kilowatts, and gives out 100 ampéres at 110 volts. The internal resistance of the dynamo is 0·04 ohm. Find (*a*) the electrical efficiency, (*b*) the mechanical efficiency, (*c*) the commercial efficiency.

45. Three cells are connected in series. Each has an E.M.F. of 1·4 volts, and a resistance of 1 ohm. The external circuit has a resistance of 5 ohms. Find the potential difference at the terminals of (*a*) the middle cell, (*b*) the external resistance.

46. A lifeboat, weighing 5·5 tons, is lifted 60 ft. in $3\frac{1}{2}$ minutes by an electric winch. The current taken is 30 ampéres on a supply voltage of 220. If the efficiency of the motor is 0·93, find the efficiency of the winch.

47. A ship has 4 sets of electric motor driven lifeboat davits. Each motor, working off a 220 volt supply line, is capable of lifting a $2\frac{1}{2}$ tons lifeboat at the rate of 6 feet per minute. The overall efficiency of each unit is 25%. What is the horse power absorbed by each unit ? What current is taken by each unit ? Give your opinion as to the value at which the circuit breaker should be adjusted.

48. A shunt wound generator has the following characteristics :—

Field Ampéres	1	3	5	7	9	11
Open Circuit Voltage ...	39	90	127	149	162	169

Draw the characteristic curve to the scales : 1 inch = 20 volts and 1 inch = 2 amperes, and find the open circuit voltage when the shunt resistance is 21 ohms. If the armature resistance is 0·02 ohm, terminal voltage at full load 140, find the ampére-output of the machine.

49. The moving coil of an ammeter is made of copper having a resistance of 2 ohms at 20°C. The resistance of the shunt is 0·0015 ohm and is constant throughout the range of temperature at which it is subjected. Find the percentage error of the ammeter reading when the temperature is 40°C. and the current through the circuit is 200 ampéres. The temperature coefficient for copper is 0·00428 per cent. degree at 0°C.

50. Find (a) graphically, (b) by calculation, the resultant of two alternating E.M.F.s of 100 and 50 volts respectively, having the same frequency, the second lagging behind the first by 45°. Plot these curves to scale for one cycle. If the resultant E.M.F. is applied to a circuit having an impedance of 39 ohms and a reactance of 15 ohms, find the resistance and the current flowing.

SOLUTIONS TO FIRST-CLASS EXAMINATION QUESTIONS

ENGINEERING SCIENCE

1. Taking the rods and coupling to be made of the same material, then, to have equal stresses in tension, their cross-sectional areas must be equal.

Let d = diameter of rods.

$$\text{Effective area of rods} = \tfrac{11}{14}(d - 0{\cdot}36)^2$$
$$= \tfrac{11}{14}(d^2 - 0{\cdot}72d + 0{\cdot}1296)$$

$$\text{Effective area of coupling} = \tfrac{11}{14}(4{\cdot}25^2 - d^2)$$
$$= \tfrac{11}{14}(18{\cdot}0625 - d^2)$$

$$\text{Area of rod} = \text{Area of coupling}$$

$$\tfrac{11}{14}(d^2 - 0{\cdot}72d + 0{\cdot}1296) = \tfrac{11}{14}(18{\cdot}0625 - d^2)$$
$$2d^2 - 0{\cdot}72d = 17{\cdot}9329$$

And from this quadratic equation, $d = 3{\cdot}18$ inches. Ans.

Longitudinal seam has 3 rivets per pitch.

$$\text{Rivet \% strength} = \frac{a\ n\ f_s}{p\ t\ f_t} \times 1\tfrac{7}{8} \times 100$$

$$= \frac{11 \times 19 \times 19 \times 3 \times 8 \times 64 \times 23 \times 15 \times 100}{14 \times 16 \times 16 \times 49 \times 65 \times 28 \times 8}$$

$$= 82{\cdot}31\%.$$

% strength of drilled plate at outer row

$$\times 100$$

$$\times 100 = 80{\cdot}61\%$$

$6\tfrac{1}{8}$

% strength of seam along the inner row

$$= \frac{p - [illegible]}{p} \times 100 + \% \text{ strength of 1 rivet in shear}$$

$$= \frac{6\frac{1}{8} - 2\frac{3}{8}}{[illegible]} \times 100 \quad [illegible] \quad \frac{82{\cdot}31}{3} = 88{\cdot}65\%$$

Least value = 80·61% = Strength of longitudinal seam.
Circumferential seam has 2 rivets per pitch.

$$\text{Rivet \% strength} = \frac{a\ n\ f_s}{p\ t\ f_t} \times 100$$

$$= \frac{11 \times 19 \times 19 \times 2 \times 32 \times 64 \times 23 \times 100}{14 \times 16 \times 16 \times 97 \times 65 \times 28}$$

$$= 59{\cdot}13\%$$

% strength of drilled plate

$$= \frac{p - [illegible]}{p} \times 100$$

$$= \frac{[illegible]}{3\frac{1}{32}} \times 100 = 60{\cdot}83\%$$

Least value = 59·13% = strength of circumferential seam. For its purpose, the circumferential seam is stronger than the longitudinal seam, considering therefore, the longitudinal seam :

Working pressure

$$= \frac{2\ t \times \text{working stress}}{\text{D}} \times \text{long. seam strength}$$

$$= \frac{2 \times 65 \times 28 \times 2240}{64 \times 4{\cdot}5 \times 57} \times \frac{80{\cdot}61}{100}$$

$$= 400{\cdot}3 \text{ lb. per sq. inch.} \quad \text{Ans.}$$

3. Volume of tank = $(0{\cdot}5236 \times 15^3) + (0{\cdot}7854 \times 15^2 \times 15)$ cubic inches.

$= 0{\cdot}5236 \times 15^3 + 0{\cdot}7854 \times 15^3$

$= 1{\cdot}309 \times 15^3$ cubic inches.

$$\text{Weight of sea water displaced} = \frac{1 \cdot 309 \times 15^3 \times 64}{1,728} = 163 \cdot 6 \text{ lb.}$$

Weight of cast iron inside tank $= 163 \cdot 6 - 55 = 108 \cdot 6$ lb.
Ans.

1 cubic foot of cast iron can exert a downward force of $(450 - 64) = 386$ lb. when immersed in sea water.

$\therefore$ Weight of cast iron necessary below vessel

$= 108 \cdot 6 \times \frac{450}{386} = 126 \cdot 6$ lb. Ans.

4.
$$\text{Centrifugal force} = \frac{1,392 \times (2 \cdot 3 \times 2 \times 3 \cdot 1416 \times \frac{100}{60})^2}{32 \cdot 2 \times 2 \cdot 3}$$

$= 10,900$ lb.

Maximum load on bolts $= 10,900 + 1,392$ lb. $= 12,292$ lb.

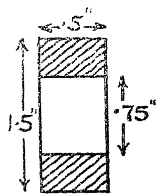
Load per bolt $= \frac{12,292}{2} = 6,146$ lb.

$$\text{Diameter of bolt} = \sqrt{\frac{6,146}{8,000 \times 0 \cdot 7854}} = 0 \cdot 989 \text{ in.} \quad \text{Ans.}$$

5. Resisting moment of section of lever where connection is made

$$\frac{B \times (D^3 - d^3)}{D \times 6} \times \text{Stress}$$

$$\frac{0 \cdot 5\ (1 \cdot 5^3 - 0 \cdot 75^3)}{1 \cdot 5 \times 6} \times p$$



Bending moment $=$ Weight $\times$ 18 inch lb.

$$\therefore \text{Weight} = \frac{0 \cdot 5\ (1 \cdot 5^3 - 0 \cdot 75^3) \times 5,000}{1 \cdot 5 \times 18 \times 6} = 45 \cdot 64 \text{ lb.}$$
Ans.

6. Load (lb.) $\times 0 \cdot 09 \times 2 \times 3 \cdot 1416 \times \frac{7}{12} \times 102 = 16 \times 33,000$

$$\text{Load} = \frac{16 \times 33,000 \times 12}{0 \cdot 09 \times 2 \times 3 \cdot 1416 \times 7 \times 102}$$

$= 15,700$ lb. Ans.

7. $C\ F \propto W \times V^2$, and Stress $\propto$ $C\ F$

$$\frac{\text{Stress}}{W \times V^2} = \text{constant}$$

For rotors of similar material, W will be the same in each case, being the weight per unit volume of the material.

$$\frac{2{,}300}{(3\cdot5 \times 700)^2} = \frac{\text{Stress}}{(8 \times 320)^2}$$

Stress due to centrifugal force $= 2{,}510$ lb. per square inch. Ans.

8. Area of tank metal $= 2\ (4 \times 1\cdot25 + 4 \times \frac{2}{3} + 1\cdot25 \times \frac{2}{3})$ $= 17$ square feet.

Weight of tank $= \dfrac{17 \times 18}{16} = 19\cdot125$ lb.

Weight of water displaced by tank $= 4 \times \frac{2}{3} \times 1\frac{1}{4} \times 64$
$= 213\cdot333$ lb.

Downward pull to submerge tank $= 213\cdot333 - 19\cdot125$
$= 194\cdot208$ lb.

Weight of iron required

$$= 194\cdot208 \times \frac{480}{(480 - 64)} = 224\cdot1 \text{ lb.} \quad \text{Ans.}$$

9. Percentage strength of longitudinal seam $= 79\cdot29$.

To be of equal strength, the circumferential seam percentage strength needs to be only half of $79\cdot29 = 39\cdot645\%$ of the solid plate.

To be 40% stronger, strength $= \frac{140}{100} \times 39\cdot645 = 55\cdot5\%$

Considering drilled plate efficiency :—

$$\frac{p - d}{p} = 0\cdot555, \qquad p$$

$$0\cdot445p = 1\cdot1875 \qquad \therefore p = 2\cdot67 \text{ inches} \quad \ldots \text{(i)}$$

Considering rivet efficiency :-

$$\frac{a\ n\ f_s}{p\ t\ f_t} = 0{\cdot}555 \qquad p = \frac{a\ n\ f_s}{0{\cdot}555\,t\,f_t}$$

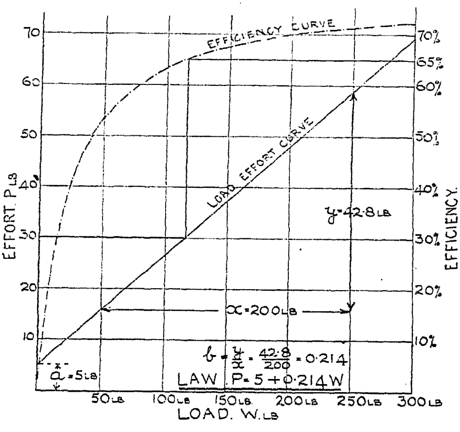
$$p = \frac{11 \times 19 \times 19 \times 2 \times 23 \times 32}{14 \times 16 \times 16 \times 0{\cdot}555 \times 35 \times 28} = 3 \text{ ins.} \qquad \text{(ii)}$$

The pitch can be 2·67 inches or 3 inches. Ans.

Either pitch gives a minimum percentage circumferential seam strength 40% stronger (for its purpose) than the minimum percentage strength of the longitudinal seam.

10.

Load W lb.	50	100	150	200	250	300
Effort P lb.	15·6	26·3	37	47·7	58·3	69·3
M.A. $= \frac{W}{P}$	3·206	3·802	4·054	4·193	4·288	4·329
Effy. % $= \frac{\text{M.A.}}{\text{V.R.}} \times 100$	53·4	63·4	67·6	69·9	71·5	72·2



From the graph,

$a = 5$

$b = 0{\cdot}214$

∴ the law connecting load and effort is

$P = 5 + 0{\cdot}214W.$

Ans. (*a*)

When the load is 267·2 lb.

$P = 5 + 0{\cdot}214 \times 267{\cdot}2$

$= 62{\cdot}18$ lb. Ans. (*b*)

Referring to the diagram, through Effort = 30 lb., draw a horizontal line to intercept the Load Effort curve, and erect a vertical to intercept the Efficiency curve.

This reads 65% on the Efficiency scale, and is the efficiency of the machine when the Effort exerted

11. Sectional area of swelled end of stay
$= (3{\cdot}25^2 \times 0{\cdot}7854) - (3{\cdot}25 \times 0{\cdot}75) = 5{\cdot}857$ square inches.

Sectional area of body of stay
$= 2{\cdot}5^2 \times 0{\cdot}7854 = 4{\cdot}908$ square inches.

Area of cotter $\times$ 2 $\times$ Shearing strength = Least area of stay $\times$ Tensile strength.

$D \times 0{\cdot}75 \times 2 \times 23 = 4{\cdot}908 \times 28$

$$\text{Depth of cotter} = \frac{4{\cdot}908 \times 28}{0{\cdot}75 \times 2 \times 23} = 3{\cdot}984 \text{ inches.} \quad \text{Ans.}$$

12. Load on stay $= 9{\cdot}5 \times \frac{27}{2} \times 180$ lb.

Reaction of ends of girder $= 9{\cdot}5 \times \frac{27}{2} \times 180 \times \frac{1}{2}$ lb.

Bending moment at centre of girder
$= 9{\cdot}5 \times \frac{27}{2} \times 180 \times \frac{1}{2} \times \frac{27}{2}$ inch lb.

$$\text{Resisting moment} = \frac{2 \times 0{\cdot}625 \times D^2}{6} \times 10{,}000$$

and since B. M. = R. M.

$$\therefore D^2 = 9{\cdot}5 \times \tfrac{27}{2} \times 180 \times \tfrac{1}{2} \times \tfrac{27}{2} \times \tfrac{6}{1} \times \frac{1}{2 \times 0{\cdot}625 \times 10{,}000}$$

$D = 8{\cdot}648$ inches. Ans.

13. Velocity of Ratio

$$\frac{\text{Radius of force circle}}{\text{Radius of weight circle}} \times \frac{\text{Product of teeth in driven wheels}}{\text{Product of teeth in driving wheels}}$$

$$= \frac{24}{6{\cdot}5} \times \frac{60 \times 75}{12 \times 15}$$

Force $\times$ Velocity ratio $\times$ Efficiency = Weight.

$$\text{Weight lifted} = \frac{200 \times 24 \times 60 \times 75 \times 0{\cdot}65}{6{\cdot}5 \times 12 \times 15} = 12{,}000 \text{ lb.}$$

Ans.

Twisting moment (foot lb.)

$$= \frac{\text{Foot lb. of work done at thrust per minute}}{2 \times \pi \times \text{revols. per min.}}$$

$$\text{T.M. (inch tons)} = \frac{\text{Work done}}{2 \times \pi \times \text{Revs.}} \times \frac{12}{2240}$$

$$\frac{40{,}000 \times \dfrac{14{\cdot}5 \times 6{,}080}{60}}{2 \times 3{\cdot}1416 \times 75} \times \qquad = 668{\cdot}3 \text{ inch tons.}$$

Ans.

15. Pressure valve lifts at, before water collects

$$= \frac{650}{4{\cdot}5^2 \times 0{\cdot}7854} = 40{\cdot}87 \text{ lb. per square inch.}$$

Taking specific gravity of cast iron as 7·2, then the effective weight of the cast iron, when totally submerged in fresh water is $\frac{7{\cdot}2 - 1}{7{\cdot}2} = \frac{6{\cdot}2}{7{\cdot}2}$ of its weight in air.

$$\text{Load on valve due to weights} = \frac{6{\cdot}2}{7{\cdot}2} \times 40{\cdot}87 = 35{\cdot}2 \text{ lb. per sq. inch.}$$

Load due to 2 feet 4 inches of water ... $= 1{\cdot}0$ lb.

Valve lifts at 36·2 lb.

40·87 lb. square inch and 36·2 lb. square inch. Ans.

16. True weight $= \sqrt{6{\cdot}5 \times 5{\cdot}5} = 5{\cdot}98$ lb. Ans.

Let x and y be the length of the two arms, and W be the true weight.

When W is hung from x arm, let W_1 be hung from y arm.

Then $W\,x = W_1\,y$ ... ... (1)

When W is hung from y arm, let W_2 be hung from x arm to preserve the balance.

Then $W\,y = W_2\,x$ ... ... (2)

Multiply equations (1) and (2) together

$W\,x \times W\,y = W_1\,y \times W_2\,x$

$W^2\,xy = W_1\,W_2\,xy$

Cancel xy, $W^2 = W_1\,W_2$

and $W = \sqrt{W_1\,W_2}$

17. 4 minutes 31 seconds = 271 seconds.

Speed with current $= \frac{3600}{271} = 13\cdot28$ knots.

7 minutes 19 seconds = 439 seconds.

Speed against current $= \frac{3600}{439} = 8\cdot2$ knots.

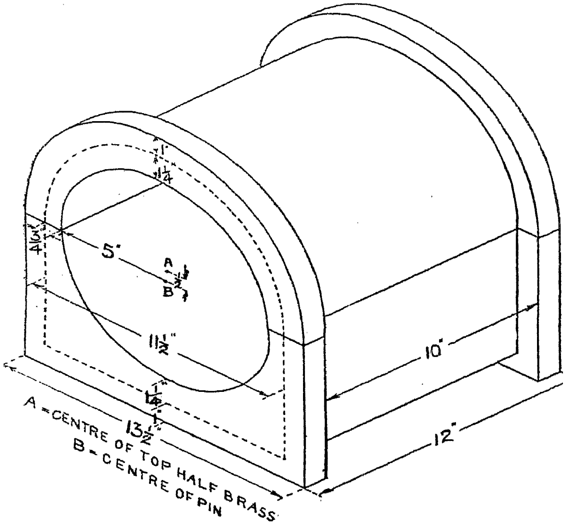
True speed = $13\cdot28 + 8\cdot2$ $10\cdot74$ knots. Ans.

$10\cdot74$ knots $= 10\cdot74 \times \frac{6080}{5280} = 12\cdot37$ statute miles per hour. Ans.

18. Since the brass is 11·5 inches wide and the pin is 10 inches diameter

$$\therefore \text{Thickness at sides} = \frac{11\cdot5 - 10}{2} = 0\cdot75 \text{ inch.}$$

Since brass is 1·25 inches thick in the crown, therefore the centre of the semi-cylindrical top half and of the top flanges may be taken as being $1\cdot25 - 0\cdot75 = 0\cdot5$ inch above centre of pin hole.



Volume of semi-cylindrical part of body

cubic inches.

$$\frac{\pi \times (5\cdot75)^2 \times 10}{2} = 519\cdot55$$

Volume of rectangular part of body

$$= 6\cdot75 \times 11\cdot5 \times 10 = 776\cdot25$$

$$\text{Volume of top flanges} = \frac{\pi \times (6\cdot75)^2 \times 2}{2} = 143\cdot2$$

$$\text{Volume of bottom flanges} = 7\cdot75 \times 13\cdot5 \times 2 = 209\cdot25$$

$$\text{Total volume, if solid} = 1648\cdot25$$

$$\text{Volume of pin hole} = \pi \times 5^2 \times 12 = 942\cdot85$$

$$\text{Nett volume} = 705\cdot4$$

Weight $= 705\cdot4 \times 0\cdot3 = 211\cdot62$ lb. Ans.

19.

$$\text{Mean outside diameter} = \frac{10 + 13}{2} = 11\cdot5 \text{ inches.}$$

Mean inside diameter $= 11\cdot5 - 4 = 7\cdot5$ inches.

Volume of body of column

$= (11\cdot5^2 - 7\cdot5^2)\ 0\cdot7854 \times 144 \quad = 8595$ cu. ins.

Volume of top flange

$= (15^2 - 10^2)\ 0\cdot7854 \times 2\cdot25 \quad = \ 221$,,

Volume of bottom flange

$= \{(19 \times 15) - (13^2 \times 0\cdot7854)\}\ 2\cdot5 = 380\cdot75$,,

$9196\cdot75$,,

$$\text{Weight} = \frac{9196\cdot75}{1{,}728} \times 450 = 2{,}395 \text{ lb. Ans.}$$

20.

$$\text{Time taken on one double stroke} = \frac{60}{4\cdot8} = 12\cdot5 \text{ seconds.}$$

Since speed of forward stroke is one-quarter of the speed of the return stroke, the time taken on the forward stroke must be four times as much as that taken on return stroke.

$\therefore$ Time for one forward stroke $= \frac{4}{5}$ of $12{\cdot}5 = 10$ seconds.
and time for one return stroke $= \frac{1}{5}$ of $12{\cdot}5 = 2{\cdot}5$ seconds.
Work done on each stroke $= 21 \times 112 \times 0{\cdot}11 \times 7$ foot lb.
This is done in $2{\cdot}5$ seconds on the return stroke.

$$\therefore \text{H.P. of engine} = \frac{21 \times 112 \times 0{\cdot}11 \times 7 \times 60}{2{\cdot}5 \times 33{,}000}$$

$= 1{\cdot}317$ H.P. Ans.

21. 1st beam. Bending moment $= \frac{480}{2} \times \frac{12}{2} = 1{,}440$ inch lb.

$$\text{Resisting moment} = \frac{1 \times 1^2}{6} \times \text{stress.}$$

$\therefore$ Stress when beam breaks $= 1{,}440 \times 6 = 8{,}640$ lb. per square inch.

Stress allowed in second beam $= \frac{8640}{5} = 1{,}728$ lb. per square inch.

2nd beam. Bending moment $= 3 \times 112 \times 108$ inch lb.

$$\text{Resisting moment} = \frac{B \times (3\ B)^2}{6} \times 1{,}728$$

$$\frac{9\ B^3}{6} \times 1{,}728 = 3 \times 112 \times 108$$

Breadth $= \sqrt[3]{14} = 2{\cdot}41$ inches. Ans.

22. Length of cylindrical portion $= 50 - 8 = 42$ feet.

$$\text{Volume} = \left(\frac{\pi}{6} \times 8^3 \times \tfrac{1}{2}\right) + \left(\frac{\pi}{4} \times 8^2 \times 42 \times \tfrac{1}{2}\right) \text{ cu. ft.}$$

$$\pi \times 8^2 \left(\tfrac{8}{6} + \tfrac{42}{4}\right) = \pi \times 32 \times \tfrac{71}{6} \text{ cubic feet.}$$

$$\text{Displacement} = \frac{3{\cdot}1416 \times 32 \times 71 \times 1024}{6 \times 16 \times 2{,}240} = 33{\cdot}99 \text{ tons. Ans.}$$

23. Load on beam $= (2 \times 10) + 4 = 24$ tons.

Reaction of supports $= 12$ tons.

Bending Moment at centre $= (12 \times 5) - (10 \times 2{\cdot}5)$
$= 35$ foot tons $= 35 \times 12$ inch tons.

$$\text{Resisting Moment} = \frac{\text{B.} \times (10{\cdot}5)^2}{6} \times 4, \text{ and R.M.} = \text{B.M.}$$

$$\therefore \text{Breadth} = \frac{35 \times 12 \times 6}{10{\cdot}5 \times 10{\cdot}5 \times 4} = 5{\cdot}715 \text{ inches.}$$ Ans.

24. $$\text{Load on spring} = \frac{\text{Compression} \times \text{C} \times d^4}{\text{D}^3 \times \text{N}},$$ and the load is the same in each case.

$$\frac{0{\cdot}7 \times 30 \times 13^4}{3^3 \times 11} = \frac{\text{Comp.} \times 30 \times 13^4}{(3{\cdot}25)^3 \times 13}$$

$$\text{Compression} = \frac{0{\cdot}7 \times (3{\cdot}25)^3 \times 13}{3^3 \times 11} = 1{\cdot}052 \text{ inches.}$$ Ans.

25. Specific gravity of brass

$$= \frac{\text{Wt. in air}}{\text{Wt. in air} - \text{wt. in water}}$$

$$= \frac{23}{23 - 20{\cdot}26} = 8{\cdot}397.$$ Ans.

Water displaced by brass and cork $= (23 + 1{\cdot}6) - 15$
$= 9{\cdot}6$ grams.

Water displaced by brass only $= 23 - 20{\cdot}26$
$= 2{\cdot}74$ grams.

$\therefore$ Water displaced by cork only $= 9{\cdot}6 - 2{\cdot}74$
$= 6{\cdot}86$ grams.

Specific gravity of cork

Wt. of cork

Wt. of equal vol. of water

$$\frac{1{\cdot}6}{} = 0{\cdot}2332. \quad \text{Ans.}$$

26. Force to pull up $= \mu$ W cos. α + W sin. α

Force to pull down $= \mu$ W cos. α — W sin.

$$\begin{aligned} 2464 &= \mu \text{ W} \times 0{\cdot}9971 + \text{W} \times 0{\cdot}077 \\ 58 &= \mu \text{ W} \times 0{\cdot}9971 - \text{W} \times 0{\cdot}077 \\ 2406 &= 2 \times \text{W} \times 0{\cdot}077 \end{aligned}$$

by subtraction

$$\text{W} = \frac{2406}{2 \times 0{\cdot}077} = 15{,}620 \text{ lb. or } 6{\cdot}974 \text{ tons.} \quad \text{Ans.}$$

27. Let d = diameter of rods.

Effective area of rods $= \frac{11}{14}\ (d - 0{\cdot}2)^2$

$\frac{11}{14}$ $\quad$ $0{\cdot}4$

Area of a hexagon $= 2{\cdot}598$ side^2

$\therefore$ Area of coupling $= 2{\cdot}598 \times 1{\cdot}7^2 - \frac{11}{14}\ d^2$

Area of rod = area of coupling.

$$\begin{aligned} \tfrac{11}{14}\ (d^2 - 0{\cdot}4\ d + 0{\cdot}04) &= 2{\cdot}598 \times 1{\cdot}7^2 - \tfrac{11}{14}\ d^2 \\ \tfrac{11}{14}(d^2 + d^2 - 0{\cdot}4d + 0{\cdot}04) &= 2{\cdot}598 \times 1{\cdot}7^2 \\ 2\ d^2 - 0{\cdot}4\ d + 0{\cdot}04 &= 2{\cdot}598 \times 1{\cdot}7^2 \times \tfrac{14}{11} \\ d^2 - 0{\cdot}2\ d + 0{\cdot}02 &= 2{\cdot}598 \times 1{\cdot}7^2 \times \tfrac{7}{11} \\ d^2 - 0{\cdot}2\ d &= 4{\cdot}778 - 0{\cdot}02 \\ d^2 - 0{\cdot}2\ d &= 4{\cdot}758 \end{aligned}$$

And from this quadratic equation, $d = 2{\cdot}284$ inches. Ans.

28. Component of 480 lb. causing motion of barge =
$480 \text{ Cos. } 30° = 480 \times 0{\cdot}866 = 415{\cdot}68$ lb.

$$\text{Horse power exerted on barge} = \frac{415{\cdot}68 \times 60}{33{,}000} = 0{\cdot}756$$
Ans.

29. Since weight per foot run is the same, the sectional area of the two shafts must be the same.

$$\therefore\ 14^2 - d^2 = (12{\cdot}125)^2, \qquad d = 7 \text{ inches.}$$

Resisting moment of solid shaft = Resisting moment of hollow shaft.

$$\frac{(12{\cdot}125)^3}{5{\cdot}1} \times q = \frac{14^4}{14 \times 5{\cdot}1} \times 8{,}000$$

$$\text{Stress} = \frac{(14^4 - 7^4) \times 8{,}000}{(12{\cdot}125)^3 \times 14} = 11{,}545 \text{ lb. per sq. inch.}$$
Ans.

30. The condition for equilibrium of the lever is that the sum of the moments of the weights of the valve, of the lever and of the movable weight, about the fulcrum, must equal the moment of the load on the valve due to steam pressure.

If the steam pressure is increased, the upward moment is increased. The downward moment must be increased by a similar amount. This can be done only by shifting the movable weight, since the moments of the weight of valve and lever are constant.

$\therefore$ By moments about fulcrum :—

Increase in up moments = Increase in down moments.

$$10 \times 8 \times 3{\cdot}25 = 60 \times x$$

$$x = \frac{10 \times 8 \times 3{\cdot}25}{60} = 4{\cdot}\dot{3} \text{ inches. Ans.}$$

31. Let B = breadth of flanges in inches.

Then resisting moment $= B \times 1 \times 12 \times 5$

Bending moment, neglecting weight of joist

$= 3{\cdot}5 \times 12 \times 12$ inch tons.

$\therefore B \times 1 \times 12 \times 5 = 3{\cdot}5 \times 12 \times 12$

$$B = \frac{3{\cdot}5 \times 12 \times 12}{12 \times 5} = 8{\cdot}4 \text{ inches. Ans.}$$

Area of section of joist

$= (B \times 1)\,2 + (10 \times 1) = 2\,B + 10$ square inches.

$$\text{Volume of joist} = \frac{(2\,B + 10)\,144}{1{,}728} = \frac{B + 5}{6} \text{ cu. feet.}$$

$$\text{Weight of joist} = \frac{(B + 5)\,490}{6 \times 2{,}240} = \frac{(B + 5)\,7}{6 \times 32} \text{ tons.}$$

Bending moment, with weight of joist

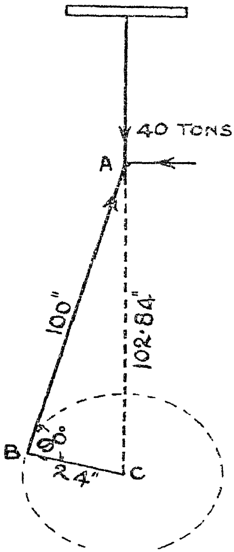
$$= 3{\cdot}5 \times 144 + \frac{(B + 5)\,7 \times 72}{6 \times 32} \text{ inch tons.}$$

$$= 3{\cdot}5 \times 144 + \frac{(B + 5)\,21}{} \text{ inch tons.}$$

$$\therefore B \times 1 \times 12 \times 5 = 3{\cdot}5 \times 144 + \frac{(B + 5)\,21}{8}$$

Multiply by $\frac{8}{3}$, $160\,B = 1344 + 7\,B + 35$

$153\,B = 1{,}379.$ $B = 9{\cdot}01$ inches. Ans.



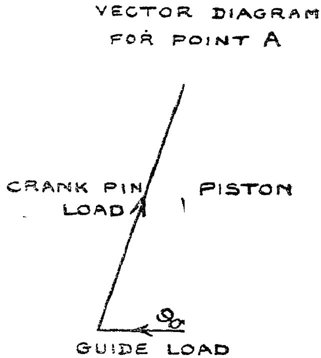
32.

Maximum twisting moment occurs on the shaft when the connecting rod and crank are at right angles.

When in this position :—

A C = $\sqrt{100^2 + 24^2}$

= 102·84 inches.



Triangle A B C is similar to vector diagram

.·. A B represents load on the piston and A C represents load on the crank.

$$\text{Load on crank} = \frac{102 \cdot 84}{100} \times 40 = 41 \cdot 136 \text{ tons.}$$

Twisting moment on shaft = 41·136 × 2,240 × 24 inch lb.

$$\text{and } 41 \cdot 136 \times 2{,}240 \times 24 = \frac{13^3}{5 \cdot 1} \times \text{stress.}$$

$$\text{Stress} = \frac{41 \cdot 136 \times 2{,}240 \times 24 \times 5 \cdot 1}{13^3} = 5{,}135 \text{ lb. per sq. inch. Ans.}$$

33. Load on spring when compression is 1 inch
= $(3 \cdot 5)^2 \times 0 \cdot 7854 \times 180$ lb.

Load when compression is 1·5 inches

= $(3 \cdot 5)^2 \times 0 \cdot 7854 \times 180 \times 1 \cdot 5$ lb.

.·. Average load lifted

$$1 + 1\cdot5$$

$$= (3\cdot5)^2 \times 0\cdot7854 \times 180 \times$$

$$= (3\cdot5)^2 \times 0\cdot7854 \times 180 \times 1\cdot25 \text{ lb.}$$

$$\text{Work done} = \frac{(3\cdot5)^2 \times 0\cdot7854 \times 180 \times 1\cdot25 \times 0\cdot5}{12}$$

$= 90\cdot22$ foot lb. Ans.

34. Mean circumference $= \pi \times 3\cdot25 = 10\cdot21$ ins.
Pitch of coils $= \frac{17}{15} = 1\cdot133$ ins.
Length of one coil $= \sqrt{(10\cdot21)^2 + (1\cdot133)^2} = 10\cdot27$ ins.
Length of 15 coils $= 15 \times 10\cdot27 = 154\cdot1$ ins.
$= 12$ ft. $10\cdot1$ ins. Ans.

35. Weight of chain on one side $= 40 \times 6 = 240$ lb.
Weight of chain on other side $= 10 \times 6 = 60$ lb.
$\therefore$ Pull on short end to start motion $= 180$ lb. This pull must gradually decrease until there is 4 feet of chain more on long end than on short end, when pull will be $4 \times 6 = 24$ lb.

$$\therefore \text{ Average pull exerted} = \frac{180 + 24}{2} = 102 \text{ lb.}$$

$$\text{Pull is exerted through } \frac{(40 - 10) - 4}{2} = 13 \text{ feet.}$$

$\therefore$ Work done $= 102 \times 13 = 1{,}326$ foot lb. Ans.

36.
$$150 = \frac{9{,}900 \times}{3 \times 39} \qquad 60\,t$$

$$150 = \frac{1{,}100\,t}{13}\left(\frac{300\,t - 38}{60\,t}\right)$$

$$150 = \frac{110\,(300\,t - 38)}{13 \times 6}$$

$$300\,t - 38 = \frac{150 \times 13 \times 6}{110} = 106\cdot\dot{3}\dot{6}$$

$300\,t = 144\cdot\dot{3}\dot{6}$
$t = 0\cdot4812$ inch. Ans.

37. When turbine is at rest the load on each pedestal = 16·5 tons, and when running, the load is 22 tons on one pedestal and 11 tons on the other.

The twisting moment is then $5{\cdot}5 \times 3{\cdot}5 \times 2$ foot tons $= 5{\cdot}5 \times 3{\cdot}5 \times 2 \times 2{,}240$ foot lb.

T.M. (foot lb.) $\times\ 2\ \pi \times$ revs. = H.P. $\times$ 33,000.

$$\text{H.P.} = \frac{5{\cdot}5 \times 3{\cdot}5 \times 2 \times 2{,}240 \times 2 \times 3{\cdot}1416 \times 100}{33{,}000}$$

= 1,642. Ans.

38. Circumference of propeller $= 14 \times 3{\cdot}1416 = 43{\cdot}9824$ feet, and this would be the velocity of the blade tip per second (one revolution per second) if the ship did not advance through the water.

$$\text{But advance of ship per sec.} = \frac{11 \times 6{,}080}{3{,}600} = 18{\cdot}58 \text{ ft.}$$

$\therefore$ Velocity of blade tip $= \sqrt{(43{\cdot}98)^2 + (18{\cdot}58)^2}$
= 47·75 feet per second. Ans.

39. Load on plunger $= 17 \times 38$ lb.
Theoretical pressure in jack

$$\frac{\text{Load}}{\text{Area}} \quad \frac{17 \times 38}{(0{\cdot}625)^2 \times 0{\cdot}7854} \text{ lb. per square inch.}$$

Effective press. = Theoretical press. $\times$ 0·63 lb. per sq. inch.
Effective pressure $\times$ Area of ram = Load lifted.

$$\frac{17 \times 38}{(0{\cdot}625)^2 \times 0{\cdot}7854} \times 0{\cdot}63 \times D^2 \times 0{\cdot}7854 = 4{\cdot}2 \times 2{,}240$$

$$D^2 \quad \frac{4{\cdot}2 \times 2{,}240 \times (0{\cdot}625)^2 \times 0{\cdot}7854}{17 \times 38 \times 0{\cdot}63 \times 0{\cdot}7854} = 9$$

Diameter of ram = 3 inches. Ans.

40. Diameter of bolts

$$\frac{D^4}{D \times 4} \qquad R \qquad \sqrt{16 \times 4 \times 7 \times 12}$$

Note, $16^4 - 8^4 = (16^2 + 8^2)(16^2 - 8^2)$
$= (16^2 + 8^2) \times (16 + 8)(16 - 8)$

$$\text{Diameter} = \sqrt{\frac{320 \times 24 \times 8}{16 \times 4 \times 7 \times 12}} = 3{\cdot}381 \text{ inches. Ans.}$$

41.

$$850 \text{ lb.} = \text{Force due to friction} + \frac{3{\cdot}5 \times 2{,}240}{20}$$

subtract

$$x \text{ lb.} = \text{Force due to friction} - \frac{3{\cdot}5 \times 2{,}240}{20}$$

$$850 - x = 0 + \frac{3{\cdot}5 \times 2{,}240}{20} \times 2$$

$850 - x = 784$

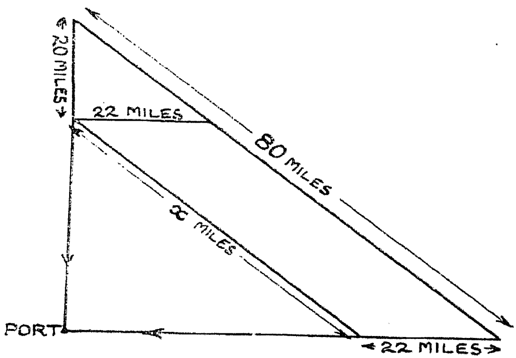
$66 = x$

Force required down the plane $= 66$ lb. Ans.

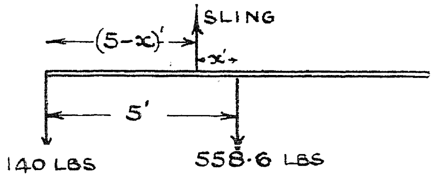
42. Let the ships be x miles apart after 2 hours.

Then $x = 80 - \sqrt{(22)^2 + (20)^2}$

$= 80 - 29{\cdot}73 = 50{\cdot}27$ miles. Ans.



43. Weight of beam $= \dfrac{10 \times 3{\cdot}25 \times 5{\cdot}5 \times 450}{144} = 558{\cdot}6$ lb.



By moments about sling.

$140\ (5 - x) = 558{\cdot}6\ x$

$700 - 140\ x = 558{\cdot}6\ x$

$700 = 698{\cdot}6\ x, \qquad x = 1{\cdot}002$ feet.

The beam is assumed to be of uniform section, and if the sling is moved 1·002 feet towards the centre it will balance.

44. Let x = speed of slow ship in knots.

then $x + 4$ = speed of fast ship.

$\dfrac{240}{x}$ = time taken by slow ship in hours.

$\dfrac{240}{x+4}$ = time taken by fast ship.

The slow ship takes 10 hours longer than the fast ship to complete the voyage.

$$\therefore \frac{240}{x} - \frac{240}{x+4} = 10$$

divide by 10, $\dfrac{24}{x} - \dfrac{24}{x+4} = 1$

L.C.M. is $x\ (x + 4)$, $\dfrac{24\ (x + 4) - 24\ x}{x\ (x + 4)} = 1$

$$\frac{24\ x + 96 - 24\ x}{x^2 + 4\ x}$$

$x^2 + 4\,x = 96, \qquad x^2 + 4\,x - 96 = 0.$

This is an easy quadratic equation, and the factors can be readily found.

$(x + 12)\,(x - 8) = 0$

If $x - 8 = 0$, then $x = 8$

and if $x + 12 = 0$, then $x = -12$.

The positive answer must be taken, and the speeds of the ships are 8 and 12 knots. Ans.

45. Find the force that would be required on the end of the 40 inch toggle in order to lift the weight.

$$F \times \frac{40 \times 2 \times 3{\cdot}1416}{0{\cdot}25} \times \tfrac{40}{100} = 5 \times 2{,}240$$

$F = 27{\cdot}85$ lb.

The force actually applied is 40 lb., which is $40 - 27{\cdot}85 = 12{\cdot}15$ lb. more than is necessary to lift the weight.

The second force must therefore act in opposition to the force on the 40 inch bar, and since it acts at the end of a 30 inch bar,

force required $= 12{\cdot}15 \times \frac{40}{30} = 16{\cdot}2$ lb. Ans.

46. Strength at A
$= (6 - 0{\cdot}75)\ 0{\cdot}5 \times 27$
$= 70{\cdot}875$ tons.

Strength of solid plate
$= 6 \times 0{\cdot}5 \times 27 = 81$ tons.

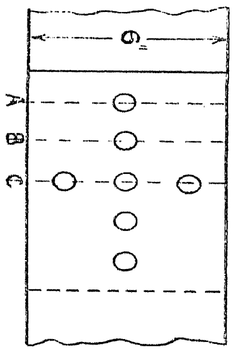
$\therefore$ Greatest possible joint

$$\text{efficiency} = \frac{70{\cdot}875}{81} = 0{\cdot}875$$

Strength of 1 rivet
$= 0{\cdot}75^2 \times 0{\cdot}7854 \times 23$
$= 10{\cdot}16$ tons.

Number of rivets required

$$= \frac{70{\cdot}875}{10{\cdot}16} = 7 \text{ (nearly).}$$



Strength through B $= 70 \cdot 875 + 10 \cdot 16 = 81 \cdot 035$ tons.
Strength through C
$= (6 - 3 \times 0 \cdot 75)\ 0 \cdot 5 \times 27 + 2 \times 10 \cdot 16$
$= 50 \cdot 625 + 20 \cdot 32 = 70 \cdot 945$ tons.
Number of rivets $= 7$. Joint strength $87 \cdot 5$ per cent. Ans.

47. Reaction of each support $= \frac{17}{2} = 8 \cdot 5$ tons.
The maximum bending moment occurs at mid-span, and it is equal to :—
$(8 \cdot 5 \times 6) - (6 \times 3) = 33$ foot tons $= 33 \times 12$ inch tons.
Bending moment = Resisting moment.

$$33 \times 12 = \frac{4 \times D^2}{6} \times 5$$

$$\frac{33 \times 12 \times 6}{4 \times 5} \quad 118 \cdot 8$$

Depth $= \sqrt{118 \cdot 8} = 10 \cdot 9$ inches. Ans.

48. Let D = internal diameter in inches then, $D + 0 \cdot 552$ = external diameter.
$\{ (D + 0 \cdot 552)^2 - D^2 \}\ 0 \cdot 7854 \times \frac{1}{144} \times 10 \times 8 \cdot 9 \times 62 \cdot 5 = 244$

$$D^2 + 1 \cdot 104\ D + 0 \cdot 3047 - D^2 = \frac{244 \times 144}{0 \cdot 7854 \times 10 \times 8 \cdot 9 \times 62 \cdot 5}$$

$= 8 \cdot 042$
$1 \cdot 104\ D = 8 \cdot 042 - 0 \cdot 3047$

$$D = \frac{7 \cdot 7373}{1 \cdot 104} = 7 \cdot 008 \text{ inches.} \quad \text{Ans.}$$

49. Let R = Resultant speed of ship.
By the cosine rule :—
$R^2 = 3^2 + 16^2 - 2 \times 3 \times 16 \times \cos. 135°$
$= 9 + 256 + 67 \cdot 89$
$= 332 \cdot 89$
R $=$ $\bar{9} = 18 \cdot 24$ knots.
Distance travelled in 95 minutes

$$\frac{18{\cdot}24 \times 95}{60}$$

$$= 28{\cdot}88 \text{ nautical miles.} \quad \text{Ans.}$$

Let $\theta°$ be the angle between original course and resultant course.

By the Sine rule :—

$$\frac{3}{\text{Sin. } \theta} = \frac{18{\cdot}24}{\text{Sin. } 135°} \qquad \text{Sin } \theta = \frac{3 \times 0{\cdot}7071}{18{\cdot}24} = 0{\cdot}1163$$

$$\theta = 6° \ 41'.$$

Resultant direction is 6° 41′ South of West or 83° 19′ West of South } Ans.

50. $0{\cdot}384$ statute mile $= 0{\cdot}384 \times \frac{5280}{6080} = 0{\cdot}3335$ nautical mile.

Average velocity $= \dfrac{\text{Distance}}{\text{Time}}$, and since the ship starts from rest,

final velocity $= 2 \times$ average velocity

$$2 \times 0{\cdot}3335 \text{ nautical miles per minute.}$$

$$\frac{2 \times 0{\cdot}3335}{5} \times 60 = 8{\cdot}004 \text{ knots.} \quad \text{Ans.}$$

$s = \frac{1}{2}\, a\, t^2 \quad \therefore$ Acceleration $= \dfrac{2\, s}{t^2} = \dfrac{2 \times 0{\cdot}3335}{5^2}$ nautical miles per min.^{2}

$v^2 = 2\, a\, s \;\therefore$ Velocity after travelling $0{\cdot}25$ nautical mile,

$$\sqrt{2 \times 2 \times 0{\cdot}3335 \times 0{\cdot}25} \text{ nautical miles per minute.}$$

$$= \tfrac{2}{5} \times 0{\cdot}5 \times 60 \sqrt{0{\cdot}3335} = 6{\cdot}924 \text{ knots.} \quad \text{Ans.}$$

51.

Plate per cent. $= \dfrac{-\,d}{p} \times 100$, and it is independent of plate thickness.

Plate per cent. for all thicknesses $= \dfrac{2{\cdot}75 - 0{\cdot}875}{2{\cdot}75} \times 100$

$= 68{\cdot}18$ per cent.

Rivet per cent. for 0·5 inch plate

$$\frac{(0{\cdot}875)^2 \times 0{\cdot}7854 \times 2}{2{\cdot}75 \times 0{\cdot}5} \times \tfrac{23}{28} \times 100 = 71{\cdot}84 \text{ per cent.}$$

The rivet per cent. varies inversely as the plate thickness.

$\therefore$ Rivet per cent. for 0·625 inch plate $= \dfrac{71{\cdot}84 \times 0{\cdot}5}{0{\cdot}625}$

$= 57{\cdot}47$ per cent.

and Rivet per cent. for 0·75 inch plate $= \dfrac{71{\cdot}84 \times 0{\cdot}5}{0{\cdot}75}$

$= 47{\cdot}87$ per cent.

$$\text{Working Press.} = \frac{2\,t}{\text{D}} \times \frac{\text{Tensile strength}}{\text{F}} \times \frac{\text{Least per cent.}}{100}$$

If any one of the rivet percentages is used, together with the corresponding plate thickness, the W.P. obtained must be the same because

$71{\cdot}84 \times 0{\cdot}5 = 57{\cdot}47 \times 0{\cdot}625 = 47{\cdot}87 \times 0{\cdot}75$;

the percentage and the plate thickness being the only variable quantities in the working pressure formula. It is therefore clear that 68·18 per cent., the plate percentage calculated on the 0·5 inch plate, will give the safe working pressure.

$$\frac{2 \times 0{\cdot}5}{84} \quad \frac{28 \times 2{,}240}{5} \times \frac{68{\cdot}18}{100}$$

$= 101{\cdot}8$ lb. per square inch. Ans.

52. $V^2 = u^2 - 2\,g\,s$, and when the bodies attain their greatest height $V = 0$, and $u^2 = 2\,g\,s$, or $s = \dfrac{u^2}{\quad}$

$$\text{Height attained by first body} \quad \frac{100^2}{64 \cdot 4} = 155 \cdot 28 \text{ ft. Ans.}$$

$$\text{Height attained by second body} = \frac{200^2}{64 \cdot 4} = 621 \cdot 12\text{ft. Ans.}$$

The bodies must be at the same distance above the ground when they meet, and this distance equals $u\,t - \frac{1}{2}\,g\,t^2$..

If the time taken to meet is "t" seconds by the first, it is $(t - 2)$ seconds by the second body.

$$100\,t - \tfrac{1}{2} \times 32 \cdot 2 \times t^2 = 200\,(t - 2) - \tfrac{1}{2} \times 32 \cdot 2\,(t - 2)^2$$

$$100\,t - 16 \cdot 1\,t^2 = 200\,t - 400 - 16 \cdot 1\,(t^2 - 4\,t + 4)$$

$$100\,t - 16 \cdot 1\,t^2 = 200\,t - 400 - 16 \cdot 1\,t^2 + 64 \cdot 4\,t - 64 \cdot 4$$

$$464 \cdot 4 = 164 \cdot 4\,t$$

$$t = 2 \cdot 824 \text{ seconds.}$$

Time to meet is 2·824 seconds after first is projected upwards and 0·824 second after second body is projected. Ans.

Height above ground $= 100 \times 2 \cdot 824 - 16 \cdot 1 \times (2 \cdot 824)^2 = 154$ feet. Ans.

53. Let internal diameter $= d$ feet.

$$\{(2^2 - d^2)\,0 \cdot 7854 \times 1\tfrac{11}{12} + 2^2 \times 0 \cdot 7854 \times \tfrac{1}{12}\} \times 450 = 2^2 \times 2 \times 0 \cdot 7854 \times 64$$

$$\{(2^2 - d^2)\,23 + 2^2\} \times \tfrac{450}{12} = 2^2 \times 2 \times 64$$

$$92 - 23\,d^2 + 4 = 13 \cdot 65\dot{3}$$

$$23\,d^2 = 82 \cdot 34\dot{6}$$

$$d \doteq 1 \cdot 892 \text{ feet.}$$

$$\text{Thickness} = \frac{2 - 1 \cdot 892}{2} \times 12 = 0 \cdot 648 \text{ inch. Ans.}$$

Since the weight of the vessel itself is equal to the weight of the water it displaces when floating, the tension in the wire when it is submerged will be equal to the weight of the water it can hold.

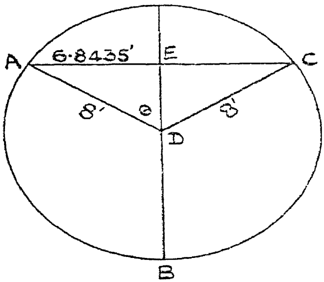
Tension $= 1{\cdot}892^2 \times 0{\cdot}7854 \times 1\frac{1}{12} \times 64 = 345$ lb. Ans.

54.

Breadth of water level $= \dfrac{157{\cdot}4}{11{\cdot}5} = 13{\cdot}687$ feet.

Sin. $\theta = \dfrac{6{\cdot}8435}{8} = 0{\cdot}8554. \qquad = 58^\circ\ 48'.$

Angle in sector A B C D
$= 360^\circ - 2(58^\circ 48') = 242^\circ\ 24'$
$= 242{\cdot}4^\circ.$



Area of sector

$= 16^2 \times 0{\cdot}7854 \times \dfrac{242{\cdot}4}{360}$

$= 135{\cdot}4$ square feet.

D E $= 8$ Cos. $58^\circ\ 48'$.

Area of triangle A D C $= 6{\cdot}8435 \times 8$ Cos. $58^\circ\ 48'$

$= 6{\cdot}8435 \times 8 \times 0{\cdot}5180 = 28{\cdot}36$ square feet.

Area of segment A B C E $= 135{\cdot}4 + 28{\cdot}36 = 163{\cdot}76$ sq. ft.

Weight of water

$= (163{\cdot}76 - 16^2 \times 0{\cdot}7854 \times 0{\cdot}32) \times 11{\cdot}5 \times \dfrac{62{\cdot}5}{2{,}240}$ tons.

$= (163{\cdot}76 - 64{\cdot}33) \times 11{\cdot}5 \times \dfrac{62{\cdot}5}{2{,}240}$ tons.

$= 99{\cdot}43 \times 11{\cdot}5 \times \dfrac{62{\cdot}5}{2{,}240} = 31{\cdot}91$ tons. Ans.

55. The area of an equilateral triangle $= 0{\cdot}433 \times \text{side}^2$

If the side of the smaller triangle is x inches, then the side of the greater is $(x + 2)$ inches.

$0{\cdot}433\ (x + 2)^2 - 0{\cdot}433\ x^2 = 24{\cdot}5$

$x^2 + 4\ x + 4 - x^2 = 56{\cdot}582$

$4\ x = 52{\cdot}582$

$x = 13{\cdot}145$ inches.

The sides are 13·145 inches and 15·145 inches. Ans.

56. Let the angle of the incline $= \theta$

Then

$$\begin{array}{rl} 2{,}240 &= 9{\cdot}25 \times 2{,}240 \times \mu \text{ Cos. } \theta + 9{\cdot}25 \times 2{,}240 \text{ Sin. } \theta \\ 94 &= 9{\cdot}25 \times 2{,}240 \times \mu \text{ Cos. } \theta - 9{\cdot}25 \times 2{,}240 \text{ Sin. } \theta \\ \hline \end{array}$$

by subtraction :

$2{,}146 = 2 \times 9{\cdot}25 \times 2{,}240 \times \text{Sin. } \theta$

$$\text{Sin. } \theta = \frac{2{,}146}{2 \times 9{\cdot}25 \times 2{,}240} = \frac{1}{19{\cdot}31}$$

The plane rises 1 foot in 19·31 feet. Ans.

57. Let $w =$ weight of valve in lb.

$x =$ distance from valve to fulcrum in inches.

By moments about the fulcrum :—

$0 \times (21 + x) + 6\ (7{\cdot}5 + x) + w\ x = 3 \times 8 \times x$

when W on lever $= 0$ lb.

$62 \times (21 + x) + 6\ (7{\cdot}5 + x) + w\ x = 65 \times 8 \times x$

when W on lever $= 62$ lb.

Subtracting the first equation from the second,

$62\ (21 + x) = 62 \times 8 \times x$

$21 + x = 8\ x$

$\therefore\ x = 3$ inches. Ans.

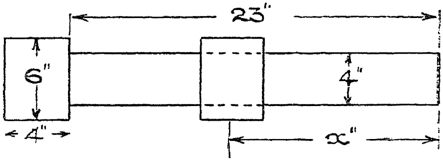
Substituting $x = 3$ in the first equation

$(0 \times 24) + (6 \times 10{\cdot}5) + 3\ w = 3 \times 8 \times 3$

$3\ w = 72 - 63 = 9$

$\therefore\ w = 3$ lb. Ans.

58. Let the mid-width of the ring be at x inches from the point of the bolt.



By moments about point of bolt.

Dist. of C.G. from point

$$= \frac{\text{Sum of moments of volumes about point}}{\text{Total volume}}$$

$$15 = \frac{(6^2 \times 0{\cdot}7854 \times 4 \times 25) + (4^2 \times 0{\cdot}7854 \times 23 \times 11{\cdot}5) + (6^2 - 4^2)0{\cdot}7854 \times 4 \times x}{(6^2 \times 0{\cdot}7854 \times 4) + (4^2 \times 0{\cdot}7854 \times 23) + (6^2 - 4^2)\,0{\cdot}7854 \times 4}$$

Eliminating 0·7854 and cross-multiplying

$$(6^2 \times 4 \times 25) + (4^2 \times 23 \times 11{\cdot}5) + (6^2 - 4^2) \times 4 \times x$$
$$= 15\,\{(6^2 \times 4) + (4^2 \times 23) + (6^2 - 4^2)\,4\}$$

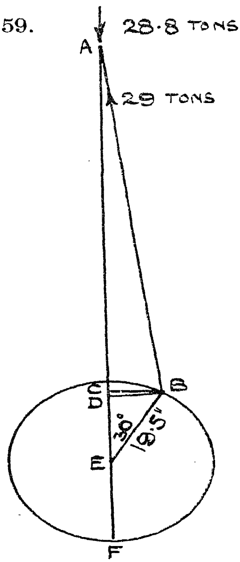
$3{,}600 + 4{,}232 + 80\,x = 8{,}880$

$80\,x = 1{,}048$

$x = 13{\cdot}1$ inches.

Mid-width of the ring is at 13·1 inches from the point of bolt Ans.

59.



CB = 19·5 × 0·5 = 9·75 inches.

EC = 19·5 × 0·866 = 16·887 inches.

Triangle A B C is similar to the vector diagram for point A.

A B represents to scale the load on the crank pin.

A C represents to the same scale the load on piston, and B C represents the load on the guide.

Load on guide $= \sqrt{(29)^2 - (28{\cdot}8)^2}$ = 3·4 tons, this is represented by 9·75 inches.

$$A\,B = \frac{29}{3{\cdot}4} \times 9{\cdot}75 = 83{\cdot}15 \text{ ins.}$$

= connecting rod length,

$$A\,C = \frac{28{\cdot}8}{3{\cdot}4} \times 9{\cdot}75 = 82{\cdot}58 \text{ ins.}$$

C D = 83·15 — 82·58 = 0·57 inch.

D F = F E + E C — C D

= 19·5 + 16·887 — 0·57

= 35·817 inches.

Load on guide = 3·4 tons.
Length of rod = 83·15 inches.
Piston is 35·817 inches from bottom. } Ans.

60. Area of opening through valve $\dfrac{D \times \pi \times L}{144}$ sq. feet.

Volume of water passing per minute

$$= \frac{D \times \pi \times L}{144} \times V \times 60 \text{ cubic feet.}$$

Weight of water per minute

$$= \frac{D \times \pi \times L}{144} \times V \times 60 \times 62{\cdot}5 \text{ lb.}$$

$= 81{\cdot}81$ D L V lb.

Weight of water per minute $= 81{\cdot}81 \times 3 \times 0{\cdot}0625 \times 12$
$= 184{\cdot}05$ lb. Ans.

61. Let D = external diameter of collar in inches,
then $4 \times (\text{shaft diameter})^3 = D^3$

Shaft diameter $= \dfrac{D}{}$

Area of each liner $= \dfrac{41{,}800}{5 \times 70}$ square inches.

Area of each collar $= \dfrac{41{,}800}{5 \times 70} \times \frac{3}{2}$ square inches.

1,800 × 3
× 70 × 2

$$D^2 \quad \frac{41{,}800 \times 3}{5 \times 70 \times 2 \times 0{\cdot}7854} = 228{\cdot}1$$

$D^2 \quad \dfrac{D^2}{2{\cdot}52} \quad 228{\cdot}1$, multiply throughout by $2{\cdot}52$

$$2{\cdot}52\ D^2 - D^2 = 228{\cdot}1 \times 2{\cdot}52$$
$$1{\cdot}52\ D^2 = 228{\cdot}1 \times 2{\cdot}52$$
$$\sqrt{\frac{228{\cdot}1 \times 2{\cdot}52}{1{\cdot}52}} = 19{\cdot}45 \text{ inches. Ans.}$$

62. Head due to pressure of 60 lb. per square inch $= 60 \times 2{\cdot}3 = 138$ feet.

Head producing velocity $= 138 - 15 - 12 = 111$ feet.

Velocity of water passing into tank $= \sqrt{2\,g\,h}$

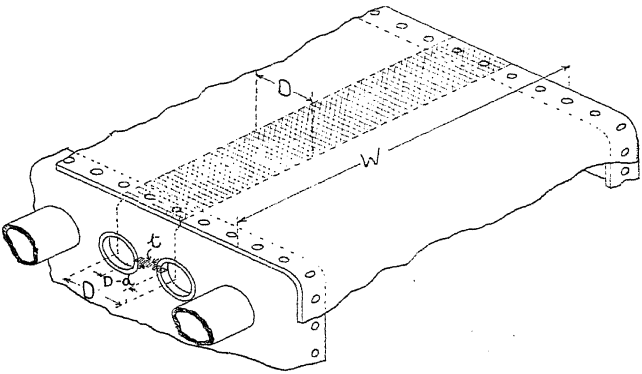
$$\sqrt{64{\cdot}4 \times 111} = 84{\cdot}53 \text{ feet per second.}$$

$$\text{Time taken} = \frac{\text{Volume of tank in cubic feet}}{\text{Volume flowing in per second}}$$

$$= \frac{15^2 \times 0{\cdot}7854 \times 12}{84{\cdot}53 \times (0{\cdot}25)^2 \times 0{\cdot}7854} = 511 \text{ seconds} = 8 \text{ mins. } 31 \text{ secs. Ans.}$$

63. Internal diameter of tubes $= 3 - 2 \times 0{\cdot}16 = 2{\cdot}68$ inches.
Thickness of tube plate $= \frac{3}{4}$ inch $= \frac{3}{4} \times 32$
$= 24$ thirty-seconds of an inch.

$$\text{W.P.} = \frac{875\ (4{\cdot}25 - 2{\cdot}68) \times 24}{33 \times 4{\cdot}25} = 235 \text{ lb. per sq. inch. Ans.}$$



Consider a strip of the top of the combustion chamber D inches wide and W inches long.

The total load on this strip is :—

$$\text{W.P.} \times W \times D \text{ lb.}$$

Since the function of the girder stays is to transmit this load on to the front and back plates of the combustion chamber, then each plate must support half of the load.

Load carried by tube plate

$$W \times D \times \text{working pressure}$$

But working pressure

$$= \frac{875 \times (D - d)\, t \times 32}{W \times D}, \ t \text{ being inches.}$$

Substitute, and we have—

Load carried

$$\frac{W \times D}{2} \quad \frac{875 \times (D - d) \times t \times 32}{W \times D}$$

$$= 875\ (D - d) \times t \times 16$$

$$= 14000\ (D - d) \times t$$

This load is carried by a piece of the tube plate having a sectional area of $(D - d) \times t$ sq. inches, and the tube plate is subjected to compressive stress.

Area $\times$ stress = Load carried

$(D - d) \times t \times \text{stress} = 14000\ (D - d) \times t$

$\therefore$ stress = 14000 lb. per sq. inch. Ans.

The formula is designed to limit the compressive stress in the tube plate to 14000 lb. per sq. inch, and this stress is independent of the value of W, D, d or t.

64. Force up incline = 11 cwts. = W Sin. $\theta + \mu$ W Cos. θ

Force on incline = 9 cwts. = W Sin. $\theta - \mu$ W Cos. θ

20 cwts. = 2 W Sin. θ, by addition

$$\text{Sin. } \theta = \frac{20}{2 \times 10 \times 20} = \tfrac{1}{20} = 0{\cdot}05,\ 0{\cdot}05 = \text{Sin. } 2^\circ\ 52'$$

The angle of the plane is $2^\circ\ 52'$, or it rises 1 foot in 20 feet. Ans.

Also, by subtraction of the equations, 2 cwts. = 2 μ WCos.θ

$$\mu = \frac{2}{2 \times 20 \times 10 \times 0{\cdot}9987} = \frac{1}{200 \times 0{\cdot}9987}$$

$$\text{Total friction} = 10 \times 2{,}240 \times \frac{1}{200 \times 0{\cdot}9987} \text{ lb.}$$

$$\therefore \text{ Friction per ton} = \frac{10 \times 2{,}240}{10 \times 200 \times 0{\cdot}9987} = 11{\cdot}214 \text{ lb.}$$

Ans.

65. Sectional area

$= 6\text{ T} + \text{T}(10 - 2\text{ T}) + 7\text{ T}$

$= 23\text{ T} - 2\text{ T}^2$ sq. inches.

$(23\text{ T} - 2\text{ T}^2) \times 12 \times 0{\cdot}26 = 80$

$23\text{ T} - 2\text{ T}^2 = 25{\cdot}64$

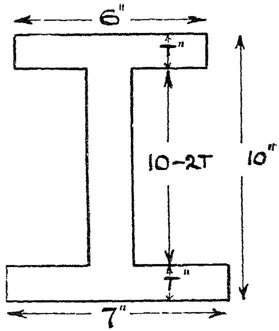
$2\text{ T}^2 - 23\text{ T} + 25{\cdot}64 = 0$

$\text{T} = \dfrac{23 \pm}{}$

$= \dfrac{23 \pm 17{\cdot}99}{4}$

$\text{T} = 10{\cdot}29$ or $1{\cdot}25$

Thickness = 1·25 inches.



By moments about bottom flange :—

Distance of C.G. above bottom flange

$$= \frac{(7 \times 1{\cdot}25 \times 0{\cdot}625) + (7{\cdot}5 \times 1{\cdot}25 \times 5) + (6 \times 1{\cdot}25 \times 9{\cdot}375)}{(7 \times 1{\cdot}25) + (7{\cdot}5 \times 1{\cdot}25) + (6 \times 1{\cdot}25)}$$

$= 4{\cdot}787$ inches. Ans.

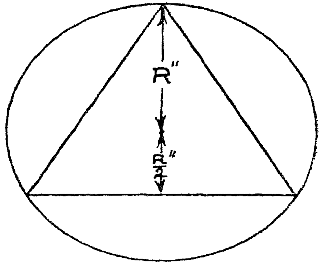
66. C.G. of the inscribed equilateral triangle must coincide with the centre of the circle.

Let R = radius of the circle in inches.

Then $\frac{3}{2}$ R = perpendicular height of the triangle.

Area of circle = π R^2 square inches.

Perimeter of circle = 2 π R inches.



Side of triangle $= \frac{2}{\sqrt{3}} \times \frac{3}{2}\,R = \sqrt{3}\,R$ inches.

Area of triangle $= \sqrt{3}\,R \times \frac{3}{2}\,R \times \frac{1}{2} = \frac{3\sqrt{3}}{\;}$ sq. ins

Perimeter of triangle $= 3\sqrt{3}\,R$ inches.

The *number* of square inches

$= \pi R^2 -$

The *number* of inches $= 2\pi R - 3\sqrt{3}\,R = R\,(2\pi - 3\sqrt{3})$
and since the *number* is the same in both cases

$$\therefore\ R^2\left(\pi - \frac{3\sqrt{3}}{2}\right) = R\,(2\pi - 3\sqrt{3})$$

Dividing each side by R, and by $\pi - \frac{3\sqrt{3}}{2}$

R = 2 inches.

Side of triangle $= 2\sqrt{3}$ inches $= 2 \times 2{\cdot}54 \times \sqrt{3} = 8{\cdot}798$ centimetres. Ans.

67. D B = 1 Sin. 55° = 0·8192
A D $= \sqrt{(3{\cdot}9)^2 - (0{\cdot}8192)^2} = 3{\cdot}813$
Load on crank pin

$$= 5{,}230 \times \frac{3{\cdot}9}{0{\cdot}8192} = 24{,}900 \text{ lb.}$$

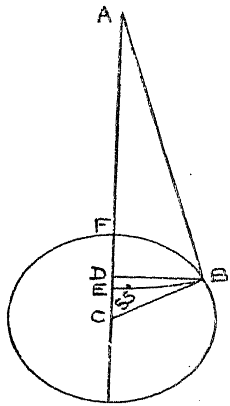
Load on piston

$$= 5{,}230 \times \frac{3{\cdot}813}{0{\cdot}8192} = 24{,}340 \text{ lb.}$$

D C = 1 Cos. 55° = 0·5736
D F = 1 — 0·5736 = 0·4264
D E = 3·9 — 3·813 = 0·087
F E = 0·4264 + 0·087 = 0·5134

Fraction of stroke $= \frac{0{\cdot}5134}{2}$

= 0·2567

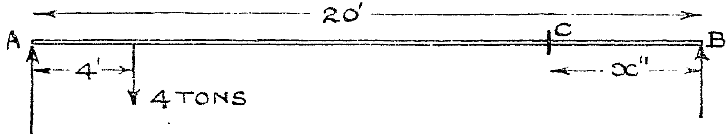


Load on crank pin = 24,900 lb.
Load on piston = 24,340 lb.
Piston has completed 25·67 per cent. of the stroke.
} Ans.

8. Neglecting the weight of the beam :—

By moments about A.

$B \times 20 = 4 \times 4 \quad \therefore B = \frac{16}{20} = 0 \cdot 8$ ton.



Bending moment at C, where the stress is 1 ton per sq. inch $= 0 \cdot 8\ x$ inch tons.

Moment of inertia of beam section

$$\frac{(6 \times 9^3) - (5 \times 7^3)}{12} = 221 \cdot 68 \text{ inch}^4$$

$$\text{Resisting moment} = \frac{I}{y} \times p = \frac{221 \cdot 68}{4 \cdot 5} \times 1$$

Bending moment = Resisting moment

$$0 \cdot 8\ x = \frac{221 \cdot 68}{4 \cdot 5} \times 1, \quad x = 61 \cdot 58 \text{ inches. Ans.}$$

$$\text{Area of section of beam} = \frac{(6 \times 9) - (5 \times 7)}{144} = \frac{19}{144} \text{ sq.}$$

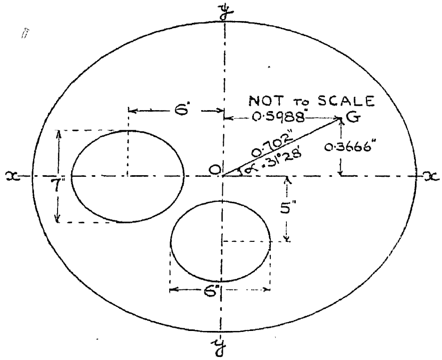
$$\text{Weight of beam} = \frac{19}{144} \times \frac{20}{1} \times \frac{480}{2240} = 0 \cdot 577 \text{ ton.}$$

Reaction at B $= 0 \cdot 8 + 0 \cdot 2885 = 1 \cdot 0885$ tons.

$$\text{Bending moment at C} = 1 \cdot 0885\ x - \frac{0 \cdot 577\ x}{240} \times \frac{x}{2} \text{ inch tons.}$$

$$1 \cdot 0885\ x - \frac{0 \cdot 577\ x^2}{480} \quad \frac{221 \cdot 68}{4 \cdot 5} \times 1$$

Solving this quadratic equation, $x = 47 \cdot 82$ inches. Ans.



69. Taking moments about axis $\overline{x\ x}$,

$$\text{C.G. from } \overline{x\ x} = \frac{\frac{\pi}{4} \times 24^2 \times 0 - \frac{\pi}{4} \times 7^2 \times 0 - \frac{\pi}{4}}{\frac{\pi}{4} \times 24^2 - \frac{\pi}{4} \times 7^2 - \frac{\pi}{4} \times}$$

$$\text{,, ,,} = -\ 0{\cdot}3666 \text{ inch.}$$

The minus sign indicates that C.G. is above $\overline{x\ x}$.

Taking moments about $\overline{y\ y}$,

$$\text{C.G. from } \overline{y\ y} = \frac{\frac{\pi}{4} \times 24^2 \times 0 - \frac{\pi}{4} \times 7^2 \times 6 - \frac{\pi}{4} \times 6^2 \times 0}{\frac{\pi}{4} \times 24^2 - \frac{\pi}{4} \times 7^2 - \frac{\pi}{4} \times}$$

$$\text{,, ,,} = -\ 0{\cdot}5988 \text{ inch.}$$

The minus sign indicates that C.G. is to the right of $\overline{y\ y}$.

$$\text{Tan } \alpha = \frac{0{\cdot}3666}{0{\cdot}5988}, \quad \alpha = 31^\circ\ 28'$$

$$\text{O G} = \frac{0{\cdot}3666}{\text{Sin. } 31^\circ\ 28'} \quad 0{\cdot}702 \text{ inch.}$$

The centre of gravity is at 0·702 inch from the centre of the plate, in a direction of 31° 28′ to the axis $\overline{x\ x}$. Ans.

70. Useful work $= 43 \times 0{\cdot}8888 = 38{\cdot}2184$ foot lb.

Useful work done = weight in lb. $\times$ 4 feet.

$$\therefore \text{Weight lifted} = \frac{38{\cdot}2184}{4} = 9{\cdot}5546 \text{ lb.}$$

Work done in stretching the spring $= 43 - 38{\cdot}2184$ $= 4{\cdot}7816$ ft. lb. and this equals average weight on the spring $\times$ stretch of spring.

$$\text{Stretch of spring} = \frac{4{\cdot}7816 \times 2 \times 12}{9{\cdot}5546} = 12{\cdot}01 \text{ inches.}$$

$$\text{Weight to stretch spring 1 inch} = \frac{9{\cdot}5546}{12{\cdot}01} = 0{\cdot}795 \text{ lb.}$$ Ans.

Weight lifted $= 9{\cdot}55$ lb. Ans.

71. (1) Plate strength in outer rows $= (4{\cdot}5 - d)\ 28 \times 1$ $= 126 - 28\ d$, tons.

(2) Rivet strength $= d^2 \times 0{\cdot}7854 \times 23 \times 4 = 72{\cdot}26 d^2$, tons.

(3) Strength of middle row =

$(4{\cdot}5 - 2\ d)\ 28 \times 1 + d^2 \times 0{\cdot}7854 \times 23$

$= 126 - 56\ d + 18{\cdot}06\ d^2$ tons.

Equating (1) to (2), $126 - 28\ d = 72{\cdot}26\ d^2$ and solving the quadratic, $d = 1{\cdot}141$ inches.

Substituting in (1) Plate strength $= 126 - 28 \times 1{\cdot}141$ $= 94{\cdot}05$ tons

Substituting in (2) Rivet strength $= 72{\cdot}26 \times (1{\cdot}141)^2$ $= 94{\cdot}05$ tons.

Substituting in (3) Middle row =

$126 - 56 \times 1{\cdot}141 + 18{\cdot}06 \times (1{\cdot}141)^2 = 85{\cdot}61$ tons.

Least strength with this diameter of rivet $= 85{\cdot}61$ tons.

Equating (1) to (3).

$126 - 28\ d = 126 - 56\ d + 18{\cdot}06\ d^2$

and from this $d = 1{\cdot}55$ inches.

Substituting in (1). Plate strength $= 126 - 28 \times 1{\cdot}55$ $= 82{\cdot}6$ tons.

This diameter of rivet will therefore give a joint strength less than that for the first case, and it need not be considered any further.

Equating (2) to (3).

$72 \cdot 26\ d^2 = 126 - 56\ d + 18 \cdot 06\ d^2$, and solving this quadratic, $d = 1 \cdot 094$ inches.

Substituting in (1). Plate strength $= 126 - 28 \times 1 \cdot 094$
$= 95 \cdot 38$ tons.

Substituting in (2). Rivet strength $= 72 \cdot 26 \times (1 \cdot 094)^2$
$= 86 \cdot 4$ tons.

Substituting in (3). Middle row $=$

$126 - 56 \times 1 \cdot 094 + 18 \cdot 06 \times (1 \cdot 094)^2 = 95 \cdot 38$ tons.

Least strength with this rivet diameter is 86·4 tons, and this will give the best joint strength.

Strength of solid plate $= 4 \cdot 5 \times 28 \times 1 = 126$ tons.

Percentage strength of joint $= \dfrac{86 \cdot 4}{126} \times 100 = 68 \cdot 56$ per cent.

Diameter of rivet $= 1 \cdot 094$ inches. Strength of joint $=$
68·56 per cent. Ans.

72. 9,000 feet per minute $= \frac{9000}{60} = 150$ feet per sec.

Let t = time in seconds to meet,

Distance fallen by first body $= \frac{1}{2}\ g\ t^2$

Distance risen by second body $= 150\ t - \frac{1}{2}\ g\ t^2$

$\therefore \frac{1}{2}\ g\ t^2 + 150\ t - \frac{1}{2}\ g\ t^2 = 525$

$t = \frac{525}{150} = 3 \cdot 5$ seconds. Ans.

Height above ground $= 525 - \frac{1}{2} \times 32 \cdot 2 \times 3 \cdot 5^2 = 327 \cdot 775$
feet. Ans.

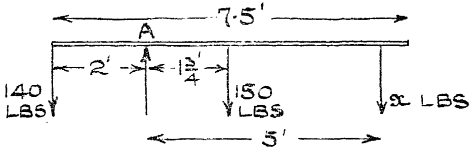
Velocity of 1st body $= 32 \cdot 2 \times 3 \cdot 5 \quad = 112 \cdot 7$ feet per sec.

Velocity of 2nd body $= 150 - 32 \cdot 2 \times 3 \cdot 5 = \ \ 37 \cdot 3$,, ,,

Velocity of impact $= 150$,, ,,

The sum of the velocities of the bodies at any instant is 150 feet per second.

73. Weight of bar $= 20 \times 7{\cdot}5 = 150$ lb.



By moments about A,

$(x \times 5) + (150 \times 1\tfrac{3}{4}) = (140 \times 2)$

$5\,x = 280 - 262{\cdot}5$

$5\,x = 17{\cdot}5, \qquad x = 3{\cdot}5$ lb. Ans.

74. Let the length of the lever be x inches $= \dfrac{x}{12}$ feet.

Weight of lever $= \dfrac{5\,x}{12}$ lb.

By moments about the fulcrum.

When weight $=$ O

$$(\mathrm{O} \times x) + \left(\frac{5x}{12} \times \frac{x}{2}\right) + (2{\cdot}5 \times 2{\cdot}5) = 5 \times (2{\cdot}5)^2 \times 0{\cdot}7854 \times 2{\cdot}5. \quad (1)$$

When weight $=$ W

$$(\mathrm{W} \times x) + \left(\frac{5x}{12} \times \frac{x}{2}\right) + (2{\cdot}5 \times 2{\cdot}5) = 60 \times (2{\cdot}5)^2 \times 0{\cdot}7854 \times 2{\cdot}5. \quad (2)$$

Subtracting $\quad \mathrm{W} \times x = 55 \times (2{\cdot}5)^2 \times 0{\cdot}7854 \times 2{\cdot}5$

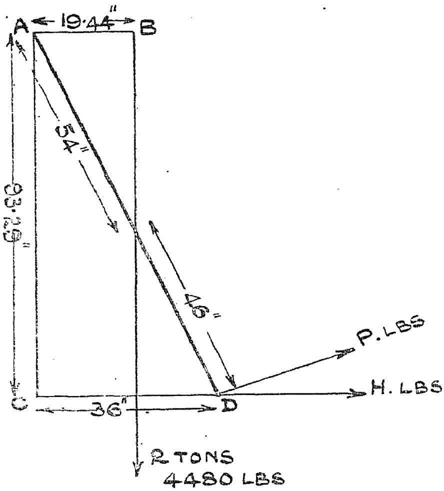
From equation (1) $\quad \dfrac{5\;x^2}{24} = 55{\cdot}09$

$x = 16{\cdot}26$ inches.

$$\mathrm{W} = \frac{55 \times 2{\cdot}5^2 \times 0{\cdot}7854 \times 2{\cdot}5}{16{\cdot}26} = 41{\cdot}5 \text{ lb.} \quad \text{Ans.}$$

75. A B $= \frac{54}{100} \times 36 = 19{\cdot}44$ inches.

Least force will be that which acts at right angles to the rod.



By moments about A,

$P \times 100 = 4{,}480 \times 19{\cdot}44$

$P = 870{\cdot}912$ lb. Ans.

To find the horizontal force,

$A\ C = \sqrt{100^2 - 36^2} = 93{\cdot}29$ inches.

By moments about A,

$H \times 93{\cdot}29 = 4{,}480 \times 19{\cdot}44$

$H = 933{\cdot}7$ lb.

Least force $= 870{\cdot}9$ lb. Horizontal force $= 933{\cdot}7$ lb. Ans.

76. Weight of 1 cubic inch of cast iron $= 0{\cdot}26$ lb.

Let N = revolutions per second.

Centrifugal force due to 1 cubic inch

$$\frac{0{\cdot}26 \times (7 \times 3{\cdot}1416 \times}{32{\cdot}2 \times 3{\cdot}5} \text{ lb.}$$

Total force tending to rupture the rim = C. F. due to 1 cubic inch × Diameter (inches)

$$= \frac{0{\cdot}26 \times (7 \times 3{\cdot}1416 \times N)^2}{32{\cdot}2 \times 3{\cdot}5} \times 84 \text{ lb.}$$

Resisting force $=$ 2 (square inches) $\times$ Tensile stress.

$$\frac{0{\cdot}26 \times (7 \times 3{\cdot}1416 \times N)^2}{32{\cdot}2 \times 3{\cdot}5} \times 84 = 2 \times 9 \times 2{,}240$$

$N = 20{\cdot}74$ revolutions per second. Ans.

C B $= 24$ Sin. $65°$
$= 24 \times 0{\cdot}9063 = 21{\cdot}75$ inches.
A C $= \sqrt{100^2 - (21{\cdot}75)^2} = 97{\cdot}61$ inches.

When A C represents load on piston to scale,
C B represents load on guide to same scale,

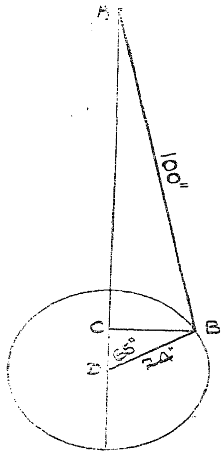
Load on guide

$$= \frac{21{\cdot}75}{97{\cdot}61} \times 22 \times 2{,}240 \text{ lb.}$$

Force opposing motion

$$= \frac{21{\cdot}75}{97{\cdot}61} \times 22 \times 2{,}240 \times 0{\cdot}072$$

$= 790{\cdot}5$ lb. Ans.



78. Let R_1 = left hand support.
R_2 = right hand support.

Moments about R_1 :—

$(10 \times 5) + (6 \times 16) = R_2 \times 20$, $R_2 = 7{\cdot}3$ tons.

$10 + 6 = R_1 + 7{\cdot}3$, $R_1 = 8{\cdot}7$ tons.

$$\left.\begin{array}{lll} M_{\text{at centre}} & = 7{\cdot}3 \times 10 - 6 \times 6 & = 37 \text{ ft. tons.} \\ M_{\text{under 10 tons load}} & = 8{\cdot}7 \times 5 & = 43{\cdot}5 \quad ,, \\ M_{\text{under 6 tons load}} & = 7{\cdot}3 \times 4 & = 29{\cdot}2 \quad ,, \end{array}\right\} \text{Ans.}$$

Maximum bending moment is $43{\cdot}5$ ft. tons.

$$\text{Stress } p \quad \frac{6\ M}{B\ D^2} \quad \frac{6 \times 43{\cdot}5 \times 12}{5 \times 12^2} = 4{\cdot}35 \text{ tons per sq. in.} \quad \text{Ans.}$$

79.

$$\text{Area} = \frac{\text{Volume}}{\text{Length}} = \frac{159 \times 2240}{0{\cdot}9 \times 62{\cdot}5 \times 50} \text{ sq. ft.} \qquad \text{(i)}$$

Let x = breadth at bottom. Area by Simpson's rule :-

Ordinates	Simpson's Multipliers	Functions of Ordinates
29·7	1	29·7
31·2	4	124·8
30·6	2	61·2
30·1	4	120·4
x	1	x

Sum = $336{\cdot}1 + x$

Common interval

$$= \frac{4{\cdot}5}{4} \text{ ft.}$$

Area

$$(336{\cdot}1 + x) \times \frac{4{\cdot}5}{4 \times 3} \qquad \ldots \text{(ii)}$$

From (i) and (ii) :—

$$(336{\cdot}1 + x) \times \frac{4{\cdot}5}{4 \times 3} = \frac{159 \times 2240}{0{\cdot}9 \times 62{\cdot}5 \times 50}$$

$$x = \frac{159 \times 2240 \times 4 \times 3}{0{\cdot}9 \times 62{\cdot}5 \times 50 \times 4{\cdot}5} - 336{\cdot}1$$

$$= 1{\cdot}6 \text{ feet. Ans.}$$

80.

Area of equilateral triangle $\times$ side^2

$$\text{Side} = \frac{\overline{\text{Area} \times 4}}{\sqrt{3}}$$

$$\text{Perpendicular height} = \frac{\sqrt{3}}{2} \times \text{side}$$

$$\times \sqrt{\text{Area} \times 4} \qquad \overline{\text{Area}} \times 4 \times 3$$

$$= \sqrt{\text{Area}} \times$$

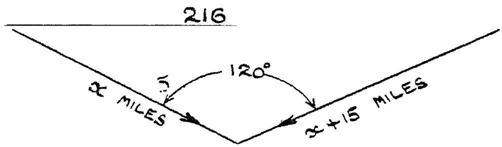
Radius $= \frac{1}{3}$ of perpendicular height

$$= \frac{1}{3} \times \sqrt{\text{Area} \times \sqrt{3}} =$$

$$= \sqrt{\frac{20{\cdot}84 \times 1{\cdot}732}{}} = 2{\cdot}002 \text{ centimetres.}$$

Area of circle $= 3{\cdot}1416 \times (2{\cdot}002)^2 \times (0{\cdot}3937)^2 = 1{\cdot}952$ sq. ins. Ans.

81. Let the ships be x miles, and $x + 15$ miles from the port



Then :—

$$216^2 = x^2 + (x + 15)^2 - 2 \times x \times (x + 15) \times \text{Cos. } 120^\circ$$

Note, Cos. $120^\circ = - 0{\cdot}5$

$$216^2 = x^2 + (x + 15)^2 + 2 \times x \times (x + 15) \times 0{\cdot}5$$

$$46{,}656 = x^2 + x^2 + 30\,x + 225 + x^2 + 15\,x$$

$$3\,x^2 + 45\,x - 46{,}431 = 0$$

$$x^2 + 15\,x - 15{,}477 = 0$$

$$\frac{-15 \pm \sqrt{225 + 61{,}908}}{2} \qquad \frac{-15 \pm 249{\cdot}26}{2}$$

taking the positive value, $x = 117{\cdot}13$.

The ships are 117·13 miles, and 132·13 miles from port. Ans.

82. Let the first compression be 1, and the load on the spring be P lb., then the second compression is 1·64, and the load on the spring is 1·64 P lb.

Let the weight of the valve, etc., be W lb.

$$1\cdot64\ P + W = 185\ (3\cdot75^2 \times 0\cdot7854)\ \ldots\ (1)$$
$$P + W = 115\ (3\cdot75^2 \times 0\cdot7854)\ \ldots\ (2)$$

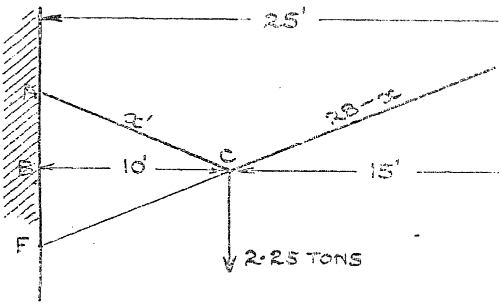
subtracting $\quad 0\cdot64\ P = 70\ (3\cdot75^2 \times 0\cdot7854)$

$$P = \frac{70}{0\cdot64}(3\cdot75^2 \times 0\cdot7854) = (109\cdot375)\ (3\cdot75^2 \times 0\cdot7854)$$

From equation (2) :—

$$\begin{aligned} W &= 115\ (3\cdot75^2 \times 0\cdot7854) - 109\cdot375\ (3\cdot75^2 \times 0\cdot7854) \\ &= 5\cdot625\ (3\cdot75^2 \times 0\cdot7854) \\ &= 62\cdot1 \text{ lb.} \quad \text{Ans.} \end{aligned}$$

83. Since the cover hangs from a block on the wire the tension must be the same throughout, and the two parts of the wire must make equal angles with the horizontal.



Triangles A B C and C D E are similar, and the ratio of their sides is 10 : 15 = 1 : 1·5

$x : 28 - x :: 10 : 15, \quad x = (28 - x) \times \frac{10}{15}$
$3\ x = 56 - 2\ x, \qquad x = 11\cdot2$ feet.

A B = $\qquad\qquad = 5\cdot04$ feet.

D E = A B × 1·5, or the difference between D E and A B is $5\cdot04 \times \frac{1}{2} = 2\cdot52$ feet. Ans.

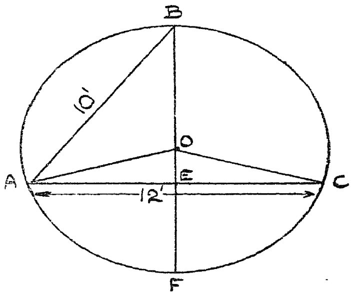
Triangle A C F is similar to the vector diagram for point C.
A F $=$ 10·08 feet and represents to scale 2·25 tons.
A C $=$ 11·2 feet and represents to the same scale the tension.

$$\text{Tension in wire} = \frac{11{\cdot}2}{10{\cdot}08} \times 2{\cdot}25 = 2{\cdot}5 \text{ tons.} \quad \text{Ans.}$$

84. $BE = \sqrt{10^2 - 6^2} = 8$ feet.
$BE \times EF = AE \times EC$

$$EF = \frac{6 \times 6}{8} = 4{\cdot}5 \text{ feet.}$$

$\therefore$ Diameter of the circle $= 8 + 4{\cdot}5 = 12{\cdot}5$ feet.
Sine $ABE = \frac{6}{10} = 0{\cdot}6$



$\therefore$ Angle $ABE = 36^\circ\ 52'$.
Angle $AOB = 180^\circ - 2\ (36^\circ\ 52') = 106^\circ\ 16'$
Angle in sector $ABCO = 2\ (106^\circ\ 16') = 212^\circ\ 32'$
$= 212{\cdot}53^\circ$.

$$\text{Length of Arc} = \frac{212{\cdot}53}{360} \times 12{\cdot}5 \times 3{\cdot}1416 = 23{\cdot}18 \text{ feet.}$$

$= 23$ feet $2{\cdot}16$ inches. Ans.

85. K nautical miles in N minutes,

$$\frac{K \times 60}{N} \text{ nautical miles per hour.}$$

$$\frac{K \times 60}{N} \; \frac{6080}{5280} \text{ statute miles per hour.}$$

$$= 69{\cdot}\dot{0}\dot{9}\,\frac{K}{N} \text{ statute miles per hour.}$$

1 kilometre $= \dfrac{1{,}000 \times 39{\cdot}37}{12}$ feet, and 1 statute mile $= 5{,}280$ feet.

$\therefore$ Number of kilometres in 1 mile $= \dfrac{5{,}280 \times 12}{1{,}000 \times 39{\cdot}37}$

$= 1{\cdot}609.$

K nautical miles in N minutes

$$= 69{\cdot}\dot{0}\dot{9}\,\frac{K}{N} \times 1{\cdot}609 = 111{\cdot}2\,\frac{K}{N} \text{ kilometres per hour.}$$

$$69{\cdot}\dot{0}\dot{9}\,\frac{K}{N} \text{ statute miles per hour.}$$

Ans.

$$111{\cdot}2\,\frac{K}{N} \text{ kilometres per hour.}$$

Statute miles per hour $= 69{\cdot}\dot{0}\dot{9} \times \dfrac{\;}{4{\cdot}8} = 14{\cdot}37.$ Ans.

Kilometres per hour $= 111{\cdot}2 \times \dfrac{1}{4{\cdot}8} = 23{\cdot}17.$ Ans.

3. Let x = diameter of small pulley in inches.

Then $x + 0{\cdot}75$ = diameter of large pulley.

Velocity ratio $= \dfrac{2\,D}{D - d} \qquad \dfrac{2\,(x + 0{\cdot}75)}{0{\cdot}75}$

Efficiency $= \dfrac{\text{Mechanical advantage}}{\text{Velocity Ratio}}$, $0{\cdot}39 = \dfrac{11}{\dfrac{2\,(x + 0{\cdot}75)}{0{\cdot}75}}$

$$x + 0{\cdot}75 = \frac{11 \times 0{\cdot}75}{2 \times 0{\cdot}39} = 10{\cdot}577 \text{ inches.}$$

Diameters of the pulleys are 10·577 inches and 9·827 inches. Ans.

87.
$$\frac{\text{H.P.}}{D^3 \times \text{Revs.} \times \text{Stress}} = \text{constant.}$$

$$\frac{1{,}000}{9^3 \times 85 \times \text{Stress}} = \qquad 1{,}500$$

$$\times\ 93 \times \text{Stress}$$

the stress cancels. D

$$d = \frac{D}{2} \qquad D \qquad D$$

$$\frac{D^4}{16}$$

D

1,000 1,500

$$\times\ 85 \qquad \tfrac{15}{16}\,D^3 \times 93$$

$$= \sqrt[3]{\frac{9^3 \times 85 \times 1{,}500 \times 16}{1{,}000 \times 15 \times 93}} = 10{\cdot}215$$

The shaft is 10·215 inches diameter outside and 5·1075 inches diameter inside. Ans.

88.
$$8 \text{ kilometres} = \frac{8 \times 1{,}000 \times 39{\cdot}37}{12 \times 6{,}080} = 4{\cdot}317 \text{ nautical miles.}$$

$$= 2\,a\,s$$

$$\text{Acceleration} = \frac{\quad}{2\,s} = \frac{17^2}{2 \times 4{\cdot}317} = 33{\cdot}47 \text{ nautical}$$

miles per hour per hour.

$$\frac{33{\cdot}47 \times 6{,}080}{3{,}600 \times 3{,}600} = 0{\cdot}0157 \text{ feet per sec. per sec.} \quad \text{Ans.}$$

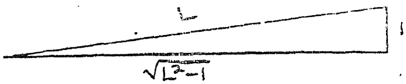
$$\text{Time to attain maximum speed} = \frac{\text{Maximum speed}}{\text{Acceleration}}$$

$$= \frac{17 \times 60}{33{\cdot}47} = 30{\cdot}5 \text{ minutes.} \quad \text{Ans.}$$

Distance in 7 minutes or $\frac{7}{60}$ hour $= \frac{1}{2}\, a\, t^2$

$$\frac{33{\cdot}47 \times 7 \times 7}{2 \times 60 \times 60} = 0{\cdot}2277 \text{ nautical mile.} \quad \text{Ans.}$$

9. Force up incline = Force due to incline + Friction force.
Force to hold on incline = Force due to incline — Friction force.



Let W = weight of truck in lb.

$$1{,}000 = \frac{\quad}{L} W + \frac{1}{L} \times 0{\cdot}0056\ W \ldots (1)$$

$$800 = \frac{\quad}{L} W - \frac{L^2 - 1}{L} \times 0{\cdot}0056\ W \ldots (2)$$

By addition :—

$$1{,}800 = \frac{2}{L} W$$

$\therefore$ 2 W = 1,800 L, and W = 900 L

Substitute W = 900 L in equation (1)

$$1{,}000 = \frac{900\ L}{L} + \frac{\sqrt{L^2 - 1}}{L} \times 0{\cdot}0056 \times 900\ L$$

$$1{,}000 = 900 + \sqrt{L^2 - 1} \times 5{\cdot}04$$

$$5{\cdot}04 \sqrt{L^2 - 1} = 100$$

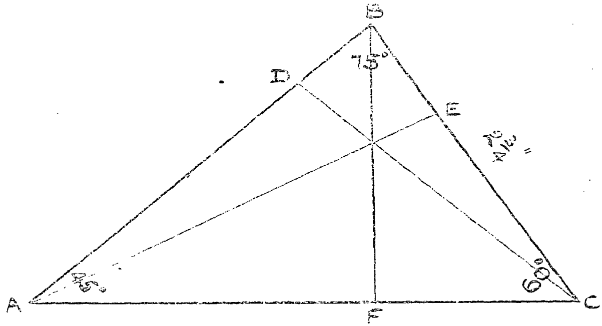
from which L = 19·865

$$W = \frac{19{\cdot}865 \times 900}{2{,}240} = 7{\cdot}981 \text{ tons.}$$

Rise of the incline is 1 in 19·865. Ans.

Weight of the truck is 7·981 tons. Ans.

90. The C.G. of a triangle is at a distance of one-third of the perpendicular height from each side.



B F $= 2\frac{3}{4} \times$ Sin. 60° $=$ 2·3815 inches.

$$\text{Distance of C.G. from side AC} = \frac{2{\cdot}3815}{3} = 0{\cdot}7938 \text{ inch.}$$

F C $= 2\frac{3}{4} \times$ Cos. 60° $=$ 1·375 inches.

A F $=$ F B $=$ 2·3815 inches.

$\therefore$ A C $=$ 1·375 $+$ 2·3815 $=$ 3·7565 inches.

A E $=$ 3·7565 $\times$ Sin. 60° $=$ 3·2531 inches.

$$\text{Distance of C.G. from side B C} = \frac{3{\cdot}2531}{3} = 1{\cdot}0843 \text{ ins.}$$

C D $=$ 3·7565 $\times$ Sin. 45° $=$ 2·6558 inches.

$$\text{Distance of C.G. from side A B} = \frac{2{\cdot}6558}{3} = 0{\cdot}8853 \text{ inch.}$$ Ans.

Distance of C.G. from side A C $=$ 0·7938 inch. Ans.

Distance of C.G. from side B C $=$ 1·0843 inches. Ans.

91. Let D $=$ depth of beam in inches $= \dfrac{D}{12}$ feet.

Weight of beam $= \frac{4}{12} \times \dfrac{D}{12} \times 10 \times 450 = 125\text{ D lb.}$

Reactions at ends $= 750 + 62{\cdot}5\text{ D lb.}$

Bending moment at centre,

$= 60\,(750 + 62{\cdot}5\,D) - (30 \times 62{\cdot}5\,D)$ inch lb.

Resisting moment $= \dfrac{4 \times D^2}{6} \times 1{,}000$

and B.M. $=$ R.M.

$\therefore\ 60\,(750 + 62{\cdot}5\,D) - (30 \times 62{\cdot}5\,D) = \dfrac{4\,D^2}{6} \times 1{,}000$

Cancel 10 throughout and multiply by 6

$(36 \times 750) + (36 \times 62{\cdot}5\,D) - (18 \times 62{\cdot}5\,D) = 4\,D^2 \times 100$

$27{,}000 + (18 \times 62{\cdot}5\,D) = 400\,D^2$

$27{,}000 + 1{,}125\,D = 400\,D^2$

$16\,D^2 - 45\,D - 1{,}080 = 0$

Solving the quadratic, and taking the positive value

$D = 9{\cdot}74$ inches. Ans.

92. Let x and y be the sides.

$$\begin{aligned}
\text{Then}\quad 2\,x + 2\,y &= 49{\cdot}5 \\
x + y &= 24{\cdot}75 \\
x^2 + 2\,xy + y^2 &= (24{\cdot}75)^2 \\
x^2 \qquad\quad + y^2 &= (20{\cdot}02)^2 \\
\hline
\text{Subtract}\qquad 2\,xy &= (24{\cdot}75)^2 - (20{\cdot}02)^2
\end{aligned}$$

$x^2 - 2\,xy + y^2 = 20{\cdot}02^2 - (24{\cdot}75 + 20{\cdot}02)\,(24{\cdot}75 - 20{\cdot}02)$

$= 400{\cdot}8004 - 211{\cdot}7612 = 189{\cdot}0383$

$x - y = \sqrt{189{\cdot}0383} = 13{\cdot}75$

$$\left.\begin{aligned} x + y &= 24{\cdot}75 \\ x - y &= 13{\cdot}75 \end{aligned}\right\}\text{Add}$$

$2\,x = 38{\cdot}5, \qquad x = 19{\cdot}25$

$y = 24{\cdot}75 - 19{\cdot}25 = 5{\cdot}5$

The sides are 19·25 feet and 5·5 feet. Ans.

93. Let D = diameter, also length, in centimetres.

Volume of oil displaced $= 0{\cdot}7854 \times D^2 \times D = 0{\cdot}7854\, D^3$ cu. cm.

Weight of oil displaced $= 0{\cdot}7854 \times 0{\cdot}83 \times D^3$ grams. But the weight of the oil displaced = the weight of the cylinder.

$$\therefore\ 0{\cdot}7854 \times 0{\cdot}83 \times D^3 = 32{,}000$$

$$D = \frac{32{,}000}{0{\cdot}7854 \times 0{\cdot}83} = 36{\cdot}61 \text{ centimetres.}$$

$$\frac{36{\cdot}61}{2{\cdot}54 \times 12} = 1{\cdot}201 \text{ feet.}$$

Note, 2·54 centimetres = one inch.

Since 1 c.c. of fresh water weighs 1 gram, and the Sp. G. of cast iron is 7·21, ∴ 1 c.c. of cast iron weighs 7·21 grams.

$$\text{Volume of metal in cylinder} = \frac{32{,}000}{7{\cdot}21} \text{ c.c.}$$

but volume of cylinder $= 0{\cdot}7854\ D^3$ c.c.
$= 0{\cdot}7854 \times (36{\cdot}61)^3$ c.c.

$$\therefore \text{Vol. of enclosed part} = \{0{\cdot}7854 \times (36{\cdot}61)^3\} - \frac{32{,}000}{7{\cdot}21} \text{ c.c.}$$

$= 38{,}540 - 4{,}438 = 34{,}102$ c.c.

12 inches $= 2{\cdot}54 \times 12 = 30{\cdot}48$ centimetres.

$$\therefore \text{Vol. enclosed} = \frac{34{,}102}{(30{\cdot}48)^3} = 1{\cdot}205 \text{ cu. feet.}$$

Diameter is 1·201 feet. Ans.

Volume enclosed is 1·205 cubic feet. Ans.

94. Base $= \sqrt{(8{\cdot}275)^2 - (2{\cdot}29)^2} = 7{\cdot}95$ metres.
260 kilos. $= 260 \times 2{\cdot}2$ lb.

8·275

7·95

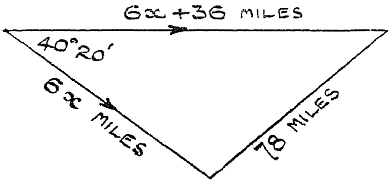
$$\text{Force required} = \left(\frac{2{\cdot}29}{8{\cdot}275} \times 260 \times 2{\cdot}2\right) + \left(14 \times \frac{7{\cdot}95}{8{\cdot}275} \times \frac{260 \times 2{\cdot}2}{2240}\right) \text{ lb.}$$

$$\frac{260 \times 2{\cdot}2}{8{\cdot}275} \quad 29 + \frac{14 \times 7{\cdot}95}{2240}\Big)$$

$$\frac{260 \times 2{\cdot}2}{8{\cdot}275} (2{\cdot}29 + 0{\cdot}04968) = 161{\cdot}7 \text{ lb.} \quad \text{Ans.}$$

95. Let x = Speed of slow ship.

Then $x + 6$ = Speed of fast ship.



$78^2 = (6x + 36)^2 + (6x)^2 - 2 \times 6x \times (6x + 36) \times \text{Cos. } 40° \, 20'$

Divide throughout by 6×6

$13^2 = (x + 6)^2 + x^2 - 2x \times (x + 6) \times 0{\cdot}7623$

$169 = x^2 + 12x + 36 + x^2 - (2x^2 + 12x) \, 0{\cdot}7623$

$169 = 2x^2 + 12x + 36 - 1{\cdot}5246 \, x^2 - 9{\cdot}1476 \, x$

$0{\cdot}4754 \, x^2 + 2{\cdot}8524 \, x - 133 = 0$

Divide throughout by $0{\cdot}4754$

$$6x - 280 = 0, \qquad \frac{133}{0{\cdot}4754} = 280 \text{ (Approx.)}$$

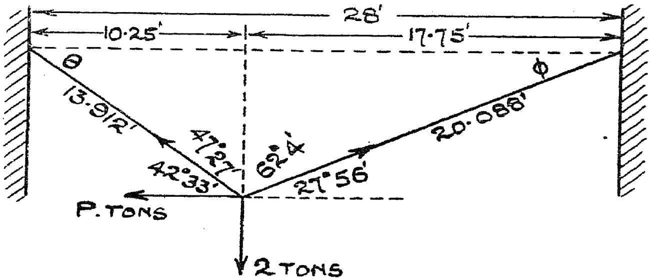
$(x + 20)(x - 14) = 0$

Taking the positive value, $x = 14$.

The speeds are 20 knots and 14 knots. Ans.

6. Let x = short length of the wire

then $34 - x$ = long length of the wire.



$$x^2 - (10{\cdot}25)^2 = (34 - x)^2 - (17{\cdot}75)^2$$
$$x^2 - 105{\cdot}0625 = 1{,}156 - 68\,x + x^2 - 315{\cdot}0625$$
$$68\,x = 946$$
$$x = \frac{946}{68} = 13{\cdot}912 \text{ feet.}$$
$$34 - x = 34 - 13{\cdot}912 = 20{\cdot}088 \text{ feet.}$$

$$\text{Cos. } \theta = \frac{10{\cdot}25}{13{\cdot}912} = 0{\cdot}7368, \therefore \theta = 42^\circ\ 33'$$

$$\text{Cos. } \phi = \frac{17{\cdot}75}{20{\cdot}088} = 0{\cdot}8836, \quad = 27^\circ\ 56'$$

Since the wire is continuous around the block, the tension must be the same throughout.

Vertical component of T in left hand wire + Vertical component of T in right hand wire = 2 tons (weight of cover).

$$\text{T Cos. } 47^\circ\ 27' + \text{T Cos. } 62^\circ\ 4' = 2$$
$$0{\cdot}6763\ \text{T} + 0{\cdot}4685\ \text{T} = 2$$
$$1{\cdot}1448\ \text{T} = 2$$
$$\text{T} = 1{\cdot}747 \text{ tons.}$$

P + horizontal component of T in left hand wire
= horizontal component of T in right hand wire.

$$\text{P} + 1{\cdot}747 \text{ Cos. } 42^\circ\ 33' = 1{\cdot}747 \text{ Cos. } 27^\circ\ 56'$$
$$\text{P} = 1{\cdot}747\ (0{\cdot}8836 - 0{\cdot}7368)$$
$$= 1{\cdot}747 \times 0{\cdot}1468 = 0{\cdot}2564 \text{ ton.}$$

Tension in wire is 1·747 tons. Ans.

Horizontal force is 0·2564 ton. Ans.

97. Let the external diameter be x feet and the internal diameter be y feet.

$(x^2 - y^2)\ 0{\cdot}7854 \times 12 \times 490 = 3450 \quad \ldots \quad (1)$

$\{(0{\cdot}95\,x)^2 - (1{\cdot}05\,y)^2\} \times 0{\cdot}7854 \times 12 \times 490 = 1{,}725 \ldots (2)$

From (1) $\quad x^2 - y^2 = 0{\cdot}747 \quad \ldots \quad \ldots \quad \ldots \quad (3)$

From (2) $\quad 0{\cdot}9025\ x^2 - 1{\cdot}1025\ y^2 = 0{\cdot}3735$

Multiply (1) by $1{\cdot}1025$

$$1{\cdot}1025\ x^2 - 1{\cdot}1025\ y^2 = 0{\cdot}8235675$$
$$0{\cdot}9025\ x^2 - 1{\cdot}1025\ y^2 = 0{\cdot}3735$$

$$0{\cdot}2\ x^2 = 0{\cdot}4500675$$

$$= \sqrt{\frac{0{\cdot}4500675}{0{\cdot}2}} = 1{\cdot}5 \text{ feet.}$$

Substitute in (3) $\quad 2{\cdot}25 - y^2 = 0{\cdot}747$

$y^2 = 1{\cdot}503$ and $y = 1{\cdot}225$ feet.

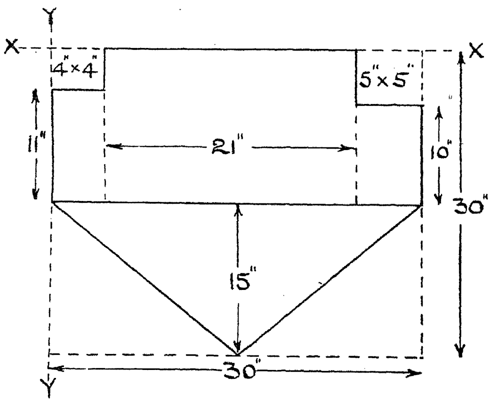
External diameter is 1·5 feet. Internal diameter is 1·225 feet. Ans.

98. By moments about $x\ x$.

Distance of C.G. below $x\ x$

$$= \frac{\text{Sum of moments of volume about } x\ x}{\text{Sum of volumes}}$$

Note, the thickness of the triangular portion is twice the thickness of the rest of the plate.



Distance of C.G. from $x\ x$ =

$$(4 \times 11 \times 1 \times 9{\cdot}5) + (21 \times 15 \times 1 \times 7{\cdot}5) + (5 \times 10 \times 1 \times 10) + \left(\frac{30 \times 15}{2} \times 2 \times 20\right)$$

$$30 \times 15$$

$$\frac{12280{\cdot}5}{859} = 14{\cdot}29 \text{ inches.}$$

By moments about $y\ y$

Distance of C.G. from $y\ y$ =

$$\frac{(4 \times 11 \times 1 \times 2) + (21 \times 15 \times 1 \times 14{\cdot}5) + (5 \times 10 \times 1 \times 27{\cdot}5) + \left(\frac{30 \times 15}{2} \times 2 \times 15\right)}{859}$$

$$\frac{12780{\cdot}5}{859} = 14{\cdot}88 \text{ inches.}$$

C.G. is 14·29 inches from $x\ x$, and 14·88 inches from $y\ y$.
Ans.

99. Sectional area of bar $= 3\,t + t\,(2 - t) = 5\,t - t^2$ sq. ins.

$$\text{Stress} = \frac{15}{5\ t\ -} \text{ tons per square inch.}$$

$$\text{Strain} = \frac{0{\cdot}087}{144}$$

$$\frac{\text{Stress}}{\text{Strain}} \quad \frac{15}{5\,t - t^2} \times \frac{144}{0{\cdot}087} = 13{,}500$$

$5\,t - t^2 = 1{\cdot}839$. $\quad t^2 - 5\,t + 1{\cdot}839 = 0$.

Solving the quadratic, $t = 4{\cdot}6$ or $0{\cdot}4$. Both the values are positive, but only 0·4 is applicable to the problem. Thickness is 0·4 inch. Ans.

100. Let x knots and $x - 3$ knots be the speeds.

Then combined speed $= x + x - 3 = 2\,x - 3$ knots.

Distance the ships are apart when the second steamer starts is $425 - 4\,x$.

Time to meet after second steamer starts

$$\frac{\text{Distance apart}}{\text{Combined speed}} = \frac{425 - 4x}{2x - 3} \text{ hours.}$$

Distance run by second steamer = Time × Speed

$$= \frac{425 - 4x}{2x - 3} \times (x - 3)$$

and this distance is 150 miles.

$$\therefore \frac{425 - 4x}{2x - 3} \times (x - 3) = 150$$

$$(425 - 4x)(x - 3) = 150(2x - 3)$$

$$425x - 1{,}275 - 4x^2 + 12x = 300x - 450$$

$$-4x^2 + 137x - 825 = 0$$

$$4x^2 - 137x + 825 = 0$$

Solving the quadratic, $x = 26{\cdot}45$, or $7{\cdot}79$. Both the values are positive, and both are applicable to the problem. The speeds of the steamers may be 26·45 and 23·45 knots or 7·79 and 4·79 knots. Ans.

Elongation = Strain × Length.

$= 0{\cdot}0019 \times 19 \times 12 = 0{\cdot}4332$ inch.

Let S = Side of section in inches, $0{\cdot}433\ S^2$ = Sectional area.

$$\frac{\text{Stress}}{\text{Strain}} = \text{modulus}$$

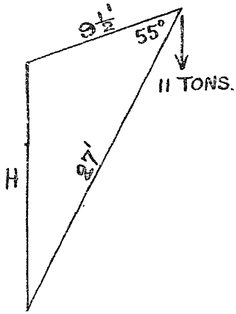
$$\frac{\dfrac{23}{0{\cdot}433\ S^2}}{0{\cdot}0019} = 7{,}500 \quad \therefore S^2 = \frac{23}{0{\cdot}433 \times 0{\cdot}0019 \times 7{,}500}$$

S = 1·93 inches.

$$\text{Stress} = \frac{23}{0{\cdot}433 \times 1{\cdot}93^2} = 14{\cdot}25 \text{ tons per square inch.}$$

The side of the section is 1·93 inches; the stress is 14·25 tons per square inch and the elongation is 0·4332 inch. Ans.

102.



Let H = height of crane post.

By Cosine rule :—

$H^2 = (9\frac{1}{2})^2 + 27^2 - 2 \times 9\frac{1}{2} \times 27 \text{ Cos. } 55^\circ$

$\therefore$ H = 22·9 feet.

Load in jib

$= \dfrac{27}{22{\cdot}9} \times 11 = 12{\cdot}92$ tons. Ans.

Load in tie

$= \dfrac{9\frac{1}{2}}{22{\cdot}9} \times 11 = 4{\cdot}56$ tons. Ans.

103. Force on level $= 8 \times 310 = 2{,}480$ lb.

If θ = angle of incline, then force on incline

$= 8 \times 310 \text{ Cos. } \theta + 310 \times 2{,}240 \text{ Sine } \theta$

Cos. θ is practically 1, and Sine $\theta = \frac{1}{75}$.

$\therefore$ Force on incline $= 2{,}480 + \dfrac{310 \times 2{,}240}{75}$

$= 11{,}739$ lb.

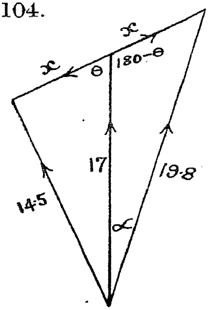
As horse power is constant, then force $\times$ speed is constant.

$\therefore 2{,}480 \times 100 = 11{,}739 \times$ new speed.

New speed $= 21{\cdot}12$.

$\therefore$ Reduction in speed $= 100 - 21{\cdot}12 = 78{\cdot}88$ per cent.
Ans.

104.



Let x be the speed of the current.

$$\left.\begin{aligned}(19{\cdot}8)^2 &= 17^2 + x^2 + 2 \times 17 \times x \text{ Cos. } \theta \\ (14{\cdot}5)^2 &= 17^2 + x^2 - 2 \times 17 \times x \text{ Cos. } \theta\end{aligned}\right\} \text{Add}$$

$(19{\cdot}8)^2 + (14{\cdot}5)^2 = 2 \times 17^2 + 2\,x^2$

$x^2 = 12{\cdot}145$, $x = 3{\cdot}48$ knots. Ans.

Substitute in first equation

$(19{\cdot}8)^2 = 17^2 + 12{\cdot}145 + 2 \times 17 \times 3{\cdot}48 \text{ Cos.}\theta$

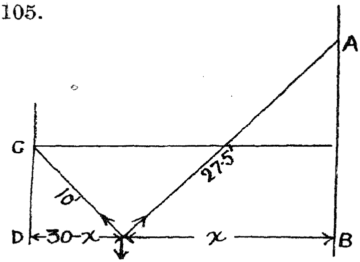
Cos. $\theta = 0{\cdot}7682$, $\theta = 39^\circ\,48'$ to course of ship.

Angle ship is deflected from course :—

$$\text{Sin. } \alpha = \frac{3 \cdot 48}{19 \cdot 8} \text{ Sine } \theta$$

$$\therefore \alpha = 6^\circ\ 30'. \quad \text{Ans.}$$

105.



Since the tension in each wire is the same then the inclination of each to the horizontal is the same

$$\frac{x}{27 \cdot 5} = \frac{30 - x}{10}$$

$$10\,x = 825 - 27 \cdot 5\,x$$

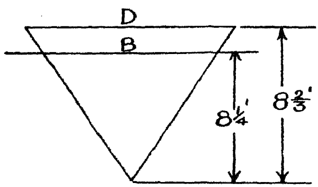
$$x = 22 \text{ feet.} \quad \text{Ans}$$

$$A\,B = \sqrt{(27 \cdot 5)^2 - 22^2} = 16 \cdot 5 \text{ feet}$$

$$C\,D = \frac{10}{27 \cdot 5} \times 16 \cdot 5 = 6 \text{ feet}$$

Difference in height $= 16 \cdot 5 - 6 = 10 \cdot 5$ feet. Ans.

106.



$$B^2 \times 1\tfrac{1}{4} \times \frac{8\frac{1}{4}}{3} = 15 \cdot 7 \times 35$$

$$\therefore B = 15 \cdot 95 \text{ feet.}$$

$$D = B \times \frac{8\frac{2}{3}}{8\frac{1}{4}} = 16 \cdot 76 \text{ feet.}$$

Ans.

107. $$\frac{1{,}450\,(t - 1)^2}{(30 + 24)\,D} \quad \frac{50}{D}\,[10\,(t - 1) - 30]$$

$$\therefore \frac{29\,(t - 1)^2}{54} = 10\,t - 40$$

$\therefore\ 29\ t^2 - 58\ t + 29 = 540\ t - 2{,}160$

$\therefore\ 29\ t^2 - 598\ t + 2{,}189 = 0$

Solving the quadratic, $t = 15{\cdot}83$ or $4{\cdot}79$
Plates of a boiler other than tubes must not be less than $\frac{5}{16}$ thick.

$$\therefore\ t = \frac{15{\cdot}83}{32} = 0{\cdot}495 \text{ inch.} \quad \text{Ans.}$$

108. Tension in wire = Weight of coal in air — Weight of water displaced + Weight of water absorbed.

$$\text{Tension} = \frac{19 \times 13 \times 9}{1{,}728} \times 79{\cdot}6 - \frac{19 \times 13 \times 9}{1{,}728}$$

$$\times \frac{1{,}024}{16} \quad \frac{19 \times 13 \times 9}{1{,}728} \times 79{\cdot}6 \times \frac{5}{100}$$

$$\frac{19 \times 13 \times 9}{1{,}728} [79{\cdot}6 - 64 + 3{\cdot}98]$$

Tension $= 25{\cdot}19$ lb. Ans.

109. Let x = base of triangle.

$$\text{Perpendicular height} = \frac{x}{2} \tan.\ 40^\circ$$

$$\text{Area} = \frac{x}{2} \times \frac{x}{2} \tan 40^\circ = 144$$

$$\therefore\ x = \frac{24}{\sqrt{\tan\ 40^\circ}}$$

$$\text{Radius of circle} = \frac{x}{2} \tan 20^\circ$$

$$\frac{12 \tan 20^\circ}{\sqrt{\tan\ 40^\circ}}$$

$$\text{Area circle} = \pi\ r^2 = \frac{3{\cdot}1416 \times 144 \tan^2 20^\circ}{\tan 40^\circ}$$

$= 71{\cdot}43$ sq. inches. Ans.

$$\text{Diameter} = \sqrt{\frac{71{\cdot}43}{0{\cdot}7854}} = 9{\cdot}52 \text{ inches. Ans.}$$

Area remaining $= 144 - 71{\cdot}43 = 72{\cdot}57$ sq. inches. Ans.

110. Time for faster ship to reach port $= \frac{126}{14} = 9$ hours.
Time for slower ship $= 9 - 3 = 6$ hours.
Distance of slower ship from port $= 6 \times 10{\cdot}5 = 63$ miles.
Let x = distance apart.
$\therefore\ x^2 = 126^2 + 63^2 - 2 \times 126 \times 63 \times \text{Cos. } 75°$
$\therefore\ x = 125{\cdot}37$ miles. Ans.

111. Weight of fresh water displaced

$$= \left(\frac{\pi}{\;} \times 3^2 \times 3{\cdot}5 + \frac{\;}{6} \times \frac{3^3}{2}\right) 62{\cdot}5$$

$= 1{,}989$ lb.

Weight of oil $= 1{,}989 - 375 = 1{,}614$ lb.

Let h = depth of oil in cylindrical part of tank.

$$\left(\frac{\pi}{4}\, 3^2\, h + \frac{\pi}{6} \times \frac{3^3}{2}\right) 62{\cdot}5 \times 0{\cdot}9 = 1,$$

$h = 3{\cdot}061$ feet.

$\therefore$ depth of oil $= 3{\cdot}061 + 1{\cdot}5 = 4{\cdot}561$ feet. Ans.

112. Stress due to centrifugal force $= \dfrac{12\, w\, V^2}{g}$

or $\dfrac{w\, R^2\, N^2}{245}$

Using the second expression :—

$$\text{Stress} = \frac{0{\cdot}26 \times (7{\cdot}25)^2 \times 584^2}{245}$$

$= 19{,}040$ lb. sq. inch $= 8{\cdot}51$ tons sq. inch. Ans.

113. Working pressure also equals

$$\frac{2\ t}{D} \times \frac{\text{tensile strength} \times \text{eff. of seam}}{\text{factor of safety}}$$

where t is in inches.

$$\frac{(T - 12)\ 90}{D} \qquad \frac{2\ T}{100\ D} \times \frac{21{\cdot}5 \times 2{,}240 \times 0{\cdot}56}{9}$$

$$\therefore\ T - 12 = \frac{21{\cdot}5 \times 2{,}240 \times 0{\cdot}56 \times 2\ T}{100 \times 9 \times 90}$$

$\therefore\ T - 12 = 0{\cdot}6659\ T$

$T = 35{\cdot}92.$

$\therefore$ Thickness $= 0{\cdot}3592$ inch. Ans.

114. Weight of the water $=$ [illegible] $= 24{\cdot}5$ grams.

Weight of oil in mixture at first $= 46{\cdot}54 - 24{\cdot}5 = 22{\cdot}04$ grams.

$$\therefore\ \text{Sp. Gravity of oil} = \frac{22{\cdot}04 \times 1{\cdot}028}{24{\cdot}5} = 0{\cdot}925. \quad \text{Ans.}$$

115.

$a\ b = \frac{\quad}{\text{Cos. } 30^\circ} = 2{\cdot}309$ feet.

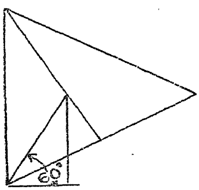
$b\ c = 2 \tan 30^\circ = 1{\cdot}1545$ feet.

Height of C.G. after tilting

$= a\ b$ Sine 60°

$= 2{\cdot}309 \times 0{\cdot}866$

$= 2$ feet.



Height C.G. is raised

$= 2 - 1{\cdot}1545 = 0{\cdot}8455$ foot.

Weight of stone

$$= \frac{0{\cdot}433 \times 4^2 \times 4 \times 62{\cdot}5 \times 2{\cdot}32}{2{,}240} \text{ tons.}$$

$\therefore$ Work done

$$\frac{0{\cdot}433 \times 16 \times 4 \times 62{\cdot}5 \times 2{\cdot}32 \times 0{\cdot}8455 \times 12}{2{,}240}$$

$= 18{\cdot}19$ inch tons. Ans.

116.

$$175 = \frac{100}{324 + 256}(t - 1)^2 + 0{\cdot}084\ t$$

$$\frac{175 \times 580}{100} = (t - 1)^2 + \frac{0{\cdot}084 \times 580\ t}{100}$$

$\therefore\ 1{,}015 = t^2 - 2\ t + 1 + 0{\cdot}04872\ t$

$\therefore\ t^2 - 1{\cdot}5128\ t = 1{,}014.$

$\therefore\ t - 0{\cdot}7564 = \pm \qquad = \pm\ 31{\cdot}85$

$\therefore\ t = 32{\cdot}606$

$$\therefore \text{ Thickness} = \frac{32{\cdot}6}{32} = 1 \text{ inch (approx.)} \quad \text{Ans.}$$

117.

$$19{\cdot}5 \text{ knots} = \frac{19{\cdot}5 \times 6{,}080}{60} = 1{,}976 \text{ feet per min.}$$

$$\text{Thrust H.P.} = \frac{32{,}000 \times 1{,}976}{33{,}000}$$

$$\text{T.M. in foot lb.} = \frac{\text{H.P.} \times 33{,}000}{2\ \pi \text{ Revs.}}$$

$$\therefore \text{ Revs.} = \frac{\text{H.P.} \times 33{,}000}{\text{T.M.} \times 2\ \pi}$$

$$\therefore \text{ Revs.} = \frac{32{,}000 \times 1{,}976 \times 33{,}000 \times 12 \times 7}{33{,}000 \times 600 \times 2{,}240 \times 2 \times 22}$$

$= 89{\cdot}8$ revs. per min. Ans.

118. Surface of tank $= 4{\cdot}5^2 \times 5$

Weight of tank $= 4{\cdot}5^2 \times 5 \times 7{\cdot}5 = 759{\cdot}4$ lb.

Weight of water displaced $= 4{\cdot}5^2 \times 4 \times 64 = 5{,}184$ lb.

Weight of oil $= 5184 - 759{\cdot}4 = 4424{\cdot}6$ lb.

$$\text{Cubic feet of oil} = \frac{4424{\cdot}6}{62{\cdot}5 \times 0{\cdot}89}$$

$$\therefore \text{Depth} = \frac{4424{\cdot}6}{62{\cdot}5 \times 0{\cdot}89 \times 4{\cdot}5^2} = 3{\cdot}928 \text{ feet. Ans.}$$

119.
$$\text{C.G.} = \frac{\text{Sum of moments of areas}}{\text{Whole area}}$$

$$\therefore 6{\cdot}6 = \frac{(b+2)\,1\tfrac{1}{2} \times \tfrac{3}{4} + 11 \times 1\tfrac{1}{2} \times 7 + b \times 1\tfrac{1}{2} \times 13\tfrac{1}{4}}{(b+2)\,1\tfrac{1}{2} + 11 \times 1\tfrac{1}{2} + b \times 1\tfrac{1}{2}}$$

$$\therefore 6{\cdot}6\,(1\tfrac{1}{2}\,b + 3 + 16{\cdot}5 + 1\tfrac{1}{2}\,b) = \tfrac{9}{8}\,b + \tfrac{9}{4} + 115{\cdot}5 + 19{\cdot}875\,b$$

$1{\cdot}2\,b = 10{\cdot}95$

$b = 9{\cdot}125$ inches. Ans.

120. Volume of the iron $= \frac{1}{450}$ cu. feet.

$$\text{Weight of equal volume of water} = \frac{62{\cdot}5}{450} = 0{\cdot}1388 \text{ lb.}$$

Weight of equal volume of oil $= 1 - 0{\cdot}882 = 0{\cdot}118$ lb.

$$\therefore \text{Sp. G. of oil} = \frac{0{\cdot}118}{0{\cdot}1388} = 0{\cdot}85$$

$$\text{Sp.G.} = \frac{140}{130 + \text{B}^\circ} \qquad \therefore \text{Beaumé} = \frac{140}{0{\cdot}85} - 130$$

$= 34{\cdot}8^\circ$. Ans.

121.
If D = diam. of circle, then $\dfrac{D}{\sqrt{2}}$ = side of square.

$$\frac{\pi}{4} D^2 \quad \frac{D^2}{2} = \pi D - \frac{4 D}{\sqrt{2}}$$

$0{\cdot}7854\ D^2 - 0{\cdot}5\ D^2 = 3{\cdot}1416\ D - 2{\cdot}828\ D.$

$0{\cdot}2854\ D = 0{\cdot}3136.$

$D = 1{\cdot}1$ inches. Ans.

122. Let initial velocity $= u$

$18^2 = u^2 - 2\ g \times 200$

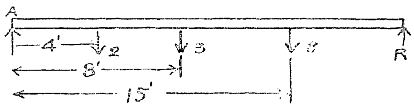
$\therefore u = \sqrt{18^2 + 64{\cdot}4 \times 200} = 114{\cdot}9$ feet per sec. Ans.

Total height $= \dfrac{}{2\ g} \quad \dfrac{(114{\cdot}9)^2}{64{\cdot}4} = 205$ feet.

$\therefore$ Distance passed through $= 2 \times 205 = 410$ feet. Ans.

123. Weight of beam $= (8 \times 9 - 7\frac{3}{8} \times 8)\ \dfrac{18 \times 490}{144 \times 2{,}240}$

$= 0{\cdot}3555$ ton.



By moments about A :—

$18\ R = 2 \times 4 + 3 \times 8 + 8 \times 15 + 0{\cdot}3555 \times 9$

$\therefore R = 8{\cdot}622$ tons.

Area of section of rivets $= (\frac{5}{8})^2 \times \frac{11}{14} \times 7 = 2{\cdot}15$ sq. inches.

$\therefore$ Shearing stress $= \dfrac{8{\cdot}622}{2{\cdot}15} = 4{\cdot}01$ tons sq. inch.

Ans.

124. Change of velocity $= V_1 - V_2 = 4{\cdot}1 \times 32{\cdot}2 = 132$ feet per sec. ... ... (1)

Average velocity $= \dfrac{V_1 + V_2}{2} \quad \dfrac{\text{distance}}{\text{time}} \quad \dfrac{350}{4{\cdot}1}$

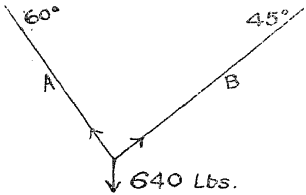
$\therefore V_1 + V_2 = \dfrac{2 \times 350}{4{\cdot}1} = 170{\cdot}8$.

Subtracting (1) from (2), $2\ V_2 = 38{\cdot}8$

$\therefore V_2 = 19{\cdot}4$ feet per sec.

$$\therefore \text{ Distance } = \frac{(19 \cdot 4)^2}{2 \times 32 \cdot 2} = 5 \cdot 84 \text{ feet. Ans.}$$

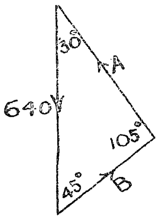
125.



Tension in A

$$= 640 \; \frac{\text{Sine } 45°}{\text{Sine } 105°}$$

$= 469$ lb. Ans.



Tension in B

$$= 640 \times \frac{\text{Sine } 30°}{\text{Sine } 105°}$$

$= 332$ lb. Ans.

126. Area of valve $= (3\frac{1}{2})^2 \times 0 \cdot 7854 = 9 \cdot 622$ sq. inches.
Load on spring for 110 lb. $= 110 \times 9 \cdot 622 - 37$
Load on spring for 180 lb. $= 180 \times 9 \cdot 622 - 37$
Increased load on spring $= 70 \times 9 \cdot 622$

$$\therefore \text{ per cent. increase in load } = \frac{70 \times 9 \cdot 622}{110 \times 9 \cdot 622 - 37} \times 100$$

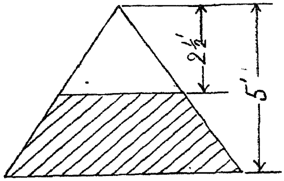
$= 66$ per cent. Ans.

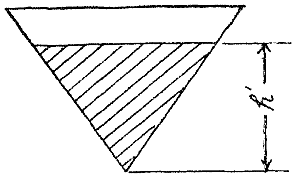
127. Similar volumes vary as the cube of their corresponding dimensions.
Let V = vol. of similar cone of unit height.

Volume of cone 5 ft. height $= V \times \frac{5^3}{1^3}$

" " $2\frac{1}{2}$ ft. " $= V \times \frac{\quad}{1^3}$

" " h " $= V \times \frac{h^3}{1^3}$





$$V\,5^3 - V\,(2\tfrac{1}{2})^3 = V$$
$$- h^3$$

$\therefore\ h = 2\frac{1}{2}\sqrt[3]{2^3 - 1^3} = 2\frac{1}{2}\sqrt[3]{}$
$= 4{\cdot}782$ feet. Ans.

128.
$$\text{Stress on longitudinal section} = \frac{P\ D}{2\ T} = \frac{350 \times 90}{2 \times 1\frac{1}{2}}$$

$= 10{,}500$ lb. per sq. inch.
Stress on circumferential section
$= 5{,}250$ lb. per sq. inch. } Ans.

Stress normal to seam at 45° (see Chap. 16)
$= 10{,}500 \text{ Cos.}^2\ 45° + 5{,}250 \text{ Sine}^2\ 45°$
$= 5{,}250 + 2{,}625$
$= 7{,}875$ lb. per sq. inch. Ans.

129.
$$\text{Stress in iron} = \frac{\text{Stress in steel}}{10}$$

Difference of expansion per unit length = Sum of strains (see Chap. 16).

300 (0·0000067 — 0·000006)

Stress steel Stress iron

E for steel E for iron

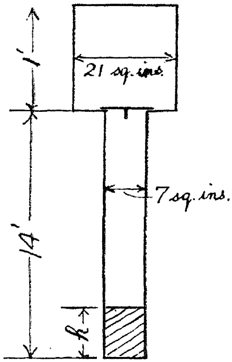
12,000 10 × 6,000

S_s

10,000

. Stress in steel = 10,000 × 300 × 0·0000007
2·1 tons per sq. inch. Ans.

130.

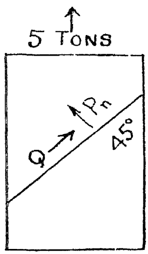


$p_1 = 34$ feet head of water.
$p_2 = (34 - h)$ feet head of water.
$v_1 = 14 \times 12 \times 7$
$v_2 = 14 \times 12 \times 7 + 21 \times 12 - h \times 7 \times 12$

$\therefore 34 \times 14 \times 12 \times 7 = (34 - h)(14 \times 12 \times 7 + 21 \times 12 - h \times 7 \times 12)$
$\therefore 34 \times 14 = (34 - h)(17 - h)$
$\therefore 34 \times 14 = 34 \times 17 - 51\,h + h^2$
$h^2 - 51\,h + 102 = 0$

Solving the quadratic, $h = 2{\cdot}09$ feet. Ans.

131.



Stress in direction of axis $= (0{\cdot}75)^2 \times 0{\cdot}7854$

Normal stress on plane at 45°

$(0{\cdot}75)^2 \times 0{\cdot}7854 \quad \times \text{Cos.}^2\ 45°$

$= 5{\cdot}658$ tons per sq. inch. Ans.

Shear stress on plane at 45°

$$= \frac{5}{(0{\cdot}75)^2 \times 0{\cdot}7854} \times \text{Cos. } 45° \text{ Sine } 45°$$

$= 5{\cdot}658$ tons per sq. inch. Ans.

132. Greatest working stress $= \frac{28}{5} = 5{\cdot}6$ tons per sq. inch.
= Stress in longitudinal section.

Stress on circumferential section $= \dfrac{5{\cdot}6}{2} = 2{\cdot}8$ tons per sq. inch.

Stress on seams at 45° $= 5{\cdot}6 \text{ Cos.}^2\ 45° + 2{\cdot}8 \text{ Sine}^2\ 45°$.
$= 4{\cdot}2$ tons per sq. inch. Ans.

133. Load = Stress in copper × Area of copper + Stress in steel × Area of steel.

$$\therefore\ 10 = S_c \times 1{\cdot}53 + S_s \times 0{\cdot}885$$

Strain is the same for each $\quad \dfrac{S_c}{E_c} \qquad \dfrac{S_s}{E_s}$

$$\therefore\ S_c = \frac{S_s\ E_c}{E_s}$$

$$= \frac{S_s \times 6{,}700}{13{,}400} = \frac{S_s}{2}$$

$$\therefore\ 10 = \frac{S_s}{2} \times 1{\cdot}53 + \qquad 0{\cdot}885$$

$$S_s\ (0{\cdot}765 + 0{\cdot}885) = 1{\cdot}65\ S_s$$

$\therefore$ Stress in steel

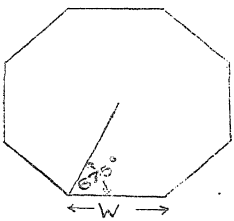
$$= \frac{10}{1{\cdot}65} = 6{\cdot}06 \text{ tons per sq. inch.}$$

Ans.

Stress in copper

$$= \frac{6{\cdot}06}{} = 3{\cdot}03 \text{ tons per sq. inch.}$$

134.



$$\text{Area of octagon} = \frac{W}{} \times \frac{W}{} \tan 67{\cdot}5^\circ \times 8$$

$$= 4{\cdot}829\ W^2$$

Weight of bar

$$= (4{\cdot}829\ W^2 - \frac{\pi}{4}\ 6^2) \times 66 \times 0{\cdot}283,$$

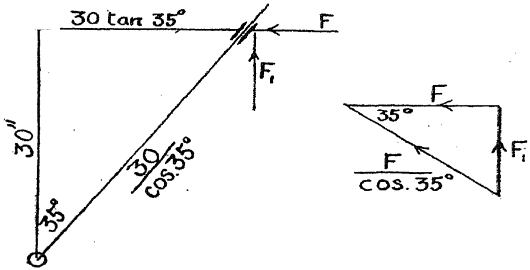
and this equals 900 lb.

$$\therefore\ 4{\cdot}829\ W^2 = \frac{900}{66 \times 0{\cdot}283} + \frac{\pi}{4}\ 6^2$$

$$= 76{\cdot}45.$$

$\therefore$ W = 3·97 inches. Ans.

135. Let F be the force on one ram. Since the connection of the rams to the tiller is a sliding connection, there must be a force F_1 acting at right angles to the line of the stroke of the rams.



The twisting moment on the rudder stock is :—
$30\ F + 30\ F_1 \tan 35°$,
$F_1 = F \tan 35°$.
$\therefore$ T.M. $= 30\ F + 30\ F \tan^2 35°$.
$= 30\ F\ (1 + \tan^2 35^2)$.

$$\text{Now } 1 + \tan^2 35° = 1 + \frac{\text{Sin}^2 35°}{\text{Cos}^2 35°} \quad \frac{\text{Cos}^2 35° + \text{Sin}^2 35°}{\text{Cos}^2\ 35°} \quad \frac{1}{\text{Cos}^2\ 35°}$$

$$\text{T.M.} = \frac{30\ F}{\text{Cos.}^2\ 35°}$$

Alternatively :—The resultant of F and F_1 must act at right angles to the tiller, since the connection is not a fixed one. The twisting moment is the resultant force multiplied by the length of the tiller for that position of the rudder.

From the force diagram, the resultant force $= \dfrac{F}{\text{Cos.}\ 35°}$

The effective length of the tiller is $\dfrac{30}{\text{Cos.}\ 35°}$

$$\therefore \text{T.M.} = \frac{F}{\text{Cos.}\ 35°} \times \frac{30}{\text{Cos.}\ 35°} \quad \frac{30\ F}{\text{Cos.}^2\ 35°} \text{ (as before)}$$

F = Press. $\times\ 12^2 \times 0{\cdot}7854$

$$\therefore \text{T.M.} = \frac{P \times 12^2 \times 0{\cdot}7854 \times 2 \times 30}{\text{Cos.}^2\ 35°}$$

$$= \frac{\pi}{16} \times 10{,}000 \times 18^3$$

$\therefore$ P = 1,133 lb. per sq. inch. Ans.

136. Vol. of foam required $= 50 \times 75 \times \frac{1}{2} = 1{,}875$ cu. ft.

Vol. of liquid to form this $= \frac{1875}{8}$

$$\text{Vol. of each tank} = \frac{1{,}875}{8 \times 2 \times 0{\cdot}95} = d^2 \times 0{\cdot}7854 \times 7$$

$\therefore\ d = 4{\cdot}737$ feet. Ans.

Vol. discharged per second theoretically

$= \text{area} \times \text{velocity}$

$$= \frac{2^2 \times 0{\cdot}7854}{144} \sqrt{64{\cdot}4 \times 36}$$

$= 1{\cdot}05$ cu. feet.

$$\therefore\ \text{Theoretical time} = \frac{1{,}875}{1{\cdot}05 \times 60 \times 8} = 3{\cdot}72 \text{ minutes.}$$

$$\therefore\ \frac{\text{Theoretical time}}{\text{Actual time}} = \frac{3{\cdot}72}{15} = \frac{1}{4{\cdot}033} \quad \text{Ans.}$$

137. The principle involved is the same as that for the Venturi water meter, explained on page 440.

Let a_1 be the area of the large part of the pipe in sq. feet. Let p_1 be the pressure in lb. per sq. foot, and v_1 be the velocity in feet per sec.

Let a_2, p_2 and v_2 be the corresponding conditions at the small part.

By Bernouilli's theorem,

$$\frac{p}{w} + h + \frac{v^2}{2\,g} \text{ is constant}$$

$$\therefore\ \frac{p_1}{w} + h_1 + \frac{{v_1}^2}{2\,g} = \frac{p_2}{w} + h_2 + \frac{{v_2}^2}{2\,g} \quad \ldots \quad (1)$$

Now $h_1 = h_2$, since the pipe is horizontal.

$$p_1 - p_2 = \frac{8 \times 62{\cdot}5}{12} \text{ lb. per sq. foot}$$

$$\frac{p_1 - p_2}{w} \quad \frac{8 \times 62{\cdot}5}{12 \times 62{\cdot}5}$$

Also $a_1\,v_1 = a_2\,v_2$, because the quantity flowing past any section is the same

$= v_1 \times$

$$\frac{a_1}{a_2} = \frac{2^2 \times 0{\cdot}7854}{1^2 \times 0{\cdot}7854} = 4$$

From (1) $\dfrac{p_1 - p_2}{w} = \dfrac{v_2^2 - v_1^2}{2\ g} = \dfrac{v_1^2 \times \left(\dfrac{a_1}{a_2}\right)^2 - v_1^2}{2\ g}$

$2\ g \qquad v_1 = \sqrt{-3} \times 15$ feet per sec.

Quantity passing $=$ area $\times$ velocity

$= 0{\cdot}7854 \times 2^2 \times \sqrt{\dfrac{4 \times 32{\cdot}2}{45}} \times 6{\cdot}25 \times 60$ galls. per min.

$= 1993{\cdot}2$ gallons per min. Ans.

138. Weight of beam $= 12{\cdot}75 \times \dfrac{11 \times (3{\cdot}75)^2}{14 \times 144} \times 490 = 479{\cdot}1$ lb.

C.G. of beam is at $\dfrac{12{\cdot}75}{2}$ feet from wall.

Maximum stress is caused by maximum bending moment, and this takes place at the wall.

$M_{wall} = 479{\cdot}1 \times \dfrac{12{\cdot}75}{2} \times 12 + W \times 12{\cdot}75 \times 12$ inch lb.

$= 12{\cdot}75 \times 12 \qquad 479{\cdot}1$

$M \qquad p$

I for a circular section $= \dfrac{\pi}{64}\ D^4$ inch^4 units.

y for a circular section $= \dfrac{D}{2}$

$\therefore\ p = \dfrac{M\ y}{I} = \dfrac{32\ M}{\pi\ D^3}$

$\therefore\ 9{\cdot}2 \times 2{,}240 = \dfrac{32 \times 12{\cdot}75 \times 12 \times (239{\cdot}55 + W)}{\pi \times (3{\cdot}75)^3}$

$239{\cdot}55 + W = \dfrac{9{\cdot}2 \times 2{,}240 \times \pi \times (3{\cdot}75)^3}{32 \times 12{\cdot}75 \times 12} = 697{\cdot}15$

$\therefore\ W = 697{\cdot}15 - 239{\cdot}55 = 457{\cdot}6$ lb. Ans.

139. V.R. = No. of ropes supporting bottom block = 6

$$\text{M.A.} = \frac{\text{Load lifted}}{\text{effort applied}} \quad \frac{2{,}000}{520}$$

$$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}} = \frac{2{,}000}{520 \times 6}$$

$= 0{\cdot}641$, or $64{\cdot}1\%$. Ans.

Let x = efficiency of each sheave

then $x^6 = 0{\cdot}641$ and $x = \sqrt[6]{0{\cdot}641}$

$= 0{\cdot}9285$, or $92{\cdot}85\%$. Ans.

Total load on top hook $= 2{,}000 + 520 + 60 + 60$

$= 2{,}640$ lb. Ans.

140. Accelerating force $= 56 - 25 = 31$ lb.

Total mass accelerated $= 56 + 25 = 81$ lb.

$$\text{Accelerating force} = \frac{\text{mass} \times \text{acceleration}}{g}$$

$$\therefore \text{Acceleration} = \frac{31 \times 32{\cdot}2}{81} = 12{\cdot}32 \text{ ft. per sec.}^2$$

Velocity after 5 seconds $= 5 \times 12{\cdot}32$

$= 61{\cdot}6$ ft. per sec.

$$\text{Average velocity during the 5 secs.} = \frac{0 + 61{\cdot}6}{2}$$

$= 30{\cdot}8$ ft. per sec.

Distance travelled = average velocity × time

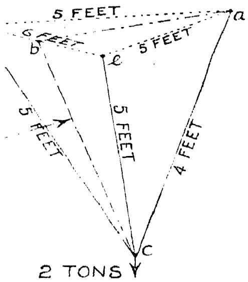
$= 30{\cdot}8 \times 5$

$= 154$ feet. Ans.

or :— $s = \frac{1}{2} a t^2$

$$= \frac{12{\cdot}32 \times 5^2}{} = 154 \text{ feet. Ans.}$$

141.

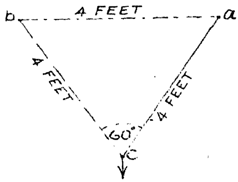


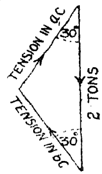
$a\ b = \sqrt{5^2 - 3^2} = 4$ ft.

$b\ c = \sqrt{5^2 - 3^2} = 4$ ft.

Consider an imaginary chain $b\ c$ to take the place of chains $d\ c$ and $e\ c$.

On the plane $a\ b\ c$, we have :—

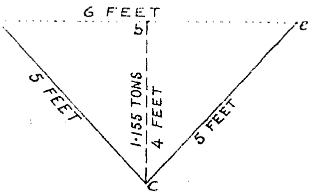




$$\text{Tension in } a = \frac{1 \text{ ton}}{\text{Cos. } 30^\circ} = \frac{1}{0{\cdot}866} = 1{\cdot}155 \text{ tons.}$$

Tension in imaginary chain $b\ c = 1{\cdot}155$ tons also.

Now look in the direction of arrow on to plane $d\ e\ c$, and we see the two 5 ft. chains taking the load of the imaginary chain between them :—



Thus, the 5 ft. chains $d\ c$ and $e\ c$, each is subjected to a tension of

$$\tfrac{1}{2} \text{ of } \frac{1{\cdot}155 \times 5}{4} = 0{\cdot}722 \text{ ton.}$$

Therefore :—

Tension in the 4 ft. chain $= 1{\cdot}155$ tons. Ans.

Tension in each 5 ft. chain $= 0{\cdot}722$ ton. Ans.

142. Co-efficient of discharge $= 0{\cdot}97 \times 0{\cdot}64 = 0{\cdot}6208$

$\therefore$ Quantity of water discharged $= 0{\cdot}6208 \times$ Area of hole $\times \sqrt{2\,g\,h}$ cubic feet per second. (See page 437).

$\therefore$ Weight per minute

$$= 0{\cdot}6208 \times \frac{0{\cdot}7854 \times (0{\cdot}5)^2}{144} \times \sqrt{64{\cdot}4 \times 6} \times 62{\cdot}5 \times 60 \text{lb. per minute.}$$

$= 62{\cdot}4$ lb. per minute. Ans.

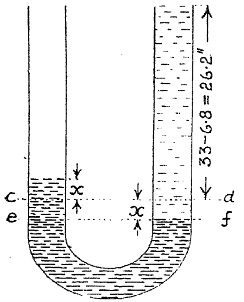
143. See page 286 for proof of formula.

$$\delta = \frac{64\ W\ R^3\ N}{C\ d^4} \quad \therefore\ W = \frac{\delta \times C \times}{64 \times R^3 \times N}$$

$$\text{Load } W = \frac{1 \times 12 \times 10^6 \times (0{\cdot}5)^4}{64 \times (1{\cdot}5)^3 \times 10}$$

$= 347{\cdot}3$ lb. Ans.

144.



As the tube is of uniform bore, the level of mercury will rise x inches in left leg, if it is depressed x inches in right leg by the water.
$c\ d$ is the original mercury level.

Now consider level $e\ f$, the mercury below $e\ f$ is in equilibrium. Therefore $2\ x$ inches of mercury in left leg balance $(26{\cdot}2 + x)$ inches of water in right leg.

Pressure on $e\ f$ in left leg = Pressure on $e\ f$ in right leg.

$$\begin{aligned} 2\,x \times 13{\cdot}6 &= (26{\cdot}2 + x) \times 1 \\ 27{\cdot}2\,x &= 26{\cdot}2 + x \\ 27{\cdot}2\,x - x &= 26{\cdot}2 \\ 26{\cdot}2\,x &= 26{\cdot}2 \\ x &= 1 \end{aligned}$$

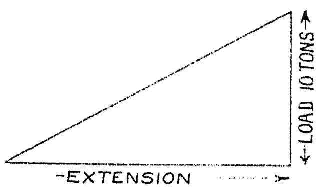
Hence, column of water is $26{\cdot}2 + 1 = 27{\cdot}2$ ins. high.

Weight of water

$$= 27{\cdot}2 \text{ cu. ins.} \times \frac{62{\cdot}5}{1{,}728} = 0{\cdot}983 \text{ lb.} \quad \text{Ans.}$$

145. Assuming the 10 tons load to be gradually applied, then average force to stretch the

$$\text{bar} = \frac{0 + 10}{2} = 5 \text{ tons.}$$



$$\text{Strain} = \frac{\text{Stress}}{E}$$

Total elongation = length × strain

$$= 8 \times 12 \times \frac{10}{0{\cdot}7854 \times (1{\cdot}25)^2 \times 13{,}000}$$

Work done = Average force × distance

$$= \frac{5 \times 8 \times 12 \times 10}{0{\cdot}7854 \times (1{\cdot}25)^2 \times 13{,}000}$$

= 0·3009 inch ton.
or 673·9 inch lb. Ans.

146. Height $= \frac{2{,}935}{N^2}$ feet.

Height at 100 r.p.m.

$$= \frac{2{,}935}{100^2} = 0{\cdot}2935 \text{ foot.}$$

Height at 120 r.p.m.

$$= \frac{2{,}935}{120^2} = 0{\cdot}2038 \text{ foot.}$$

$\therefore$ Change in height = 0·2935 − 0·2038 = 0·0897 foot
= 1·0764 inches. Ans.

147. Total pressure on water side = H A w
= 10 × 20 × 30 × 62·5 = 375,000 lb.

Centre of pressure is at $\frac{20}{3}$ feet above base.

Total pressure on oil side = 7·5 × 15 × 30 × 0·77 × 62·5
= 162,500 lb.

Centre of pressure is at $\frac{15}{3}$ = 5 feet above base.

Resultant pressure = 375,000 — 162,500 = 212,500 lb.
= 94·86 tons. Ans.

Taking moments about base of bulkhead :—

$375{,}000 \times \frac{20}{3} - 162{,}500 \times 5 = 212{,}500 \times x$

$5\,(375{,}000 \times \frac{4}{3} - 162{,}500) = 212{,}500 \times x$

$$\therefore \frac{5 \times 337{,}500}{212{,}500} = x$$

$7{\cdot}941 = x$

Resultant centre of pressure is at 7·941 feet above base.
Ans.

148. Weight of oil displaced = 32 kilograms.
Volume of oil displaced

$$= \frac{32 \times 2{\cdot}2}{0{\cdot}83 \times 62{\cdot}5} \times 1{,}728 \text{ cu. ins.}$$

= 2,344 cu. ins., and this is the external volume of the sphere.

$$\text{Vol. of sphere} = \frac{\pi}{6} D^3$$

$$\therefore \text{ External diam.} = \sqrt[3]{\quad} \qquad \frac{\times\ 6}{\pi}$$

= 16·48 ins.

Weight of material = 32 × 2·2 lb.

∴ Actual volume of metal

$$\frac{32 \times 2{\cdot}2}{0{\cdot}26} = 270{\cdot}7 \text{ cu. ins.}$$

∴ internal volume = 2,344 — 270·7 = 2073·3 cu. ins.

$$\text{Internal diam.} = \sqrt[3]{\frac{2073{\cdot}3 \times 6}{\pi}} = 15{\cdot}82 \text{ ins.}$$

$$\left.\begin{array}{l}\therefore \text{ External diameter} = 16{\cdot}48 \text{ inches} \\ \text{Internal diameter} = 15{\cdot}82 \text{ inches}\end{array}\right\} \text{Ans.}$$

Alternative solution :—
Let D = diam. of sphere in centimetres.

Weight of oil displaced $= \dfrac{\pi}{6} D^3 \times 0{\cdot}83$ grams, because 1 c.c. of oil weighs 0·83 gram.
But weight of sphere = weight of oil displaced.

$$\therefore \frac{\quad}{6} D^3 \times 0{\cdot}83 = 32 \times 1{,}000$$

$$D^3 = \frac{32{,}000 \times 6}{\pi \times 0{\cdot}83} \text{ and } D = \sqrt[3]{\frac{32{,}000 \times 6}{\pi \times 0{\cdot}83}}$$

$$= 41{\cdot}92 \text{ cms., or } \frac{41{\cdot}92}{2{\cdot}54} = 16{\cdot}5 \text{ inches.}$$

Sp. Gr. of cast iron $= 7{\cdot}21$, therefore 1 c.c. of cast iron weighs $7{\cdot}21$ grams.

Let d = internal diam. of sphere.

$$\frac{\pi}{\ }\left\{(41{\cdot}92)^3 - d^3\right\} \times 7{\cdot}21 = 32{,}000$$

$$(41{\cdot}92)^3 \qquad \frac{32{,}000 \times 6}{\pi \times 7{\cdot}21}$$

$$73{,}640 - d^3 = 8{,}478$$

$d^3 = 65{,}162, \quad d = \sqrt[3]{65{,}162} = 40{\cdot}25$ cms., or $15{\cdot}84$ ins.

External diam. is $41{\cdot}92$ cms., or $16{\cdot}5$ ins.

Internal diam. is $40{\cdot}25$ cms., or $15{\cdot}84$ ins. Ans.

149. $$\frac{T}{\ } = \frac{q}{\ } \quad \therefore T = \frac{J\,q}{\ }$$

J for a hollow shaft $= \frac{\ }{32}(D^4 - d^4)$

and $r = \dfrac{D}{2}$

$$\therefore T = \frac{\pi (D^4 - d^4) \times 2 \times q}{32 \times D} = \frac{\pi}{16} \times \frac{(D^4 - d^4) \times q}{D}$$

or $q = \dfrac{T \times 16 \times D}{\times (D^4 - d}$

$$\frac{1{,}600 \times 16 \times 15}{\pi \times (15^4 - 7^4)} \qquad \frac{1{,}600 \times 16 \times 15}{\pi \times (15^2 + \qquad (15^2 - 7^2)}$$

$$\frac{1{,}600 \times 16 \times 15}{\pi \times 274 \times 176} = 2{\cdot}534 \text{ tons per sq. inch. Ans.}$$

$$\frac{q}{r} = \frac{C\,i}{l}$$

$$\therefore i = \frac{q\,l}{C\,r} \text{ radians.}$$

$$\therefore i = \frac{2{\cdot}534 \times 20 \times 15 \times 2}{5{,}730 \times 15} \times 57{\cdot}3 \text{ degrees}$$

1·0136 degrees. Ans.

150. Let a_1, p_1, h_1 and v_1 be the conditions at the larger section of the pipe, and let a_2, p_2, h_2 and v_2 be the conditions at the smaller section.

Now $h_1 = h_2$, because the pipe is horizontal.

$$\frac{p_1}{w} + \frac{v_1^2}{2\,g} = \frac{p_2}{w} + \frac{v_2^2}{2\,g}$$

w

$$(p_1 - p_2) = \quad \times w = \frac{45^2 - 4^2}{64{\cdot}4} \times 62{\cdot}5$$

$$= \frac{49 \times 41 \times 62{\cdot}5}{64{\cdot}4}$$

But $(p_1 - p_2)$ = difference in pressure at the two sections in lb. per sq. foot.

$\therefore$ difference in pressure in lb. per sq. inch

$$= \frac{49 \times 41 \times 62{\cdot}5}{64{\cdot}4 \times 144} = 13{\cdot}54 \text{ lb. per sq. inch.}$$

$\therefore$ pressure at the section where the velocity is 45 feet per sec.

$= 25 - 13{\cdot}54 = 11{\cdot}46$ lb. per sq. inch. Ans.

151. $$\text{V.R.} = \frac{2 \times \pi \times 19}{0{\cdot}375} = 318{\cdot}4. \quad \text{Ans.}$$

Linear law is $P = a + b\,W$

(1) $30 = a + b \times 2{,}300$
(2) $10 = a + b \times 500$ subtract

$20 = \quad b \times 1{,}800$

$$\therefore b = \frac{20}{1{,}800} = \frac{1}{90}$$

From (2), $10 = a + \frac{1}{90} \times 500$

$\therefore\ a = 10 - \frac{500}{90} = 4\frac{4}{9}$

Putting these values into the linear law,

$P = 4\frac{4}{9} + \frac{1}{90}\,W$

When W = 3,000 lb. :—

$$P = 4\tfrac{4}{9} + \tfrac{3000}{90} = 37\tfrac{7}{9}$$

$\therefore$ Effort to lift 3,000 lb. $= 37\frac{7}{9}$ lb. Ans.

$$\text{M.A.} = \frac{W}{\ } \quad \frac{3{,}000}{37\tfrac{7}{9}}$$

$$\text{Efficiency} = \frac{\text{M.A.}}{\text{V.R.}} \quad \frac{3{,}000}{37\tfrac{7}{9} \times 318{\cdot}4}$$

$= 0{\cdot}2494$, or $24{\cdot}94\%$. Ans.

152. Centrifugal Force $= \dfrac{\ }{r\,g} \quad \dfrac{W\,(\omega\, r)^2}{r\,g} \quad \dfrac{W\,\omega^2\,r}{g}$

Referring to sketch of Porter governor on page 180
Moments about o :—

$$\text{C.F.} \times h = w \times r + \frac{W}{2} \times 2\,r$$

$$w \times \omega^2 \times r \qquad \times h = w\,r + W\,r \qquad (r \text{ cancels})$$

$$= \left(\frac{w + W}{\ }\right) \qquad \text{feet.}$$

At 250 revs. per min. :—

$$\text{Height} = \left(\frac{3 + 40}{\ }\right) \times \frac{32{\cdot}2 \times 60^2}{250^2 \times (2\ \pi)^2} \text{ feet}$$

$$= \frac{43 \times 32{\cdot}2 \times 60 \times 60}{3 \times 250 \times 250 \times 4 \times \pi^2}$$

$= 0{\cdot}6738$ foot.

or $8{\cdot}085$ inches. Ans.

153. See page 161 for definition of Impulse.

Kinetic Energy of bullet

$$\frac{W \quad v^2}{2 \; g} \qquad \frac{1 \times 1{,}500^2}{16 \times 2 \times 32{\cdot}2} \text{ ft. lb.}$$

Average force in lb.

$$\frac{\text{Change of K.E. in ft. lb.}}{\text{Distance in feet}}$$

$$\frac{1{,}500^2 \times 12}{16 \times 2 \times 32{\cdot}2 \times 8} \text{ lb.} = 3{,}275 \text{ lb.} \quad \text{Ans.}$$

Alternative solution :—

Impulse of a force = Force × time during which the force acts, and Impulse of a force = Change of momentum produced.

Time to arrest motion of bullet

$$\frac{\text{Distance}}{\text{Average velocity}} \qquad \frac{\frac{2}{3}}{750} \text{ second}$$

$$\therefore \text{ Force} \times \frac{2}{3 \times 750} \qquad \frac{1 \times 1{,}500}{16 \times 32{\cdot}2}$$

$$\text{Force} = \frac{3 \times 750 \times 1{,}500}{2 \times 16 \times 32{\cdot}2} = 3{,}275 \text{ lb.} \quad \text{Ans.}$$

154. % Strength of drilled plate at outer row

$$= \frac{P - d}{} \times 100$$

$$\times 100 = 84{\cdot}66\%. \quad \text{Ans.}$$

% Strength of (5) rivets in pitch

$$\frac{a \times n \times f_s \times}{\times t \times f_t} \times 100$$

$$= \frac{\frac{1}{14} \times (1\frac{7}{16})^2 \times 5 \times 23 \cdot 5 \times 1\frac{7}{8}}{9\frac{3}{8} \times 1\frac{7}{16} \times 28} \times 100 = 94 \cdot 77\%. \text{ Ans.}$$

% strength of joint at middle row $\cdot\, 2\,d$ + % strength of one rivet.

$9\frac{3}{8} - 2\frac{7}{8}$ $\times$ 100 + 94·77

$= 69 \cdot 33 + 18 \cdot 95 = 88 \cdot 28\%$. Ans.

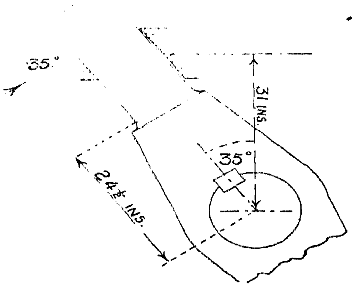
% Strength at inner row

$$= \frac{P - 2\,d}{P} + \text{\% strength of 3 rivets}$$

$= 69 \cdot 33 + 3 \times 18 \cdot 95 = 126 \cdot 18\%$. Ans.

The strength of a joint is the strength at its weakest part, this is the drilled plate at the outer row, therefore, strength of joint compared with solid plate $= 84 \cdot 66\%$. Ans.

155.



Total force of ram (P)

$$= \frac{\pi}{4} \times 12^2 \times 1{,}100 \text{ lb.}$$

Force applied at right angles to tiller =

$$\frac{P}{\text{Cos. } 35^\circ}$$

$$\frac{\pi \times 144 \times 1{,}100}{4 \times 0 \cdot 8192} \text{ lb.}$$

Leverage from block to rudder stock centre

$$= \frac{31}{\text{Cos. } 35^\circ} = 37 \cdot 84 \text{ ins.}$$

Length from block to section considered

$= 37 \cdot 84 - 24 \cdot 5$

$= 13 \cdot 34$ inches.

Bending Moment at this section

$$\frac{\pi \times 144 \times 1{,}100}{4 \times 0 \cdot 8192} \times 13 \cdot 34 \text{ inch lb.}$$

$$\frac{M}{I} = \frac{p}{y}, \quad I = \frac{\pi}{64} D^4, \quad y = \frac{D}{2}$$

$$\frac{M\ y}{I} \quad \frac{32\ M}{\pi}$$

$$\overline{32\ M}$$

$$\frac{\times\ \pi \times 144 \times 1{,}100 \times 13{\cdot}34}{\pi \times 4 \times 0{\cdot}8192 \times 14{,}250} \qquad 11{\cdot}31 \text{ ins.}$$

Diameter of tiller arms at $24\frac{1}{2}$ inches from rudder stock centre = 11·31 inches. Ans.

156. Consider *vertical* fall of water, acceleration due to gravity = 32·2 ft. per sec. per sec. Let t = time to reach ground, i.e., to fall through a vertical distance of 3 feet.

$$= \tfrac{1}{2}\, g\, t^2 \quad \therefore 3 = 16{\cdot}1\, t^2 \quad \therefore t = \frac{3}{16\cdot} = 0{\cdot}4316 \text{ sec.}$$

The *horizontal* component of the velocity of the jet of water is constant, the horizontal space passed over in 0·4316 sec. is 15 feet.

Horizontal velocity

$$\frac{\text{distance}}{\text{time}} \quad \frac{15.}{0{\cdot}4316} = 34{\cdot}75 \text{ ft. per sec.}$$

This is the velocity of efflux.

Neglecting friction, $34{\cdot}75 = \sqrt{2\ g\ h}$ where h = head of water above orifice (see page 437).

$$\therefore h = \frac{(34{\cdot}75)^2}{2 \times 32{\cdot}2} = 18{\cdot}75 \text{ feet.}$$

Total head of water above base of tank = 18·75 + 3 = 21·75 feet.

$\therefore$ Pressure on base = 21·75 × 62·5 = 1,359 lb. per sq. ft. Ans.

157. See page 180 for proof that the height of a simple conical pendulum is $\dfrac{2{,}935}{N^2}$. Since $h = \dfrac{2{,}935}{N^2}$, then the height varies inversely as the (revolutions)^2

$$\text{At 75 r.p.m. } h = \frac{2{,}935}{75^2} = 0{\cdot}5217 \text{ foot.}$$

$$= 6{\cdot}26 \text{ inches. Ans.}$$

$$\text{At 70 r.p.m. } h = \frac{2{,}935}{70^2} = 0{\cdot}599 \text{ foot.}$$

$$= 7{\cdot}188 \text{ inches.}$$

$\therefore$ Change in height $= 7{\cdot}188 - 6{\cdot}26 = 0{\cdot}928$ inch. Ans.

158.

DENSITY 0·5

Imagine the hydrometer to have a uniform cross section over its whole length, and let the equivalent length of the bulb and keel be x divisions.

Now length of hydrometer $\times$ section $\times$ density of liquid $=$ weight of hydrometer.

The section does not vary, and the weight remains the same independent of the density of the liquid.

$\therefore$ length $\times$ density $=$ constant

$(12 + x)\ 0{\cdot}5 = x \times 1$

$6 + 0{\cdot}5\,x = x$ and $x = 12$

When floating at 4 divisions from bottom, length $= 12 + 4 = 16$

$16 \times$ density $= 12 \times 1$, because length $\times$ density is constant.

$\therefore$ density $= \frac{12}{16} = 0{\cdot}75$. Ans.

If the question means that the mark to indicate a density of 1 is to be taken as the first mark, then at the 4th mark from the bottom the equivalent length will be $12 + 3 = 15$

$15 \times$ density $= 12 \times 1$.

$\therefore$ density $= \frac{12}{15} = 0{\cdot}8$. Ans.

159. Stress in copper = half stress in steel.

Difference in expansion per unit of length = Sum of the strains (see page 235).

$$\therefore\ 85\,(0{\cdot}0000095 - 0{\cdot}0000067) = \frac{S_s}{E_s} + \frac{S_c}{E_c}$$

$$\frac{S_s}{30 \times 10^6} \qquad \frac{S_s}{15 \times 10^6} \qquad \frac{S_s}{15 \times 10^6}$$

$$\frac{S_s}{15 \times 10^6} = 85\,(0{\cdot}0000095 - 0{\cdot}0000067)$$

$\therefore$ stress in steel $= 85 \times 0{\cdot}0000028 \times 15 \times 10^6$
3,570 lb. per sq. inch. Ans.

and stress in copper $= \dfrac{3{,}570}{2} = 1{,}785$ lb. per sq. inch
Ans.

Strain due to stress $= \dfrac{3{,}570}{30{,}000{,}000}$ $0{\cdot}000119$ for each.

Extension per unit length due to stress, and change of temperature :— .

For steel $= 0{\cdot}0000067 \times 85 + 0{\cdot}000119 = 0{\cdot}0006885$.
Ans.

Or for copper $= 0{\cdot}0000095 \times 85 - 0{\cdot}000119 = 0{\cdot}0006885$.
Ans.

160. Let v = initial velocity of water, in ft. per sec.

Weight of water striking plate per second

$0{\cdot}7854 \times 0{\cdot}75^2 \times v \times 62{\cdot}5$
lb.

$$\text{Momentum} = \text{Mass} \times \text{Velocity}$$

Change of momentum per sec.

$$= \frac{0{\cdot}7854 \times 0{\cdot}75^2 \times v \times 62{\cdot}5 \times v}{144 \times 32{\cdot}2}$$

Change of momentum per sec. = Force

$$\frac{0{\cdot}7854 \times 0{\cdot}75^2 \times 62{\cdot}5 \times v^2}{144 \times 32{\cdot}2} = 30$$

$$v = \sqrt{\frac{30 \times 144 \times 32{\cdot}2}{0{\cdot}7854 \times 0{\cdot}75^2 \times 62{\cdot}5}}$$

$= 70{\cdot}98$ ft. per sec. Ans. (*a*)

$$\text{Kinetic Energy of jet} = \frac{W\,v^2}{2\,g}$$

$$= \frac{0{\cdot}7854 \times 0{\cdot}75^2 \times 70{\cdot}98 \times 62{\cdot}5 \times 70{\cdot}98^2}{144 \times 2 \times 32{\cdot}2} \text{ ft. lb. per sec.}$$

$$= \frac{0{\cdot}7854 \times 0{\cdot}75^2 \times 70{\cdot}98 \times 62{\cdot}5 \times 30 \times 144 \times 32{\cdot}2 \times 60}{144 \times 2 \times 32{\cdot}2 \times 0{\cdot}7854 \times 0{\cdot}75^2 \times 62{\cdot}5} \text{ ft. lb. per min.}$$

$= 63{,}880$ ft. lb. per min. Ans. (*b*)

If the question set states that the jet pressure is 30 lb. per square inch, then the jet velocity would be,

$$\sqrt{\frac{2 \times 32{\cdot}2 \times 30 \times 144}{62{\cdot}5}} = 66{\cdot}72 \text{ ft. per sec.}$$

and the energy per minute would be 53,040 ft. lb.

161. Turning Moment (M) = (see page 174).

ϕ = Angular acceleration in radians per sec. per sec.

$$= \frac{250 \times 2\,\pi}{60 \times 15}$$

I_p = polar moment of inertia

$= 600$ lb. ft.^{2} units.

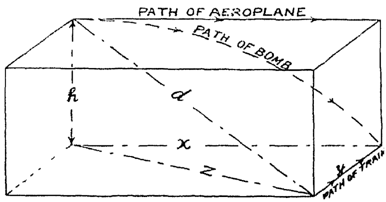
$$\therefore \text{ Turning Moment } = \frac{250 \times 2\pi \times 600}{60 \times 15 \times 32\cdot2}$$

$= 32\cdot53$ ft. lb. Ans.

162. 240 m.p.h. $=$ 352 ft. per sec.

60 m.p.h. $=$ 88 ft. per sec.

The time taken for the bomb to reach the ground is the same as if it fell vertically through 2,000 feet from rest.



$s = \frac{1}{2} at^2$

$$\therefore t = \sqrt{\frac{\quad}{16\cdot1}} = 11\cdot15 \text{ secs.} \quad \text{Ans. } (a)$$

Distance x is the distance travelled by the aeroplane while the bomb was falling (if the plane does not alter course).

$$x = 352 \times 11\cdot15 = 3924\cdot8 \text{ feet.}$$

Distance y is the distance travelled by the train while the bomb was falling.

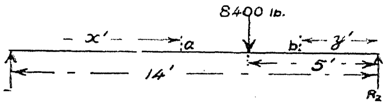
$$y = 88 \times 11\cdot15 = 981\cdot2 \text{ feet.}$$

$$\therefore d = \sqrt{x^2 + y^2 + h^2}$$

$$= \sqrt{3924\cdot8^2 + 981\cdot2^2 +}$$

$= 4{,}512$ feet. Ans. (b)

163.



Moments about R_1,

$8400 \times 9 = R_2 \times 14$

$\therefore R_2 = 5400$ lb.

and $R_1 = 3{,}000$ lb.

Let "a" (at x feet from R_1) and "b" (at y feet from R_2) be the positions where the maximum stress (i.e., stress at the outer fibres of the material) is 2,000 lb. per sq. inch.

$M_{at\ a} = 3{,}000 \times x \times 12$ inch lb.,

$$\frac{6\ M}{B\ D^2} \qquad \therefore 2{,}000 = \frac{6 \times 3{,}000 \times x \times 12}{5 \times 9^2}$$

$$\therefore x = \frac{2000 \times 5 \times 9^2}{6 \times 3000 \times 12}$$

$= 3\frac{3}{4}$ feet.

$M_{at\ b} = 5400 \times y \times 12$ inch lb.

$$p = \frac{6\ M}{} \qquad \therefore 2000 = \frac{6 \times 5400 \times y \times 12}{5 \times 9^2}$$

$$\therefore y = \frac{2000 \times 5 \times 9^2}{6 \times 5400 \times 12}$$

$= 2\frac{1}{12}$ feet.

At 3 ft. 9 ins. from left hand support, and also at 2 ft. 1 inch from right hand support, the maximum stress in the material is 2,000 lb. per sq. inch. Ans.

164. Circumference of thread $= \pi \times 2{\cdot}5$ inches.

Fraction of one turn that nut is tightened $= \dfrac{0{\cdot}625}{\pi \times 2{\cdot}5}$

and since the pitch of thread is $\frac{1}{6}$ inch, this fraction is

equivalent to $\dfrac{0{\cdot}625}{\pi \times 2{\cdot}5 \times 6}$ inch.

Additional extension of the bolt is one half of this,

$$\frac{0{\cdot}625}{\pi \times 2{\cdot}5 \times 6 \times 2}$$

$$\text{Strain} = \frac{\text{Extension}}{\text{Length}} = \frac{0{\cdot}625}{\pi \times 2{\cdot}5 \times 6 \times 2 \times 24}$$

Stress = E × Strain

$$= 30 \times 10^6 \times \frac{0{\cdot}625}{\pi \times 2{\cdot}5 \times 6 \times 2 \times 24}$$

= 8,290 lb. per sq. inch.

Additional stress = 8,290 lb. per sq. inch. Ans.

165. As the taper is equally divided between the two edges, it is the same case as two inclined planes placed back to back as explained on page 197.

Horizontal force to pull a body up an incline
$= W \tan (\phi + \alpha)$

$\therefore$ Force to drive cotter in $= 2 \{W \tan (\phi + \alpha)\}$

And Force to drive cotter out $= 2 \{W \tan (\phi - \alpha)\}$

$$\text{Now, } \tan \alpha = \frac{\frac{1}{2}}{8} = \tfrac{1}{16} = 0{\cdot}0625$$

$\therefore \alpha = 3° \; 34'$

$\tan \phi = \mu = 0{\cdot}2$

$\therefore \phi = 11° \; 19'.$

$\phi + \alpha = 11° \; 19' + 3° \; 34' = 14° \; 53'$

$\tan 14° \; 53' = 0{\cdot}2657$

$\phi - \alpha = 11° \; 19' - 3° \; 34' = 7° \; 45'$

$\tan 7° \; 45' = 0{\cdot}1361$

Force to drive in $= 2 \{W \tan (\phi + \alpha)\}$

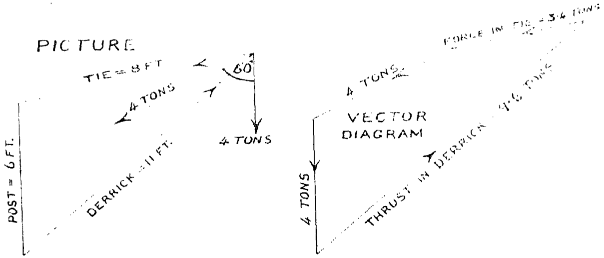
$\therefore 600 = 2 \times W \times 0{\cdot}2657$

$$\therefore W = \frac{600}{2 \times 0{\cdot}2657} = 1{,}129 \text{ lb.}$$

Force holding parts together is 1,129 lb. Ans. (*a*)

$$\text{Force to drive out} = 2\{W \tan(\phi - \alpha)\}$$
$$= 2 \times 1{,}129 \times 0{\cdot}1361$$
$$= 307{\cdot}3 \text{ lb.} \quad \text{Ans. } (b)$$

166.



On the Vector diagram :

Force in tie measures 3·4 tons (tensile). Ans.

Force in derrick measures 9·6 tons (compressive). Ans.

Work done $= 4 \times 2240 \times 2$ ft. lb. per sec.

$$\therefore \text{H.P.} = \frac{4 \times 2240 \times 2}{550}$$

$$\text{I.H.P. of winch} = \frac{4 \times 2240 \times 2 \times 100 \times 100}{550 \times 68 \times 86}$$

$= 55{\cdot}7.$ Ans.

167. Let speed of slow ship $= x$ knots,

then speed of fast ship $= (x + 3)$ knots.

Working in hours of time :—

Time for fast ship $+ 12 =$ Time for slow ship

$$\frac{1065}{x +} = \frac{1065}{x} \text{ multiply by } x \times (x + 3)$$

$$1065\ x + 12\ x\ (x + 3) = 1065\ (x + 3)$$
$$1065\ x + 12\ x^2 + 36\ x = 1065\ x + 3195$$
$$\therefore\ 12\ x^2 + 36\ x - 3195 = 0$$
$$\text{or, } 4\ x^2 + 12\ x - 1065 = 0$$
$$\text{from which, } x = 14\tfrac{7}{8}$$

Speeds of ships are $14\frac{7}{8}$ knots and $17\frac{7}{8}$ knots. Ans.

168. 30 m.p.h. = 44 ft. per sec.

$$\text{Accelerating force} = \frac{W \times a}{g}$$

$$150 \times 2240 \times 44$$

$$32{\cdot}2 \times 1{\cdot}5 \times 60$$

$$= 5{,}102 \text{ lb.}$$

Force to overcome friction = 16 × 150 = 2,400 lb.

$\therefore$ Total force applied = 5102 + 2400 = 7502 lb. or 3·349 tons. Ans.

H.P. at 44 ft. per sec.

$$\text{Force (lb.)} \times \text{distance in ft. per sec.}$$

$$550$$

$$= \frac{2400 \times 44}{550}$$

$$= 192 \text{ H.P.} \quad \text{Ans.}$$

169. Deflection $\delta = \dfrac{W\ l^3}{3\ E\ I}$, this is for a simple cantilever with a concentrated load W at the free end. W is the force applied at the end of the steam pipe by the steam range expansion, and the maximum bending moment occurs at the neck of pipe near the stop valve.

$$M_{max.} = W \times l$$

$$M\ l^2$$

$$3\ E\ I$$

$$3\ E\ I\ \delta$$

$$\text{or } M =$$

$$\frac{M}{I} \qquad p$$

$$\therefore p = \frac{M\,y}{I} = \frac{3 \times E \times I \times \delta \times}{l^2 \times 2 \times I}$$

$$= \frac{3 \times 29 \times 10^6 \times 0 \cdot 3 \times 6}{120^2 \times 2}$$

$= 5437 \cdot 5$ lb. per sq. in. Ans.

170. Load in steel + Load in copper ... = 20 tons.

$\text{Stress}_s \times \text{Area}_s + \text{Stress}_c \times \text{Area}_c \ldots = 20$

$\text{Strain}_s \times E_s \times \text{Area}_s + \text{Strain}_c \times E_c \times \text{Area}_c = 20$

$\text{Strain} \times \text{Area}\ (E_s + E_c) = 20$

Because strain and area are the same for each.

$$\text{Strain} = \frac{20}{1 \cdot 25 \times 19500}$$

Stress in steel = Strain × E

in steel = Strain × E × Area

$$= \frac{20 \times 13500 \times 1 \cdot 25}{1 \cdot 25 \times 19500}$$

$$= \frac{20 \times 135}{195} = 13 \cdot 85 \text{ tons.} \quad \text{Ans.}$$

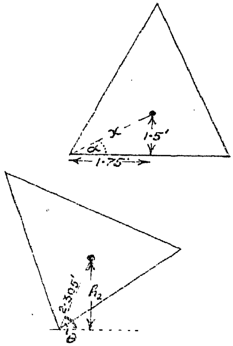
And similarly,

$$\text{Load in copper} = \frac{20 \times 60}{195} = 6 \cdot 15 \text{ tons.} \quad \text{Ans.}$$

171. Steel weighs 490 lb. per cu. ft.

Weight of cone $= 0 \cdot 7854 \times (3\frac{1}{2})^2 \times 6 \times \frac{1}{3} \times 490$
$= 9432 \cdot 5$ lb.

C.G. of cone is at $\frac{1}{4}$ height $= 1 \cdot 5$ ft. above base.



$$\text{Tan } \alpha = \frac{1\cdot5}{1\cdot75} = 0\cdot8571$$

$$\alpha = 40° \ 36'$$

$$\frac{1\cdot5}{\text{Sin } 40° \ 36'} \qquad \frac{1\cdot5}{0\cdot6507}$$

$$= 2\cdot305 \text{ ft.}$$

$$\theta = 40° \ 36' + 40° = 80° \ 36'$$

$$h_2 = 2\cdot305 \times \text{Sin } 80° \ 36'$$

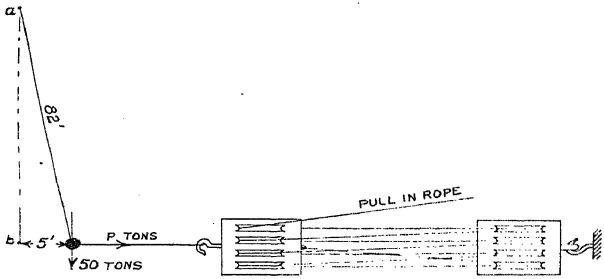
$$= 2\cdot305 \times 0\cdot9866$$

$$= 2\cdot275 \text{ ft.}$$

Therefore C.G. has been raised a vertical distance of 2·275 — 1·5 = 0·775 ft.

Work done = 9432·5 × 0·775 = 7310 ft. lb. Ans.

172.



$$a\,b = \sqrt{82^2 - 5^2} = 81\cdot84 \text{ ft.}$$

Moments about a,

$$P \times 81\cdot84 = 50 \times 5$$

$$\therefore P = \frac{50 \times 5}{81\cdot84} = 3\cdot054 \text{ tons.}$$

The blocks will probably be arranged as shown in sketch, the boiler being pulled in the same direction as the effort is applied, then velocity ratio = 9.

$$\text{Efficiency} = \frac{\text{Mechanical advantage}}{\text{Velocity ratio}}$$

$$\therefore \text{ M.A.} = 9 \times 0{\cdot}7 = 6{\cdot}3$$

$$\text{M.A.} = \frac{\text{Load}}{\text{Effort}} \quad \therefore \text{ Effort} = \frac{3{\cdot}054 \times 2{,}240}{6{\cdot}3}$$

$$= 1{,}086 \text{ lb. Ans.}$$

173. See page 182 for proof of formula.

30 vibrations per minute = $\frac{1}{2}$ vibration per second,

$\therefore$ time for one vibration = 2 seconds.

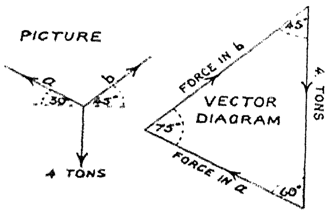
$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$2 = 2\pi\sqrt{\frac{l}{32{\cdot}2}} \quad \text{square both sides}$$

$$4 = \frac{4\pi^2 l}{32{\cdot}2}$$

$$\therefore l = \frac{2^2 \times 32{\cdot}2}{2^2 \times \pi^2} = 3{\cdot}262 \text{ feet. Ans.}$$

174.



By sine rule :—

$$\frac{\text{Force in } a}{\text{Sin. } 45^\circ} = \frac{4}{\text{Sin. } 75^\circ}$$

$$\text{Force in } a = \frac{4 \times 0{\cdot}7071}{0{\cdot}9659} = 2{\cdot}929 \text{ tons.}$$

$$\frac{\text{Force in } b}{\text{Sin. } 60^\circ} = \frac{4}{\text{Sin. } 75^\circ} \quad \therefore \text{ Force in } b = \frac{4 \times 0{\cdot}866}{0{\cdot}9659}$$

$$= 3{\cdot}587 \text{ tons.}$$

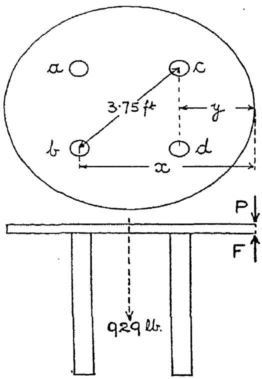
Factor of safety of a

$$\frac{6{,}700 \times 3^2}{2{\cdot}929 \times 2{,}240} = 9{\cdot}192. \quad \text{Ans.}$$

Factor of safety of b

$$\frac{6{,}700 \times 3^2}{3{\cdot}587 \times 2{,}240} = 7{\cdot}505. \quad \text{Ans.}$$

175.



Total weight $= 801 + (4 \times 32)$
$= 929$ lb.

$a\ c = 3{\cdot}75 \sin 45° = 2{\cdot}652$ feet.

$$x = \frac{2{\cdot}652}{2} + 2{\cdot}5 = 3{\cdot}826 \text{ feet.}$$

If the force F is applied vertically upwards, then the table tilts on legs a and b.

Taking moments about the line $a\ b$

$$F \times x = 929 \times \frac{2{\cdot}652}{2}$$

$$F = \frac{929 \times 1{\cdot}326}{3{\cdot}826} = 321{\cdot}9 \text{ lb.} \quad \text{Ans.} \quad (a)$$

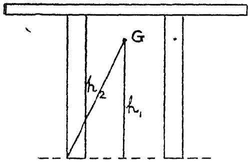
If the force P is applied vertically downwards, then the table tilts on legs c and d.

$y = 2{\cdot}5 - 1{\cdot}326 = 1{\cdot}174$ feet.

Taking moments about the line $c\ d$

$$P \times y = 929 \times 1{\cdot}326$$

$$P = \frac{929 \times 1{\cdot}326}{1{\cdot}174} \qquad (b)$$



Height of C.G. of table from floor

$$= \frac{\text{Summation of moments of weights}}{\text{Summation of weights}}$$

$$= \frac{(801 \times 48{\cdot}5) + (128 \times 24)}{929} = 45{\cdot}12 \text{ ins.}$$

a c

1·326 feet = 15·91 inches

$h_2 = \sqrt{(45{\cdot}12)^2 + (15{\cdot}91)^2} = 47{\cdot}84$ inches.

When the table has been tilted until its C.G. is vertically above the line joining the feet of the legs, it would then fall completely over.

Rise of C.G. $= h_2 - h_1 = 47{\cdot}84 - 45{\cdot}12 = 2{\cdot}72$ inches.

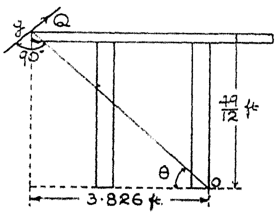
929

Work done 210·6 ft. Ans. (*c*)

The work done would be the same for both cases (*a*) and (*b*).

The forces F and P might be applied at the end of a diameter passing through two opposite legs, causing the table to tilt about the bottom of one leg. F and P would be greater than the values already found.

The question, as given, is not definite. If the *least* force acting in an upward direction, and the *least* force acting in a downward direction, was asked for then a solution would be as follows:—



The least force Q, acting in an upward direction, that would cause the table to tilt, about the line passing through the bottom of two legs, would be applied at right angles to the line *g o*.

$$\text{Tan } \theta = \frac{\quad}{12 \times 3{\cdot}826},$$

from which $\theta = 46° \ 52'$

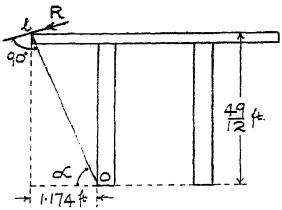
$$g\,o = \frac{49}{12 \sin 46° \ 52'},$$

596 feet.

By moments around o

$$Q \times 5{\cdot}596 = 929 \times 1{\cdot}326$$

$$Q = \frac{929 \times 1{\cdot}326}{5{\cdot}596} = 220{\cdot}1 \text{ lb.} \quad \text{Ans.}$$



The least force R, acting in a downward direction, that would cause the table to tilt, about the line passing through the bottom of two legs, would be applied at right angles to the line $l\ o$.

$$\text{Tan } \alpha = \frac{49}{12 \times 1{\cdot}174}$$

from which $\alpha = 73° \ 57'$

$$l\ o = \frac{49}{12 \sin 73° \ 57'} = 4{\cdot}248 \text{ feet.}$$

By moments around o

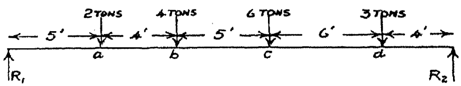
$$R \times 4{\cdot}248 = 929 \times 1{\cdot}326$$

$$R = \frac{929 \times 1{\cdot}326}{4{\cdot}248} = 290 \text{ lb.} \quad \text{Ans.}$$

The work done in tilting the table to a position of unstable equilibrium would still be 210·6 ft. lb. In whatever manner the overturning moment is applied, provided the table tilts about a line through the bottom of two legs, the C.G. is raised 2·72 inches, and the work done must be the same.

The student must observe carefully the question given in order to determine the manner in which the tilting force is to be applied.

176.



Moments about R_1 :—

$$(2 \times 5) + (4 \times 9) + (6 \times 14) + (3 \times 20) = R_2 \times 24$$

$$\therefore \; R_2 = \frac{190}{24} = 7\tfrac{11}{12} \text{ tons.}$$

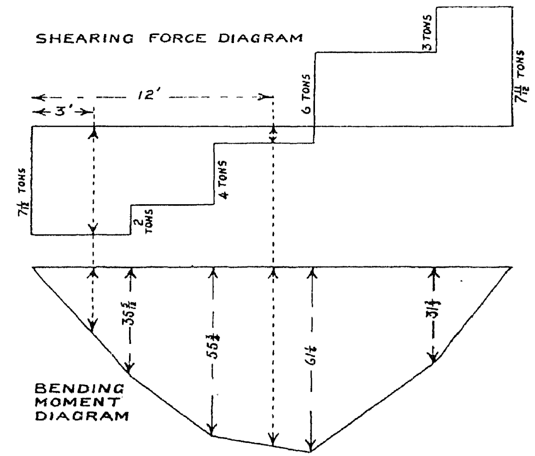
$2 + 4 + 6 + 3 = R_1 + R_2 \quad \therefore \; R_1 = 7\tfrac{1}{12}$ tons.

$M_{at\ a} = R_1 \times 5 = 7\tfrac{1}{12} \times 5 = 35\tfrac{5}{12}$ ft. tons.

$M_{at\ b} = (7\tfrac{1}{12} \times 9) - (2 \times 4) = 55\tfrac{3}{4}$ ft. tons.

$M_{at\ c} = (7\tfrac{11}{12} \times 10) - (3 \times 6) = 61\tfrac{1}{6}$ ft. tons.

$M_{at\ d} = 7\tfrac{11}{12} \times 4 = 31\tfrac{2}{3}$ ft. tons.

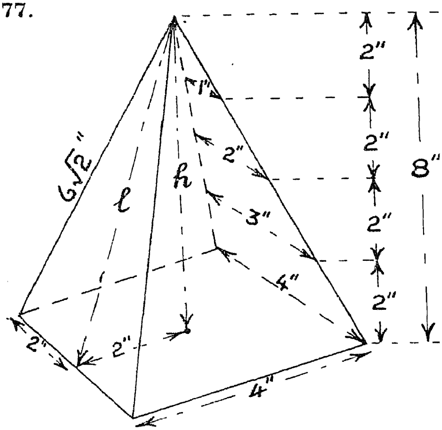


From the diagrams :—

S.F. at 3 ft. from left measures $7\tfrac{1}{12}$ tons.
B.M. at 3 ,, ,, ,, $21\tfrac{1}{4}$ ft. tons.
S.F. at 12 ,, ,, ,, $1\tfrac{1}{12}$ tons.
B.M. at 12 ,, ,, ,, 59 ft. tons.

The student should draw the diagrams to the scale stated in the question at the examination. For purposes of reproduction, the diagrams given here are to a smaller scale.

177.



$= (6\sqrt{2})^2$
$6^2 \times 2 - 2^2$

$6^2 \times 2 - 2^2 - 2^2 = 64$

$h = 8$ inches vertical height.

The widths across the slant face will be 4, 3, 2, 1 and 0 inches respectively, at vertical heights of 0, 2, 4, 6 and 8 inches respectively from the base. The common interval is 2 inches. Putting the cross-sectional areas through Simpson's rule :—

Width	Cross Section Areas	Simpson's Multi-pliers	Functions for Volume	Distances from Bottom	Functions for Moments
4	16	1	16	0	0
3	9	4	36	2	72
2	4	2	8	4	32
1	1	4	4	6	24
0	0	1	0	8	0
		Sum =	64	Sum	128

$$\text{Volume} = \frac{64 \times 2}{3} = 42\tfrac{2}{3} \text{ cu. inches.} \quad \text{Ans.}$$

C.G. from bottom

$$= \frac{\Sigma \text{ Moments of volumes}}{\Sigma \text{ volumes}}$$

$$= \frac{128 \times 2 \times 3}{3 \times 64 \times 2} = 2 \text{ inches.} \quad \text{Ans.}$$

178. Let u feet per sec. be the velocity at which the body passes the higher point

$$s = u\,t + \tfrac{1}{2}\,g\,t^2$$
$$1{,}542 = 7{\cdot}6\,u + 16{\cdot}1 \times (7{\cdot}6)^2$$
$$7{\cdot}6\,u = 1{,}542 - 16{\cdot}1 \times (7{\cdot}6)^2 = 612{\cdot}2$$
$$u = \frac{612{\cdot}2}{7{\cdot}6} = 80{\cdot}55 \text{ ft. per sec.}$$

80·55 ft. per sec. is the velocity of the body after it has fallen from rest through a distance s

From the formula, $v^2 = 2\,g\,s$

$$\frac{(80{\cdot}55)^2}{2\,g} = 100{\cdot}8 \text{ feet.}$$

The body was released from a height of 100·8 feet above the higher point. Ans.

Velocity after 12 secs. $= 12 \times 32{\cdot}2 = 386{\cdot}4$ feet per sec. Ans.

Acceleration is the time rate of change of velocity, or the change of velocity per unit of time.

179. 30 inches head of fresh water

$$\frac{30 \times 62{\cdot}5 \times 144}{1{,}728}$$

$$\frac{30 \times 62{\cdot}5}{12} \text{ lb. per sq. foot.}$$

Let a_1 be the area of entrance of the meter ; v_1 be the velocity ; p_1 be the pressure and h_1 the head.

Let a_2 ; v_2 ; p_2 ; h_2 be the conditions at the throat.

By Bernouilli's theorem,

$$\frac{p_1}{w} + \frac{v_1^2}{2\,g} = \frac{p_2}{w} + \frac{v_2^2}{2\,g}$$

Now $h_1 = h_2$, because the meter is horizontal.

$$\therefore \frac{p_1 - p_2}{w} = \frac{v_2^2 - v_1^2}{2\,g} \quad \ldots \quad \ldots \quad \ldots \quad (1)$$

$$\frac{p_1 - p_2}{w} = \frac{30 \times 62{\cdot}5}{12 \times 62{\cdot}5} = \frac{5}{2}$$

$a_1\,v_1 = a_2\,v_2$, because the quantity passing the entrance equals the quantity passing the throat.

$$a_1\,v_1 = a_2\,v_2$$

$$v_2 = v_1 \times \frac{a_1}{a_2} = v_1 \times \frac{12^2 \times \frac{\pi}{4}}{4^2 \times \frac{\pi}{4}} = 9\,v_1$$

$\therefore\ v_2^2 = (9\,v_1)^2 = 81\,v_1^2$. Substitute in (1)

$$\frac{5}{2} = \frac{81\,v_1^2 - v_1^2}{2\,g}$$

$$\frac{5}{2} = \frac{80\,v_1^2}{64{\cdot}4},\quad v_1 = \sqrt{\frac{5 \times 64{\cdot}4}{2 \times 80}} = 1{\cdot}419 \text{ ft. per sec.}$$

$\therefore$ gallons per minute

$$= \frac{12^2 \times 0{\cdot}7854}{144} \times 1{\cdot}419 \times 60 \times 6{\cdot}25$$

$= 417{\cdot}9$ gallons. Ans.

180. Linear law : $P = a + b\,W$

$$\begin{aligned}(1)\quad 25 &= a + b \times 240 \\ (2)\quad 7 &= a + b \times 24\end{aligned} \quad \text{subtract}$$

$$18 = 216\,b$$

$$\therefore b = \frac{18}{216} = \frac{1}{12}$$

From (2), $7 = a + 24\,b \qquad a = 7 - 2 = 5.$

$\therefore$ Linear law is $P = 5 + \frac{1}{12}\,W$

For 720 lb. load, $P = 5 + \frac{1}{12} \times 720 = 65$ lb. Ans. (*a*)

$$\text{M.A.} = \frac{W}{P} = \frac{720}{65}$$

$$\text{Eff.} = \frac{\text{M.A.}}{\text{V.R.}} = \frac{720}{65 \times 20} = 0{\cdot}3692 = 36{\cdot}92\%. \quad \text{Ans.}$$

181. Suppose a car does 25 miles for the consumption of 1 gallon of petrol, then for 1 mile the consumption will be $\frac{1}{25}$ gallon. That is the reciprocal of the miles per gallon is the gallons per mile.

Let W = weight of car in tons,

then W $\times$ miles per gallon = 33

$$\therefore \text{ miles per gallon} = \frac{33}{W}$$

$$\text{and gallons per mile} = \frac{W}{33}$$

$$\therefore \text{ pints per mile} = \frac{8\,W}{33}$$

$$\text{B.H.P.} = \frac{W \times 162 \times \text{miles per hour} \times 5{,}280}{33{,}000 \times 60}$$

Pints per mile $\times$ miles per hour = pints used per hour,

$$\text{and } \frac{\text{pints used per hour}}{\text{B.H.P.}} = \text{pints used per B.H.P. hour.}$$

$\therefore$ pints used per B.H.P. hour

$$\frac{\frac{8\,W}{33} \times \text{miles per hour} \times 33{,}000 \times 60}{W \times 162 \times \text{miles per hour} \times 5{,}280}$$

$$\frac{8 \times 33{,}000 \times 60}{33 \times 162 \times 5{,}280} = 0{\cdot}561. \quad \text{Ans.}$$

Alternative solution.

Let W = wt. of car in tons.

Let x = miles per gal., then $W \times x = 33$.

Let m.p.h. stand for miles per hour.

Consumption of petrol

$$= \frac{1}{x} \text{ gallons per mile} = \frac{8}{x} \text{ pints per mile.}$$

Consumption per hour

$$= \frac{8}{x} \times \text{m.p.h. (pints)} \quad \ldots \quad \ldots \quad \ldots \quad \text{(i.)}$$

Frictional resistance = W × 162 lb.

Work done per hour = W × 162 × m.p.h. × 5,280 ft. lb.

∴ Horse power

$$\frac{W \times 162 \times \text{m.p.h.} \times 5{,}280}{33{,}000 \times 60} \quad \ldots \quad \text{(ii.)}$$

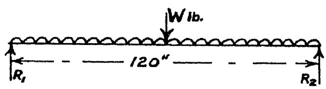
Consumption in pints per H.P. per hour = (i.) ÷ (ii.)

$$\frac{8 \times \text{m.p.h.} \times 33{,}000 \times 60}{x \times W \times 162 \times \text{m.p.h.} \times 5{,}280}$$

$$\frac{8 \times 33{,}000 \times 60}{33 \times 162 \times 5{,}280}$$

= 0·561 pint per B.H.P. per hour. Ans.

182.



Weight of beam is a uniformly distributed load.

Wt. of beam = $0{\cdot}7854\,(20^2 - 14^2) \times 120 \times 0{\cdot}26 = 5{,}008$ lb.

Each reaction will be half of the total load.

$$R_1 = R_2 = \frac{5{,}008 + W}{2}$$

$$M_{centre} = \frac{l}{2} - \frac{5{,}008}{2} \times \frac{l}{4}$$

$$\frac{5{,}008 + W}{2} \times 60 - \frac{5{,}008}{2} \times 30$$

$$= 30 \left\{ (5{,}008 + W) - \frac{5{,}008}{2} \right\}$$

$$= 30\ (2{,}504 + W)$$

$$\frac{M}{I} = \frac{p}{y} \quad \therefore p = \frac{M\ y}{I} = \frac{M \times D \times 64}{2 \times \pi\ (D^4 - d^4)}$$

$$32\ M\ D$$

$$p$$

Note. $D^4 - d^4 = (D^2 + d^2)\ (D^2 - d^2) = 596 \times 204$

$$700 = \frac{32 \times 30\ (2{,}504 + W) \times 20}{\pi \times 596 \times 204}$$

$$\therefore (2{,}504 + W) = \frac{700 \times \pi \times 596 \times 204}{32 \times 30 \times 20} = 13{,}930 \text{ lb.}$$

$$\therefore W = 13{,}930 - 2{,}504$$

$$= 11{,}426 \text{ lb., or } 5{\cdot}102 \text{ tons. Ans.}$$

183. Volume of bunker $= 1{,}077 \times 43$ cu. feet.

Cross section area $= \dfrac{1{,}077 \times 43}{56}$ square feet.

Let x = width across the top

$0{\cdot}5\ x$ = width across the bottom

$\frac{3}{7}\ x$ = depth.

To find area by Simpson's rule :-

Ordinates	Simpson's Multipliers	Functions of Ordinates
x	1	x
45	4	180
40	2	80
34	4	136
$0{\cdot}5\ x$	1	$0{\cdot}5\ x$

Common interval

$$= \frac{3}{7 \times 4} x$$

$$= \frac{3}{28} x$$

One-third of common interval

$$= \frac{x}{28}$$

$$\text{Sum} = 396 + 1{\cdot}5\ x$$

Area = Sum of functions $\times$ $\frac{1}{3}$ common interval

$$\frac{1{,}077 \times 43}{56} = (396 + 1{\cdot}5\ x) \times \frac{x}{28}$$

$$\frac{1{,}077 \times 43 \times 28}{56} = 396\ x + 1{\cdot}5\ x^2$$

$$23155{\cdot}5 = 396\ x + 1{\cdot}5\ x^2$$

$$\therefore\ 1{\cdot}5\ x^2 + 396\ x - 23155{\cdot}5 = 0$$

or $x^2 + 264\ x - 15{,}437 = 0$

from which $x = 49{\cdot}25$

Width of bunker across the top $= 49{\cdot}25$ feet. Ans.

184. Thrust horse power

$$= \frac{\text{Force (lb.)} \times \text{distance in ft. per min.}}{33{,}000}$$

$$\frac{32{,}000 \times 14{\cdot}3 \times 6{,}080}{33{,}000 \times 60}$$

$$\text{B.H.P.} = \frac{32{,}000 \times 14{\cdot}3 \times 6{,}080}{33{,}000 \times 60 \times 0{\cdot}7}$$

$$= 2{,}007.$$

$$\text{T (inch lb.)} = \frac{63{,}000 \times \text{H.P.}}{\text{R.P.M.}}$$

$$\text{R.P.M.} = \frac{63{,}000 \times \text{H.P.}}{\text{T}} \quad \frac{63{,}000 \times 2{,}007}{547 \times 2{,}240}$$

$= 103{\cdot}2$ revs. per min. Ans. (*a*)

$$\text{I.H.P.} = \frac{\text{B.H.P.}}{\text{Mech. Eff.}} \quad \frac{2{,}007}{0{\cdot}83} \quad 2{,}418. \quad \text{Ans. } (b).$$

185. $$\text{K.E.} = \frac{W\,r^2\,N^2}{5{,}870} = \frac{6 \times 2{,}240 \times 4{\cdot}5^2 \times 120^2}{5{,}870}$$

$= 667{,}700$ ft. lb. Ans. (*a*).

Friction force at surface of bearing $= \mu\,W$

$= 0{\cdot}01 \times 6 \times 2{,}240$ lb.

Ft. lb. of energy lost per rev.

$=$ Force (lb.) $\times$ Circumference (ft.)

$= 0{\cdot}01 \times 6 \times 2{,}240 \times \pi \times \frac{8}{12}$

$= 281{\cdot}6$ ft. lb. Ans. (*b*).

Number of revs. to come to rest

$$= \frac{\text{Total energy to be lost}}{\text{Energy lost per rev.}}$$

$$\frac{667{,}700}{281{\cdot}6}$$

$= 2{,}372$ revolutions. Ans. (*c*).

186. The twisting moment transmitted is to be the same.

Let q = stress allowed in solid shaft,

then $1{\cdot}3\ q$ = stress allowed in hollow shaft.

$$\text{T of solid shaft} = \frac{\pi \times D^3 \times q}{16}$$

$$\text{T of hollow shaft} = \frac{\pi \times (D^4 - d^4) \times q}{16 \times D}$$

$$\frac{\pi \times D^3 \times q}{16} \quad \frac{\pi \times (D^4 - d^4) \times 1{\cdot}3 \times}{16 \times D}$$, 16 and q cancel

$$\frac{(D^4 - d^4) \times 1{\cdot}3}{D}$$

$$\frac{D^4}{1{\cdot}3}$$

$$d^4 = D^4 - \frac{D^4}{1{\cdot}3}$$

$$d^4 = 0{\cdot}2308\ D^4$$

$$d = \sqrt[4]{0{\cdot}2308} \times D$$

$$d = 0{\cdot}6931\ D$$

∴ Ratio of inside to outside diam. = 0·6931 : 1. Ans. (a)

Wt. of hollow shaft = $0{\cdot}7854\ (D^2 - d^2) \times l \times$ wt. per cu. in.

Wt. of solid shaft = $0{\cdot}7854 \times D^2 \times l \times$ wt. per cu. in.

Ratio of Hollow : Solid

$$= D^2 - d^2 : D^2 = D^2 - (0{\cdot}6931\ D)^2 : D^2$$

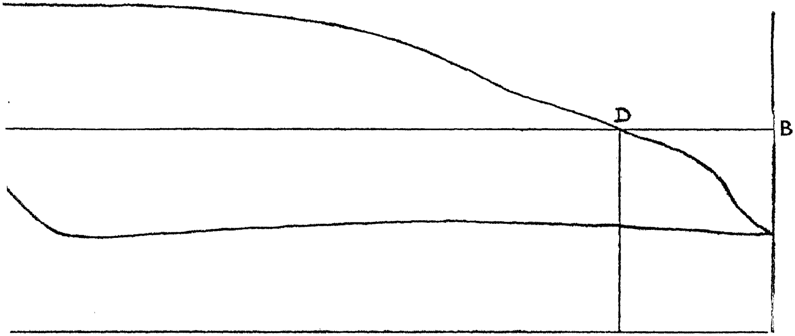
$$= 0{\cdot}5196\ D^2 : D^2 = 0{\cdot}5196 : 1$$

$$= 51{\cdot}96 : 100$$

∴ Hollow shaft is 100 — 51·96 = 48·04% lighter. Ans. (b).

SOLUTIONS TO FIRST-CLASS EXAMINATION QUESTIONS.

HEAT AND HEAT ENGINES.



Length of card A B = 4·75 inches.

Length of C D = 3·75 inches.

Pressure at point D = 1·29 × 80 = 103·2 lb. gauge.

= 118·2 lb. per square inch absolute.

Weight of 1 cubic foot of steam at 118·2 lb. per square inch

118·2 + 1
= 0·271 lb.
118·2
410

Volume of steam used per stroke

$$\frac{3\cdot75}{4\cdot75} \times \frac{26^2 \times 0\cdot7854}{144} \times 4 \text{ cu. feet.}$$

Weight of steam per hour

$$\frac{3{\cdot}75}{4{\cdot}75} \times \frac{26^2 \times 0{\cdot}7854}{144} \times 4 \times 65 \times 2 \times 0{\cdot}271 \times 60$$

$= 24{,}620$ lb. $= 11$ tons. Ans.

2. Heat given up by 1 lb. of steam

$= 966 - 0{\cdot}7\,(162 - 212) + (162 - 125)$
$= 1{,}038$ B.T.U.

Heat absorbed by 1 lb. of circulating water $= (115 - 57)$
$= 58$ B.T.U.

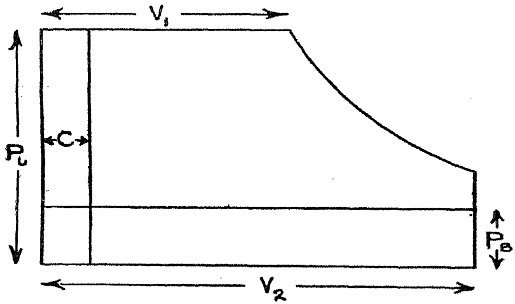
Steam per day $= 1{,}500 \times 15{\cdot}5 \times 24$ lb.

Circulating water per day $= \dfrac{1{,}500 \times 15{\cdot}5 \times 24 \times 1{,}038}{58 \times 2{,}240}$

$= 4{,}458$ tons. Ans.

3. Full area $= P_1\,V_1\left(1 + \log_e \dfrac{V_2}{V_1}\right)$

Nett area $= P_1\,V_1\left(1 + \log_e \dfrac{V_2}{V_1}\right) - P_1C - P_b\,(V_2 - C)$



Average height of nett area, and therefore the mean effective pressure

$$\frac{\log_e \dfrac{V_2}{V_1}\Big) - P_1\,C - P_b\,(V_2 - C)}{V_2 - C}$$

$$= \frac{\quad}{\text{Stroke}} \left(1 + \log_e \frac{V_2}{V_1}\right) - \frac{P_1 C}{\text{Stroke}} - P_b$$

$$\frac{V_2}{V_1} = \frac{48 + 4{\cdot}8}{26 + 4{\cdot}8} = 1{\cdot}714, \text{ and } \log_e 1{\cdot}714 = 0{\cdot}5382$$

$P_1 = 180 + 15 = 195$, and $P_b = 55 + 15 = 70$

$$\text{M.E.P.} = \frac{195 \times 30{\cdot}8}{48}(1 + 0{\cdot}5382) - \frac{195 \times 4{\cdot}8}{48} - 70$$

$= 192{\cdot}5 - 19{\cdot}5 - 70 = 103$ lb. per square inch. Ans.

$$\text{I.H.P.} = \frac{26^2 \times 0{\cdot}7854 \times 103 \times 4 \times 2 \times 65}{33{,}000} = 862. \text{ Ans.}$$

4. Let stroke be 100 units in length.

Distance piston is from end of stroke when compression starts $= 100 - 86 = 14$ units. Clearance $= 8$ units.

$p_1 v_1 = p_2 v_2$
$(42 + 15)(14 + 8) = p_2 \times 8$
$57 \times 22 = 8\, p_2$

$p_2 = 156{\cdot}75$ lb. per square inch absolute
$= 141{\cdot}75$ lb. gauge. Ans.

Heat units to form 1 lb. of steam before starting to blow $= (380 - 160) + 966 - 0{\cdot}7\,(380 - 212) = 1068{\cdot}4$ B.T.U.

$$\text{Blow out} = \frac{3{\cdot}5}{18} = \tfrac{7}{36} \text{ of the feed}$$

and amount left to form steam $= \tfrac{29}{36}$ of the feed.

∴ Full feed to form 1 lb. of steam when blowing has started $= \tfrac{36}{29}$ lb.

Heat units to form 1 lb. of steam when blowing $= \tfrac{36}{29}\,(380 - 160) + 966 - 0{\cdot}7\,(380 - 212) = 1121{\cdot}5$ B.T.U.

Total consumption increases proportionally to the heat units required to form 1 pound of steam.

$$\therefore \text{Consumption} = \frac{1121{\cdot}5}{1068{\cdot}4} \times 20 = 21 \text{ tons (nearly).}$$

Ans.

$p_1 \, v_1 = p_2 \, v_2$

$(145 + 15)$ (cut-off $+ 4) = (102 + 15)$ $(0 \cdot 9$ of $40 + 4)$

160 (cut-off $+ 4) = 117 \times 40$

Cut-off $= \dfrac{117 \times 40}{160} - 4 = 29 \cdot 25 - 4 = 25 \cdot 25$ inches of stroke. Ans.

7. Area of one diagram $= \dfrac{\quad}{2}$ square inches.

Average height $= \dfrac{3 \cdot 5}{2 \times 4 \cdot 5} \quad \dfrac{3 \cdot 5 \times 80}{2 \times 4 \cdot 5}$

$= \frac{280}{9}$ lb. square inch.

I.H.P. $= \dfrac{280 \times 20^2 \times 0 \cdot 7854 \times 2 \times 2 \times 80}{9 \times 33{,}000} = 94 \cdot 78.$ Ans.

8. Absolute steam pressure $= 124 + 15 = 139$ lb. per sq. inch.

Relative volume $= \dfrac{16 \times (139 + 1{,}600)}{139 + 1} \quad \dfrac{16 \times 1{,}739}{140}$

$= 198 \cdot 7$

$0 \cdot 1875$ cubic inch of water originated from

$0 \cdot 1875 \times 198 \cdot 7 = 37 \cdot 25$ cubic inches of steam.

$\therefore$ Cut-off takes place at $37 \cdot 25 - 1 \cdot 25$

$= 36$ inches of stroke. Ans.

9. Absolute pressure in condenser $= 29 \cdot 8 - 23 = 6 \cdot 8$ inches of mercury

$\dfrac{6 \cdot 8}{30} \times 14 \cdot 7 = 3 \cdot 332$ lb. square inch absolute.

$3 \cdot 332 - 1 \cdot 04 = 2 \cdot 292$ lb. per square inch lost due to air. Ans.

10. Cut off takes place at 0·6 stroke, when the crank will have passed through a greater angle than 90° from the centre. Boiler pressure is acting on piston when crank is at right angles to the line of stroke, and this is therefore the position when the maximum load comes on the guide.

Distance from centre of crosshead to shaft centre in this position $= \sqrt{9^2 \quad 2^2} = \sqrt{77} = 8{\cdot}776$ feet.

Load on guide $= \dfrac{2}{8{\cdot}776} \times 24^2 \times 0{\cdot}7854 \times 180$ lb.

Area of guide $= \dfrac{-}{8{\cdot}776} \times 24^2 \times 0{\cdot}7854 \times \frac{180}{100}$

$= 185{\cdot}6$ square inches. Ans.

11. Fall in temperature $= (360 - 100) = 260$ F°.

Strain $= 0{\cdot}0000067 \times 260$ inch per inch of length.

Stress = Modulus × Strain $= 13{,}500 \times 0{\cdot}0000067 \times 260$

$= 23{\cdot}517$ tons per square inch. Ans.

$= 52{,}670$ lb. per square inch. Ans.

12. Mean pressure of H.P. referred to L.P.

$$\frac{26^2}{71^2} \times 73 = 9{\cdot}788 \text{ lb. per sq. inch.}$$

Mean press. of M.P. referred to L.P. $\dfrac{42^2}{71^2} \times 27 = 9{\cdot}444$ lb.

Mean pressure in L.P. $= 10{\cdot}5$ lb.

Mean pressure all referred to L.P. $= 29{\cdot}732$ lb.

When vacuum drops 4 inches, reduction in mean pressure = 2 lb.

∴ Second mean pressure $= 29{\cdot}732 - 2 = 27{\cdot}732$ lb. per square inch.

$$\frac{\text{Mean pressure}}{\text{Revs.}^2} = \text{constant.}$$

$$\therefore \text{ New revs.} = 72\sqrt{\frac{27{\cdot}732}{29{\cdot}732}} \quad 69{\cdot}53 \text{ revs. per min.}$$

Ans.

13. The maximum stress will occur in the rod at the bottom of the thread, and on the up stroke.

Effective area of under side of piston
$= (67^2 - 7^2)\ 0{\cdot}7854$ square inches.

Effective area of upper side of piston
$= (67^2 - 5^2)\ 0{\cdot}7854$ square inches.

Pressure on under side $= 12 + 15 = 27$ lb. per square inch absolute.

Pressure on upper side $= 5$ lb. per square inch absolute.

$$\text{Stress} = \frac{(67^2 - 7^2)\ 0{\cdot}7854 \times 27 - (67^2 - 5^2)\ 0{\cdot}7854 \times 5}{5^2 \times 0{\cdot}7854}$$

lb. per square inch.

$$= \frac{(74 \times 60 \times 27) - (72 \times 62 \times 5)}{25} = 3{,}902 \text{ lb. per sq. inch.}$$

Ans.

14. Assume compound engine develops 100 H.P. for 33 tons consumption. The triple engine would develop 121·67 H.P. for 33 tons.

$$\text{Consum. for 118 H.P.} = \frac{118}{121{\cdot}67} \times 33 = 32 \text{ tons.} \quad \text{Ans.}$$

15. $$\text{H.P.} = \frac{\text{Cu. ft. steam used per min.} \times \text{effec. press. per sq.ft.}}{33{,}000}$$

$$\frac{2{,}700 \times 50 \times 144}{60 \times 33{,}000} = 9{\cdot}82 \text{ horse power.} \quad \text{Ans.}$$

16.

Steam used per minute $= \dfrac{2{,}500 \times 15}{60}$ lb. $= \frac{2500}{4}$ lb.

This steam condenses to the same weight of water, the volume of which is $\dfrac{2{,}500}{4 \times 62{\cdot}5}$ cubic feet.

H.P. of pumps $\dfrac{2{,}500}{4 \times 62{\cdot}5} \times 180 \times 144 \times \frac{1}{33000}$

$= 7{\cdot}85$ horse power. Ans.

17.

Steam used per day $= \dfrac{1{,}500 \times 15 \times 24}{2{,}240} = \dfrac{6{,}750}{28}$

$= 241{\cdot}07$ tons.

Blow out $= \dfrac{\text{Feed density}}{\text{Boiler density}} = \dfrac{0{\cdot}08}{3{\cdot}5} = \frac{8}{350}$ of the feed.

$\therefore$ Weight of steam used $= 1 - \frac{8}{350} = \frac{342}{350}$ of the feed.

Actual blow out per day $= 241{\cdot}07 \times \frac{8}{342} = 5{\cdot}64$ tons.

Fresh water in condenser from steam condensed
$= 241{\cdot}07 - 10 = 231{\cdot}07$ tons.

Let x = tons leaking per day

$(231{\cdot}07 \times 0) + (x \times 1) = (231{\cdot}07 + x)\ 0{\cdot}08$

$x - 0{\cdot}08\,x = 231{\cdot}07 \times 0{\cdot}08$

$$x = \frac{231{\cdot}07 \times 0{\cdot}08}{0{\cdot}92} = 20{\cdot}09 \text{ tons.}$$

Amount available at hotwell
$= 231{\cdot}07 + 20{\cdot}09$ .. $=$ 251·16 tons.

Amount necessary as feed
$= 241{\cdot}07 + 5{\cdot}64$ .. $=$ 246·71 tons.

Amount passing to bilges . 4·45 tons.

Blow out $= 5{\cdot}64$ tons per day. Ans.

Leakage $= 20{\cdot}09$ tons per day. Ans.

To bilges $= 4{\cdot}45$ tons per day. Ans.

18. Water heated per hour $= 2{,}500 \times 14{\cdot}5$ lb.

Heat imparted per lb. $= 168 - 124 = 44$ B.T.U.

$\therefore$ Equivalent horse power of heater

$$\frac{2{,}500 \times 14{\cdot}5 \times 44}{2{,}545} = 626{\cdot}7. \quad \text{Ans.}$$

19. Heat to form 1 lb. of steam before leakage occurs

$= (370 - 120) + 966 - 0{\cdot}7\ (370 - 212)$

$= 1105{\cdot}4$ B.T.U.

To maintain a density of $\dfrac{3{\cdot}4}{32}$ with feed at $\dfrac{0{\cdot}3}{32}$

blow out must be, $\dfrac{0{\cdot}3}{3{\cdot}4} = \frac{3}{34}$ of the feed.

Amount left to form steam $= 1 - \frac{3}{34} = \frac{31}{34}$ of the feed.

If the engines are worked at the same power, for each 1 lb. of steam formed $\frac{34}{31}$ lb. of feed must be supplied and $\frac{3}{31}$ lb. will be blown out.

$\therefore$ Heat to form 1 lb. of steam when blowing

$= \frac{34}{31}\ (370 - 120) + 966 - 0{\cdot}7\ (370 - 212)$

$= 1129{\cdot}6$ B.T.U.

New consumption $= 35 \times \dfrac{1129{\cdot}6}{1105{\cdot}4} = 35{\cdot}76$ tons. Ans.

20. Solids per gallon $= 1{,}870 + 4 + 93 = 1{,}967$ grains.

Final boiler density $= 1{,}967 \times 3$ grains per gallon. This is due to sodium chloride, since the other substances will have precipitated.

Number of times boiler water has changed, and sea water

has been added $= \dfrac{1{,}967 \times 3}{1{,}870} = 3{\cdot}155$ times.

Tons of sea water that have entered boiler $= 40 \times 3{\cdot}155$ Tons.

Scale forming matter in this

$$= \frac{40 \times 3{\cdot}155 \times 2{,}240 \times 97}{10 \times 7{,}000} = 391{\cdot}7 \text{ lb.} \quad \text{Ans.}$$

21. Assume 100 lb. of steam are generated by boiler, and of this x lb. are used by heater, and $(100 - x)$ by the engines.

Heat before passing through heater = Heat afterwards.

$x\{966 - 0{\cdot}7\,(382 - 212) + 382\} + (100 - x)125 = 100 \times 190$

$1{,}229\,x + 12{,}500 - 125\,x = 19{,}000$

$1{,}104\,x = 6{,}500$

$$\frac{6{,}500}{1{,}104} = 5{\cdot}88 \text{ lb.}$$

$\therefore$ Steam used by heater is 5·88 per cent. of the steam generated. Ans.

22. When the engine is on the top centre the opening to exhaust at the bottom end is 2 inches. Since there is + 0·25 inch exhaust lap at this end, the valve must have moved 2 + 0·25 = 2·25 inches from its mid-travel. The outside edge of the valve at the top end must have moved a similar distance.

$\therefore$ Lap + lead at top end = 2·25 inches.

Lap = 2·25 — 0·12 = 2·13 inches. Ans.

23. Heat of formation of wet steam per lb.

$= (381 - 160) + 0{\cdot}94\,\{966 - 0{\cdot}7\,(381 \quad 212)\}$

$= 1017{\cdot}838$ B.T.U.

Extra heat to superheat the steam = Heat to dry the steam + Heat to superheat.

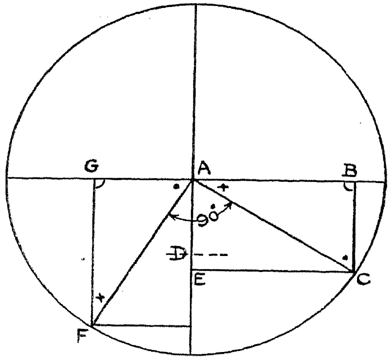
$= 0{\cdot}06\,\{966 - 0{\cdot}7\,(381 - 212)\} + 0{\cdot}48 \times 50$

$= 74{\cdot}862$ B.T.U.

$\therefore$ Extra heat and extra fuel required

$$= \frac{74{\cdot}862}{1017{\cdot}838} \times 100 = 7{\cdot}35 \text{ per cent.} \quad \text{Ans.}$$

24. Draw a circle having a diameter equal to the valve travel. On the vertical centre line set down a distance A D from the centre equal to the steam lap ; and a further distance D E to represent the lead. Since the diagram is not to be used as a means of obtaining a graphical solution, it is not essential that it shall be drawn to scale.



Through E draw the horizontal line E C. C is the centre of the eccentric sheave when the engine is on the dead centre. Complete the right angled triangle A B C.

A C $= \frac{1}{2}$ travel $= 1{\cdot}6$ inches.

B C $=$ Lap $+$ Lead $= 0{\cdot}7 +$ lead.

When the engine has moved through 90°, the eccentric centre has taken up the position marked F. Complete the right angled triangle A F G.

The sum of the angles B A C, C A F, F A G is two right angles, and since C A F is a right angle, therefore B A C and F A G together equal one right angle. But angles B A C and A C B together equal one right angle, since the sum of the angles of any triangle is 180°, and angle A B C is a right angle by construction. Subtract the common angle B A C, and angle F A G equals angle A C B. Also, angle G F A equals angle B A C, and F A $=$ A C. $\therefore$ the triangles A F G and A B C are equal in all respects. Side F G $=$ side A B.

But F G = port opening for the position of eccentric + steam lap = 0·7 + 0·7 = 1·4 ins.; and A B = 1·4 ins.

In triangle A B C, A C = 1·6 inches and A B = 1·4 inches.

$\therefore$ B C = $\sqrt{(1\cdot6)^2 - (1\cdot4)^2} = \sqrt{0\cdot6} = 0\cdot775$

$0\cdot7 + \text{lead} = 0\cdot775 \quad \therefore \text{lead} = 0\cdot075$ inch. Ans.

25. Assume an engine of 1,000 I.H.P., and let x = steam per I.H.P. per hour.

Power required to drive the pumps = 0·008 × 1,000 = 8 H.P.

Steam used by the engines per minute $= \dfrac{1{,}000 \times x}{60}$ lb.

The volume of this steam when condensed to water

$$\frac{1{,}000 \times x}{60 \times 62\cdot5} \text{ cubic feet.}$$

Number of cubic feet pumped per minute × Pressure per square foot pumped against = Work done by the pumps

$$\frac{1{,}000 \times x}{60 \times 62\cdot5} \times 210 \times 144 = 8 \times 33{,}000 \times$$

$$x = \frac{8 \times 33{,}000 \times 42 \times 60 \times 62\cdot5}{1{,}000 \times 210 \times 144 \times 100}$$

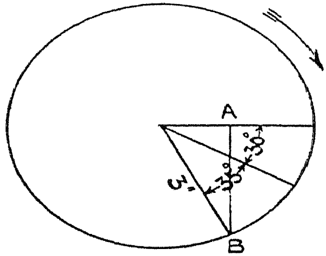
= 13·75 lb. of steam per I.H.P. per hour. Ans.

26. This problem may be worked by a method similar to that explained in the solution to example No. 24.

Half travel of valve $= \sqrt{(1\cdot5)^2 + (3\cdot5)^2} = \sqrt{14\cdot5} = 3\cdot808$ ins.

Valve travel = 2 × 3·808 = 7·616 inches. Ans.

27. A B = displacement of valve from its mid-travel.



$\frac{\text{A B}}{}$ = Sin.

A B = 3 × 0·9063 = 2·7189 inches. Ans.

28. Let W = weight of second quantity, and T be its temperature.

Then W (T — 32) = 20 (120 — 32)

W × T — 32 W = 1,760

Heat lost by second quantity = Heat gained by first quantity.

W (T — 150) = 20 (150 — 120)

W × T — 150 W = 600
W × T — 32 W = 1,760

Subtract, — 118 W = — 1,160

W = $\frac{1160}{118}$ = 9·83 lb.

9·83 (T — 32) = 1,760

T — 32 = 179·04

T = 211·04° F.

Weight is 9·83 lb. and the temperature is 211·04°F. Ans.

9. $\frac{\text{Effective Press. per sq. ft.} \times \text{Cu. ft. steam per minute}}{33{,}000}$ = H.P.

$$\frac{p \times 144 \times 2{,}700}{33{,}000 \times 60} = \frac{33{,}000 \times 60 \times 15}{144 \times 2{,}700}$$

p = 76·39 lb. per sq. inch.

Back pressure = 80 — 76·39 = 3·61 lb. per square inch. gauge. Ans.

30. Let x = the dryness fraction of the steam.
Heat required to form 1 lb. of wet steam
$= (394 - 200) + x\{966 - 0\cdot7\,(394 - 212)\}$ B.T.U.
Heat required to form 1 lb. of dry steam
$= (394 - 200) + 966 - 0\cdot7\,(394 - 212)$ B.T.U.
$109\,\{194 + 838\cdot6\,x\} = 100\,\{194 + 838\cdot6\}$
$21{,}146 + 91407\cdot4\,x = 103{,}260$
$91407\cdot4\,x = 82{,}114$

$$x = \frac{82{,}114}{91407\cdot4} \quad 0\cdot898. \quad \text{Ans.}$$

31. Let W = weight of the water in lb.
Heat before mixing = Heat after mixing.
$$2 \times 0\cdot095\,(350 - 32) + 4\,\{2 \times 0\cdot095\,(350 - 32)\}$$
$$= 2 \times 0\cdot095\,(84 - 32) + W\,(84 - 32)$$
$$5 \times 2 \times 0\cdot095\,(350 - 32)$$
$$= 2 \times 0\cdot095\,(84 - 32) + W\,(84 - 32)$$
$$302\cdot1 = 9\cdot88 + 52\,W$$
$$W = \frac{292\cdot22}{52} = 5\cdot62 \text{ lb.}$$

Heat in copper $\times$ 4 = Heat in water
$$4 \times 2 \times 0\cdot095\,(350 - 32) = 5\cdot62\,(T - 32)$$
$$T = \frac{4 \times 2 \times 0\cdot095 \times 318}{5\cdot62} + 32$$
$$= 43 + 32 = 75^\circ \text{ F.}$$

The weight of the water is 5·62 lb., and its temperature is 75° F. Ans.

32. Heat to form 1 lb. of steam before blowing down was resorted to $= (380 - 140) + 966 - 0\cdot7\,(380 - 212)$
$= 1088\cdot4$ B.T.U.

Heat to form 1 lb. of steam when blowing $= 1088\cdot4 \times \frac{102}{100}$
1110·168 B.T.U.

Extra heat $= 21{\cdot}768$ B.T.U. which is the sensible heat in the water that is blown out for each pound of steam formed.

$$\text{Weight blown out per 1 lb. of steam formed} = \frac{21{\cdot}768}{240}$$

$= 0{\cdot}0907$ lb.

$\therefore$ Weight of feed water is $1{\cdot}0907$ lb. for each 1 lb. of steam formed.

$$\frac{\text{Feed density}}{\text{Boiler density}} = \text{the fraction of the feed that is blown out,}$$

$$\frac{\text{F. D.}}{4} = \frac{0{\cdot}0907}{1{\cdot}0907}$$

$$\text{Feed density} = \frac{4 \times 0{\cdot}0907}{1{\cdot}0907} = 0{\cdot}332 \text{ of the sea density.}$$

Ans.

33. Let x = clearance in inches of the stroke.

$$14{\cdot}7 \times (18 + x)^{\frac{4}{3}} = 114 \times (x)^{\frac{4}{3}}$$

$$\left(\frac{18 + x}{x}\right)^{\frac{4}{3}} = \frac{114}{14{\cdot}7}, \text{ and raising each side to the } \tfrac{3}{4} \text{ power,}$$

$$\frac{18 + x}{x} = \left(\frac{114}{14{\cdot}7}\right)^{\frac{3}{4}} = 4{\cdot}648$$

$$4{\cdot}648\, x = 18 + x$$

$$\therefore x = \frac{18}{3{\cdot}648} = 4{\cdot}935 \text{ inches.} \quad \text{Ans.}$$

34. Weight of salt water fed in $= 1{\cdot}25 \times \frac{1027}{1000} = 1{\cdot}28375$ tons.

Number of times the water has been changed

$$\frac{\text{Increase in density}}{\text{Feed density}} = \frac{70 - 27}{27}$$

$$\frac{\text{Tons fed in}}{\text{Tons in evap.}}$$

$$\therefore \text{Tons in evaporator} = \frac{1{\cdot}28375 \times 27}{43} = 0{\cdot}8063 \text{ ton.}$$

Ans.

35. Isothermal compression, $p \times v =$ constant.

$15 \times 1{,}728 = 315 \times v_2$

$$\frac{15 \times 1{,}728}{315} = 82\cdot28 \text{ cubic inches. Ans.}$$

Adiabatic compression, $p \times v^{1\cdot4} =$ constant.

$15 \times 1{,}728^{1\cdot4} = 315 \times v_2^{1\cdot4}$

$1\cdot4 \log. v_2 = \log. 15 + 1\cdot4 \log. 1{,}728 - \log. 315$

$1\cdot4 \log. v_2 = 3\cdot2103$

$\log. v_2 = 2\cdot2931$

$v_2 = 196\cdot4$ cubic inches. Ans.

36. Combustion of carbon, $C + O_2 = CO_2$

$12 + 32 = 44$

or 1 lb. of C requires $\frac{32}{12} = 2\cdot666$ lb. of oxygen.

Combustion of Hydrogen, $H_2 + O = H_2O$

$2 + 16 = 18$

or 1 lb. of H requires $\frac{16}{2} = 8$ lb. of oxygen.

Available hydrogen $= 0\cdot13 \quad \frac{0\cdot02}{} = 0\cdot1275$ lb. per 1 lb. of oil.

Oxygen to consume the carbon $= 0\cdot85 \times 2\cdot666 = 2\cdot2661$ lb.

Oxygen to consume the hydrogen $= 0\cdot1275 \times 8 = 1\cdot02$ lb.

Oxygen required $= 3\cdot2861$ lb.

Air required $= 3\cdot2861 \times \frac{100}{23} = 14\cdot29$ lb. Ans.

37. Latent heat of water $= 143$ B.T.U. per lb.

$= 143 \times \frac{5}{9} = 79\cdot44$ Centigrade H.U. per lb.

$$\text{Resulting temperature} = \frac{\text{Total heat units}}{\text{Total weight}}$$

R.T. =

$$\frac{(10 \times 2\cdot2 \times 45) + (6 \times 2\cdot2 \times 65) - (4 \times 2\cdot2 \times 79\cdot44) - (4 \times 2\cdot2 \times 25 \times 0\cdot5) + (4 \times 2\cdot2 \times 0)}{(10 + 6 + 4) \times 2\cdot2}$$

Each of the above quantities is multiplied by 2·2 to bring kilos. to pounds, but since it is common to each quantity it cancels out.

$$\text{R.T.} = \frac{450 + 390 - 317{\cdot}76 - 50 - 0}{20} = \frac{472{\cdot}24}{20}$$

$= 23{\cdot}612°$ C.

$23{\cdot}612°$ C. $= (23{\cdot}612 \times \frac{9}{5}) + 32 = 74{\cdot}5°$ F Ans.

This problem may also be solved by converting all the temperatures to Fahrenheit, and all the kilograms to pounds.

$-25°$ C. $= (-25 \times \frac{9}{5}) + 32 = -45 + 32 = -13°$ F.

$45°$ C. $= (45 \times \frac{9}{5}) + 32 = 113°$ F.

$65°$ C. $= (65 \times \frac{9}{5}) + 32 = 149°$ F.

R.T. =

$$\frac{(10 \times 2{\cdot}2 \times 113) + (6 \times 2{\cdot}2 \times 149) - (4 \times 2{\cdot}2 \times 143) - (4 \times 2{\cdot}2 \times 45 \times 0{\cdot}5) + (4 \times 2{\cdot}2 \times 32)}{(10 + 6 + 4) \times 2{\cdot}2}$$

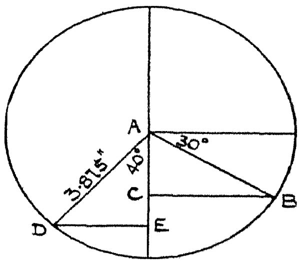
Note that in raising the temperature of the ice from $-13°$F. to the melting point of $+32°$F., it has to be raised through 45° F.

$$\text{R.T.} = \frac{1{,}130 + 894 - 572 - 90 + 128}{20}$$

$= 74{\cdot}5°$ F., as before.

38. Valve travel 7·75 inches.

B is the position of the eccentric centre when the engine is on the dead centre. D is the position of the eccentric centre when the engine is 100° past the centre.



$$\text{A C} = \text{lap} + \text{lead} = \frac{\text{Travel}}{2} \times \text{Sin. } 30°$$

$= 1{\cdot}9375$ inches.

A E = displacement of the valve from mid-travel when eccentric centre is at D, and is equal to the port opening + lap = 1·25 + lap.

$$\text{But A E} = \frac{\text{Travel}}{} \times \text{Cos. } 40^\circ = 3{\cdot}875 \times 0{\cdot}766 = 2{\cdot}968 \text{ ins.}$$

$\therefore\ 1{\cdot}25 + \text{lap} = 2{\cdot}968$

$\text{Lap} = 2{\cdot}968 - 1{\cdot}25 = 1{\cdot}718$ inches.

and $\text{Lead} = 1{\cdot}9375 - \text{lap}$

$= 1{\cdot}9375 - 1{\cdot}718 = 0{\cdot}2195$ inch. Ans.

39.

$$13^3 \quad \frac{24 \times 195 \times D^2}{1{,}110\left(2\right.}$$

$$13^3 \times 1{,}110\left(2 + \right. \qquad 24 \times 195 \times D^2$$

$d^2 = 26^2 = 676$

$$13^3 \times 1{,}110\left(\frac{1{,}352 + D^2}{676}\right) = 24 \times 195 \times D^2$$

$$1{,}352 + D^2 = \frac{24 \times 195 \times D^2 \times 676}{13^3 \times 1{,}110}$$

$1{,}352 + D^2 = 1{\cdot}297\ D^2$

$$0{\cdot}297\ D^2 = 1{,}352, \text{ and } D^2 = \frac{1{,}352}{0{\cdot}297} \qquad \frac{26^2 \times 2}{0{\cdot}297}$$

$$\frac{LP^2}{HP^2} \qquad D^2 \qquad \frac{26^2 \times 2}{0{\cdot}297} \qquad \frac{1}{26^2}$$

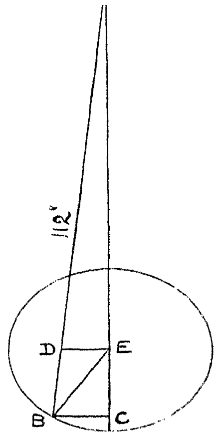
$$\text{Number of expansions} \quad \frac{6{\cdot}75}{0{\cdot}6} = 11{\cdot}25. \quad \text{Ans.}$$

Note that it is not essential to find the actual diameter of the L.P. cylinder.

40. Twisting moment $=$ Piston load $\times$ D E

B C $= 26$ Sin. $45° = 26 \times 0{\cdot}707$
$= 18{\cdot}382$ inches.

C E $=$ B C $= 18{\cdot}382$ inches.



$= 110{\cdot}5$ inches.

A E $= 110{\cdot}5 - 18{\cdot}382 = 92{\cdot}118$ inches

In the similar triangles A D E and A B C

A E : D E :: A C : B C

A E $\times$ B C

A C

$$= \frac{92{\cdot}118 \times 18{\cdot}382}{110{\cdot}5} = 15{\cdot}33 \text{ inches.}$$

T.M. $= 47 \times 2{,}240 \times 15{\cdot}33$ inch lb.

Resisting moment $\times$ stress.
5·1

and R.M. $=$ T.M.

$$\therefore \text{ Stress } = \frac{47 \times 2{,}240 \times 15{\cdot}33 \times 5{\cdot}1}{13 \times 13 \times 13} = 3{,}747 \text{ lb. per sq. inch. Ans.}$$

41. Law is of the form $p\ v^n =$ constant.
Take volume swept out by the piston as 100
Then first volume $= 107{\cdot}5$, and second volume $= 7{\cdot}5$
$15 \times (107{\cdot}5)^n = 515 \times (7{\cdot}5)^n$
Log. $15 + n$ log. $107{\cdot}5 =$ log. $515 + n$ log. $7{\cdot}5$
n (log. $107{\cdot}5$ — log. $7{\cdot}5$) $=$ log. 515 — log. 15

$$n = \frac{\text{log. } 515 - \text{log. } 15}{\text{log. } 107{\cdot}5 - \text{log. } 7{\cdot}5} \qquad \frac{2{\cdot}7118 - 1{\cdot}1761}{2{\cdot}0315 - 0{\cdot}8751}$$

$$\frac{1{\cdot}5357}{1{\cdot}1564} = 1{\cdot}328$$

The law is $p\ v^{1{\cdot}328} =$ constant. Ans.

42. The evaporation of part of the water is due to the release of sensible heat when the pressure and temperature fall.

Let the vessel hold W lb. of water at first.

Sensible heat given up $= W(350 - 212) = (138\,W)$ B.T.U.

Weight of water evaporated

$= \dfrac{138\ W}{966}$ lb., 966 being the latent heat per 1 lb. at 212°F.

or, alternatively :—Let w lb. of water be evaporated.

Total heat before release of pressure = Total heat after.

$$W \times 350 = (W - w)\,212 + w\,(966 + 212)$$

$$350\ W = 212\ W - 212\ w + 966\ w + 212\ w$$

$$350\ W - 212\ W = 966\ w$$

$$w = \frac{138\ W}{966}\ \text{lb.}$$

The weight of water now left in the vessel is $W - w$ lb.

Let this be raised in temperature to T° F.

Sensible heat given up when pressure is released

$= (W - w)\ (T - 212)$ B.T.U.

Weight of water evaporated

$\dfrac{(W - w)\ (T - 212)}{966}$ lb., and this must be equal to w lb.

$$- w)\ (T - 212) = 966\ w$$

Substitute $w = \dfrac{138\ W}{966}$

$$\left.\frac{138\ W}{966}\right)(T - 212) = 966 \times \frac{138\ W}{966}$$

$$\left.\frac{966\ W - 138\ W}{966}\right)(T - 212) = 138\ W$$

$$W\left(\tfrac{828}{966}\right)(T - 212) = 138\ W, \qquad W \text{ cancels.}$$

$$T \quad 212 = \frac{138 \times 966}{828} = 161$$

$$T = 161 + 212 = 373°F. \quad \text{Ans.}$$

In order that the water in the vessel shall attain a temperature of 350°F., it must be subjected to a pressure of about 135 lb. per sq. inch absolute. It is assumed that this pressure is due to air contained in the vessel, and that no steam is formed until the escape valve is opened. This reduces the pressure to atmospheric pressure and the temperature falls to 212°F., causing some water to evaporate owing to release of sensible heat. To attain the temperature of 373°F., the water must be subjected to a pressure of about 180 lb. per sq. inch absolute.

43. Displacement of valve from mid-travel when the crank is 90° past the centre = lap + port opening.

$$\overline{\quad - (1{\cdot}25)^2} = \sqrt{7{\cdot}4375} = 2{\cdot}727 \text{ inches.}$$

$$\text{Port opening} = 2{\cdot}727 - 1{\cdot}125 = 1{\cdot}602 \text{ inches.} \quad \text{Ans.}$$

44. Load on piston $= (50{\cdot}8)^2 \times 0{\cdot}7854 \times 6{\cdot}35 \times 2{\cdot}2$ lb.

$$\text{Stroke} = \frac{974 \times 0{\cdot}03937}{12} \text{ feet.}$$

I.H.P. =

$$\frac{(50{\cdot}8)^2 \times 0{\cdot}7854 \times 6{\cdot}35 \times 2{\cdot}2 \times 974 \times 0{\cdot}03937 \times 85 \times 8}{33{,}000 \times 12 \times 2}$$

$= 932{\cdot}6$ Ans.

Fuel per I.H.P. per hour.

$$\frac{3{\cdot}6 \times 2{,}240}{24 \times 932{\cdot}6} \quad 0{\cdot}36 \text{ lb.} \quad \text{Ans.}$$

45. Steam used per day $= 79 \times 8\cdot5 = 671\cdot5$ tons.

Total feed per day $= 671\cdot5 + 40 = 711\cdot5$ tons.

Feed density $\dfrac{40}{711\cdot5}$

$$\text{Feed density} = \frac{160}{711\cdot5} = 0\cdot225 \text{ of sea density.} \quad \text{Ans.}$$

46. Blow out $= \dfrac{0\cdot55}{3\cdot7} \quad \frac{55}{370}$ of feed.

$\therefore$ Water evaporated $= \dfrac{370 \qquad 55}{370}$ of feed

$\therefore$ Feed $= \frac{370}{315} \times$ water evaporated.

$\frac{370}{315}\,(T - 133) + 966 - 0\cdot7\,(T - 212)$

$= 1\cdot043\,\{T - 133 + 966 - 0\cdot7\,(T - 212)\}$

$\therefore 1\cdot174\,(T - 133) - 1\cdot043\,(T - 133)$

$= (1\cdot043 - 1)\,\{(966 - 0\cdot7\,(T - 212)\}$

$\therefore 0\cdot131\,(T - 133) = 0\cdot043\,\{966 - 0\cdot7\,(T - 212)\}$

$\therefore 0\cdot131\,T - 17\cdot423 = 47\cdot919 - 0\cdot0301\,T$

$\therefore 0\cdot1611\,T = 65\cdot342$

$T = 405°$ F. Ans.

47. $15\,(22 + 4\cdot3)^n = 190 \times 4\cdot3^n$

$\therefore \log. 15 + n \log. 26\cdot3 = \log. 190 + n \log. 4\cdot3$

$\therefore n\,(\log. 26\cdot3 - \log. 4\cdot3) = \log. 190 - \log. 15$

$\therefore 0\cdot7865\,n = 1\cdot1027$

$\therefore n = 1\cdot402.$ Ans.

48. $3\cdot 11$ C°. $= 3\cdot 11 \times \frac{9}{5}$ F°.

Calorific value $\times$ weight of fuel $=$ Equivalent wt. of water $\times$ Rise in temp.

Cal. Value $\times\ 0\cdot 81 = (2{,}270 + 412) \times 3\cdot 11 \times \frac{9}{5}$

$\therefore$ Calorific value $= 18580$ B.T.U. per lb. Ans.

Calorific value $= \dfrac{2682 \times 3\cdot 11}{0\cdot 81} = 10310$ gram calories per gram. Ans.

49. Let d = dia. for 3 valves, and $d + \frac{3}{4}$ = dia. for 2 valves.

$2\,(d + \frac{3}{4})^2 = 3\,d^2$

$\therefore 2\,d^2 + 3\,d + \frac{9}{8} = 3\,d^2$

$\therefore d^2 - 3\,d = \frac{9}{8}$

$\therefore d - 1\cdot 5 = \pm\sqrt{\frac{9}{8} + \frac{9}{4}} = \pm\sqrt{\frac{27}{8}}$

$= \pm\ 1\cdot 84.$

$\therefore d = 1\cdot 5 \pm 1\cdot 84 = 3\cdot 34$ inches.

Total safety valve area $= (3\cdot 34)^2 \times 0\cdot 7854 \times 3 = 26\cdot 3$ sq. inches

Total heating surface $= 95 \times 36 = 3{,}420$ sq. feet.

Absolute boiler pressure $= \dfrac{1\cdot 5 \times 3{,}420}{26\cdot 3} = 195$ lb. sq. in.

$= 180$ lb. sq. inch gauge. Ans.

50. $15 \times v_1^{1\cdot 35} = 150 \times v_2^{1\cdot 3}$

$1\cdot 35$

$= \frac{150}{15} = 10$

$\therefore \dfrac{\quad}{v_2} = 10^{\overline{1\cdot 35}} = 5\cdot 504$

$21 + C$

$= 5\cdot 504$

$\therefore 4\cdot 504\ C = 21$

$\therefore C = 4\cdot 66$ inches. Ans.

51. Oil used per I.H.P. per hour $= 0 \cdot 43 \times 0 \cdot 8 = 0 \cdot 344$ lb.

$$\text{Thermal efficiency} = \frac{2,545}{0 \cdot 344 \times 20,000} \times 100 = 37 \text{ per cent.}$$

$100 - 37 = 63$ per cent. loss, and this is due to heat in exhaust gases and cooling water.

$\therefore$ Loss in water $= 63 - 30 = 33$ per cent. Ans.

52. Thermal efficiency estimated on I.H.P. $= 100 - 29 - 27 = 44$ per cent.

23 per cent. of this is lost in friction, $\therefore$ 77 per cent. is useful work.

$\therefore$ Useful work $= 0 \cdot 77 \times 44 = 33 \cdot 88$ per cent. of heat in fuel. Ans.

53. $383 \cdot 5°$ F. $= 195 \cdot 3°$ C. $212°$ F. $= 100°$ C.
Height of water varies as the volume.

$$\therefore \text{Height} = 84 \times \frac{1 + 0 \cdot 0000119 \times 191 \cdot 3^{1 \cdot 8}}{1 + 0 \cdot 0000119 \times 96^{1 \cdot 8}}$$

$$= 84 \times \frac{1 + 0 \cdot 1522}{1 + 0 \cdot 0441}$$

$= 92 \cdot 69$ inches.

$\therefore$ Difference $= 92 \cdot 69 - 84 = 8 \cdot 69$ inches. Ans.

54. Clearance volume expressed in c.m. of stroke.

$$\frac{6,000}{26 \cdot 5^2 \times 0 \cdot 7854} = 10 \cdot 88 \text{ c.m.}$$

$$15 \times (53 + 10 \cdot 88)^{1 \cdot 38} = p_2 \times 10 \cdot 88^{1 \cdot 38}$$

$$\therefore p_2 = 15 \times \left(\frac{63 \cdot 88}{10 \cdot 88}\right)^{1 \cdot 38} = 172 \cdot 5 \text{ lb. sq. inch absolute.}$$

$= 157 \cdot 5$ lb. sq. inch gauge. Ans.

55. Mechanical efficiency $= \frac{4550}{5660} = 0{\cdot}804$. Ans.

Thermal efficiency $= 1 - 0{\cdot}3 - 0{\cdot}32 = 0{\cdot}38$.

$\therefore$ Overall efficiency $= 0{\cdot}804 \times 0{\cdot}38 = 0{\cdot}3055$. Ans.

Oil per I.H.P. per hour $= \dfrac{21{\cdot}5 \times 2{,}240}{24 \times 5{,}660} = 0{\cdot}3545$.

Thermal efficiency $= \dfrac{2{,}545}{0{\cdot}3545 \times \text{Calorific value}}$

$\therefore$ Calorific value $= \dfrac{2{,}545}{0{\cdot}3545 \times 0{\cdot}38}$

$= 18{,}900$ B.T.U. per lb. Ans.

56. Steam used per day $= \dfrac{1{,}500 \times 14 \times 24}{2{,}240}$ 225 tons.

Fresh water returned to hotwell $= 0{\cdot}98 \times 225 = 220{\cdot}5$ tons

Let x = tons leaking per day.

$220{\cdot}5 \times 0 + x \times 1 = (220{\cdot}5 + x)\ 0{\cdot}15$

$0{\cdot}85\,x = 220{\cdot}5 \times 0{\cdot}15$

$x = 39$ tons.

Total water in hotwell $= 220{\cdot}5 + 39 = 259{\cdot}5$ tons.

Water necessary as feed $= 225$ tons.

$\therefore$ Overflow to bilges $= 259{\cdot}5 - 225 = 34{\cdot}5$ tons. Ans.

57. Lap + lead $= 3\frac{9}{16}$ Sine $44° 44' = 2{\cdot}508$ inches.

$\therefore$ lap $= 2{\cdot}508 - 0{\cdot}4375 = 2{\cdot}0705$ inches. Ans.

Max. opening to steam = throw — lap

$= 3{\cdot}5625 - 2{\cdot}0705 = 1{\cdot}492$ inches.

Sine of angle of advance due to lap $= \dfrac{\quad}{3{\cdot}5625} = 0{\cdot}5812$

= Sine $35° 32'$.

$\therefore$ Steaming angle $= 180° - 2\ (35°\ 32') = 108°\ 56'$.

Angle of preadmission $= 44° 44' - 35° 32' = 9° 12'$

At cut off, angle of crank from dead centre
$= 180^\circ\ 56' - 9^\circ\ 12' = 99^\circ\ 44'.$

$\therefore$ Cut off occurs at $\dfrac{1 + \text{Sine } 9^\circ\ 44'}{}$ 0·5845 stroke.
Ans.

58. Vol. at 55° I $\dfrac{+\ 460}{500} + \dfrac{500}{55 + 460}\Bigg]$
$= 1{\cdot}00045.$

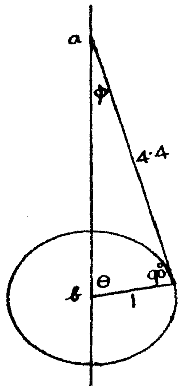
Vol. at 375° F. $= \frac{1}{2}\left[\dfrac{375 + 460}{500} + \dfrac{500}{375}\right.$
$= 1{\cdot}1344.$

Increase per unit vol. $= 1{\cdot}1344 - 1{\cdot}00045 = 0{\cdot}13395.$

Actual increase vol. $= \dfrac{0{\cdot}13395 \times 9 \times 2{,}240}{62{\cdot}5}$

Difference in water level $= \dfrac{0{\cdot}13395 \times 9 \times 2{,}240}{62{\cdot}5\ (10^2 - 3\frac{1}{4}^2)\ 0{\cdot}7854}$
$= 0{\cdot}615$ foot $= 7{\cdot}38$ inches. Ans.

59.



Assuming down stroke.

Tan $\theta = 4{\cdot}4 \quad \therefore \ \theta = 77^\circ\ 12'$

$a\ b = \dfrac{4{\cdot}4}{\text{Sin. } 77^\circ\ 12'} = 4{\cdot}512$

Distance piston is from top

$\dfrac{4{\cdot}4 + 1 - 4{\cdot}512}{2}$

$= 0{\cdot}445$ of stroke.

$= 2{\cdot}0025$ feet from top. Ans.

Twisting moment $= \dfrac{\text{Piston load}}{\text{Cos. } \phi} \times 2{\cdot}25$

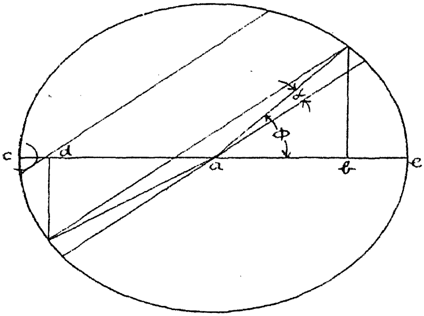
Piston load $= \dfrac{800{,}000 \times \text{Cos. } 12^\circ\ 48'}{2{\cdot}25}$

$= 346{,}700$ lb. $= 154{\cdot}8$ tons. Ans.

60. Valve travel $= (1\frac{7}{16} + 2)\ 2 = 6\frac{7}{8}''$. Ans.

Sine of angle of advance $= \dfrac{1\frac{9}{16}}{3\frac{7}{16}} = 0{\cdot}4545$

$\therefore\ \theta = 27°\ 2'$.



Sine $\propto\ = \dfrac{\frac{}{16}}{3\frac{7}{16}} = 0{\cdot}0545$. $\qquad \propto\ = 3°\ 8'$.

$\phi = 27°\ 2' + 3°\ 8' = 30°\ 10'$.

$a\ b = 3\frac{7}{16}$ Cos. $30°\ 10' = 2{\cdot}972''$

$c\ b = 3{\cdot}4375 + 2{\cdot}972 = 6{\cdot}4095''$.

Fraction of stroke at release $= \dfrac{6{\cdot}4095}{6{\cdot}875} = 0{\cdot}9323$.
Ans.

$27°\ 2' - 3°\ 8' = 23°\ 54'$

$d\ a = 3\frac{7}{16}$ Cos. $23°\ 54' = 3{\cdot}143$.

$e\ c = 3{\cdot}4375 + 3{\cdot}143 = 6{\cdot}5805$.

Fraction of stroke at compression $= \dfrac{6{\cdot}5805}{6{\cdot}875} = 0{\cdot}957$.
Ans.

61. Volume of 1 lb. of air at 32°F. and 14·7 lb. per sq. in.

$$= \frac{1}{0{\cdot}0807} = 12{\cdot}39 \text{ cu}$$

$\therefore$ 5 lb. of air at 70°F. and 14·7 lb. per sq. inch

$= 5 \times 12{\cdot}39 \times \dfrac{530}{492} = 66{\cdot}75$ cu. feet.

$1 \times (66{\cdot}75)^{1{\cdot}4} = 25 \times v_2^{\,1{\cdot}4}$

$\therefore\ v_2 = 6{\cdot}698$ cu. feet. Ans.

$\dfrac{p\ v}{} = \text{constant}\ \therefore\ \dfrac{1 \times 66{\cdot}75}{530} = \dfrac{25 \times 6{\cdot}698}{T_2}$

$T_2 = 1330°$F. abs. $= 870°$F.

62. Diam. of shaft $= \dfrac{345}{25{\cdot}4 \times 12}$ feet.

H.P. expended on friction per min.

$$= \frac{29{\cdot}4 \times 2{,}240 \times 8 \times 0{\cdot}014 \times 345 \times \pi \times 85}{25{\cdot}4 \times 12 \times 33{,}000} \qquad 67{\cdot}53. \text{ Ans.}$$

$$\text{Centigrade heat units} = \frac{67{\cdot}53 \times 42{\cdot}42 \times 5}{9} \qquad 1{,}592 \text{ Ans.}$$

63. Latent Heat is the heat given to, or taken from a substance which changes its physical state, while its temperature remains unchanged. It is upon the pressure that the temperature of evaporation depends, and the latent heat is different for all different temperatures. The higher the pressure, the higher the temperature of evaporation, and the lower the latent heat value.

Let x = dryness fraction.

Total Heat required per lb. to produce wet steam
$= (328 - 230) + x\,\{966 - 0{\cdot}7\,(328 - 212)\}$
$= 98 + 884{\cdot}8\,x$ B.T.U.

Total Heat required per lb. to produce dry steam
$= 98 + 884{\cdot}8$ B.T.U.

$(98 + 884{\cdot}8\,x) \times \frac{105}{100} = 98 + 884{\cdot}8.$
$102{\cdot}9 + 929{\cdot}04\,x = 982{\cdot}8.$

$$x = \frac{982{\cdot}8 - 102{\cdot}9}{929{\cdot}04} = 0{\cdot}9472. \text{ Ans.}$$

64. Fuel per hour $= 3{,}600 \times 0{\cdot}37$
Gases per hour $= 3{,}600 \times 0{\cdot}37 \times 19$
Note, 1 lb. of fuel + 18 lb. of air = 19 lb. of gases.

Available heat
$= 3{,}600 \times 0{\cdot}37 \times 19 \times (620 - 420) \times 0{\cdot}203$
$= 1{,}028{,}000$ B.T.U. per hour.

Heat utilised in forming steam $= 1{,}028{,}000 \times 0{\cdot}8 =$ 822,400 B.T.U.

Heat per lb. of steam =
$390 - 120 + 0{\cdot}96\,[966 - 0{\cdot}7\,(390 - 212)]$
$= 1{,}077{\cdot}7$ B.T.U.

$$\therefore \text{Steam formed} = \frac{822{,}400}{1077{\cdot}7} = 764 \text{ lb. per hour. Ans.}$$

65. Losses are 5 per cent. of input to generator, $\therefore$ output is 95 per cent. of input.

$\therefore$ Losses are $\frac{5}{95}$ of output.

$\therefore$ loss of power $= \frac{5}{95} \times 7{,}500$ kilowatts.

1 kilowatt hour $= 3{,}410$ B.T.U.

$\therefore$ Loss $= \frac{5}{95} \times 7{,}500 \times \frac{3410}{60}$ B.T.U. per minute.

Weight of air under given conditions

$$= 35{,}000 \times 0{\cdot}0807 \times \frac{492}{530} \times \frac{34{\cdot}5}{34} \text{ lb.}$$

$$\therefore \text{ Rise temp.} \times 0{\cdot}24 \times 35{,}000 \times 0{\cdot}0807 \times \frac{492}{530} \times \frac{34{\cdot}5}{34}$$

$$= \frac{5}{95} \times 7{,}500 \times \frac{3410}{60}$$

$\therefore$ Rise temp. $= 35{\cdot}16$ F°.

Temperature of windings $= 55°$ C. $= 131°$ F.

$\therefore$ Difference in temperature of windings and final temperature of air :—

$= 131 - 105{\cdot}16 = 25{\cdot}84$ F°. Ans.

66. Weight of fuel per working stroke

$$\frac{3{,}600}{6 \times 60 \times 85} \times 0{\cdot}35.$$

$= 0{\cdot}04119$ lb.

Weight of air taken in per stroke

$= 0{\cdot}0807 \times 10 \times \frac{17}{15} \times \frac{492}{545}$

$= 0{\cdot}8255$ lb. Ans. (*b*).

See Chapter XXIV.

Weight of air required per lb. of fuel

$= 11{\cdot}6 \times 0{\cdot}84 + 34{\cdot}8 \times 0{\cdot}15$

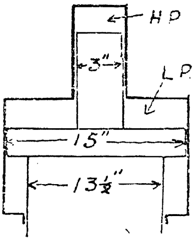
$= 14{\cdot}96$ lb.

Air required per working stroke $= 14{\cdot}96 \times 0{\cdot}04119$

$= 0{\cdot}6161$ lb. Ans. (*a*).

Excess air $= 0{\cdot}8255 - 0{\cdot}6161 = 0{\cdot}2094$ lb. Ans. (*c*).

67.



Vol. of each bottle

$= 0{\cdot}5236 \times 1^3 + 0{\cdot}7854 \times 1^2 \times 7.$

$= 6{\cdot}0214$ cu. feet.

Vol. of 3 bottles and pipes

$= 3 \times 6{\cdot}0214 \times 1{\cdot}1$

$= 19{\cdot}87$ cubic feet.

Vol. of free air to be pumped $= 19{\cdot}87 \times \frac{1215}{15} - 19{\cdot}87$
$= 1{,}590$ cu. ft.

$$\text{Vol. of free air per stroke} = \frac{(15^2 - 3^2)\ 0{\cdot}7854 \times 0{\cdot}91}{144}$$

$= 1{\cdot}072$ cu. feet.

$$\therefore \text{Time} = \frac{1{,}590}{1{\cdot}072 \times 150} = 9{\cdot}883 \text{ minutes. Ans.}$$

68. In the previous question (No. 67), the cylinders of the air compressor are 3 inches, $13\frac{1}{2}$ inches, and 15 inches diameter, and with the given size of middle pressure cylinder, the compressor must be arranged tandem fashion, as indicated by the diagram.

In question 68 the middle pressure cylinder is $6\frac{1}{2}$ inches, and the compressor cannot have the tandem arrangement. Only the size of the L. P. cylinder is required in solving the problem.

Vol. of free air taken in by L. P. per stroke

$$= \frac{15^2 \times 0{\cdot}7854 \times 15 \times 0{\cdot}9}{1{,}728} = 1{\cdot}381 \text{ cu. feet.}$$

Half of this is delivered to the air reservoirs $= 0{\cdot}6905$ cu. ft.

Now $p \times v =$ constant, since temperature is constant.

$250 \times \text{vol.} = 15 \times 12{\cdot}4$

$$\therefore \text{vol. of 1 lb. of air at 250 lb. per sq. inch} = \frac{15 \times 12{\cdot}4}{250}$$

and weight of 1,000 cu. ft. $= \dfrac{1{,}000 \times 250}{15 \times 12 \cdot 4}$ lb.

By similar reasoning, the weight of 1,000 cu. ft. at 350 lb. per sq. inch $= \dfrac{1{,}000 \times 350}{15 \times 12 \cdot 4}$ lb.

$\therefore$ Weight of air to supply

$$\frac{1{,}000 \times 350}{15 \times 12 \cdot 4} - \frac{1{,}000 \times 250}{15 \times 12 \cdot 4}$$

$$= \frac{1{,}000 \times 100}{15 \times 12 \cdot 4}$$

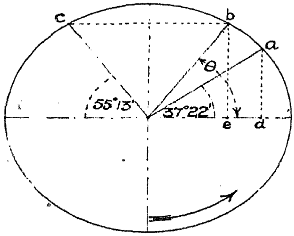
Weight supplied per stroke $= \dfrac{0 \cdot 6905}{12 \cdot 4}$ lb.

$\therefore$ Number of strokes $= \dfrac{1{,}000 \times 100}{15 \times 12 \cdot 4} \times \dfrac{12 \cdot 4}{0 \cdot 6905}$

$$= \frac{1{,}000 \times 100}{15 \times 0 \cdot 6905}$$

Time $= \dfrac{1{,}000 \times 100}{15 \times 0 \cdot 6905 \times 180} = 53 \cdot 66$ minutes. Ans.

69.



When the engine is on the centre the displacement of the valve from its mid-travel = lap + lead.

This is represented by $a\,d$

$a\,d$ = lap + lead = 3·5 sin. 37° 22′

$\therefore$ lap = 3·5 sin 37° 22′ − 0·25 = 1·8745 inches.

When the port is 1 inch open to steam the displacement of the valve from its mid-travel = 1 + 1·8745 = 2·8745 ins. = $b\,e$

The eccentric centre may be at b or c, in other words, there are two positions of the eccentric, and of the crank, when the port is 1 inch open to steam.

$$\text{Sin } \theta = \frac{2{\cdot}8745}{3{\cdot}5} \qquad \theta = 55^\circ\ 13'$$

$55^\circ\ 13' - 37^\circ\ 22' = 17^\circ\ 51'$

$180^\circ - 55^\circ\ 13' - 37^\circ\ 22' = 87^\circ\ 25'$

The steam port is 1 inch open when the crank has moved through $17^\circ\ 51'$, or $87^\circ\ 25'$ from the centre. Ans.

70. Latent heat at 395° F. $= 966 - 0{\cdot}7\ (395 - 212)$
$= 837{\cdot}9$ B.T.U. per lb.

Latent heat at 133° F. $= 966 - 0{\cdot}7\ (133 - 212)$
$= 1021{\cdot}3$ B.T.U. per lb.

Latent heat at $101{\cdot}7^\circ$ F. $= 966 - 0{\cdot}7\ (101{\cdot}7 - 212)$
$= 1043{\cdot}2$ B.T.U. per lb.

1st case—

Total heat above 32° F. in steam at 395° F. when $0{\cdot}94$ dry
$= (395 - 32) + 0{\cdot}94 \times 837{\cdot}9 = 1150{\cdot}63$ B.T.U. per lb.
Total heat above 32° F. in steam at 133° F. and $0{\cdot}755$ dry
$= (133 - 32) + 0{\cdot}755 \times 1021{\cdot}3 = 872{\cdot}08$ B.T.U. per lb.
$\therefore$ heat drop $= 1150{\cdot}63 - 872{\cdot}08 = 278{\cdot}55$ B.T.U. per lb.
and this represents the work done per 1 lb. of steam.

2nd case—

Total heat above 32° F. in steam at 235 lb. per sq. inch and superheated 200 F°.

$= (395 - 32) + 837{\cdot}9 + 200 \times 0{\cdot}56 = 1312{\cdot}9$ B.T.U.
Total heat above 32° F. in steam at $101{\cdot}7^\circ$ F. and $0{\cdot}828$ dry
$= (101{\cdot}7 - 32) + 0{\cdot}828 \times 1043{\cdot}2 = 933{\cdot}47$ B.T.U. per lb.
$\therefore$ heat drop $= 1{,}312{\cdot}9 - 933{\cdot}47 = 379{\cdot}43$ B.T.U. per lb., and this represents the work done per 1 lb. of steam.

Increase $= 379{\cdot}43 - 278{\cdot}55 = 100{\cdot}88$ B.T.U.

$$\text{Increase per cent.} = \frac{100{\cdot}88}{278{\cdot}55} \times 100 = 36{\cdot}2\%. \quad \text{Ans.}$$

71. Latent heat of water = 143 B.T.U. per lb. = $143 \times \frac{5}{9}$ = $79\frac{4}{9}$ C.H.U. per lb.

All the weights are given in kilograms, but it can make no difference to the final result if the weights are considered as pounds.

$$\text{Final temp.} = \frac{\text{Total heat above } 0^\circ \text{ C.}}{\text{Total weight}}$$

$$\text{Final Temp.} = \frac{(91 \cdot 5 \times 54) + (97 \cdot 5 \times 0) - (31 \cdot 5 \times 25 \times 0 \cdot 5) - (31 \cdot 5 \times 79\frac{4}{9})}{91 \cdot 5 + 97 \cdot 5 + 31 \cdot 5}$$

$$\text{Final temp.} = \frac{4{,}941 + 0 - 393 \cdot 75 - 2502 \cdot 5 + 0}{220 \cdot 5}$$

$$\frac{2044 \cdot 75}{220 \cdot 5} = 9 \cdot 27^\circ \text{ C.} \quad \text{Ans.}$$

72. Assume the atmospheric pressure is 15 lb. per sq. in. abs. Final pressure = 450 + 15 = 465 lb. per sq. in. abs.

Isothermal compression—

$p\ v$ = constant

$15 \times 3 \times 1{,}728 = 465 \times v_2$

$$v_2 = \frac{15 \times 3 \times 1{,}728}{465} = 167 \cdot 23 \text{ cu. ins.} \quad \text{Ans.}$$

Adiabatic compression—

$p\ v^{1 \cdot 4}$ = constant

$15 \times (3 \times 1{,}728)^{1 \cdot 4} = 465 \times v_2^{1 \cdot 4}$

$(3 \times 1{,}728)^{1 \cdot 4} = 31 \times v_2^{1 \cdot 4}$

$1 \cdot 4 \log. 5{,}184 = \log. 31 + 1 \cdot 4 \log. v_2$

$\therefore\ 1 \cdot 4 \log. v_2 = 1 \cdot 4 \log. 5{,}184 - \log. 31$

$$\log. v_2 = \log. 5{,}184 - \frac{\log. 31}{1 \cdot 4}$$

$= 3 \cdot 7146 - 1 \cdot 0653 = 2 \cdot 6493$

v_2 = 446 cu. inches. Ans.

73.* See pages 350 and 351.

Heat to form 1 lb. of steam

$= (380 - 145) + 966 - 0{\cdot}7\,(380 - 212) + 0{\cdot}54\,(720 - 380)$

$= 235 + 966 - 117{\cdot}6 + 183{\cdot}6 = 1{,}267$ B.T.U.

$$\text{Boiler efficiency} = \frac{\text{Heat in the steam formed}}{\text{Heat in the coal burnt}}$$

$$= \frac{1{,}267 \times 816}{10{,}110 \times 160}$$

$= 0{\cdot}6393$, or $63{\cdot}93\%$. Ans.

Equivalent evaporation =

$$\frac{\text{Heat in the steam formed per 1 lb. of coal}}{966}$$

$$= \frac{1{,}267 \times \frac{816}{160}}{966} = 6{\cdot}69 \text{ lb.} \quad \text{Ans.}$$

74. $1{\cdot}34 \times \text{C.V.} = (1{,}634 + 223) \times (20{\cdot}4 - 14{\cdot}3)$

$$\text{C.V.} = \frac{1{,}857 \times 6{\cdot}1}{1{\cdot}34} = 8{,}452 \text{ gram calories per gram.}$$ Ans.

8,452 gram calories per gram

$$= \frac{8{,}452}{252} \text{ B.T.U. per gram}$$

$$= \frac{8{,}452}{252} \times \frac{1{,}000}{2{\cdot}2} = 15{,}250 \text{ B.T.U. per lb.}$$ Ans.

See page 401 for diagram and description of Bomb Calorimeter.

75. Friction H.P. at thrust,

$$= \frac{12 \times 2{,}240 \times 0{\cdot}05 \times 2\,\pi \times 7{\cdot}5 \times 77}{33{,}000 \times 12} = 12{\cdot}32$$

Heat equivalent of friction H.P. $= 12{\cdot}32 \times 2{,}545$ B.T.U. per hour.

Let x = gallons of oil required per hour. 1 gallon weighs $10 \times 0{\cdot}92 = 9{\cdot}2$ lb.

Heat gained by oil = Heat given out at bearings

$$x \times 9{\cdot}2 \times 10 \times 0{\cdot}48 = 12{\cdot}32 \times 2{,}545$$

$$\frac{12{\cdot}32 \times 2{,}545}{9{\cdot}2 \times 10 \times 0{\cdot}48} = 710 \text{ gallons. Ans.}$$

76. Referred mean pressure $= \left\{\frac{140}{7}(1 + \log_e 7) - 3\right\} 0{\cdot}68$

" " $= 38$ lb. per sq. inch

$$1{,}200 = \frac{38 \times \text{L.P.}^2 \times 0{\cdot}7854 \times 600}{33{,}000}$$

$$\text{L.P.}^2 = \frac{1{,}200 \times 33{,}000}{38 \times 0{\cdot}7854 \times 600} = 2{,}211$$

$$\text{L.P.} = \sqrt{2{,}211} = 47{\cdot}01 \text{ inches.}$$

$$\text{H.P.}^2 = \frac{2{,}211}{3{\cdot}5} = 631{\cdot}6, \quad \text{H.P.} = \sqrt{631{\cdot}6} = 25{\cdot}13 \text{ ins.}$$

Practical cylinder diameters are, H.P. = 25 ins.
L.P. = 47 ins.

77. $170°$ C. $= 170 \times \frac{9}{5} + 32 = 338°$ F.

$45°$ C. $= 45 \times \frac{9}{5} + 32 = 113°$ F.

Latent heat at $338°$ F. $= 966 - 0{\cdot}7\,(338 - 212)$

$= 877{\cdot}8$ B.T.U. per lb.

1 litre = 1,000 c.c. = 1,000 grams of fresh water
= 1 kilogram

$\therefore$ 4,000 litres = 4,000 kilograms.

It cannot affect the answer if it is assumed that 50 lb. of steam are blown into 4,000 lb. of water.

Heat lost by steam = Heat gained by water

Let $t°$ F. be the final temperature.

$50\ (877 \cdot 8 + 338 - t) = 4{,}000\ (t - 113)$

$877 \cdot 8 + 338 - t = 80\ t - 9{,}040$

$81\ t = 10255 \cdot 8$

$$t = \frac{10255 \cdot 8}{81} = 126 \cdot 6° \text{ F.} \\ = 52 \cdot 5° \text{ C.} \Big\} \text{Ans.}$$

Alternatively, Latent heat $= 966 - 0 \cdot 7\ (t - 212)$ B.T.U. per lb.

,, $= 966 \times \frac{5}{9} - 0 \cdot 7\ (t°c - 100)$ Centigrade Units per lb.

,, $= 536 \cdot 67 - 0 \cdot 7\ (t°c - 100)$ C.H.U. per lb.

Heat lost by steam = heat gained by water

$50\ \{536 \cdot 67 - 0 \cdot 7\ (170 - 100) + 170 - t\}$

$= 4{,}000\ (t - 45)$. Divide by 50.

$536 \cdot 67 - 49 + 170 - t = 80\ t - 3{,}600$

$81\ t = 4257 \cdot 67$

$$t = \frac{4257 \cdot 67}{81} = 52 \cdot 5° \text{ C. as before.}$$

78. Heat given up by 1 lb. of exhaust steam when condensed

$= 0 \cdot 81\ \{966 - 0 \cdot 7\ (162 - 212)\} + (162 - 120)$

$= 810 \cdot 8 + 42 = 852 \cdot 8$ B.T.U.

Steam condensed per minute $= \dfrac{7{,}800 \times 15 \cdot 3}{60} = 1{,}989$ lb.

Heat taken up by 1 lb. of circulating water

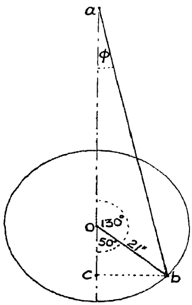
$= 84 - 55 = 29$ B.T.U.

Circulating water per minute

$$= \frac{852 \cdot 8 \times 1{,}989}{29} = 58{,}490 \text{ lb.}$$

Horse power of pump $= \dfrac{58{,}490 \times 30}{33{,}000} = 53 \cdot 17$. Ans.

79



$c\,b = 21 \sin. 50° = 16 \cdot 086$ inches

$o\,c = 21 \cos. 50° = 13 \cdot 499$ inches

Load on guide $= \sqrt{(29 \cdot 37)^2 - (28 \cdot 9)^2}$

$= 5 \cdot 233$ tons. Ans. (*a*)

$c\,b$ represents $5 \cdot 233$ tons to the same scale that $a\,b$ represents $29 \cdot 37$ tons

$$a\,b = 16 \cdot 086 \times \frac{29 \cdot 37}{5 \cdot 233} = 90 \cdot 3 \text{ ins.}$$

Connecting rod length $= 90 \cdot 3$ inches. Ans. (*b*)

$$\frac{21}{\sin.\ \phi} = \frac{90 \cdot 3}{\sin.\ 130°}$$

$$\sin.\ \phi = \frac{21}{90 \cdot 3} \times \sin.\ 130°,\ \phi = 10°\ 16'$$

$a\,c = 90 \cdot 3 \cos. 10°\ 16' = 88 \cdot 85$ inches.

Effect of angularity of con. rod $= 90 \cdot 3 - 88 \cdot 85 = 1 \cdot 45$ ins.
Piston is $13 \cdot 499 + 1 \cdot 45 = 14 \cdot 949$ inches below half stroke.
$21 - 14 \cdot 949 = 6 \cdot 051$ inches from bottom of stroke. Ans. (*c*)

80.

$$\text{B.H.P.} = \frac{(370 - 10) \times \pi \times 5 \times 300}{33{,}000} = \frac{36 \times \pi \times 5}{11}$$

Heat generated at brake $= \dfrac{36 \times \pi \times 5}{11} \times 2{,}545$ B.T.U. per hour.

Heat carried away $= \dfrac{36 \times \pi \times 5}{11} \times 2{,}545 \times 0 \cdot 85$ B.T.U. per hour.

10 C°. rise in temperature $= 10 \times \frac{9}{5} = 18$ F°. rise in temperature.

Let x = gallons of water per hour. 1 gall. weighs 10 lb.

$$10 \times x \times 18 = \frac{36 \times \pi \times 5}{11} \times 2{,}545 \times 0{\cdot}85$$

$$x = \frac{36 \times \pi \times 5 \times 2{,}545 \times 0{\cdot}85}{11 \times 10 \times 18} = 618 \text{ galls. per hour.}$$

81. Weight of moving parts per sq. inch of piston area

$$= \frac{275}{12^2 \times 0{\cdot}7854} = 2{\cdot}431 \text{ lb.}$$

Force to accelerate moving parts = mass × acceleration

$$= \frac{2{\cdot}431 \times 161}{32{\cdot}2} = 12{\cdot}155 \text{ lb.}$$

The engine is vertical, and the weight of the moving parts acts downwards.

∴ effective pressure on piston = 53·2 + 12·155 + 2·431 = 67·786 lb. per sq. inch.

$$\text{Crank effort} = \frac{\text{effective piston load} \times \text{C T}}{\text{crank length}}$$

(see page 372)

When the crank is horizontal C T = crank length

∴ Crank effort = 67·786 × 12^2 × 0·7854 = 7,668 lb. Ans.

Turning moment = 7,668 × 0·75 = 5,751 ft. lb. Ans.

82. Latent heat at 390° F. = 966 — 0·7 (390 — 212) = 841·8 B.T.U. per lb.

60° F. = 60 + 460 = 520° F. absolute.

Final absolute temperature = 520 × 1·1 = 572° F.

Final temperature = 572 — 460 = 112° F.

If the initial weight was 100 lb., then the final weight is 105 lb., and the weight of steam used is 5 lb.

Let x = dryness fraction

Heat lost by steam = Heat gained by water

$$5 \{841{\cdot}4\,x + 390 - 112\} = 100\,(112 - 60)$$

$$841{\cdot}4\,x + 278 = 1{,}040$$

$$841{\cdot}4\,x = 762$$

$$x = \frac{762}{841{\cdot}4} = 0{\cdot}9058. \quad \text{Ans.}$$

83. Initial volume of pipes and oil $= 0{\cdot}7854 \times (0{\cdot}5)^2 \times 500 \times 12$
$= 1{,}178$ cu. inches.

Change of volume of the oil $= 1{,}178\ (0{\cdot}00043 \times 25)$ cu. ins.

Coefficient of linear expansion of copper $= 0{\cdot}00001$

Coefficient of cubical expansion of copper $= 0{\cdot}00001 \times 3$
$= 0{\cdot}00003$

Change of volume of pipes $= 1{,}178\ (0{\cdot}00003 \times 25)$

$\therefore$ volume of oil released

$$= 1{,}178\ (0{\cdot}00043 \times 25) - 1{,}178\ (0{\cdot}00003 \times 25)$$
$$= 1{,}178 \times 25\ (0{\cdot}00043 - 0{\cdot}00003)$$
$$= 1{,}178 \times 25 \times 0{\cdot}0004 = 11{\cdot}78 \text{ cu. ins.} \quad \text{Ans.}$$

84. 1,400 kilograms per sq. centimetre

$$= \frac{1{,}400 \times 2{\cdot}2 \times 144}{(0{\cdot}3937)^2} \text{ lb. per sq. foot.}$$

Work done on 1 cu. foot of lead = pressure per sq. ft. × vo

$$= \frac{1{,}400 \times 2{\cdot}2 \times 144}{(0{\cdot}3937)^2} \text{ foot lb.}$$

1 B.T.U. = 778 ft. lb.

$$1 \text{ C.H.U.} = \frac{778 \times 9}{5} = 1{,}400 \text{ ft. lb. (approx.)}$$

$\therefore$ heat equivalent of the work done on 1 cu. ft. of lead

$$\frac{1{,}400 \times 2{\cdot}2 \times 144}{(0{\cdot}3937)^2 \times 1{,}400}$$

$$\frac{2{\cdot}2 \times 144}{(0{\cdot}3937)^2} \text{ C.H.U.}$$

Weight of 1 cu. ft. of lead = 712 lb.

Sp. heat of lead = 0·029

$$\therefore\ 712 \times 0{\cdot}029 \times \text{rise of temperature} = \frac{2{\cdot}2 \times 144}{(0{\cdot}3937)^2}$$

$$\text{Rise of temperature} = \frac{2{\cdot}2 \times 144}{(0{\cdot}3937)^2 \times 712 \times 0{\cdot}029}$$

$= 99$ C°. Ans.

Available hydrogen $= 12{\cdot}1\% \quad 1{\cdot}5\% \quad 11{\cdot}9125\%$.

$$\begin{aligned} \text{C.V.} &= 14{,}500 \times 0{\cdot}86 + 62{,}000 \times 0{\cdot}119125 \\ &= 19{,}856 \text{ B.T.U. per lb.} \end{aligned}$$

Oil used per I.H.P. hour $= 0{\cdot}8 \times 0{\cdot}38 = 0{\cdot}304$ lb.

$$\text{Thermal efficiency, based on I.H.P.} = \frac{2{,}545}{0{\cdot}304 \times 19{,}856}$$

$= 0{\cdot}4216$, or $42{\cdot}16\%$. Ans.

See page 399 for lower and higher calorific values.

$6{\cdot}8$ C° rise of temperature $= 6{\cdot}8 \times \frac{9}{5} = 12{\cdot}24$ F°

$0{\cdot}015 \times \text{C.V.} = (3 + 12) \times 12{\cdot}24$

$$\text{C.V.} = \frac{15 \times 12{\cdot}24}{0{\cdot}015} = 12{,}240 \text{ B.T.U. per lb.} \quad \text{Ans.}$$

This is the higher calorific value of the coal.

87. Referred mean pressure

$$\begin{aligned} &= \left\{\frac{p_1}{r}(1 + \log_e r) - p_b\right\} \times \text{diagram factor} \\ &= \{\tfrac{200}{12}(1 + \log_e 12) - 3\} \times 0{\cdot}65 \\ &= 35{\cdot}77 \text{ lb. per sq. inch.} \end{aligned}$$

$$2{,}500 = \frac{35{\cdot}77 \times \text{L.P.}^2 \times 0{\cdot}7854 \times 600}{33{,}000}$$

$$\text{L. P}^2. = \frac{2{,}500 \times 33{,}000}{35{\cdot}77 \times 0{\cdot}7854 \times 600} = 4{,}893$$

L. P. $= \sqrt{4{,}893} = 70$ inches diameter (practically)

$$\text{H. P}^2. = \frac{4{,}893}{7{\cdot}2} = 680, \qquad \text{H. P.} = \sqrt{680} = 26 \text{ inches}$$

(practically)

M. P^2. $= 680 \times 2{\cdot}7$, M. P. $= \sqrt{680 \times 2{\cdot}7} = 43$ inches

(practically)

The engine would be 26 ins. $\times$ 43 ins. $\times$ 70 ins. Ans.

88. Latent heat at 341° F. $= 966 - 0{\cdot}7\ (341 - 212)$
$= 875{\cdot}7$ B.T.U. per lb.

Latent heat at 214·7° F. $= 966 - 0{\cdot}7\ (214{\cdot}7 - 212)$
$= 964{\cdot}11$ B.T.U. per lb.

Heat before throttling = Heat after throttling.

Note that the sensible heat of the water is measured above 32° F. All tables of the properties of saturated steam give the heat from 32° F., but obviously in this question the answer would be the same, even if the 32 was omitted.

Let x = dryness fraction :—

$875{\cdot}7\ x + (341 - 32)$

$= (259 - 214{\cdot}7) \times 0{\cdot}5 + 964{\cdot}11 + (214{\cdot}7 - 32)$

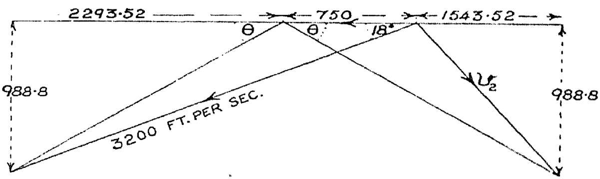
$875{\cdot}7\ x = 22{\cdot}15 + 964{\cdot}11 + 182{\cdot}7 - 309$

$875{\cdot}7\ x = 859{\cdot}96$

$$\frac{859{\cdot}96}{875{\cdot}7} = 0{\cdot}982$$

The dryness fraction was 0·982. Ans.

89.



$3{,}200 \cos. 18° = 3043{\cdot}52$ ft. per sec.

$3{,}200 \sin. 18° = 988{\cdot}8$ ft. per sec.

$3{,}043{\cdot}52 - 750 = 2293{\cdot}52$ ft. per sec.

$$\text{Tan } \theta = \frac{988{\cdot}8}{2293{\cdot}52} = 0{\cdot}4311, \qquad \theta = 23°\ 19'$$

Entrance and exit angles are 23° 19′. Ans.

$2293{\cdot}52 - 750 = 1543{\cdot}52$ ft. per sec.

$v_2 = \sqrt{(988{\cdot}8)^2 + (1543{\cdot}52)^2}$

$= 1{,}833$ ft. per sec. (absolute vel. at exit). Ans.

90. See page 315.
Total area $= \{(15 \times 7) + (12 \times 7) + (15 \times 12)\} \times 2$
$= 738$ sq. ft.
Total area $= 738 \times 144 \times (2{\cdot}54)^2$ sq. cms. $= a$
Depth of insulation $= 6 \times 2{\cdot}54$ cms. $= d$
Temperature difference $= 25 - (-5) = 30$ C°. $= \theta$
Time $= 60$ seconds $= t$

$$Q = \frac{c\,a\,t\,\theta}{d} \quad \frac{0{\cdot}0003 \times 738 \times 144 \times (2{\cdot}54)^2 \times 60 \times 30}{6 \times 2{\cdot}54}$$

gram calories.

$$= \frac{0{\cdot}0003 \times 738 \times 144 \times (2{\cdot}54)^2 \times 60 \times 30}{6 \times 2{\cdot}54 \times 252} = 96{\cdot}4 \text{ B.T.U.}$$

Ans.

91. Available hydrogen $= 13 - \frac{2}{8} = 12{\cdot}75\%$.
Oxygen to burn the carbon in 1 lb. of fuel
$= 2\frac{2}{3} \times 0{\cdot}85 = 2{\cdot}267$ lb.
Oxygen to burn the hydrogen in 1 lb. of fuel
$= 8 \times 0{\cdot}1275 = 1{\cdot}02$ lb.

Oxygen to burn 1 lb. of fuel $= 3{\cdot}287$ lb.
Theoretical air per 1 lb. of fuel $= 3{\cdot}287 \times \frac{100}{23} = 14{\cdot}29$ lb.
Ans. (*a*)
Actual air per 1 lb. of fuel $= 14{\cdot}29 \times 1{\cdot}7 = 24{\cdot}293$ lb.
Weight of gases formed per 1 lb. of fuel burnt $= 24{\cdot}293 + 1$
$= 25{\cdot}293$ lb.
$\therefore$ Weight of gases per hour $= 1{,}400 \times 25{\cdot}293 = 35{,}410$ lb.
Ans. (*b*)

92. Heat to form 1 lb. of steam
$= (395 - 169) + 966 - 0{\cdot}7\,(395 - 212) = 1{,}063{\cdot}9$ B.T.U.
Steam formed per 1 lb. of coal burnt $= \frac{80800}{8025}$ lb.
Thermal efficiency of boilers

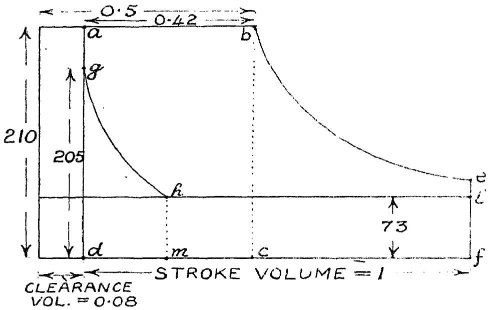
$$\frac{\text{Heat in steam formed per 1 lb. of coal}}{\text{Calorific value}}$$

$$\frac{1063{\cdot}9 \times \frac{80800}{8025}}{12{,}950} = 0{\cdot}8271, \text{ or } 82{\cdot}71\%. \quad \text{Ans.}$$

Thermal effy. $\times$ Mech. effy. $\times$ Boiler effy. $=$ Overall effy.
Thermal efficiency of engines

$$= \frac{0{\cdot}11}{0{\cdot}8 \times 0{\cdot}8271} = 0{\cdot}1662, \text{ or } 16{\cdot}62\%. \quad \text{Ans.}$$

93.



Absolute pressures are,

Initial steam $= 195 + 15 = 210$ lb. sq. inch

Back pressure $= 58 + 15 = 73$ lb. sq. inch

At end of compression $= 190 + 15 = 205$ lb. sq. in.

Let the stroke volume be 1

Then the clearance volume is 0·08, and the volume at cut-off is $0{\cdot}42 + 0{\cdot}08 = 0{\cdot}5$

Ratio of expansion

$$= \frac{1{\cdot}08}{0{\cdot}5} = 2{\cdot}16, \text{ and } \log_e 2{\cdot}16 = 0{\cdot}76935$$

Ratio of compression

$$= \frac{205}{73} = 2{\cdot}809, \text{ and } \log_e 2{\cdot}809 = 1{\cdot}032$$

Volume at h = vol. at $g \times 2{\cdot}809$

,, $= 0{\cdot}08 \times 2{\cdot}809 = 0{\cdot}2247$

$\therefore h\, l = 1{\cdot}08 - 0{\cdot}2247 = 0{\cdot}8553$

Valve closes to exhaust at 0·8553, or 85·53% of return stroke. Ans. (*a*)

Area $a\ b\ c\ d = 210 \times 0{\cdot}42 = 88{\cdot}2$ units

Area $b\ e\ f\ c = 210 \times 0{\cdot}5 \times \log_e \times$ (ratio of expansion)

$= 210 \times 0{\cdot}5 \times 0{\cdot}76935 = 80{\cdot}782$ units

$\therefore$ area $a\ b\ e\ f\ d = 88{\cdot}2 + 80{\cdot}782 = 168{\cdot}982$ units

$$\text{Gross mean pressure} = \frac{\text{area } a\ b\ e\ f\ d}{\text{stroke volume}} = \frac{168{\cdot}982}{1}$$

$= 168{\cdot}982$ lb. per sq. inch. Ans. (*b*)

Area $g\ h\ m\ d$ $= 205 \times 0{\cdot}08 \times \log_e \times$ (ratio of compression).
$= 205 \times 0{\cdot}08 \times 1{\cdot}032 = 16{\cdot}92$ units.

Area $h\ l\ f\ m$ $= 0{\cdot}8553 \times 73 = 62{\cdot}45$ units.
$16{\cdot}92 + 62{\cdot}45 = 79{\cdot}37$ units.

Nett area $= a\ b\ e\ l\ h\ g = 168{\cdot}982 - 79{\cdot}37 = 89{\cdot}612$ units.

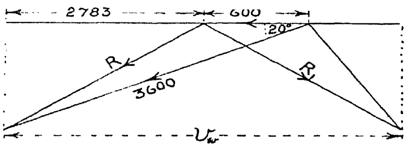
$$\text{Theoretical m.e.p.} = \frac{\text{Nett area}}{\text{stroke volume}}$$

89·612

9·612 lb. per sq. in. Ans. (c)

The actual m.e.p. will be some fraction of the theoretical m.e.p., and it may be about $0{\cdot}75 \times 89{\cdot}612$, or about 67 lb. per sq. inch. Note that the diagram factor is not assumed to be 0·75. It is something less than this, because the area under the compression curve has already been subtracted.

94.



3,600 cos. 20° = 3,383 feet per second.

3,383 — 600 = 2,783 feet per second.

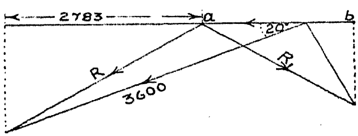
If there is no friction loss then R (relative velocity at entrance) must equal R_1 (relative velocity at exit).

Change of velocity of the steam $= v_w = 2 \times 2{,}783$ ft. per sec.

Steam supplied per sec. $= \frac{1800}{3600} = 0{\cdot}5$ lb.

Force = change of momentum per second.

$$\text{Force} = \frac{0{\cdot}5}{32{\cdot}2} \times 2 \times 2{,}783 = 86{\cdot}42 \text{ lb.} \quad \text{Ans. } (a)$$



If $R_1 = 0{\cdot}88$ R due to friction, then it follows that $a\ b = 0{\cdot}88 \times 2{,}783$.

Change of velocity of the steam

$= 2{,}783 + 0{\cdot}88 \times 2{,}783$
$= 5{,}232$ feet per sec.

$$\text{Force} = \frac{0 \cdot 5}{32 \cdot 2} \times 5{,}232 = 81 \cdot 24 \text{ lb.} \quad \text{Ans. } (b)$$

Work done per sec. $= 81 \cdot 24 \times 600$ ft. lb.

One horse power $= 550$ ft. lb. per sec.

$$\therefore \text{Horse power} = \frac{81 \cdot 24 \times 600}{550} = 88 \cdot 65. \quad \text{Ans. } (c)$$

95. Heat to be taken from 1 lb. of water at 48° F. to change to ice at 30° F.

$= (48 - 32) + 143 + (32 - 30) \times 0 \cdot 5 = 160$ B.T.U.

Total heat to be extracted per hour $= 1{,}120 \times 160$ B.T.U.

Heat taken up by 1 lb. of CO_2 when its dryness fraction increases from $0 \cdot 34$ to $0 \cdot 92 = (0 \cdot 92 - 0 \cdot 34) \times 105 \cdot 5$ $= 61 \cdot 19$ B.T.U.

$$\therefore \text{Pounds of } CO_2 \text{ required} = \frac{1{,}120 \times 160}{61 \cdot 19}$$

2,929 lb. Ans.

96. The passage of the liquid ammonia through the regulating valve is a throttling, or wire-drawing process. A wire-drawing process is a constant total heat operation, that is the total heat after wire-drawing is the same as the total heat before. Measuring the total heat above 0° F., Let x = final dryness.

Heat before = Heat after

$$70 \times 1 \cdot 1 = 14 \times 1 \cdot 1 + x\{566 - 0 \cdot 8 \times 14\}$$

$$61 \cdot 6 = 554 \cdot 8\, x$$

$$x = \frac{61 \cdot 6}{554 \cdot 8} = 0 \cdot 111. \quad \text{Ans.}$$

The law is $p\,v^n$ = constant

$$\therefore 100 \times 3^n = 67 \cdot 85 \times 4^n$$

By Logs. $\text{Log. } 100 + n \log 3 = \log 67 \cdot 85 + n \log 4$

$\text{Log } 100 - \log 67 \cdot 85 = n \log 4 - n \log 3$

$$\frac{\text{Log } 100 - \log 67 \cdot 85}{\text{Log } 4 - \log 3} \qquad \frac{2 - 1 \cdot 8315}{0 \cdot 6021 - 0 \cdot 4771} \qquad \frac{0 \cdot 1685}{0 \cdot 1250}$$

$n = 1 \cdot 35$. Ans.

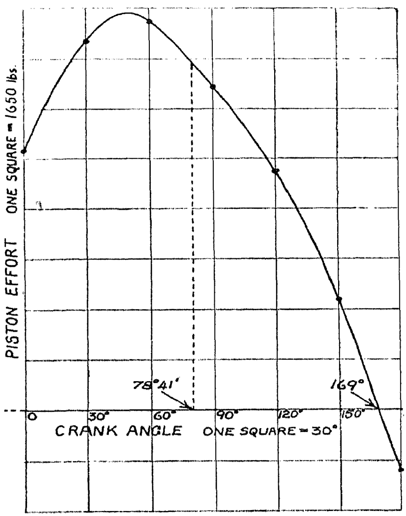
It is difficult to understand what is expected as the value of the constant. We might give the constant as $100 \times 3^{1 \cdot 35} = 440 \cdot 6$. But since the pressure is expressed in lb. per sq. inch and the volume is in cubic feet, it would be equally correct to give the constant as

$$100 \times 144 \times 3^{1 \cdot 35} = 63{,}460.$$

98.

Crank angle in degrees ...	0°	30°	60°	90°	120°	150°	180°
Nett steam load in lb.	13100	15400	14200	9800	5600	1300	—5000
Accelerating force in lb. ...	4600	3330	1450	—860	—2200	—2300	—3000
Piston effort in lb. ...	8500	12070	12750	10660	7800	3600	—2000

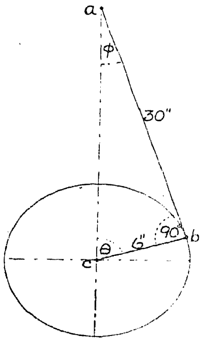
At 0°, the nett steam load is 13,100 lb. and of this 4,600 lb. is required to accelerate the moving parts. Therefore the piston effort is 13,100 — 4,600 = 8,500 lb.



At 90°, the nett steam load is 9,800, whilst the accelerating force is — 860 lb. The acceleration here is negative, or the moving parts are being retarded, and the piston effort is 9,800 — (— 860) = 10,660 lb.

At 180° the piston effort is — 5,000 — (—3,000) = — 2,000 lb.

The diagram shows piston effort plotted on a base line of crank angle turned through. The curve crosses the base line at 169°, and here the piston effort is zero.



Tan ϕ = $\frac{6}{30}$ = 0·2, ϕ (angularity of con. rod) = 11° 19′

θ = 90° — 11° 19′ = 78° 41′, and this is the crank angle when the crank and connecting rod are at 90°.

From the graph, the piston effort at 78° 41′ measures 11,470 lb.

$a\ c = \sqrt{6^2 + 30^2} = 30{\cdot}6$ inches.

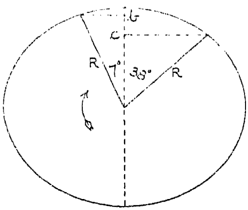
$a\ b$ represents the piston effort, and to the same scale $a\ c$ represents the force acting in the connecting rod.

$$\text{Force in connecting rod} = 11{,}470 \times \frac{\quad}{30}\ \text{lb.}$$

$$\text{Twisting moment} = 11{,}470 \times \frac{30{\cdot}6}{30} \times 6 = 70{,}200 \text{ in lb.}$$

Ans.

99.



Let the radius of the crank circle be R millimetres.

The valve closes at 45° — 7° = 38° past the top centre.

Referring to the diagram,

$a\ b$ = R — R cos. 7°
= R (1 — cos. 7°)

$a\ c$ = R — R cos. 38
= R (1 — cos. 38°)

$a\ b + a\ c$ = 134 m.m.

$\therefore$ R (1 — cos. 7° + 1 — cos. 38°) = 134

R (2 — 0·9925 — 0·7880) = 134

0·2195 R = 134

$$R = \frac{134}{0{\cdot}2195} = 610{\cdot}5 \text{ m.m.}$$

Stroke = 2 × 610·5 = 1221 m.m.

$$\frac{1221}{25{\cdot}4} = 48{\cdot}08 \text{ inches.}$$

Ans.

100. Maximum speed

$$= 750 + \frac{0{\cdot}5}{100} \text{ of } 750 = 753{\cdot}75 \text{ revs. per min.}$$

Minimum speed

$$= 750 - \frac{0{\cdot}5}{100} \text{ of } 750 = 746{\cdot}25 \text{ revs. per min.}$$

Variation of speed $= 753{\cdot}75 - 746{\cdot}25$
$= 7{\cdot}5$ revs. per min.

The flywheel is accelerated during the first 90° movement of the crank from the top centre, and retarded during the next 90°, when the crank has reached the bottom centre. On the up stroke it is accelerated for the first 90°, and retarded during the next 90° movement. It follows, therefore, that the flywheel attains its maximum speed at 90° past top, or bottom centres, because acceleration has just ceased and retardation has not yet begun. When on the centres, the flywheel attains its lowest speed, because retardation has just finished and acceleration has not yet begun. At each of these four points the acceleration is zero.

Time for crank to move through 90°, or $\frac{1}{4}$ revolution

$$\frac{60}{750 \times 4} \text{ second.}$$

Change of speed in this time $= 7{\cdot}5$ revs. per minute

$$\frac{7{\cdot}5 \times 2\pi}{60} \text{ radians per second.}$$

Angular acceleration

$$\frac{\text{Change of angular speed}}{\text{Time to change}}$$

$$\frac{7{\cdot}5 \times 2\pi}{60} \div \frac{60}{750 \times 4}$$

$$\frac{7{\cdot}5 \times 2\pi \times 750 \times 4}{60 \times 60} = 39{\cdot}27 \text{ radians per sec. per sec.} \quad \textit{Ans.}$$

101. The final temperature must be the same if we consider 9 lb. of ice are mixed with 11·5 lb. of water.

Latent heat of water $= 143 \times \frac{5}{9}$ Centigrade heat units per pound.

Let t be the final temperature in C°.

Heat gained by ice = Heat lost by water

$$9\,[(0{\cdot}5 \times 7) + (143 \times \tfrac{5}{9}) + (t - 0)] = 11{\cdot}5\,(79 - t)$$

$$31{\cdot}5 + 715 + 9\,t = 908{\cdot}5 - 11{\cdot}5\,t$$

$$\therefore\ 20{\cdot}5\,t = 162$$

$$t = \frac{162}{20{\cdot}5} = 7{\cdot}9^\circ \text{ C.}$$

$7{\cdot}9^\circ$ C. $= 7{\cdot}9 \times \frac{9}{5} + 32 = 46{\cdot}2^\circ$ F. Ans.

102. $$12 \text{ knots} = \frac{12 \times 6080}{60} = 1{,}216 \text{ ft. per minute}$$

$$\text{Thrust horse power} = \frac{51 \times 2{,}240 \times 1{,}216}{33{,}000} = 4{,}210$$

Combined efficiency of engine and propeller

$$\frac{4{,}210}{9{,}000}$$

Let x be the mechanical efficiency of the engine.

Then $0{\cdot}9\,x$ is the propeller efficiency.

$$x \times 0{\cdot}9\,x = \frac{4{,}210}{9{,}000}$$

$$x^2 = \frac{4{,}210}{8{,}100}, \text{ and } x = 0{\cdot}721$$

$$0{\cdot}9\,x = 0{\cdot}9 \times 0{\cdot}721 = 0{\cdot}6489$$

$$\left.\begin{array}{l}\text{The mechanical efficiency} = 0{\cdot}721, \text{ or } 72{\cdot}1\% \\ \text{The propeller efficiency} = 0{\cdot}6489, \text{ or } 64{\cdot}89\%\end{array}\right\}\text{Ans.}$$

103. Heat to superheat = 200 × 0·48 = 96 B.T.U. per lb.

Latent heat at 395·6° F. = 966 — 0·7 (395·6 — 212)
= 837·48 B.T.U. per lb.

Sensible heat above 32° F. = 395·6 — 32 = 363·6 B.T.U. per lb.

∴ Total heat of initial pressure steam, measured from 32°F.
= 96 + 837·48 + 363·6 = 1297·08 B.T.U. per lb.

Latent heat at 134·4° F. = 966 0·7 (134·4 — 212)
= 1020·32 B.T.U. per lb. if dry.

Latent heat when dryness is 0·94
= 1020·32 × 0·94
= 959·1 B.T.U. per lb.

Sensible heat above 32° F. = 134·4 — 32
= 102·4 B.T.U. per lb.

∴ Total heat measured from 32° F. = 959·1 + 102·4
= 1061·5 B.T.U. per lb.

Heat drop, before alteration
= 1297·08 — 1061·5
= 235·58 B.T.U. per lb.

Latent heat at 102·7° F. = 966 — 0·7 (102·7 — 212)
= 1042·51 B.T.U. per lb. if dry.

Latent heat when dryness is 0·92 = 1042·51 × 0·92
= 959·11 B.T.U. per lb.

Sensible heat above 32° F. = 102·7 — 32
= 70·7 B.T.U. per lb.

∴ Total heat measured from 32° F. = 959·11 + 70·7
= 1029·81 B.T.U. per lb.

Heat drop after alteration
= 1297·08 — 1029·81
= 267·27 B.T.U. per lb.

Additional heat drop, due to alteration
= 267·27 — 235·58 = 31·69 B.T.U. per lb.

Per cent increase in power

$$= \frac{31{\cdot}69}{235{\cdot}58} \times 100 = 13{\cdot}45\%. \quad \text{Ans.}$$

104. At 325° F.

$$V = 1 + \frac{(325 - 39{\cdot}2)^2}{711\ (697 + 325)} = 1 + 0{\cdot}1124 = 1{\cdot}1124$$

At 250° F.

$$V = 1 \qquad \frac{(250 - 39{\cdot}2)^2}{711\ (697 + 250)} = 1 + 0{\cdot}066 = 1{\cdot}066$$

Height of water varies directly as the volume

$$\therefore \frac{H_1}{V_1} = \frac{H_2}{V_2}, \quad H_2 = \frac{H_1 \times V_2}{V_1}$$

$$H_2 = \frac{48 \times 1{\cdot}1124}{1{\cdot}066} = 50{\cdot}07 \text{ inches.}$$

Difference in levels $= 50{\cdot}07 - 48 = 2{\cdot}07$ inches.

$\therefore$ level in boiler is 2·07 inches above level shown in glass. Ans.

105. Assuming one end plate only of each boiler is

$$\text{Surface lagged} = 3 \left(\pi \times 17 \times 13 + 17^2 \times \right.$$

$$= 2{,}763 \text{ sq. feet.}$$

Heat radiated per hour before lagging

$$= \text{Area} \times K\ [810^4 - 560^4] \text{ B.T.U.}$$

Heat radiated per hour after lagging

$$= \text{Area} \times K\ [610^4 - 550^4] \text{ B.T.U.}$$

Saving per hour

$$= \text{Area} \times K\ [(810^4 - 560^4) - (610^4 - 550^4)] \text{ B.T.U.}$$

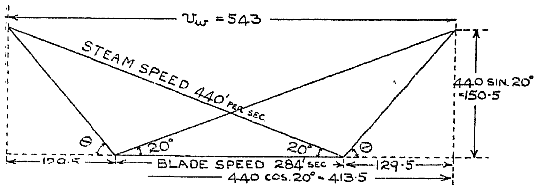
$$= \text{Area} \times K \times 28{,}517 \times 10^7 \text{ B.T.U.}$$

Saving of coal per day

$$= \frac{2{,}763 \times 16 \times 28{,}517 \times 10^7 \times 24}{10^{10} \times 12{,}500 \times 2{,}240}$$

$$= 1{\cdot}08 \text{ tons. Ans.}$$

106.



440 cos 20° = 413·5, and 413·5 — 284 = 129·5 ft. per sec.
440 sin 20° = 150·5 ft. per sec.

Tan. $\theta = \dfrac{150\cdot5}{129\cdot5}$, $\theta = 49° \; 17'$, entrance angle to moving blades. Ans. (a)

v_w = 2 × 440 cos 20° — 284 = 543 ft. per sec.

or v_w = 129·5 + 284 + 129·5 = 543 ft. per sec.

Force on blades = Change of momentum per second.

$$\frac{1 \times 543}{32\cdot2}$$

Work done per 1 lb. of steam per second

$$= \frac{1 \times 543}{32\cdot2} \times 284$$

= 4789 ft. lb. Ans. (b).

107. Latent heat at 388° F. = 966 — 0·7 (388 — 212) = 842·8 B.T.U. per lb.

Latent heat when 0·98 dry = 0·98 × 842·8
= 825·944 B.T.U. per lb.

Sensible heat measured from 32° F. = 388 — 32
= 356 B.T.U. per lb.

∴ total heat of high pressure steam, measured from 32° F. = 825·944 + 356 = 1181·944 B.T.U. per lb.

Latent heat at 338° F. = 966 — 0·7 (338 — 212)
= 877·8 B.T.U. per lb.

Sensible heat measured from 32°F. = 338 — 32
= 306 B.T.U. per lb.

∴ total heat of dry reduced pressure steam, measured from 32° F. = 877·8 + 306 = 1183·8 B.T.U. per lb.

It is seen that the total heat of the high pressure steam is less than the total heat of dry reduced pressure steam by 1183·8 — 1181·944 = 1·856 B.T.U. per lb. Therefore the wire-drawing process of the reducing valve does not quite dry the steam.

$$\text{Final wetness} = \frac{1{\cdot}856}{877{\cdot}8} \times 100 = 0{\cdot}211\%.$$

$$\text{or final dryness} = 100 - 0{\cdot}211 = 99{\cdot}789\%.$$

The final temperature of the reduced pressure steam is 338° F. Ans.

108. Let d = original diameter of sphere in inches,

$$\text{then increase in diameter} = \frac{0{\cdot}15}{100}\, d$$

Original dia. × Change of temp. × Coefficient
= Increase in dia.

$$d \times \text{Change of temp.} \times 0{\cdot}0000128 \quad \frac{0{\cdot}15}{100}$$

$$\therefore \text{Change of temp.} = \frac{0{\cdot}15}{100 \times 0{\cdot}0000128} = 117{\cdot}19\ ^\circ\text{F}.$$

Final temp. of sphere and water
= 60 + 117·19 = 177·19° F.

Fall in temp. of the water
= 200 — 177·19 = 22·81 F°.

Weight of sphere × Change of temp. × Sp. heat
= Weight of water × Change of temp.

$$\times\ d^3 \times 2{\cdot}56 \times \frac{62{\cdot}5}{1728} \times 117{\cdot}19 \times 0{\cdot}22 = 24 \times 10 \times 22{\cdot}81$$

$$\therefore d = \sqrt[3]{\frac{24 \times 10 \times 22{\cdot}81 \times 6 \times 1{,}728}{\pi \times 2{\cdot}56 \times 62{\cdot}5 \times 117{\cdot}19 \times 0{\cdot}22}} = 16{\cdot}36 \text{ inches.}$$

Ans.

109. Available hydrogen $= 12 - \dfrac{1{\cdot}5}{8} = 11\frac{13}{16}\%$.

Oxygen required for combustion

$$= 2\tfrac{2}{3} \times 0{\cdot}85 + \quad \times \frac{11\frac{13}{16}}{100}$$

$$= 3{\cdot}212 \text{ lb. per pound of fuel.}$$

Theoretical weight of air

$$= 3{\cdot}212 \times \frac{100}{23} = 13{\cdot}965 \text{ lb.} \quad \text{Ans.}$$

Actual weight of air $= 13{\cdot}965 \times 1{\cdot}5 = 20{\cdot}948$ lb.

Weight of gases resulting from the combustion of 1 pound of fuel $= 20{\cdot}948 + 0{\cdot}985 = 21{\cdot}933$ lb.

$\therefore$ heat carried away per hour

$$= 21{\cdot}933 \times 1{\cdot}2 \times 2{,}240 \times (550 - 76) \times 0{\cdot}24$$

$$= 6{,}704{,}000 \text{ B.T.U.} \quad \text{Ans.}$$

110. Latent heat at 116° F. $= 966 - 0{\cdot}7\,(116 - 212)$

$= 1033{\cdot}2$ B.T.U. per lb.

Dryness fraction $= 1 - 0{\cdot}13 = 0{\cdot}87$

Latent heat of wet steam $= 1033{\cdot}2 \times 0{\cdot}87$

$= 898{\cdot}884$ B.T.U. per lb.

Sensible heat above hotwell temp. $= 116 - 104$

$= 12$ B.T.U. per lb.

$\therefore$ heat given up by one pound of steam when condensed

$= 12 + 898{\cdot}884 = 910{\cdot}884$ B.T.U.

Heat taken up by one pound of circulating water

$= 90 - 55 = 35$ B.T.U.

$\therefore$ Pounds of water to condense one pound of steam

$$= \frac{910{\cdot}884}{35} = 26{\cdot}02, \text{ say } 26 \text{ lb.}$$

Actual weight of steam condensed per hour

$$\frac{520}{26} = 20 \text{ tons}$$

$$\text{Tube surface required} = \frac{20 \times 2{,}240}{14} = 3{,}200 \text{ sq.}$$

Surface of one tube

$$= \frac{3\ \pi}{4 \times 12} \times 12 = \frac{3\ \pi}{4} \text{ sq. feet.}$$

$\therefore$ Number of tubes required

$$= 3{,}200 \div \frac{3\ \pi}{4}$$

$$= 3{,}200 \times \frac{4}{3\ \pi} = 1{,}358. \quad \text{Ans.}$$

111. $$\text{Brake thermal efficiency} = \frac{2{,}545}{0{\cdot}47 \times 19{,}500}$$

Indicated thermal efficiency

$$\frac{2{,}545}{0{\cdot}47 \times 19{,}500 \times 0{\cdot}805} = 0{\cdot}345 \text{ or } 34{\cdot}5\%$$

Total Heat Loss $= 100 - 34{\cdot}5 = 65{\cdot}5\%$

Total Heat Loss $=$ Loss in gases $+$ Loss in water

$65{\cdot}5 =$ (Loss in water $+ 10$) $+$ Loss in water

$$\therefore \text{ Loss in water} \quad \frac{65{\cdot}5 - 10}{} \quad 7{\cdot}75\%$$

Loss in cooling water $= 27{\cdot}75\%$
Loss in exhaust gases $= 37{\cdot}75\%$ Ans.

112. Specific heat of Copper $= 0{\cdot}095$

Specific heat of Steel $= 0{\cdot}116$

Heat lost by steel = Heat gained by liquid + heat gained by calorimeter.

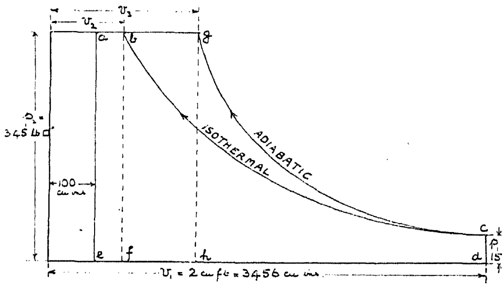
$$100\ (150 - 44{\cdot}5) \times 0{\cdot}116$$
$$= 70\ (44{\cdot}5 - 10) \times \text{Sp. Ht.} + 12\ (44{\cdot}5 - 10) \times 0{\cdot}095$$

$$11{\cdot}6 \times 105{\cdot}5 = 70 \times 34{\cdot}5 \times \text{Sp. Ht.} + 1{\cdot}14 \times 34{\cdot}5$$
$$1223{\cdot}8 = 2415\ \text{Sp. Ht.} + 39{\cdot}33$$

$$\therefore\ 2{,}415\ \text{Sp. Ht.} = 1184{\cdot}47$$

$$\text{Specific Heat} = \frac{1184{\cdot}47}{2{,}415} = 0{\cdot}4904. \quad \text{Ans.}$$

113.



Isothermal compression, $p\,v =$ constant.

$$p_1 \times v_1 = p_2 \times v_2$$

$$15 \times 3{,}456 = 345 \times v_2$$

$$\frac{15 \times 3{,}456}{345} = 150{\cdot}3 \text{ cu. inches} = 0{\cdot}08698 \text{ cu. ft.}$$

Volume delivered $= 150{\cdot}3 - 100 = 50{\cdot}3$ cu. inches. Ans. (*a*)

Adiabatic compression, $p\,v^{1\cdot4} =$ constant.

$$p_1 \times v_1^{1\cdot4} = p_2 \times v_3^{1\cdot4}$$

$$15 \times (3456)^{1\cdot4} = 345 \times v_3^{1\cdot4}$$

By logs, $\text{Log } 15 + 1{\cdot}4 \log. 3{,}456 = \log. 345 + 1{\cdot}4 \log. v_3$

Divide each term by 1·4

$$\frac{\text{Log. } 15}{1{\cdot}4} + \log. 3{,}456 = \frac{\log. 345}{1{\cdot}4} + \log. v_3$$

$$\text{Log. } v_3 = 0{\cdot}8401 + 3{\cdot}5386 - 1{\cdot}8127$$
$$= 2{\cdot}5660$$
$$v_3 = 368{\cdot}2 \text{ cu. inches} = 0{\cdot}213 \text{ cu. foot.}$$

Volume delivered $= 368{\cdot}2 \quad 100 = 268{\cdot}2$ cu. inches.
Ans. (*b*)

Referring to the diagram, the area *b c d f* represents the work done in raising the pressure from p_1 to p_2 under isothermal conditions, and the area *a b f e* is the work done in delivering the air.

Area *b c d f*

$$= p_1 v_1 \log_e \frac{v_2}{} = 144 \times 1 \quad 2 \times \log_e \frac{0{\cdot}08698}{}$$

$$= 144 \times 93{\cdot}97 \text{ ft. lb.}$$

Area *a b f e*

$$= p_2 \times \frac{50{\cdot}3}{1{,}728} = 144 \times 345 \times 0{\cdot}02912$$

$$= 144 \times 10{\cdot}04 \text{ ft. lb.}$$

Total work during compression and delivery

$$= 144 \times 104{\cdot}01 \text{ ft. lb.}$$

Again, the area *g c d h* represents the work done in raising the pressure from p_1 to p_2 under adiabatic conditions, the area *a g h e* is the work done in delivering the air.

Area *g c d h*

$$\frac{p_2 v_3 - p_1 v_1}{1{\cdot}4 - 1} = 144 \times \frac{345 \times 0{\cdot}213 - 15 \times 2}{0{\cdot}4}$$

$$= 144 \times 108{\cdot}71 \text{ ft. lb.}$$

Area $a\ g\ h\ e$

$$= p_2 \times \frac{268 \cdot 2}{1{,}728} = 144 \times 345 \times 0 \cdot 1553$$

$$= 144 \times 53 \cdot 54$$

Total work during compression and delivery

$$= 144 \times 162 \cdot 25 \text{ ft. lb.}$$

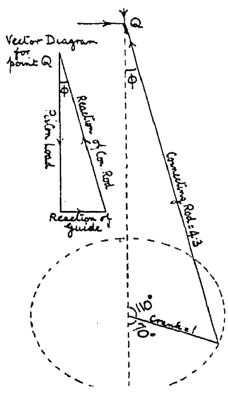
Much less work is done during the isothermal process, and it is the more economical method.

In order to keep the temperature constant during isothermal compression, heat equivalent to the work done must be taken from the air. The internal energy of the air, depending upon its temperature only, is the same after compression as it was before.

During adiabatic compression, the work done is stored as additional internal energy, and the temperature rises. The internal energy of the air is greater after adiabatic compression than it was before. More work must therefore be done during adiabatic compression than during isothermal compression.

The above statement is sufficient answer to the latter part of the question. The candidate at the examination would not be expected to calculate the work done in each case, but the calculations are given here to assist the student.

114.



Let length of crank $= 1$
then length of stroke $= 2$
and length of con. rod $= 4 \cdot 3$

$$\text{Sin } \phi = \frac{\text{Sin } 110^\circ}{4 \cdot 3} = 0 \cdot 2185$$

$$= 12^\circ\ 37'$$

Load on guide

$$= \text{Load in con. rod} \times \text{Sin } 12^\circ\ 37'$$

$$= 52{,}700 \times 0 \cdot 2185 = 11{,}520 \text{ lb. Ans.}$$

Load on piston

$$= \text{Load in con. rod} \times \text{Cos. } 12^\circ\ 37'$$

$$= 52{,}700 \times 0 \cdot 9759$$

$$= 51{,}420 \text{ lb. (effective load)}$$

Area of Piston $\times$ Effective pressure per sq. inch
$= $ Effective load

$$\therefore \text{ Effective pressure } = \frac{51{,}420}{27^2 \times \frac{\pi}{4}} = 89{\cdot}8 \text{ lb. per sq. inch.}$$

$\therefore$ Steam pressure on high pressure piston
$= 89{\cdot}8 + 70$
$= 159{\cdot}8$ lb. per sq. inch. Ans.

Distance from crosshead to shaft centre when crank is on centre $= 4{\cdot}3 + 1 = 5{\cdot}3$

Distance from crosshead to shaft centre when crank is 110° past centre $= 4{\cdot}3 \text{ Cos. } 12^\circ\ 37' - 1 \text{ Cos. } 70^\circ$
$= 4{\cdot}1964 - 0{\cdot}342$
$= 3{\cdot}8544$

Distance moved by crosshead, and by piston
$= 5{\cdot}3 - 3{\cdot}8544$
$= 1{\cdot}4456$

Fraction of stroke at cut off

$$= \frac{1{\cdot}4456}{} = 0{\cdot}7228, \text{ or } 72{\cdot}28\%. \quad \text{Ans.}$$

115. Rise in temperature of cooling water
$= 45 - 20 = 25$ C°.
$= 25 \times \frac{9}{5} = 45$ F°.

Heat carried away per hour $= 530 \times 10 \times 45$ B.T.U.

$$\text{Heat generated per hour} = \frac{530 \times 10 \times 45}{0{\cdot}85} \text{ B.T.U.}$$

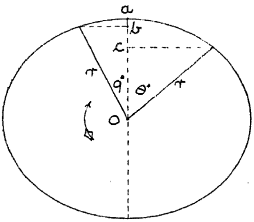
$$\text{B.H.P.} = \frac{530 \times 10 \times 45}{0{\cdot}85 \times 2{,}545} = 110{\cdot}3. \quad \text{Ans.}$$

$$\frac{\text{Effective load on brake} \times \pi \times 5 \times 70}{33{,}000} = 110$$

Effective load on brake

$$\frac{110{\cdot}3 \times 33{,}000}{\pi \times 5 \times 70} = 3{,}309 \text{ lb.} \quad \text{Ans.}$$

116.



Let r = length of the crank.
Then $2\,r$ = length of the stroke.

$$a\,c = \frac{9}{100} \text{ of } 2\,r = 0{\cdot}18\,r$$

$$c\,o = r - 0{\cdot}18\,r = 0{\cdot}82\,r$$

$$\text{Cos } \theta = \frac{0{\cdot}82\,r}{r} = 0{\cdot}82, \ \theta = 34^\circ\,55'$$

$\therefore$ valve is open for $9^\circ + 34^\circ\,55'$
$= 43^\circ\,55'$. Ans.

$$a\,b = r - r \cos.\ 9^\circ = 0{\cdot}0123\,r$$

$$a\,c = r - r \cos.\ \theta = 0{\cdot}18\,r$$

$$\therefore a\,b + a\,c = 0{\cdot}0123\,r + 0{\cdot}18\,r = 0{\cdot}1923\,r$$
$$0{\cdot}1923\,r = 4 \text{ inches.}$$

$$= 20{\cdot}8 \text{ inches.}$$
$$0{\cdot}1923$$

Stroke of engine $= 20{\cdot}8 \times 2 = 41{\cdot}6$ inches. Ans.

117. Steam supplied to coils per sec. $= \dfrac{2000}{3600} = \dfrac{5}{9}$ lb.

Heat given up by one pound of heating steam
$= 0{\cdot}95$ L.H. + S.H.

Heat given up
$= 0{\cdot}95\,[966 - 0{\cdot}7\,(382 - 212)] + (382 - 260)$
$= 804{\cdot}65 + 122 = 926{\cdot}65$ B.T.U. per lb.

Latent heat at 250° F. $= 966 - 0{\cdot}7\,(250 - 212)$
$= 939{\cdot}4$ B.T.U. per lb.

No data is given of the feed temperature to the evaporator, it is therefore assumed that the vapour is dry, and that 939·4 B.T.U. are required to form 1 lb. of vapour.

$$\text{Vapour formed per sec.} = \frac{926{\cdot}65}{939{\cdot}4} \times \frac{5}{9} \text{ lb.}$$

Let d = diameter of valve in inches.

Then $\pi\, d \times \dfrac{d}{24} \times \dfrac{1}{144}$ = area of escape in sq. feet.

Area of escape × velocity of escape = volume escaping.

$$\frac{\pi\, d^2}{24 \times 144} \times 750 = \frac{926 \cdot 65 \times 5 \times 13 \cdot 72}{939 \cdot 4 \times 9}$$

$$d = \sqrt{\frac{926 \cdot 65 \times 5 \times 13 \cdot 72 \times 24 \times 144}{\pi \times 750 \times 939 \cdot 4 \times 9}}$$ Ans.

118. $\text{I.H.P.} = \dfrac{60 \times 1{,}000}{746 \times 0 \cdot 8 \times 0 \cdot 92} = 109 \cdot 3$

8·5 kilograms per sq. cent.

$$\frac{8 \cdot 5 \times 2 \cdot 205}{(0 \cdot 3937)^2} = 120 \cdot 9 \text{ lb. per sq. inch.}$$

$$109 \cdot 3 = \frac{120 \cdot 9 \times d^2 \times 0 \cdot 7854 \times 1 \cdot 5\, d \times 120 \times 6}{33{,}000 \times 12}$$

$$\sqrt[3]{\frac{109 \cdot 3 \times 33{,}000 \times 12}{120 \cdot 9 \times 0 \cdot 7854 \times 1 \cdot 5 \times 720}} = 7 \cdot 5 \text{ inches.}$$

$7 \cdot 5 \times 25 \cdot 4 = 190 \cdot 5$ millimetres. Ans.

119. Let N = revolutions per minute.

$$\%\ \text{Mechanical efficiency} = \frac{\text{B.H.P.}}{\text{I.H.P.}} \times 100$$

$$\frac{1000 \times 100}{\text{I.H.P.}} = \sqrt{N} \times 85$$

$$\text{I.H.P.} = \frac{100000}{\sqrt{N} \times \sqrt{85}} \quad \ldots \quad \text{(i.)}$$

Also :—

$$\text{I.H.P.} = \frac{85 \times \frac{\pi}{4} \times 650^2 \times 2 \times 650 \times N \times 6}{(25 \cdot 4)^2 \times 25 \cdot 4 \times 12 \times 2 \times 33{,}000}$$

$$= 16 \cdot 95\ N \quad \ldots \quad \ldots \quad \ldots \quad \text{(ii.)}$$

$$16{\cdot}95\ N = \frac{100000}{\quad \times \sqrt{85}} \qquad \frac{100000}{16{\cdot}95 \times \sqrt{85}}$$

$$N = \sqrt[\frac{3}{2}]{\frac{100000}{16{\cdot}95 \times \sqrt{85}}} = 74{\cdot}25 \text{ revs. per min.} \quad \text{Ans.}$$

Mechanical efficiency $= \sqrt{74{\cdot}25 \times 85} = 79{\cdot}43\%$. Ans.

120. Heat supplied to cylinders, per I.H.P. per hour
$= 0{\cdot}73 \times 1{\cdot}38 \times 13{,}650$ B.T.U.

Heat equivalent of one I.H.P. hour $= 2{,}545$ B.T.U.

$\therefore$ Thermal efficiency of engine

$$\frac{2{,}545}{0{\cdot}73 \times 1{\cdot}38 \times 13{,}650}$$

$= 0{\cdot}1851$, or $18{\cdot}51\%$. Ans. (*a*).

I.H.P. for 1·38 lb. of fuel per hour $= 1$

,, ,, 1 lb. ,, ,, $\dfrac{1}{1{\cdot}38}$

,, $\frac{63}{24} \times 2240$ lb. ,, $= \dfrac{1 \times 63 \times 2{,}240}{1{\cdot}38 \times 24}$

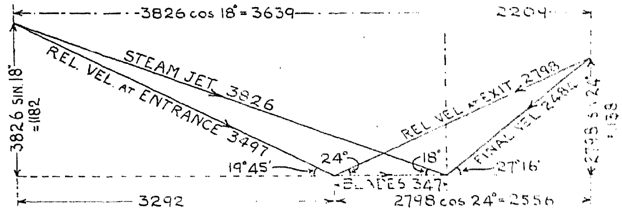
$= 4{,}260$ I.H.P. Ans. (*b*).

Overall thermal efficiency of engines and boilers
$= 0{\cdot}73 \times 0{\cdot}1851 = 0{\cdot}1351 = 13{\cdot}51\%$.

Percentage of total heat lost $= 100 - 13{\cdot}51 = 86{\cdot}49\%$.

For each lb. of fuel, this loss is $0{\cdot}8649 \times 13{,}650$
$= 11{,}805$ B.T.U. per lb. Ans. (*c*).

121.



3826 cos. 18° = 3639, and 3639 — 347 = 3292 ft. per sec.

3826 sin. 18° = 1182 ft. per sec.

$$\text{Tan } \theta = \frac{1182}{3292}, \qquad \theta = 19^\circ\ 45'$$ (entrance angle of moving blades).

Velocity of steam relative to moving blades at entrance

$$= \frac{1182}{\sin.\ 19^\circ\ 45'}$$

$$= 3497 \text{ ft. per sec.}$$

Velocity of steam relative to moving blades at exit

$$= 3497 \times 0{\cdot}8 = 2798 \text{ ft. per sec.}$$

2798 cos. 24° = 2556, and 2556 — 347 = 2209 ft. per sec.

2798 sin. 24° = 1138 ft. per sec.

$$\text{Tan } \phi = \frac{2209}{1138} \qquad \phi = 62^\circ\ 44'$$

Final absolute velocity of steam

$$\frac{2209}{\sin.\ 62^\circ\ 44'} = 2484 \text{ ft. per sec.}$$

Entrance angle of moving blades = 19° 45′
Final abs. velocity of steam = 2484 ft. per sec.
Direction is at 62° 44′ to turbine axis, or at
90° — 62° 44′ = 27° 16′ to direction of blade motion. } Ans.

122. 1 cu. ft. CO_2 gas at 14·7 lb. per sq. inch and 32°F. weighs $0{\cdot}0807 \times 1{\cdot}518$ lb.

1 cu. ft. CO_2 gas at 14·7 lb. per sq. inch and 60°F. weighs $\dfrac{0{\cdot}0807 \times 1{\cdot}518 \times 492}{520}$

$= 0{\cdot}1159$ lb.

Permeable space $= 36 \times 56 \times 20 \times 0{\cdot}75$ cu. ft.

Volume of CO_2 gas required $= 36 \times 56 \times 20 \times 0{\cdot}75 \times 0{\cdot}25$ cu. feet.

Weight of $CO_2 = 36 \times 56 \times 20 \times 0{\cdot}75 \times 0{\cdot}25 \times 0{\cdot}1159$
$= 876{\cdot}2$ lb. Ans.

Volume occupied by 3 lb. of water $= \dfrac{3}{62{\cdot}5}$ cu. ft., and this is the volume of 2 lb. of CO_2 in the liquid form.

$\therefore$ Volume of bottles $= \dfrac{3 \times 876{\cdot}2}{62{\cdot}5 \times 2} = 21{\cdot}03$ cu. feet. Ans.

123. For 1 lb. of vapour formed let x lb. be blown out. Then the feed is $1 + x$ lb.

Amount of feed $\times$ feed density = Amount blown out $\times$ Evaporator density

$$(1 + x) \times 1 = x \times 2\tfrac{1}{2}$$
$$1 = 1\tfrac{1}{2}\, x$$
$$\text{and } x = \tfrac{2}{3} \text{ lb.}$$

Therefore, for every 1 pound of vapour formed, $1\frac{2}{3}$ lb. of water is fed into the evaporator and $\frac{2}{3}$ lb. is blown out. The whole $1\frac{2}{3}$ lb. receives sensible heat, but only 1 lb. receives latent heat. Heat required to form 1 pound of vapour $= 1\frac{2}{3}\,(222 - 70) + 966 - 0{\cdot}7\,(222 - 212)$ B.T.U. $= 1212{\cdot}3$ B.T.U.

Heat required to form 24 tons of vapour

$= 1212{\cdot}3 \times 24 \times 2240$ B.T.U.

Heat given up by 1 pound of heating steam

$= 966 - 0{\cdot}7\,(384 - 212) + (384 - 234) = 995{\cdot}6$ B.T.U.

$\therefore$ Weight of heating steam required

$$= \frac{1212 \cdot 3 \times 24 \times 2240}{995 \cdot 6} \text{ lb.}$$

Weight of coal required

$$= \frac{1212 \cdot 3 \times 24 \times 2240}{995 \cdot 6 \times 7 \cdot 5 \times 2240} = 3 \cdot 896 \text{ tons.} \quad \text{Ans.}$$

124. Fuel used per I.H.P. per hour $= \dfrac{109 \cdot 5}{308} = 0 \cdot 3556$ lb.

Fuel used per B.H.P. per hour $= \dfrac{109 \cdot 5}{195} = 0 \cdot 5616$ lb.

Indicated thermal efficiency

$$= \frac{2545}{0 \cdot 3556 \times 19200} \times 100 = 37 \cdot 29\%. \quad \text{Ans.}$$

Brake thermal efficiency

$$= \frac{2545}{0 \cdot 5616 \times 19200} \times 100 = 23 \cdot 6\%. \quad \text{Ans.}$$

Friction and pumping losses $= 37 \cdot 29 - 23 \cdot 6 = 13 \cdot 69\%$.
Heat carried away by cooling water per hour

$$= 171 \times 60 \times (118 - 63)$$
$$= 564300 \text{ B.T.U.}$$

As a per cent. of the total heat supplied this is

$$= \frac{564300}{109 \cdot 5 \times 19200} \times 100 = 26 \cdot 83\%$$

The remainder, i.e., $100 - (37 \cdot 29 + 26 \cdot 83) = 35 \cdot 88\%$, is lost in exhaust gases.

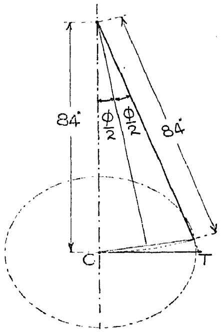
Heat balance diagram :—

Heat in Fuel = 100%

I.H.P. = 37·29% Cooling water = 26·83% Exhaust

B.H.P. = 23·6% Friction, etc. = 13·69%

125.



Torque produced in main engine shaft
$= 1600 \times 1000 \times 0{\cdot}12$ inch lb.

$$\text{Sin. } \frac{\phi}{2} = \frac{10{\cdot}5}{84} = 0{\cdot}125, \qquad \frac{\phi}{2} = 7^\circ\,11'$$

$$\phi = 14^\circ\ 22'$$

C.T. $= 84$ tan. $14^\circ\ 22' = 21{\cdot}51$ ins.
(note the isosceles triangle).

$$\therefore \text{ Vertical pull on crosshead} = \frac{\text{Torque}}{\text{C.T.}}$$

$$\frac{1600 \times 1000 \times 0{\cdot}12}{21{\cdot}51} \text{ lb.}$$

Cross sectional area of each bolt $= 0{\cdot}7854 \times (0{\cdot}875)^2$

$\therefore$ Shear stress in bolts

$$= \frac{1600 \times 1000 \times 0{\cdot}12}{21{\cdot}51 \times 0{\cdot}7854 \times (0{\cdot}875)^2 \times 2} = 7424 \text{ lb. per sq. in.}$$

Ans.

126. $\frac{3}{8}$ of $12 = 4{\cdot}5$ inches.

The law is $p\ v^n =$ constant

$\therefore\ 105 \times (4{\cdot}5 + 0{\cdot}5)^n = 30 \times (12 + 0{\cdot}5)^n$

Log $105 + n \log 5 = \log 30 + n \log 12{\cdot}5$

Log $105 - \log 30 = n (\log 12{\cdot}5 - \log 5)$

$$\frac{\text{Log } 105 - \log 30}{\text{Log } 12{\cdot}5 - \log 5} = \frac{2{\cdot}0212 - 1{\cdot}4771}{{\cdot}0969 - 0{\cdot}6990}$$

$$\frac{0{\cdot}5441}{0{\cdot}3979} = 1{\cdot}368$$

$n = 1{\cdot}368.$ Ans.

The law is $p\ v^{1\cdot368} =$ constant

$105 \times 5^{1\cdot368} = (70 + 15) \times v^{1\cdot368}$

Log $105 + 1{\cdot}368 \log 5 = \log 85 + 1{\cdot}368 \log v$

$2{\cdot}0212 + 1{\cdot}368 \times 0{\cdot}6990 = 1{\cdot}9294 + 1{\cdot}368 \log v$

$$\text{Log } v = \frac{1.0478}{1.368} = 0.7663$$

$$v = 5.838$$

Position of piston $= 5.838 - 0.5 = 5.338$ inches of stroke.

$$\text{Fraction of stroke} = \frac{5.338}{12} = 0.4448. \quad \text{Ans.}$$

127. The duty of a refrigerating machine is to extract heat from the substance to be cooled, and the measure of the success with which this is carried out is the Coefficient of Performance. It is given by the ratio $\dfrac{\text{Heat extracted}}{\text{Work expended}}$, and its value is generally greater than 1. For the ideal machine, working on the reversed Carnot cycle, the coefficient of performance $= \dfrac{T_2}{T_1 - T_2}$, where T_2 is the temperature at which the heat is taken in, i.e., the evaporator and T_1 is the temperature at which it is rejected, i.e., the condenser. The temperatures must be absolute.

Heat to be taken from each pound of water to change it into ice $= (67 - 32) + 143 + 0.5\,(32 - 27) = 180.5$ B.T.U.

A 4-ton machine can extract $4 \times 2240 \times 143$ B.T.U. in 24 hours.

$\therefore$ Weight of ice made in 24 hours

$$= \frac{4 \times 2240 \times 143}{180.5 \times 2240} = 3.169 \text{ tons.} \quad \text{Ans.}$$

$$\text{Coeff. of performance} = \frac{T_2}{T_1 - T_2}$$

$$5.8 = \frac{T_2}{530 - T_2}$$

$$5.8\,(530 - T_2) = T_2$$

$$\therefore T_2 = \frac{5.8 \times 530}{6.8} = 452.1°\text{F. abs., or } -7.9°\text{F.} \quad \text{Ans.}$$

128. The highest pressure will be attained at the end of compression. During the combustion period there is theoretically no rise in pressure, and the highest temperature is reached at the end of combustion.

The law is $p\ v^{1 \cdot 4}$ = constant.

$14 \times 13^{1 \cdot 4} = p_2 \times 1^{1 \cdot 4}$

p_2 = 507·7 lb. per sq. inch abs. (highest pressure). Ans.

$$\frac{p\ v}{\mathrm{T}} \quad \text{constant for a gas.}$$

$$\frac{14 \times 13}{560} \quad \frac{507 \cdot 7 \times 1}{\mathrm{T}_2} \quad \mathrm{T}_2 = 1562\ ^\circ\mathrm{F.\ abs.}$$

∴ Temp. at end of compression = 1562 — 460 = 1102°F.

Assume 35 lb. of air are compressed and 1 lb. of oil is used. The weight of the gases formed is 36 lb.

Heat given up by fuel = Heat gained by gases.

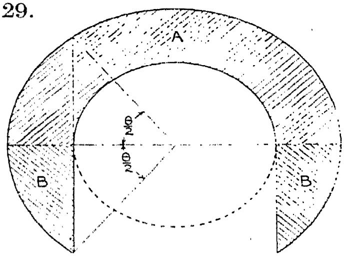
$18500 = 36 \times$ Change of temp. $\times\ 0 \cdot 2375$

$$\therefore \text{ Change of temp.} = \frac{18500}{36 \times 0 \cdot 2375} = 2164\ \mathrm{F}^\circ.$$

∴ Final temp. = 1102 + 2164 = 3266°F. (highest temp). Ans.

As the specific heat of the products of combustion was not stated, it has been assumed to be the same as that of air.

129.



The rubbing surface of each face of the shoe consists of the area of a semi-annulus A, and the area of two half segments B and B.

Area of semi-annulus

$$= \frac{\ }{4}\,[20^2 - 12^2] \times$$

= 100·53 sq. ins.

$$\mathrm{Cos.}\ \frac{\theta}{2} = \frac{6}{10} = 0 \cdot 6. \qquad \frac{\ }{2} = 53^\circ\ 8', \text{ and } \theta = 106^\circ\ 16'$$

Area of whole segment — sin θ]

10^2

,, ,, $= 44 \cdot 7$ sq. inches.

$\therefore$ Area of rubbing surface $= 100 \cdot 53 + 44 \cdot 7 = 145 \cdot 23$ sq. inches.

Total load on thrust $= 45 \times 145 \cdot 23 \times 6$ lb.

$$\text{I.H.P. of engine} = \frac{45 \times 145 \cdot 23 \times 6 \times 12 \times 6080}{33000 \times 60} \times \frac{100}{69}$$

$= 2095$. Ans.

130.
Twisting moment for a solid shaft $= \frac{\quad}{16} d^3 \times q$

$$\text{also, twisting moment} = \frac{63000 \times \text{H.P.}}{\text{revs.}}$$

$$63000 \times \text{H.P.}$$

16 revs.

$$\text{and } \frac{d^3 \times q \times \text{revs.}}{\text{H.P.}} = \text{a constant.} \qquad (1)$$

$$\text{H.P.} = \frac{p}{33000} \text{-}, \quad \therefore \text{H.P.} \propto p \times \text{revs.}$$

The mean effective pressure (p) varies directly as the abs. boiler pressure (P)

$$\therefore \text{H.P.} \propto \text{P} \times \text{revs.} \qquad \ldots (2)$$

Substituting for H.P. in (1):

$$\frac{d^3 \times q \times \text{revs.}}{\text{P} \times \text{revs.}} = \text{constant}, \quad \therefore \frac{d^3 \times q}{\quad} = \text{constant.}$$

If the stress is the same, then $\frac{\quad}{P} \doteq$ constant.

215

$\therefore\ P = 215 \times \qquad = 172 \cdot 7$ lb. per sq. inch abs.

or 157·7 lb. per sq. inch gauge. Ans.

131. Steam load on piston

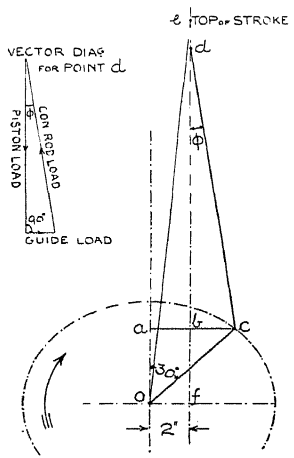
$$= \frac{\pi}{4} \times 20^2 \times 110 = 34557 \cdot 6 \text{ lb.}$$

Accelerating force = Mass × acceleration

$$,, \quad ,, \quad = \frac{1400 \times 280}{32 \cdot 2} = 12180 \text{ lb.}$$

Load on crosshead = 34557·6 — 12180 = 22377·6 lb., or 9·99 tons. Ans.

132.



$a\,c = 21 \times \sin 30° = 10 \cdot 5$ inches.

$b\,c = 10 \cdot 5 - 2 = 8 \cdot 5$ inches.

$$\text{Sin } \phi = \frac{8 \cdot 5}{72}, \qquad \phi = 6° \ 47'$$

$$\frac{\text{Guide load}}{\text{Piston load}} = \tan \phi$$

$\therefore$ Guide load

$= 20 \times 2240 \times 0 \cdot 119$ lb.

Guide pressure

$$\frac{20 \times 2240 \times 0 \cdot 119}{173}$$

$= 30 \cdot 81$ lb. per sq. inch. Ans.

$d\,b = 72 \cos 6° \ 47' = 71 \cdot 48$ inches.

$a\,o = 21 \cos 30° = 18 \cdot 186$ inches.

∴ Vertical distance from crosshead to shaft centre when the crank is 30° past the vertical $= 71{\cdot}48 + 18{\cdot}186$ $= 89{\cdot}666$ inches.

Vertical distance from crosshead to shaft centre when the piston is at its highest position (this is when crank and con. rod are in line) $= e\,f$.

$$e\,f = \sqrt{93^2 - 2^2} = 92{\cdot}977 \text{ inches}$$

∴ Distance piston has moved down its stroke $= e\,d$
$= 92{\cdot}977 - 89{\cdot}666 = 3{\cdot}311$ inches. Ans.

133. Lap + lead — exhaust lap = opening to exhaust.
Lap + 0·24 — 0·17 = 2

∴ Steam lap $= 2 - 0{\cdot}24 + 0{\cdot}17 = 1{\cdot}93$ inches. Ans.

Sine angle of advance

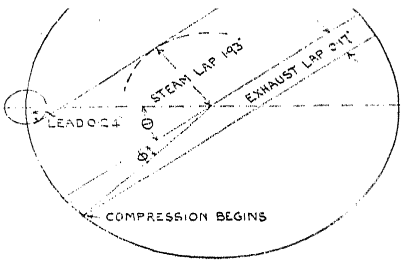
$$= \frac{\text{Lap} + \text{lead}}{\frac{1}{2}\text{ travel}} = \frac{1{\cdot}93 + 0{\cdot}24}{3}$$

Angle of advance $(\theta) = $

$$\text{Sin } \phi = \frac{\text{Exhaust lap}}{\frac{1}{2}\text{ travel}} = \frac{0{\cdot}17}{3{\cdot}75}$$

$\phi = 2° \; 36'$.

∴ At start of compression the crank is at $35° \; 22' + 2° \; 36'$ $= 37° \; 58'$ to the line of stroke. Ans.



134. Assume 100 lb. of steam is supplied by the boilers. 93·6 lb. passes through the engine and reaches the air pump as water at 126°F., and 6·4 lb. of steam are supplied to the heater.

The available heat is therefore that in 93·6 lb. of water at 126°F. + the heat in 6·4 lb. of steam. Since none of this is lost, all must appear in the boiler feed water.

$$\therefore (93{\cdot}6 \times 126) + 6{\cdot}4\,[966 \quad 0{\cdot}7\,(T - 212) + T] = 100 \times 196$$

$$11793{\cdot}6 + 6{\cdot}4\,[1114{\cdot}4 + 0{\cdot}3\,T] = 19600$$

$$1114{\cdot}4 + 0{\cdot}3\,T = \frac{19600 - 11793{\cdot}6}{6{\cdot}4} = 1219{\cdot}75$$

$$0{\cdot}3\,T = 1219{\cdot}75 - 1114{\cdot}4 = 105{\cdot}35$$

$$T = \frac{105{\cdot}35}{0{\cdot}3} = 351{\cdot}2^\circ F. \quad \text{Ans.}$$

The drainage water from the heater coils is probably higher than 126°F., and when mixed with the air pump water at 126°F., will make the hotwell water (water entering the surface heater) greater than 126°F. Less heat will have been taken from the heating steam and less has to be supplied to the feed water.

Suppose t_1 is the temp. of water from heater coils.

6·4 lb. of water at t_1 mix with 93·6 lb. of water at 126°F.

$$\text{The resulting temp. } (t_2) = \frac{6{\cdot}4\,t_1 + 93{\cdot}6 \times 126}{100}$$

Now equate heat lost by heating steam to heat given to feed water.

$$6{\cdot}4\,[966 - 0{\cdot}7\,(T - 212) + (T - t_1)] = 100\,[196 - t_2]$$

$$6{\cdot}4\,[966 - 0{\cdot}7\,(T - 212) + T] - 6{\cdot}4\,t_1 = 100 \times 196 - 100\,t_2$$

Substitute the value of t_2

$$6{\cdot}4\,[966 - 0{\cdot}7\,(T - 212) + T] - 6{\cdot}4\,t_1 = 100 \times 196 - 6{\cdot}4\,t_1 - 93{\cdot}6 \times 126$$

$$\therefore (93{\cdot}6 \times 126) + 6{\cdot}4\,[966 - 0{\cdot}7\,(T - 212) + T] = 100 \times 196, \text{ as before.}$$

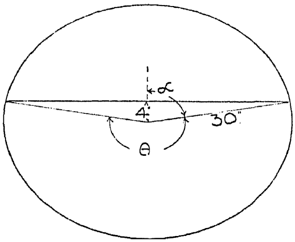
SOLUTIONS TO FIRST-CLASS EXAMINATION QUESTIONS.

NAVAL ARCHITECTURE

1. Load $= \text{H} \times \text{A} \times w$

$$= \frac{12 \times 2{\cdot}2 \times 28 \times 64}{2{,}240} = 21{\cdot}12 \text{ tons. Ans.}$$

2.



$\frac{4}{30} = \cos.\ \alpha = 0{\cdot}1333$

$\alpha = 82° \ 20'$.

$\theta = 360° - (2 \times 82° \ 20')$

$= 195° \ 20'$.

Area of under-water segment

$$= \frac{\text{R}^2}{2}(\theta - \sin.\ \theta)$$

$$\frac{2{\cdot}5^2}{2}\left(\frac{195\frac{1}{3}}{57{\cdot}3} - \text{Sin. } 195° \ 20'\right)$$

$$= 3{\cdot}125\ [3{\cdot}408 - (-0{\cdot}2644)]$$

$$= 3{\cdot}125 \times 3{\cdot}672 \text{ square feet.}$$

Weight of water displaced

$$= \frac{3{\cdot}125 \times 3{\cdot}672 \times 36 \times 1017}{2{,}240 \times 16} = 11{\cdot}72 \text{ tons.}$$

Ans.

3. Volume of compartment $= 175 \times 44 = 7{,}700$ cubic feet.

$$\text{Volume of the coal} = \frac{175 \times 2{,}240}{80} = 4{,}900 \text{ cubic feet.}$$

Volume of water $= 2{,}800$ cubic feet.

Weight of sea water $= \frac{2800}{35} + 0\cdot08$ of 175

$= 80 + 14 = 94$ tons. Ans.

4. Displacement in tons neglecting fore and aft

$$\text{portions} = \frac{20 \times 64}{3 \times 2{,}240}\left\{\begin{array}{r}25 + 30 + 4\,(140 + 327 + 353 + 185)\\ +2\,(254 + 402 + 261)\end{array}\right\}$$

$$\frac{20 \times 64}{3 \times 2{,}240} \times 5{,}909 = 1{,}126 \text{ tons.}$$

Total displacement $= 1{,}126 + 11 = 1{,}137$ tons. Ans.

$5{,}600 \times 2{,}240 \times 16 \times (1{,}026 - 1{,}009)$

$= A \times \frac{3}{12} \times 1{,}026 \times 1{,}009$

$$\text{Water plane area} = \frac{5{,}600 \times 2{,}240 \times 16 \times 17 \times 12}{3 \times 1{,}026 \times 1{,}009}$$

$= 13{,}180$ square feet. Ans.

6. Displacement in river $= 7{,}000 - 80 = 6{,}920$ tons.

$$\frac{6{,}920 \times 2{,}240 \times 16}{1{,}008} \qquad \frac{7{,}000 \times 2{,}240 \times 16}{1{,}027}$$

$$\frac{\text{Change in draught (inches)} \times 12{,}500}{12}$$

$$10 \times 2{,}240 \times 16 \left(\frac{692}{1{,}008} - \frac{700}{1{,}027} \right) = \frac{x \times 12{,}500}{12}$$

$$10 \times 2{,}240 \times 16 \left(\frac{710{,}684 - 705{,}600}{1{,}008 \times 1{,}027} \right) = \frac{x \times 12{,}500}{12}$$

$$\frac{10 \times 2{,}240 \times 16 \times 5{,}084}{1{,}008 \times 1{,}027} = \frac{x \times 12{,}500}{12}$$

$$x = \frac{10 \times 2{,}240 \times 16 \times 5{,}084 \times 12}{1{,}008 \times 1{,}027 \times 12{,}500} = 1{\cdot}69 \text{ inches.}$$

The ship has a draught greater by 1·69 inches when in the river. Ans.

7. C.G. of bulkhead is $\frac{15}{3}$ = 5 feet below surface.

$$\text{Load} = H \times A \times w = \frac{5 \times 26 \times 15 \times 62{\cdot}5}{2 \times 2{,}240} = 27{\cdot}21 \text{ tons}$$

Ans.

The Centre of Pressure is at $\frac{15}{2}$ = 7·5 feet below the surface.

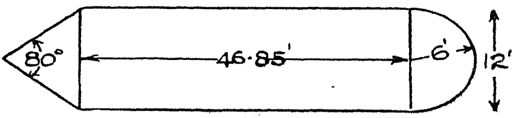
$$\text{C.G. above keel} = \frac{\text{Sum of moments of weights}}{\text{Final displacement}}$$

$$= \frac{(3{,}500 \times 15) - (480 \times 10) - (200 \times 17) + (350 \times 14)}{3{,}500 - 480 - 200 + 350} = 15{\cdot}52$$

C.G. is at 15·52 feet above the keel. Ans.

9.

$$\text{Length of bow section} = \frac{6}{\text{Tan. } 40^\circ} = \frac{6}{0{\cdot}8391} = 7{\cdot}15 \text{ ft.}$$



$$\text{Area of bow section} = \frac{7 \cdot 15 \times 12}{2} = 42 \cdot 9 \text{ square feet.}$$

$$\text{Area of middle section} = 46 \cdot 85 \times 12 = 562 \cdot 2 \text{ square feet.}$$

$$\text{Area of stern section} = \frac{3 \cdot 1416 \times 6^2}{2} = 56 \cdot 55 \text{ square ft.}$$

$$\text{Water plane area} = 661 \cdot 65 \text{ square feet.}$$

$$\text{Co-efficient of water plane area} = \frac{661 \cdot 65}{60 \times 12} = 0 \cdot 919. \quad \text{Ans.}$$

$$\text{Displacement} = \frac{661 \cdot 65 \times 5 \times 1{,}023}{16 \times 2{,}240} = 94 \cdot 36 \text{ tons. Ans.}$$

10. $$\frac{350 \times 40 \times k_1 \times \frac{1}{12}}{35} = \text{tons per inch at 1st draught.}$$

$$\frac{350 \times 40 \times k_2 \times \frac{1}{12}}{35} = \text{tons per inch at 2nd draught.}$$

$$\frac{350 \times 40 \times k_1 \times \frac{1}{12}}{35} \quad \frac{350 \times 40 \times k_2 \times \frac{1}{12}}{35} = 1$$

$$\frac{40 \times 10}{12} (k_1 -$$

$$\therefore \; k_1 - k_2 = \quad = 0 \cdot 03. \quad \text{Ans.}$$

11. Shift of C.G. due to moving 500 tons already aboard through 10 feet

$$\frac{\text{Moment of weight shifted}}{\text{Displacement}} \quad \frac{500 \times 10}{3{,}500} = \frac{10}{7} \text{ feet.}$$

$= 1\frac{3}{7}$ feet downwards.

500 tons are now put aboard at $10 + 1\frac{3}{7} = 11\frac{3}{7}$ feet above the C.G. of the ship, and the shift of the C.G.

$$\frac{\text{Moment}}{\text{Final displacement}} \quad \frac{500 \times 11\frac{3}{7}}{3{,}500 + 500} = \frac{10}{7} = 1\frac{3}{7} \text{ feet.}$$

The first operation lowers the C.G. through $1\frac{3}{7}$ feet; the second operation raises the C.G. through $1\frac{3}{7}$ feet, and the nett result is that the C.G. is not altered in position. The draught of the vessel will be increased, because the displacement is greater.

12. Let the alteration in draught be x feet.

$$1\cdot7L(D+x) + \frac{L \times B \times (D+x) \times k}{D + x} \quad (1\cdot7LD + \frac{L \times B \times D \times k}{D}$$

$$1\cdot7\,L\,(D + x) + (L \times B \times k) - 1\cdot7\,L\,D - (L \times B \qquad = 2{,}600$$

$$1\cdot7\ L\ (D + x) - 1\cdot7\ L\ D = 2{,}600$$

$$1\cdot7\ L\ (D + x - D) = 2{,}600$$

$$1\cdot7\ L \times x = 2{,}600$$

$$\frac{2{,}600}{1\cdot7 \times 670} = 2\cdot282 \text{ feet.} \quad \text{Ans.}$$

13. Let S and S — 2 be the speeds.

Consumption per voyage varies as Speed^2

$$= 1\cdot0\cdot0$$

(S

S

S — 2

$$\therefore\ S = 1\cdot132\ S - 2 \times 1\cdot132$$

$$\therefore\ S = \frac{2\cdot264}{0\cdot132} = 17\cdot15 \text{ knots.} \quad \text{Ans.}$$

Consumption per day varies as Speed^3

$$\frac{(17\cdot15)^3}{(15\cdot15)^3} \quad \frac{C}{C - 52} = 1\cdot451$$

$$\therefore\ C = 1\cdot451\ C - 52 \times 1\cdot451$$

$$\frac{52 \times 1\cdot451}{0\cdot451} = 167\cdot8 \text{ tons.} \quad \text{Ans.}$$

14. $$\text{Load on the sides} = \frac{18 \times 45 \times 18 \times 64}{2 \times 2{,}240} = 208{\cdot}3 \text{ tons}$$

$$\text{and } \frac{5 \times 45 \times 5 \times 64}{2 \times 2{,}240} = 16{\cdot}07 \text{ tons.}$$

Nett load $= 208{\cdot}3 - 16{\cdot}07 = 192{\cdot}23$ tons. Ans.

Let h = height of resulting centre of pressure from bottom of bulkhead.

By moments about bottom, $16{\cdot}07 \times \frac{5}{3} + 192{\cdot}23\,h = 208{\cdot}3 \times \frac{18}{3}$

$h = 6{\cdot}36$ feet from bottom. Ans.

15. Let D = original displacement in tons.

$$\text{Draught in river} = \frac{D \times 2{,}240 \times 16}{1{,}008 \times 13{,}000} \text{ feet.}$$

$$\text{Draught at sea} = \frac{(D - 500)\ 2{,}240 \times 16}{1{,}028 \times 13{,}000}$$

$$\frac{D \times 2{,}240 \times 16}{1{,}008 \times 13{,}000} - \frac{(D - 500)\ 2{,}240 \times 16}{1{,}028 \times 13{,}000} = 1\tfrac{3}{4}$$

$$\frac{2{,}240 \times 16}{13{,}000}\left(\frac{D}{1{,}008} - \frac{D - 500}{1{,}028}\right) =$$

$$\frac{1{,}028\,D - 1{,}008\,D + 500 \times 1{,}008}{1{,}008 \times 1{,}028} = \frac{7}{4} \times \frac{13{,}000}{2{,}240 \times 16}$$

$$20\ D = \frac{7 \times 13{,}000 \times 1{,}008 \times 1{,}028}{4 \times 2{,}240 \times 16} - 504{,}000$$

D = 7,680 tons. Ans.

16. Consumption per day varies as speed^3

$$\therefore \text{ Reduced consumption} = 21 \times \frac{16^3}{19^3} = 12{\cdot}53 \text{ tons.}$$ Ans.

Oil used $= 31 - 4 = 27$ tons.

$$\text{Distance from port} = \frac{27}{12{\cdot}53} \times 16 \times 24 = 827 \text{ miles.}$$ Ans.

17. Consumption per day varies as speed^3

Let x = normal speed in knots.

$= \frac{100}{60}$, $\quad$ 1·185

$(x - 3)^3 \qquad x - 3$

$\therefore x = 1{\cdot}185\ x - 3 \times 1{\cdot}185$

$x = 19{\cdot}22$ knots. Ans. (*c*)

Let y = distance between ports.

$y \qquad y$

$16{\cdot}22 \qquad 19{\cdot}22$

$\therefore 19{\cdot}22\ y - 16{\cdot}22\ y = 4{\cdot}5 \times 19{\cdot}22 \times 16{\cdot}22$

$$\therefore y = \frac{4{\cdot}5 \times 19{\cdot}22 \times 16{\cdot}22}{3}$$

$= 467{\cdot}4$ miles. Ans. (*a*)

Number of days at 19·22 knots $= \dfrac{467{\cdot}4}{19{\cdot}22 \times 24}$

$= 1{\cdot}014$ days.

Consumption for voyage varies as speed^2

$C \qquad (19{\cdot}22)^2$

$\qquad\qquad 1{\cdot}404$

$C - 12{\cdot}5 \qquad (16{\cdot}22)^2$

$\therefore C = 1{\cdot}404\ C - 12{\cdot}5 \times 1{\cdot}404$

$\therefore C = 43{\cdot}44$ tons per voyage

$$\therefore \text{Consumption per day} = \frac{43{\cdot}44}{1{\cdot}014} = 42{\cdot}87 \text{ tons. Ans. } (b)$$

Second part of question :—

$12\frac{1}{2}$ tons that remain must be reduction in consumption.

$$\therefore \text{Coal} = \frac{12{\cdot}5}{0{\cdot}4} = 31{\cdot}25 \text{ tons.}$$

Consumption for voyage varies as

Let x = speed in knots

$= \frac{100}{60}$

$(x - 3)^2 \qquad x - 3$

$\therefore x = 1{\cdot}291\ x - 3 \times 1{\cdot}291$

$\therefore x = 13{\cdot}31$ knots. Ans.

If y = distance in miles

$$\frac{y}{10\cdot31} - \frac{y}{13\cdot31} = 4\tfrac{1}{2}$$

$13\cdot31\ y - 10\cdot31\ y = 4\cdot5 \times 10\cdot31 \times 13\cdot31$

$y = 205\cdot7$ miles. Ans.

Days on voyage at full speed $= \dfrac{205\cdot7}{13\cdot31 \times 24} = 0\cdot643$ day.

Coal per day $= \dfrac{31\cdot25}{0\cdot643} = 48\cdot53$ tons. Ans.

18. Displacement $= \dfrac{480\cdot5 \times 48\cdot5 \times 24 \times 0\cdot72}{35}$

$= 11{,}500$ tons.

$$9{,}375 = \frac{(11{,}500)^{\frac{2}{3}} \times k^3}{275}$$

$\therefore\ 3 \log. k = \log. 9{,}375 + \log. 275 - \tfrac{2}{3} \log. 11{,}500$

$\therefore\ \log. k = \dfrac{3\cdot9719 + 2\cdot4393 - 2\cdot7071}{3}$ $\cdot2347$

$\therefore\ k = 17\cdot17$ knots. Ans.

19. Days at 17 knots $= \dfrac{1{,}300}{17 \times 24} = 3\cdot186$ days.

Days at reduced speed $= 3\cdot186 + 3\cdot5 = 6\cdot686$ days.

Reduced speed $= \dfrac{1{,}300}{6\cdot686 \times 24} = 8\cdot1$ knots.

Consumption at 17 knots $= 3\cdot186 \times 130 = 414\cdot18$ tons.

Consumption per voyage varies as speed^2

$\therefore$ Consumption $= 414\cdot18 \times \dfrac{8\cdot1^2}{17^2}$

$= 94\cdot06$ tons.

Coal remaining $= 500 - 94\cdot06 = 405\cdot9$ tons. Ans.

Coal per day $= 130 \times \dfrac{8\cdot1^3}{17^3} = 14\cdot07$ tons. Ans.

20. Thrust H.P. $= 2{,}000 \times 0{\cdot}87 \times 0{\cdot}7$

Load on thrust (lb.) × distance (feet per min.
Thrust H.P. × 33,000.

$$\therefore \text{Load} = \frac{2{,}000 \times 0{\cdot}87 \times 0{\cdot}7 \times 33{,}000}{16 \times 0{\cdot}9 \times 75{\cdot}3}$$

$= 37{,}070$ lb. Ans.

21. Let V = ship's speed in feet per min.

$$\text{Friction per sq. foot} = 0{\cdot}3 \times \frac{V^2}{600^2} \times 1{\cdot}026 \text{ lb.}$$

$$\text{Total friction} = 0{\cdot}3 \times \frac{V^2}{600^2} \times 1{\cdot}026 \times 20{,}000 \text{ lb.}$$

$$\therefore 0{\cdot}3 \frac{\quad}{600^2} \times 1{\cdot}026 \times 20{,}000 \times V = 1{,}500 \times 33{,}000 \times 0{\cdot}7$$

$$\therefore V = \frac{\times\ 0{\cdot}7 \times 600^2}{0{\cdot}3 \times 1{\cdot}026 \times 20{,}(}$$

$= 1{,}265$ feet per min. $= 12{\cdot}49$ knots. Ans.

22. For an engine driving a ship, horse power varies as speed³.
Horse power for 12 knots

$$= 19{,}500 \times \left(\frac{12}{17{\cdot}5}\right)^3 = 6{,}287.$$

$\therefore$ Horse power on each remaining shaft $= \frac{6287}{2} = 3143{\cdot}5.$

23.

$$\text{Displacement} = \frac{520 \times 55 \times 26 \times 0{\cdot}78}{35} = 16{,}570 \text{ tons.}$$

$$\text{I.H.P.} = \frac{(16{,}570)^{\frac{2}{3}} \times 14^3}{250} = 7{,}133. \quad \text{Ans.}$$

I.H.P. for 10 per cent. margin above $= 7{,}133 \times (1{\cdot}1)^3$
$= 9{,}495.$ Ans.

24.

$$17{\cdot}5 \text{ knots} = \frac{17{\cdot}5 \times 6080}{60} = 1773{\cdot}3 \text{ ft. per min.}$$

$= 29{\cdot}55$ feet per sec.

Let x lb. be the friction per sq. foot at this speed.

$x \times 33{,}000 \times 1773{\cdot}3 = 5{,}320 \times 33{,}000$

$$\therefore\ x = \frac{5{,}320}{1773{\cdot}3} = 3 \text{ lb.}$$

Friction per sq. foot at 10 ft. per sec.

$$= 3 \times \left(\frac{10}{29{\cdot}55}\right)^{1{\cdot}876}$$

$= 0{\cdot}393$ lb. per sq. foot. Ans.

Displacement $\propto$ volume $\propto$ length^3

$\therefore\ D \propto L^3$ or $L \propto D^{\frac{1}{3}}$

Surface $\propto$ Length^2

$\therefore\ S \propto L^2$ or $L \propto S^{\frac{1}{2}}$

$\therefore\ S^{\frac{1}{2}} \propto D^{\frac{1}{3}} \quad \therefore\ S \propto D^{\frac{2}{3}}$

$$D + 1{,}500)^{\frac{2}{3}} \qquad S_2$$

$$D$$

$$\frac{D + 1{,}500}{D} = (2)^{\frac{3}{2}} = 2{\cdot}828$$

$\therefore\ D + 1{,}500 = 2{\cdot}828$ D.

$\therefore\ 1{\cdot}828\ D = 1{,}500.$

$\therefore\ D = 820{\cdot}5$ tons. Ans.

26. To arrive in 96 hours, speed $= \frac{2040}{96} = 21{\cdot}25$ knots.

Consumption per day at this speed

$$= 260 \times \quad 21{\cdot}25)^{3{\cdot}2}$$

$= 315{\cdot}6$ tons.

Number of days $= 4 + 1 = 5$ days.

$\therefore$ Tons of fuel on board $= 5 \times 315{\cdot}6 = 1{,}578$ tons. Ans.

27. Total load on one side

$$= \frac{20}{2} \times 20 \times 47{\cdot}75 \times \frac{64}{2{,}240} = 272{\cdot}857 \text{ tons.}$$

Total load on other side

$$\frac{3{\cdot}5}{2} \times 3{\cdot}5 \times 47{\cdot}75 \times \frac{64}{2{,}240} = 8{\cdot}356 \text{ tons.}$$

$\therefore$ Nett load $= 272{\cdot}857 - 8{\cdot}356 = 264{\cdot}5$ tons. Ans.

Taking moments about bottom of bulkhead,

$$272{\cdot}857 \times \frac{20}{3} - 8{\cdot}356 \times \frac{3{\cdot}5}{3} = 264{\cdot}5\ x$$

$$1819{\cdot}17 - 9{\cdot}75 = 264{\cdot}5\,x$$

$$x = \frac{1809{\cdot}42}{264{\cdot}5} = 6{\cdot}84 \text{ feet.}$$

Resultant centre of pressure is at 6·84 feet above bottom. Ans.

28. See page 454 for definitions.

Disturbing moment = Restoring moment

$$20 \times 30 = \text{W.}\ \overline{\text{G M}} \sin \theta^\circ$$

$$\text{Now sin. } 3^\circ = 0{\cdot}0523, \text{ and } 3^\circ = \frac{3 \times 2\pi}{360} = 0{\cdot}05236 \text{ radian.}$$

The sine, tangent and circular measure of small angles are the same.

$$\therefore 600 = 6{,}000\ \overline{\text{G M}} \times 0{\cdot}0523$$

$$\overline{\text{G M}} = \frac{600}{6{,}000 \times 0{\cdot}0523} = \frac{1}{0{\cdot}523} = 1{\cdot}912 \text{ feet.} \quad \text{Ans.}$$

29. See pages 452 and 453 for definitions.

Volume displaced before bilging $= 200 \times 40 \times 10$ cu. feet.

The weight of the barge does not change, therefore the volume displaced after bilging must remain the same as before.

Intact water plane area afterwards

$$= (200 \times 40) - (30 \times 40 \times 0{\cdot}6)$$
$$= 8{,}000 - 720 = 7{,}280 \text{ sq. feet.}$$

Let x = new draught in feet.

Then $7{,}280\ x = 200 \times 40 \times 10$

$x = 10{\cdot}989$ feet, or 10 ft. 11·9 inches.

This is giving an answer to a degree of accuracy which would not be possible to measure on the barge itself. Also, it appears doubtful whether the loss of water plane area is 720 square feet, or whether it is $30 \times 40 = 1{,}200$ sq. ft. But the answer given is undoubtedly what is expected to the question.

30.

Areas	Simpson's Multipliers	Functions of areas
14000	1	14000
12800	4	51200
11300	2	22600
8700	4	34800
0	1	0

Sum = 122,600

$\frac{1}{3}$ common interval $= \dfrac{5{\cdot}25}{3}$ feet.

Displacement volume in fresh water $\dfrac{122{,}600 \times 5{\cdot}25}{3}$

$= 214{,}550$ cu. feet.

Tons displacement $\dfrac{214{,}550 \times 62{\cdot}5}{2{,}240}$

$= 5986{\cdot}33$ tons. Ans.

Displacement volume in sea water

$$= \frac{5986{\cdot}33 \times 2{,}240 \times 16}{1{,}028} = 208{,}706 \text{ cu. feet.}$$

Difference in volume displaced $= 214{,}550 - 208{,}706$
$= 5{,}844$ cu. feet.

For a small change in draught, caused by moving from fresh water to sea water, the water plane area may be considered to remain 14,000 sq. feet.

$\therefore$ Change of draught

$$= \frac{5{,}844}{14{,}000} = 0{\cdot}417 \text{ ft., or 5 inches.} \quad \text{Ans.}$$

31. $\overline{B\ M} = \dfrac{I}{V} = \dfrac{1{,}500{,}000}{6{,}500 \times 35} = 6{\cdot}59$ feet, i.e., the metacentre is 6·59 feet above the centre of buoyancy.

Since the metacentric height, i.e., $\overline{G\ M}$ is 4 feet

$\therefore \overline{B\ G} = 6{\cdot}59 - 4 = 2{\cdot}59$ feet.

Also the centre of buoyancy is 13 feet above the keel

$\therefore$ G is $13 + 2{\cdot}59 = 15{\cdot}59$ feet above keel. Ans.

32. Width of water plane at surface

$$= \frac{14}{18} \times 20 = \frac{140}{9} \text{ feet.}$$

$$\text{Area of wetted surface} = \frac{140 \times 14}{9 \times 2} \text{ sq. feet.}$$

Distance from free surface to C.G. of wetted bulkhead

$$= \frac{14}{3} \text{ ft.}$$

Total load on bulkhead $= H\ A\ w$

$$= \frac{14 \times 140 \times 14 \times 62{\cdot}5}{3 \times 9 \times 2 \times 2{,}240} = 14{\cdot}18 \text{ tons. Ans.}$$

A shore should be placed at the centre of pressure, which is at half the height of the wetted bulkhead = 7 feet above base. Ans.

33.

Ordinates	Simpson's Multipliers	Functions for Areas	Distances from Top	Functions for 1st Moments	Functions for 2nd Moments
16	1	16	0	0	0
15	4	60	3	180	540
13	2	26	6	156	936
8·5	4	34	9	306	2754
0	1	0	12	0	0
	Sum =	136		642	4230

Area $= \frac{3}{3} \times 136$ sq. feet.

1st Moments $= \frac{3}{3} \times 642 = 642$ feet units $\therefore H = \frac{642}{136}$ feet

$$P = H\ A\ w = \frac{642 \times 136 \times 64}{136 \times 2{,}240} = 18{\cdot}34 \text{ tons. Ans.}$$

2nd Moments $= \frac{3}{3} \times 4{,}230 = 4{,}230$ feet units.

$$\text{Centre of Pressure} = \frac{\text{2nd Moments}}{\text{1st Moments}} = \frac{4{,}230}{642}$$

$= 6{\cdot}588$ feet from top. Ans.

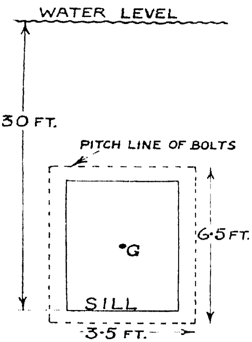
34.
$$\text{Displacement} = \frac{350 \times 45 \times 23 \times 0{\cdot}72}{35} = 7{,}452 \text{ tons.}$$
Ans. (*a*)

Area of immersed 'midship section $= 45 \times 23 \times 0{\cdot}83$ $= 859{\cdot}05$ sq. feet. Ans. (*b*)

$$\text{Prismatic coefficient} = \frac{7452 \times 35}{859{\cdot}05 \times 350} = 0{\cdot}8673. \text{ Ans. } (c)$$

or Prismatic coefficient
$$= \frac{\text{Block coefficient}}{\text{Coeff. of immersed 'midship section}} = \frac{0{\cdot}72}{0{\cdot}83} = 0{\cdot}8673.$$

35.



Centre of gravity of wetted area is at $30 - 3 = 27$ feet below water level.

Load on wetted area

$= 27 \times 3{\cdot}5 \times 6{\cdot}5 \times 64$ lb.

$$\text{Stress in bolts} = \frac{27 \times 3{\cdot}5 \times 6{\cdot}5 \times 64}{36 \times 0{\cdot}442}$$

$= 2{,}470$ lb. per sq. inch. Ans.

36. If the ship remained on an even keel, then the increase in draught would be $\frac{100}{40} = 2{\cdot}5$ inches.

Moment to cause change of trim $= 100 \times 80 = 8{,}000$ ft. tons.

Change of trim $= \frac{8000}{800} = 10$ inches.

Since the centre of flotation is at mid-length, the change of trim is divided equally between forward and aft.

Draught forward $= 23$ ft. 7 ins. $+ 2{\cdot}5$ ins. $- 5$ ins.
$= 23$ ft. $4{\cdot}5$ ins.

Ans.

Draught aft $= 23$ ft. 7 ins. $+ 2{\cdot}5$ ins. $+ 5$ ins.
$= 24$ ft. $2{\cdot}5$ ins.

37. Load on portion of bulkhead 3 feet wide
$= \frac{20}{2} \times 3 \times 20 \times 64 = 38{,}400$ lb.

Each stiffener carries 38,400 lb., since the support afforded by the bulkhead plate itself is disregarded.

The top carries $\frac{1}{3}$ of the total load, and the bottom the remainder.

Shearing force at top $= \frac{38400}{3} = 12{,}800$ lb.
Shearing force at bottom $= 25{,}600$ lb. } Ans.

The shearing force is zero at $\dfrac{l}{\sqrt{3}}$ from the top (see page 259)

The shearing force is zero at $\dfrac{20}{\sqrt{3}} = 11{\cdot}55$ feet from top. Ans.

38.

Tons per inch	Simpson's Multipliers	Functions of Areas	Multiplier for Moment	Functions of 1st Moments
30·2	1	30·2	0	0
29·6	4	118·4	3×1	$3 \times 118{\cdot}4$
28·2	2	56·4	3×2	$3 \times 112{\cdot}8$
25·5	4	102·0	3×3	$3 \times 306{\cdot}0$
21·2	2	42·4	3×4	$3 \times 169{\cdot}6$
13·6	4	54·4	3×5	$3 \times 272{\cdot}0$
0	1	0	3×6	0

Sum $= 403{\cdot}8$ $\qquad$ $3 \times 978{\cdot}8$

The displacement is the area enclosed by the tons per inch immersion curve.

Displacement $= \frac{3}{3} \times 403{\cdot}8 \times 12 = 4845{\cdot}6$ tons. Ans.
Centre of buoyancy below water line

$\frac{3}{3} \times 3 \times 978{\cdot}8 \times 12$

$$\frac{3 \times 978{\cdot}8}{403{\cdot}8} = 7{\cdot}27 \text{ feet.} \quad \text{Ans.}$$

39. Consumption $\propto$ (Speed)^3, but I.H.P. $\propto$ Consumption

$\therefore$ I.H.P. $\propto$ (Speed)^3, and $\dfrac{\text{I.H.P.}}{(\text{Speed})^3}$ = constant.

$$\frac{8,300}{14{\cdot}5^3} \qquad \frac{4,350}{S^3}$$

$$S = 14{\cdot}5 \sqrt[3]{\frac{4,350}{8,300}} = 11{\cdot}69 \text{ knots.} \quad \text{Ans.}$$

40. Parallel sinkage due to 100 tons $= \frac{100}{40} = 2{\cdot}5$ inches.

If 100 tons were put aboard above the centre of flotation, the draught would be increased by 2·5 inches.

The displacement of the vessel would be 8,000 + 100 = 8,100 tons.

Imagine, now, the 100 tons is moved through 80 feet.

Moment to cause change of trim = $100 \times 80 = 8,000$ ft. tons.

Assume the metacentric height to remain unchanged

then $8,000 = 8,100\ GM\ \theta$

$$\theta = \frac{8,000}{8,100 \times 410} = \frac{8}{3,321} \text{ radian.}$$

Distance from centre of flotation to forward
$= \frac{388}{2} + 12 = 206$ feet.

Distance from centre of flotation to aft
$= \frac{388}{2} - 12 = 182$ feet.

Change of draught forward due to angle of trim
$= 206 \times \frac{8}{3321} \times 12 = 5{\cdot}957$ inches.

Change of draught aft due to angle of trim
$= 182 \times \frac{8}{3321} \times 12 = 5{\cdot}263$ inches.

Draught forward = 23 ft. 7 ins. + 2·5 ins. − 5·96 ins.
= 23 ft. 3·54 ins.

Draught aft = 23 ft. 7 ins. + 2·5 ins. + 5·26 ins.
= 24 ft. 2·76 inches.

Ans.

41. The frictional resistance of the water upon the wetted surface of a ship's hull results in the surrounding envelope of water being dragged forward in the direction of motion of the ship. At the stern of the vessel there is a zone of water having a forward velocity, and this velocity is the "speed of the wake." It will be some fraction of the ship's speed, say x, and Dr. Froude assumed it to be 10%, or 0·1 of the ship's speed. The propeller works in water that has this forward velocity, and the speed of the propeller through the water is $V - x\,V = V\,(1 - x)$, where V is the ship's speed.

$$\text{Engine speed} = \frac{114 \times 16 \times 60}{6{,}080} = 18 \text{ knots.}$$

$$\text{Apparent slip} = 18 - 15{\cdot}5 = 2{\cdot}5 \text{ knots.}$$

$$\text{Apparent slip per cent.} = \frac{2{\cdot}5}{18} \times 100 = 13{\cdot}88\%. \quad \text{Ans.}$$

$$\text{Actual slip} = 18 - 15{\cdot}5\,(1 - 0{\cdot}1) = 4{\cdot}05 \text{ knots.}$$

$$\text{Actual slip per cent.} = \frac{4{\cdot}05}{18} \times 100 = 22{\cdot}5\%. \quad \text{Ans.}$$

42. During the first $8\frac{1}{2}$ hours the consumption is $(1{\cdot}225)^3$ times the normal amount.

Let x be the fraction of the normal speed during the remaining $(24 - 8\frac{1}{2}) = 15\frac{1}{2}$ hours.

$$\text{Then } \frac{\frac{1}{2}}{24} \times (1{\cdot}225)^3 + \frac{15\frac{1}{2}}{24} \times x^3 = 1$$

$$\therefore\ 0{\cdot}6513 + 0{\cdot}6458\,x^3 = 1$$

$$\therefore\ x = 0{\cdot}8143 \text{ of the normal speed.}$$

Distance = speed × time.

$$\therefore\ \frac{\text{[illegible]}}{24} \times 1{\cdot}225 + \frac{\text{[illegible]}}{24} \times 0{\cdot}8143 = 0{\cdot}9598 \text{ of normal distance.}$$

$\therefore$ Per cent. reduction in distance = 100 — 95·98 = 4·02. Ans.

43. Let S and (S — 2) knots be the speeds.

Let 100 be the consumption for the voyage at full speed.

Then 100 — 24 = 76 is the consumption at reduced speed.

Speed $\propto$ $\sqrt{\text{consumption}}$ per distance

$$\frac{\text{Speed}}{\sqrt{\text{consumption}}} = \text{constant}$$

$$\frac{S}{\sqrt{100}} = \frac{(S-2)}{\sqrt{76}}, \text{ and } \frac{S}{(S-2)} = \sqrt{\frac{100}{76}}$$

$$\frac{S}{(S-2)} = 1\cdot147, \quad 1\cdot147\ S - 2\cdot294 = S$$

$$0\cdot147\ S = 2\cdot294$$

$$\frac{2\cdot294}{0\cdot147} = 15\cdot61 \text{ knots. Ans.}$$

Let C be the consumption per day at full speed.

Then (C — 41) is the consumption at reduced speed.

Speed $\propto$ $\sqrt[3]{\text{consumption}}$ per unit of time

$$\frac{\text{Speed}}{\sqrt[3]{\text{consumption}}} = \text{constant}$$

$$\frac{15\cdot61}{\sqrt[3]{C}} = \frac{13\cdot61}{\sqrt[3]{C-41}}, \text{ cube each side}$$

$$\frac{(15\cdot61)^3}{C} = \frac{(13\cdot61)^3}{C-41}$$

$$\frac{C}{C-41} = \left(\frac{15\cdot61}{13\cdot61}\right)^3 = 1\cdot509$$

$$C = 1\cdot509\ C - 61\cdot84$$

$$C = \frac{61\cdot84}{0\cdot509} = 121\cdot5 \text{ tons per day. Ans.}$$

44. Total load on part of bulkhead $3\frac{1}{2}$ feet wide

$$= H \times A \times w$$

$$= 12 \times 24 \times 3\tfrac{1}{2} \times 64 = 64{,}512 \text{ lb.}$$

This load is carried by each stiffener, and the load varies from 0 at the top to a maximum at the bottom. The centre of pressure of the load is at $\frac{1}{3}$ height from the bottom.

$\therefore$ load at bottom $= \frac{2}{3}$ of $64{,}512 = 43{,}008$ lb.

43,008 lb. is the shearing force on 12 rivets securing the bracket to the tank top.

$\therefore$ Average shearing stress on each rivet

$$\frac{43{,}008}{12 \times 0{\cdot}875} = 4{,}096 \text{ lb. per sq. inch.} \quad \text{Ans.}$$

SOLUTIONS TO FIRST-CLASS EXAMINATION QUESTIONS.

ELECTRICITY

1. Resistance varies directly as length and inversely as sectional area of wire.

$R \times$

constant.

$3{\cdot}6 \times 0{\cdot}139^2 \quad 6{\cdot}08 \times D^2$

$1 \qquad 4$

$$D = \sqrt{\frac{3{\cdot}6 \times 0{\cdot}139^2 \times 4}{6{\cdot}08}} = 0{\cdot}214 \text{ inch. Ans.}$$

2. Energy supplied by dynamo $= 5 \times 0{\cdot}93 \times 746$ watts.

$$\text{Current given out by dynamo} = \frac{5 \times 0{\cdot}93 \times 746}{110} \text{ ampéres}$$

$$\text{Number of lamps} = \frac{5 \times 0{\cdot}93 \times 746}{110 \times 0{\cdot}29} = 108. \quad \text{Ans.}$$

3. Electrical energy consumed $= 150 \times 40$ watts.

$$\text{Equivalent horse power} = \frac{150 \times 40}{746}$$

$$\text{Horse power of engine} = \frac{150 \times 40 \times 100}{746 \times 92}$$

$= 8{\cdot}744$ H.P. Ans.

4. Output of dynamo $= 14 \times 0{\cdot}92 \times 746$ watts.
Candle power of lamps $= 230 \times 32$.

$$\therefore \text{Watts per candle power} = \frac{14 \times 0{\cdot}92 \times 746}{230 \times 32}$$

$= 1{\cdot}306$ watts. Ans.

5.

$$\text{Work done per minute} = \frac{360 \times 64 \times 88}{6{\cdot}25} \text{ ft.}$$

$$\text{H.P. of motor} = \frac{360 \times 64 \times 88}{6{\cdot}25 \times 0{\cdot}52 \times 33{,}000}$$

$$\text{Current} = \frac{360 \times 64 \times 88}{6{\cdot}25 \times 0{\cdot}52 \times 33{,}000} \quad \frac{746}{100}$$

$= 141$ Amps. Ans.

6.

$$\text{Current I} = \frac{110 \times 1{,}000}{110} = 1{,}000 \text{ amps.}$$

Equivalent resistance of 2 volt drop $=$ E

$= 0{\cdot}002$ ohm.

Total resistance $= 0{\cdot}0023 + 0{\cdot}00055 + 0{\cdot}002 = 0{\cdot}00485$ ohm.

Drop in voltage $=$ IR $= 1{,}000 \times 0{\cdot}00485 = 4{\cdot}85$ volts.

$\therefore$ Voltage in armature $= 110 + 4{\cdot}85 = 114{\cdot}85$ volts.

$$\text{Efficiency} = \frac{110}{114{\cdot}85} = 0{\cdot}9578 \text{ or } 95{\cdot}78\%. \quad \text{Ans.}$$

7. $122°$ F. $= (122 - 32)\frac{5}{9} = 50°$ C.

Conductivity at $15°$ C. $= 1 - 0{\cdot}003807 \times 15 + 0{\cdot}000009009 \times 15^2 = 0{\cdot}944922025$

Conductivity at $50°$C. $= 1 - 0{\cdot}003807 \times 50 + 0{\cdot}000009009 \times 50^2 = 0{\cdot}8321725$

$$\text{Resistance} \propto \frac{1}{\text{conductivity}} \times \text{length}$$

$$\text{R} \times \text{C} = \text{constant}$$

$$3{,}600 \text{ yards} ; \quad \frac{5{\cdot}6 \times 0{\cdot}94492}{1{,}000} \qquad \frac{\text{R} \times 0{\cdot}83217}{3{,}600}$$

$\therefore$ R = 22·896 ohms. Ans.

$$215 \text{ feet} ; \ \text{R} = \frac{22{\cdot}896 \times 215}{3{,}600 \times 3} = 0{\cdot}455 \text{ ohm. Ans.}$$

8.

$$\text{R}_0 \ (1 + 0{\cdot}00428 \times 30) \qquad 1{\cdot}1284$$

$$\text{R}_{15} \qquad \text{R}_0 \ (1 + 0{\cdot}00428 \times 15) \qquad 1{\cdot}0642$$

The voltage being the same in each case, current is inversely proportional to resistance.

$$\therefore \text{Current} = 2 \times \frac{1{\cdot}0642}{1{\cdot}1284} = 1{\cdot}\text{[illegible]} \quad \text{amps. Ans.}$$

9.

$$\text{Equivalent H.P.} = \frac{5{\cdot}8 \times 2{,}240 \times 100 \times 0{\cdot}45}{24 \times 2{,}545}$$

= 9·57. Ans.

$$\text{Current} = \frac{9{\cdot}57 \times 746}{220} = 32{\cdot}45 \text{ amps. Ans.}$$

$$\text{Kilowatts} = \frac{9{\cdot}57 \times 746}{1{,}000} = 7{\cdot}139. \text{ Ans.}$$

10.

$$\text{Resistance when hot} = \frac{65{\cdot}5 \times 16{\cdot}5}{(0{\cdot}004)^2 \times 0{\cdot}7854 \times 1{,}000{,}000} \times 5$$

= 429·9 ohms.

$$\text{Watts} = \frac{\text{E}^2}{\text{R}} \qquad \frac{110^2}{429{\cdot}9} = 28{\cdot}14$$

$$\therefore \text{C.P.} = \frac{28{\cdot}14}{0{\cdot}886} = 31{\cdot}76. \text{ Ans.}$$

11. Let I = current in amps.

Drop in E.M.F. in windings = I R = 0·15 I volts.

Useful E.M.F. = 220 — 0·15 I volts.

Power in watts = (220 — 0·15 I) I = 45 × 746.

$\therefore$ 0·15 I² — 220 I + 45 × 746 = 0.

Solving this quadratic, I = 1292·8 amps or 173·2 amps.

If larger value is taken efficiency of motor would be very small.

$\therefore$ I = 173·2 amps.

$$\text{Efficiency} = \frac{45 \times 746}{45 \times 746 + 173{\cdot}2^2 \times 0{\cdot}15} = 0{\cdot}8816. \quad \text{Ans.}$$

12.

$$\text{Heat given to water per second} = \frac{600 \times \text{[illegible]}}{60} \text{ calories.}$$

1 calorie = 4·19 watt secs.

$\therefore$ power = 4·19 × 50 = 209·5 watts.

$$\text{Power} = I^2 R \quad \therefore I = \sqrt{\frac{209{\cdot}5}{2{\cdot}5}}$$

= 9·154 amps. Ans.

13.

$$\text{Force on conductor} = \frac{B\ I\ l}{10} \text{ dynes.}$$

1 lb. force = 445,000 dynes.

$$\therefore \text{Force in lb.} = \frac{15{,}000 \times 100 \times 25}{10 \times 445{,}000} = 8{\cdot}426 \text{ lb.} \quad \text{Ans.}$$

14. Frequency is rapidity of cycles and is expressed as the number of cycles per second.

Frequency = Pairs of poles × Revs. per second.

$$= \frac{6 \times 1{,}000}{2 \times 60} = 50 \text{ cycles per second.} \quad \text{Ans.}$$

For the motor :—

$$50 = \frac{40 \times \text{Revs. per minute}}{2 \times 60}$$

$$\therefore \text{Speed of motor} = \frac{50 \times 2 \times 60}{40} = 150 \text{ revs. per min.}$$

15. $I^n t = K, \quad 20^n \times 8 = K, \quad 40^n \times 2{\cdot}6 = K$

$\therefore \; 40^n \times 2{\cdot}6 = 20^n \times 8$

$\therefore \; \left(\dfrac{40}{20}\right)^n = \dfrac{8}{2{\cdot}6}$

$\therefore \; 2^n = 3{\cdot}077$

$\log.\ 2 \times n = \log.\ 3{\cdot}077$

$\therefore \; n = \dfrac{\log.\ 3{\cdot}077}{\log.\ 2} \quad \dfrac{0{\cdot}4881}{0{\cdot}3010} = 1{\cdot}621$

$(40)^{1{\cdot}621} \times 2{\cdot}6 = (30)^{1{\cdot}621} \times t$

$\therefore \; t = (\frac{40}{30})^{1{\cdot}621} \times 2{\cdot}6 = 4{\cdot}146$

Battery would discharge for 4·146 hours. Ans.

16. Weight of deposit $= 39{\cdot}25 - 23{\cdot}8$
$= 15{\cdot}45$ grams.

Weight $= I \times t \times el.\ q.$

$15{\cdot}45 = I \times 30 \times 60 \times 0{\cdot}001118$

$$\frac{15{\cdot}45}{30 \times 60 \times 0{\cdot}001118} = 7{\cdot}68 \text{ amps.}$$

$\therefore$ Error of ammeter $= 7{\cdot}68 - 7{\cdot}3 = 0{\cdot}38$ ampére. Ans.

17. Flux cut per second by each conductor $= 100{,}000 \times 2 \times \frac{1440}{60}$
$= 4{\cdot}8 \times 10^6$ lines per second.

Total number of conductors $= 2 \times 480 = 960$, and all conductors are in series.

$\therefore$ Total E.M.F. $= 4{\cdot}8 \times 10^6 \times 960 \times 10^{-8}$ volts.
$= 46{\cdot}08$ volts. Ans.

18. Applied voltage = Volt drop due to resistance + back E.M.F.

$E = I\,R + e$

At no load : $100 = 1{\cdot}5 \times 0{\cdot}25 + e$

$\therefore \; e = 100 - 1{\cdot}5 \times 0{\cdot}25 = 99{\cdot}625$

Back E.M.F. at no load = 99·625 volts. Ans. (*a*)

At full load : $100 = 35 \times 0{\cdot}25 + e$

$\therefore \; e = 100 - 8{\cdot}75 = 91{\cdot}25$

Back E.M.F. at full load = 91·25 volts. Ans. (*b*)

Electrical effy. at full load $= \frac{91 \cdot 25}{100} = 91 \cdot 25\%$. Ans. (c)

Note that the above is purely the *electrical* efficiency.

Now consider the machine running light, i.e., no load, and therefore no useful work being done, then :—

E.M.F. equation : $E = I\,R + e$

Power equation : $E\,I = I^2\,R + e\,I$

$I^2\,R$ is the loss due to ohmic resistances of the motor ; $e\,I$ is the mechanical loss which is practically constant for any load, its value in this case being $99 \cdot 625 \times 1 \cdot 5$ watts.

At Full Load :—

$$\begin{aligned} \text{Total losses} &= I^2\,R + \text{mechanical losses} \\ &= 35^2 \times 0 \cdot 25 + 99 \cdot 625 \times 1 \cdot 5 \\ &= 455 \cdot 6875 \text{ watts.} \end{aligned}$$

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\text{Input} - \text{losses}}{\text{Input}}$$

$$= \frac{(100 \times 35) - 455 \cdot 6875}{(100 \times 35)} = 0 \cdot 8698 \text{ or } 86 \cdot 98\%.$$

19. See pages 463 and 467 for E.M.F. and Ohm's law. Volt drop due to internal resistance of dynamo $= I\,R = 75 \times 0 \cdot 015 = 1 \cdot 125$ volts.

Total volt drop is $105 - 100 = 5$ volts, therefore volt drop through the mains $= 5 - 1 \cdot 125 = 3 \cdot 875$ volts.

Resistance of mains

$$= \frac{E}{I} = \frac{3 \cdot 875}{75} = 0 \cdot 05167 \text{ ohm.} \quad \text{Ans.}$$

20. $\frac{R_1}{R_2} = \frac{R_0\,(1 + a\,t_1)}{R_0\,(1 + a\,t_2)}$ $R_1 = \frac{E_1}{I_1}$ and $R_2 = \frac{E_2}{I_2}$

$\frac{E_1 \times I_2}{I_1 \times E_2} = \frac{1 + a\,t_1}{1 + a\,t_2}$, E remains constant and therefore cancels.

$$\frac{I_2}{I_1} = \frac{1 + a\,t_1}{1 + a\,t_2} \qquad 1 + a\,t_2 = \frac{I_1}{I_2} \times (1 + a\,t_1)$$

$$1 + 0 \cdot 00428 \times t_2 = \frac{3}{2 \cdot 8} \times (1 + 0 \cdot 00428 \times 20)$$

$= 1 \cdot 163$

$0 \cdot 00428 \times t_2 = 1 \cdot 163 - 1$

$$\frac{0{\cdot}163}{0{\cdot}00428} = 38{\cdot}12^\circ \text{ C.}$$

Increase in temperature $= 38{\cdot}12 - 20 = 18{\cdot}12$ C°. Ans.

21. Volt drop through resistance $= I\ R$
$= 50 \times 0{\cdot}25 = 12{\cdot}5$ volts.

$\therefore$ Power lost $= E \times I = 12{\cdot}5 \times 50 = 625$ watts.

Assuming total resistance of the circuit to remain unaltered, then current flowing when supply is increased from 150 to 240

$$= \frac{240}{150} \times 50 = 80 \text{ amps.}$$

Volt drop through resistance will now be $80 \times 0{\cdot}25$ $= 20$ volts and power lost $= 20 \times 80 = 1{,}600$ watts.

Ratio of power lost for first and second cases $= 625 : 1600$, and this is also the ratio of heat generated.

$$\left.\begin{aligned} \therefore \text{ Ratio} = 625 : 1{,}600 = 1 : \tfrac{1600}{625} &= 1 : 2{\cdot}56 \\ \text{or } \tfrac{625}{1600} \quad 1 &= 0{\cdot}3906 : 1 \end{aligned}\right\} \text{Ans.}$$

22. E.M.F. $= \Phi\ Z\ n \times 10^{-8}$

$$= \frac{2{\cdot}5 \times 10^6 \times 420 \times 600}{10^8 \times 60} = 105 \text{ volts. Ans.}$$

23. Length of 1st wire $= 50 \times 100$ centimetres.
Length of 2nd wire $= 90 \times 12 \times 2{\cdot}54$ centimetres.
Let a_1 = area of cross section in sq. cms. of 1st wire and a_2 = area of cross section in sq. cms. of 2nd wire.
Let w = weight in grams per cu. cm.
Weight of 1st wire $= 50 \times 100 \times a_1 \times w = 100$ grams.

$$_1 \quad \frac{100}{50 \times 100 \times w}$$

Weight of 2nd wire $= 90 \times 12 \times 2{\cdot}54 \times a_2 \times w$

$$\frac{2 \times 1{,}000}{16 \times 2{\cdot}2} \text{ grams.}$$

$$\frac{2 \times 1{,}000}{90 \times 12 \times 2{\cdot}54 \times w \times 16 \times 2{\cdot}2}$$

$$R = \frac{s\,l}{a} \quad \therefore \text{ Ratio of resistances } = \frac{l_1}{a_1} : \frac{l_2}{a_2}$$

$$\frac{50 \times 100 \times 50 \times 100 \times w}{100} \qquad \frac{90 \times 12 \times 2{\cdot}54 \times 90 \times 12 \times 2{\cdot}54 \times w \times 16 \times 2{\cdot}2}{2 \times 1000}$$

$= 2{\cdot}50 : 1{\cdot}324$
$= 1{\cdot}888 : 1.$ Ans.

24.

$$\text{B.H.P. of motor} \times 0{\cdot}6 = \frac{2 \times 2{,}240 \times 4 \times 60}{33{,}000 \times 12}$$

$$\therefore \text{ B.H.P.} = \frac{2 \times 2{,}240 \times 4 \times 60}{33{,}000 \times 12 \times 0{\cdot}6} = 4{\cdot}525. \quad \text{Ans.}$$

Power supplied to motor $= 4{\cdot}525 \times \frac{100}{84} \times 746$ watts

$$\therefore \text{ current taken} = \frac{4{\cdot}525 \times 746}{220 \times 0{\cdot}84}$$

$= 18{\cdot}27$ ampéres. Ans.

25. Each lamp takes 60 watts for 40 candle power, which is $\frac{60}{40} = 1{\cdot}5$ watts per candle power. Ans. (*a*)
Current for each lamp

$$= \frac{\text{Watts}}{\text{Volts}} = \frac{60}{110} = 0{\cdot}\dot{5}\dot{4} \text{ amp.} \quad \text{Ans. } (b)$$

Power supplied to lamps $= 350 \times 60 = 21{,}000$ watts.
Power supplied to motor $= 15 \times \frac{100}{80} \times 746 = 13987{\cdot}5$ wts.
Total power supplied $= 21{,}000 + 13987{\cdot}5 = 34987{\cdot}5$ watts

$$= \frac{34987{\cdot}5}{746} \text{ horse power} = 46{\cdot}9$$

I.H.P. of engine allowing for an overload of 10% =
$46{\cdot}9 \times \frac{110}{100} \times \frac{100}{89} \times \frac{100}{85} = 68{\cdot}2$ I.H.P. Ans. (*c*)
Current through conducting wires allowing for the overload

$$\text{of } 10\% = \frac{34987{\cdot}5}{110} \times \frac{110}{100} = 349{\cdot}875 \text{ amps.}$$

$$\text{Cross section area} = \frac{349 \cdot 875}{1{,}000} = 0 \cdot 349875 \text{ sq. inch.}$$

$$\therefore \text{ Diameter} = \sqrt{\frac{0 \cdot 349875}{0 \cdot 7854}} = 0 \cdot 667 \text{ inch. Ans. } (d)$$

26. Flux cut per second by each conductor
$= 1{,}440{,}000 \times 2 \times \frac{1400}{60}$ lines per sec.
$\therefore$ E.M.F. induced in each conductor

$$= \frac{1{,}440{,}000 \times 2 \times 1{,}400}{10^8 \times 60} = 0 \cdot 672 \text{ volt. Ans.}$$

27. Reactance $(x) = 2 \pi f L$
$= 2 \times \pi \times 50 \times 0 \cdot 04 = 12 \cdot 57$ ohms.

Impedance $(Z) = \sqrt{R^2 + x^2}$
$= \sqrt{12^2 + (12 \cdot 57)^2} = 17 \cdot 38$ ohms. Ans.

28.

$$\text{Current supplied} = \frac{40 \times 60}{110} \text{ amps.}$$

$$\text{Resistance of 2 cables} = \frac{1 \cdot 2 \times 165 \times 2}{3 \times 1{,}000} \text{ ohms.}$$

Volt drop through cables $= I R$

$$\frac{40 \times 60 \times 1 \cdot 2 \times 165 \times 2}{110 \times 3 \times 1{,}000}$$

$= 2 \cdot 88$ volts.

Reading of voltmeter at switchboard should be
$110 + 2 \cdot 88 = 112 \cdot 88$ volts. Ans.

29. E.M.F. $= \Phi Z n \times 10^{-8}$

$$\therefore Z = \frac{220 \times 10^8 \times 60}{5 \cdot 4 \times 10^6 \times 400} \quad 611 \cdot 1$$

$\dfrac{\text{Number of conductors}}{\text{Number of poles}}$ must be an integer, therefore,

Number of conductors $= 612$. Ans.

30. From the direct current values :—

$$\therefore \quad \text{Resistance} = \frac{60^2}{300} = 12 \text{ ohms.}$$

From the alternating current values :—
The question does not state whether the 1,200 watts is the apparent power, or the true power. If it is taken as the apparent power, then,

$$E \times I = 1{,}200 \quad \therefore E \times \frac{E}{Z} = 1{,}200$$

$$\therefore Z \text{ (the impedance)} = \frac{130^2}{1{,}200} = 14{\cdot}08 \text{ ohms.}$$

$$Z^2 = R^2 + x^2 \quad \therefore x = \sqrt{Z^2 - R^2}$$

$$\therefore x \text{ (the reactance)} = \sqrt{(14{\cdot}08)^2 - 12^2} = 7{\cdot}36 \text{ ohms.}$$

Ans.

If 1,200 watts is taken as the true power, then,

$$1{,}200 \text{ watts} = \text{true power} = I^2 R \text{ watts}$$

$$I = \sqrt{\tfrac{1200}{12}} = 10 \text{ amps.}$$

$$Z = \frac{E}{I} = \frac{130}{10} = 13 \text{ ohms.}$$

$$x = \sqrt{Z^2 - R^2} = \sqrt{13^2 - 12^2} = 5 \text{ ohms.} \quad \text{Ans.}$$

or, alternatively,

$$E \times (I \times \text{power factor}) = 1{,}200$$

$$\text{power factor} = \frac{R}{Z}, \quad \text{and } I = \frac{E}{Z}$$

$$\therefore E \times \frac{E}{Z} \times \frac{R}{Z} = 1{,}200$$

13

$$\therefore x = \sqrt{13^2 - 12^2} = 5 \text{ ohms, as before.}$$

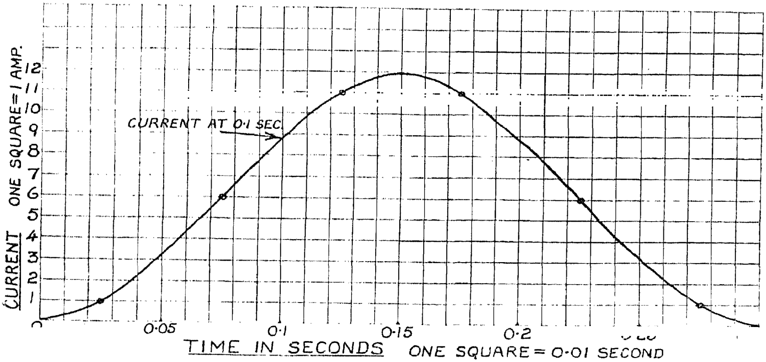
The student might note here, for his own good, that

$$\text{Power Factor} = \frac{R}{Z} = \frac{12}{13} = 0{\cdot}923$$

$$\text{Cos. Angle of lag } (\theta) = 0{\cdot}923$$

$$\therefore \theta = 22^\circ\, 38'.$$

31. The Root Mean Square value of an alternating current is equal to the number of ampéres of direct current which, if it replaced the alternating current, would have the same heating effect. (See page 484).



The current at 0·1 second is marked on the graph, and reads 8·8 ampéres.

The heating effect varies with the square of the current ; therefore to obtain the mean heating effect of this current, values at regular intervals of time are taken from the graph, these values are squared and put through Simpson's rule :

Time in Sec.	Values of I^2	Simpson's Multiplier	Functions of I^2
0	0^2	1	0
0·05	$3{\cdot}2^2$	4	40·96
0·1	$8{\cdot}8^2$	2	154·88
0·15	12^2	4	576
0·2	$8{\cdot}8^2$	2	154·88
0·25	$3{\cdot}2^2$	4	40·96
0·3	0^2	1	0

Sum = 967·68

Total multipliers =

Mean $I^2 = \dfrac{967{\cdot}68}{18}$

$= 53{\cdot}76$

Root of mean I^2

$= 7{\cdot}33$

$\therefore$ R.M.S. value

$= 7{\cdot}33$ ampéres. Ans.

32. Let d = diameter of fuse in centimetres, and l = its length in centimetres.

Surface area of fuse $= \pi d \times l$ sq. cms.

Rise of temperature $= 327 - 17 = 310$ C°.

Heat lost per sec. from the surface

$$= \frac{\pi d \times l \times 310}{4{,}000} \text{ calories}$$

Resistance of fuse $= \frac{s\ l}{} = \frac{45}{10^6}$ ohms.

Power lost $= I^2 \times R$

$$\frac{7^2 \times 45 \times l}{10^6 \times \frac{\pi}{4} \times} \text{ watts.}$$

Energy lost per sec.

$$= \frac{7^2 \times 45 \times l}{10^6 \times \frac{\pi}{4} \times d^2} \times 1 \text{ J}$$

4·19 Joules = 1 calorie

∴ Heat generated

$$\frac{7^2 \times 45 \times l}{10^6 \times \quad \times d^2 \times 4{\cdot}19} \text{ calories per sec.}$$

When the heat generated is equal to the loss of heat from the surface of the fuse, then the fuse will blow.

$$\frac{7^2 \times 45 \times}{10^6 \times \frac{\pi}{4} \times d^2 \times 4{\cdot}19} \qquad 4{,}000$$

$$\sqrt[3]{\frac{7^2 \qquad 4}{10^6 \times \pi^2 \times 4{\cdot}19 \times 310}}$$

$= 0{\cdot}1402$ c.m.

$= 1{\cdot}402$ m.m. Ans.

33. (Impedance)^2 = (Resistance)^2 + (Reactance)^2

$$Z^2 = R^2 + x^2$$

$\therefore$ Impedance $= \sqrt{3^2 + 4^2} = 5$ ohms.

Current flowing $= \dfrac{100}{5} = 20$ ampéres. Ans. (*a*)

Active E.M.F. $= \text{I R} = 20 \times 3 = 60$ volts. Ans. (*b*)

Inductive E.M.F. $= \text{I } x = 20 \times 4 = 80$ volts. Ans. (*c*)

34. Power given out by heater element

$$= \text{I}^2 \text{ R} = \text{I}^2 \times 16 \text{ watts}$$

Energy given out by heater element

$$= \text{I}^2 \text{ R} \times \text{time}$$

$$= \text{I}^2 \times 16 \times 600 \text{ watt seconds.}$$

4·19 watt seconds, or Joules = 1 calorie

$\therefore$ heat given out by heater

$$= \frac{\text{I}^2 \times 16 \times 600}{4 \cdot 19} \text{ calories.}$$

1 litre = 1,000 grams. Rise in temp. = (100 — 15) = 85 C°

$\therefore$ heat given to water = 1,000 × 85 calories.

As no efficiency of the heater is given it is assumed to be 100%.

Heat given out by heater = Heat given to the water

$$\frac{\text{I}^2 \times 16 \times 600}{4 \cdot 19} = 1{,}000 \times 85$$

$$\sqrt{\frac{4 \cdot 19 \times 1{,}000 \times 85}{16 \times 600}} \quad 6 \cdot 09 \text{ ampéres.} \quad \text{Ans.}$$

35. Volts lost due to internal resistance = I × R

$$= 75 \times 0 \cdot 015 = 1 \cdot 125 \text{ volts.}$$

Volts lost through mains = (106 — 100) 1·125

$$= 4 \cdot 875 \text{ volts.}$$

$$\text{Resistance of mains} = \frac{4{\cdot}875}{75} = 0{\cdot}065 \text{ ohm.} \quad \text{Ans.}$$

Equivalent horse power lost

$$= \frac{4{\cdot}875 \times 75}{746} = 0{\cdot}4901. \quad \text{Ans.}$$

36. $$\text{Force per conductor} = \frac{\mathrm{B\,I}\,l}{10} \text{ dynes}$$

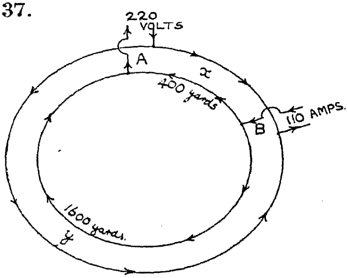
$$= \frac{3300 \times 20 \times 16 \times 2{\cdot}54}{10} \text{ dynes.}$$

Total force on all conductors

$$= \frac{3300 \times 20 \times 16 \times 2{\cdot}54 \times 246}{10} \times \frac{2{\cdot}2}{981 \times 1000}$$

$= 147{\cdot}6$ lb. Ans.

37.



Total length of mains
$= 2000 \times 2$
$= 4000$ yards.

Resistance of mains

$$\frac{0{\cdot}032 \times 4000}{1000}$$

$= 0{\cdot}128$ ohm.

Resistance of mains is proportional to the length.

$\therefore$ Resistance of mains from A to B in the direction of x
$= \frac{1}{5}$ of $0{\cdot}128$ ohm.

and resistance of mains from A to B in the direction of y
$= \frac{4}{5}$ of $0{\cdot}128$ ohm.

Now the current passing is inversely proportional to the resistance,

$\therefore$ current in x is four times current in y

Current supplied to B, via $x = \frac{4}{5}$ of $110 = 88$ amps. Ans.

Current supplied to B, via $y = \frac{1}{5}$ of 110 = 22 amps. Ans.

Voltage drop from A to B, via x

$$= \text{I R} = 88 \times \frac{0{\cdot}128}{5} = 2{\cdot}2528 \text{ volts.}$$

Voltage drop from A to B, via y

$$= \text{I R} = \frac{22 \times 4 \times 0{\cdot}128}{} = 2{\cdot}2528 \text{ volts.}$$

$\therefore$ voltage at B = 220 — 2·2528 = 217·7472 volts. Ans.

38. Voltage across shunt = 150 volts.

$$\text{Current through shunt} = \frac{\text{E}}{\text{R}} = \frac{150}{30} = 5 \text{ amps.}$$

Current through armature conductors = 75 + 5 = 80 amps.

Voltage drop across armature = I × R = 80 × 0·06 = 4·8 volts.

Internal voltage = 150 + 4·8 = 154·8 volts.

Input power = 154·8 × 80 watts.

Output power = 150 × 75 watts.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} \quad \frac{150 \times 75}{154{\cdot}8 \times 80} = 0{\cdot}9086, \text{ or } 90{\cdot}86\%. \quad \text{Ans.}$$

39. Heat generated at brake per hour

$$= \frac{748 \times 10 \times 10}{0{\cdot}85} \text{ centigrade heat units.}$$

Heat generated at brake per hour

$$\frac{748 \times 10 \times 10}{0{\cdot}85} \times \tfrac{9}{5} \text{ B.T.U.}$$

One horse power hour = 2545 B.T.U.

$\therefore$ Equivalent horse power

$$= \frac{748 \times 10 \times 10 \times 9}{0{\cdot}85 \times 5 \times 2545} = 62{\cdot}23$$

$$\text{Power output} = \frac{62{\cdot}23 \times 746}{1000} = 46{\cdot}42 \text{ kilowatts.}$$

Ans. (*a*)

$$\text{Power input} = \frac{110 \times 500}{1000} = 55 \text{ kilowatts.}$$

$$\therefore \text{Efficiency} = \frac{46{\cdot}42}{55} = 0{\cdot}844, \text{ or } 84{\cdot}4\%. \quad \text{Ans.}$$

40. E.C.E. of copper = $31{\cdot}8 \times 0{\cdot}00001044$

Weight of deposit = $I \times t \times$ E.C.E.

$0{\cdot}49 = I \times 30 \times 60 \times 31{\cdot}8 \times 0{\cdot}00001044$

$$I = \frac{0{\cdot}49}{30 \times 60 \times 31{\cdot}8 \times 0{\cdot}00001044} = 0{\cdot}82 \text{ an}$$

Error of ammeter = $0{\cdot}82 - 0{\cdot}8 = 0{\cdot}02$ a

The error is upon the correct reading of 0·82 amp.

$$\therefore \text{Error per cent.} = \frac{0{\cdot}02}{0{\cdot}82} \times 100 = 2{\cdot}439\%. \quad \text{Ans.}$$

41. Horse power output

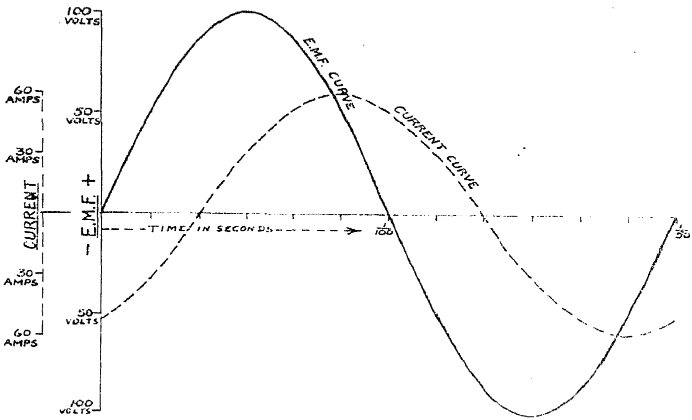
$$= \frac{\text{Torque (ft. lb.)} \times 2\,\pi \times \text{revs. per minute}}{33{,}000}$$

$$= \frac{144 \times 2\,\pi \times 400}{33{,}000}$$

$$\text{Horse power input} = \frac{10{\cdot}965}{0{\cdot}9} = 12{\cdot}19$$

$$\text{Current taken} = \frac{\text{watts}}{\text{volts}} = \frac{12{\cdot}19 \times 746}{500} = 18{\cdot}18 \text{ ampéres. Ans.}$$

42. See page 483 for method of constructing sinusoidal curves.



43. 1 pint $= \frac{5}{4}$ lb. $= \dfrac{5 \times 1000}{4 \times 2\cdot2}$ grams.

Heat to be supplied to the water =

$$\frac{5 \times 1000}{4 \times 2\cdot2} \times (100 - 15) \text{ gram calories.}$$

Current taken by heater $= \dfrac{E}{R}$

Energy dissipated per sec. $= I^2 R = (\frac{8}{5})^2 \times 150 = 384$ watt seconds.

4·19 watt sec. = 1 gram calorie.

Heat dissipated per sec. $= \dfrac{384}{4\cdot19}$ gram calories.

Heat absorbed per sec. $= \dfrac{384}{4\cdot19} \times 0\cdot8$ gram calories

$$\therefore \text{ time} = \frac{5 \times 1000 \times 85}{4 \times 2\cdot2} \quad \frac{384 \times 0\cdot8}{4\cdot19}$$

= 658·8 seconds.
= 10 minutes 58·8 secs. Ans.

44. Total power input $= 13 \cdot 5 \times 1000 = 13500$ watts.

Total power output $= 100 \times 110 = 11000$ watts.

Total loss of power $= 13500 - 11000 = 2500$ watts.

Power lost due to internal resistance $= I^2 R$
$= 100^2 \times 0 \cdot 04 = 400$ watts.

Power lost due to friction, windage, &c. $= 2500 - 400$
$= 2100$ watts.

$$\text{Electrical efficiency} = \frac{11000}{11000 + 400}$$

$= 0 \cdot 9649$, or $96 \cdot 49\%$. Ans. (*a*)

$$\text{Mechanical efficiency} = \frac{13500 - 2100}{13500}$$

$= 0 \cdot 8445$, or $84 \cdot 45\%$. Ans. (*b*)

$$\text{Commercial efficiency} = \frac{11000}{13500}$$

$= 0 \cdot 8149$, or $81 \cdot 49\%$. Ans. (*c*)

or, commercial effy. = Electrical effy. × Mechanical effy.
$= 0 \cdot 9649 \times 0 \cdot 8445 = 0 \cdot 8149$.

45. Total E.M.F. $= 1 \cdot 4 \times 3 = 4 \cdot 2$ volts.

Total resistance $= (3 \times 1) + 5 = 8$ ohms.

$$\therefore \text{Current flowing} = \frac{E}{R} = \frac{4 \cdot 2}{8} = 0 \cdot 525 \text{ amp.}$$

Volts lost in middle battery $= I\,R = 0 \cdot 525 \times 1 = 0 \cdot 525$ volt.

P.D. across its terminals = E.M.F. generated − E.M.F. lost.

P.D. across its terminals $= 1 \cdot 4 - 0 \cdot 525 = 0 \cdot 875$ volt. Ans. (*a*).

P.D. across external resistance $= I\,R = 0 \cdot 525 \times 5$
$= 2 \cdot 625$ volts. Ans. (*b*)

46.

$$\text{Useful horse power} = \frac{5{\cdot}5 \times 2240 \times 60}{3{\cdot}5 \times 33000} = 6{\cdot}4.$$

$$\text{Horse power supplied to the motor} = \frac{220 \times 30}{746}$$

$$= 8{\cdot}847.$$

Combined efficiency of motor and winch

$$= \frac{6{\cdot}4}{8{\cdot}847} = 0{\cdot}7235$$

$0{\cdot}7235$ = Effy. of motor × Effy. of winch.

$$\therefore \text{ Effy. of winch} = \frac{0{\cdot}7235}{0{\cdot}93} = 0{\cdot}7779, \text{ or } 77{\cdot}79\%.$$ Ans.

47 $$\text{Useful horse power of each motor} = \frac{2{\cdot}5 \times 2240 \times 6}{33000}$$

and this is 25%, or $\frac{1}{4}$ of the horse power supplied.

∴ horse power supplied to each motor

$$= \frac{2{\cdot}5 \times 2240 \times 6}{33000} \times \frac{4}{1}$$

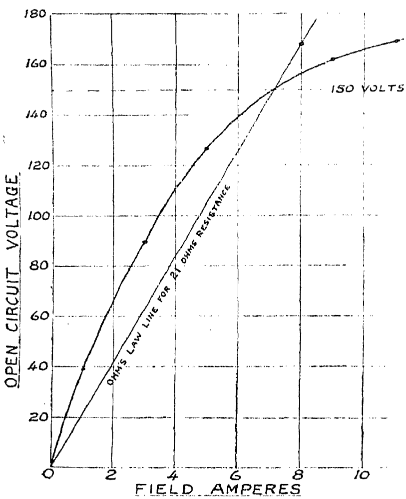
$$= 4{\cdot}073. \quad \text{Ans. } (a)$$

Watts supplied to each motor $= 4{\cdot}073 \times 746$.

$$\therefore \text{ Current supplied} = \frac{4{\cdot}073 \times 746}{220} = 13{\cdot}81 \text{ amps.}$$ Ans. (*b*)

Circuit breakers may be adjusted to allow for 75% overload. If each unit has its own circuit breaker, then it could be set to open at $13{\cdot}81 \times \frac{175}{100} = 24{\cdot}17$ amps.

48.



The relation between the open circuit voltage (which is the generated E.M.F.) and the field ampéres (which is the current passing through the shunt windings causing excitation of the field magnets) is shown by the characteristic curve plotted as in the graph. When the shunt resistance is 21 ohms, the relation between the volt drop across the shunt windings and the current flowing through is 21. For instance, 168 volts ÷ 8 amps. = 21 ohms. or 147 volts ÷ 7 amps, = 21 ohms, and so on. Taking any such ratio (say 168 volts and 8 amps) a point is plotted on the graph and a straight line drawn through it from zero. This is ohm's law line for 21 ohms.

The two graphs intersect at a point where the open circuit voltage is 150. Here, then, because the E.M.F. generated is 150 volts and the required volt drop across the shunt is 150, is a fixed condition of equality.

With the external circuit switched in, the terminal voltage is given as 140, so now there will be 140 volts across the shunt and the current flowing through the shunt will be $140 \div 21 = 6\frac{2}{3}$ ampéres. Referring to the graph, when $6\frac{2}{3}$ ampéres pass through the shunt windings, the generated E.M.F. is 146 volts, hence there must be $146 - 140 = 6$ volts drop across the armature windings. The current through the armature will be 300 ampéres, this being 6 volts ÷ 0·02 ohm. ; of this, $6\frac{2}{3}$ ampéres pass around the shunt, and the remainder, which is $300 - 6\frac{2}{3} = 293\frac{1}{3}$ ampéres pass to the external circuit.

$$\left.\begin{array}{ll}\text{Open circuit voltage} & = 150 \\ \text{Ampére output} & = 293\tfrac{1}{3}\end{array}\right\} \text{Ans.}$$

49. At a temperature of 20°C., equivalent resistance of ammeter and shunt (in parallel) :—

$$\frac{1}{R} = \frac{1}{2} + \frac{1}{0{\cdot}0015} = \frac{0{\cdot}0015 + 2}{2 \times 0{\cdot}0015} = \frac{2{\cdot}0015}{2 \times 0{\cdot}0015}.$$

$$\therefore\ R = \frac{2 \times 0{\cdot}0015}{2{\cdot}0015} \text{ ohms.}$$

Volt drop across ammeter and shunt $= I\,R$

$$= 200 \times \frac{2 \times 0{\cdot}0015}{2{\cdot}0015}$$

Current through ammeter only

$$= \frac{E}{R} = \frac{200 \times 2 \times 0{\cdot}0015}{2{\cdot}0015 \times 2} \text{ amps.} \qquad \text{(i)}$$

Resistance of ammeter when temp. is 40°C. :—

$$\frac{R_1}{R_2} \quad \frac{1 + a\,t_1}{1 + a\,t_2} \qquad \frac{2 \times (1 + 0{\cdot}00428 \times 40)}{(1 + 0{\cdot}00428 \times 20)}$$

$$= 2{\cdot}158 \text{ ohms.}$$

Equivalent resistance of ammeter and shunt when at 40°C. :—

$$\frac{1}{R} \quad \frac{1}{2{\cdot}158} \quad \frac{1}{0{\cdot}0015} \quad \frac{2{\cdot}1595}{2{\cdot}158 \times 0{\cdot}0015}$$

$$\therefore\ R = \frac{2{\cdot}158 \times 0{\cdot}0015}{2{\cdot}1595} \text{ ohms.}$$

Volt drop across ammeter and shunt

$$\frac{200 \times 2{\cdot}158 \times 0{\cdot}0015}{2{\cdot}1595}$$

Current through ammeter only

$$\frac{200 \times 2{\cdot}158 \times 0{\cdot}0015}{2{\cdot}1595 \times 2{\cdot}158} \text{ amps.} \qquad \text{(ii)}$$

Difference between current through ammeter at 20°C. and that through at 40°C. :—

$$\frac{200 \times 0{\cdot}0015}{2{\cdot}0015} - \frac{200 \times 0{\cdot}0015}{2{\cdot}1595}$$

$$= 200 \times 0{\cdot}0015 \left(\frac{1}{2{\cdot}0015} - \frac{1}{2{\cdot}1595}\right)$$

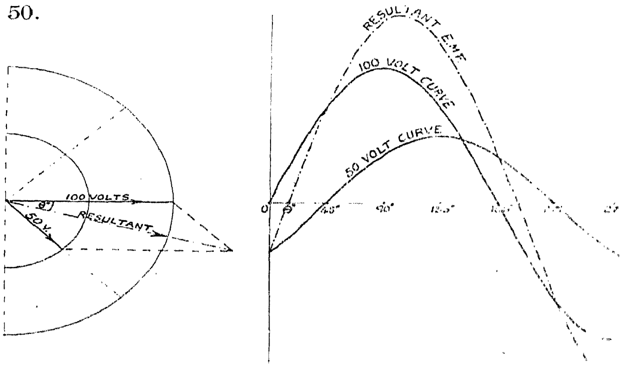
$$= \frac{200 \times 0{\cdot}0015 \times 0{\cdot}158}{2{\cdot}0015 \times 2{\cdot}1595} \text{ amps.}$$

Percentage difference compared with original current

$$\frac{200 \times 0{\cdot}0015 \times 0{\cdot}158 \times 2{\cdot}0015 \times 100}{2{\cdot}0015 \times 2{\cdot}1595 \times 200 \times 0{\cdot}0015}$$

$$\frac{0{\cdot}158 \times 100}{2{\cdot}1595} = 7{\cdot}317\%. \quad \text{Ans.}$$

50.



The method of determining the resultant is explained on page 484. Note that to obtain the points for plotting the resultant E.M.F., the instantaneous values from the 100 volt curve and the 50 volt curve are added together.

By measurement of vector diagram. Resultant E.M.F. = 140 volts. Ans.

By calculation (employing cosine rule) :—

$$R^2 = 100^2 + 50^2 - 2 \times 100 \times 50 \times \cos. 135^\circ$$
$$= 10000 + 2500 + 7071$$
$$= 19571$$

$$\therefore R = \sqrt{19571} = 139{\cdot}9 \text{ volts. Ans.}$$

$$\text{Cos } \theta = \frac{100^2 + (139{\cdot}9)^2 - 50^2}{2 \times 100 \times 139{\cdot}9} = 0{\cdot}9675$$

$$\theta = 14^\circ\ 40'$$

The resultant E.M.F. lags 14° 40′ behind the E.M.F. of 100 volts.

$$\text{Impedance}^2 = \text{Resistance}^2 + \text{Reactance}^2$$

$$\therefore \text{Resistance} = \sqrt{39^2 - 15^2} = 36 \text{ ohms. Ans.}$$

$$\text{Current flowing} = \frac{E}{Z} = \frac{139{\cdot}9}{39} = 3{\cdot}586 \text{ amps. Ans.}$$